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A latency-reduced successive cancellation list decoder for polar codes

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Dear editor,

Polar codes [1] provably achieve the capacity of binary-input memoryless channels under successive cancellation (SC) decoder. Cyclic redundancy check (CRC) aided successive cancellation list (CA-SCL) decoder [2] is an enhanced version of SC decoder and its block error rate (BLER) is comparable to the state-of-art low-density parity check code. However, the complexity of SCL algorithm grows linearly with list size L. When decoding polar codes with large L, the decoding latency can hardly meet stringent latency demand in next generation wireless networks. Hence, reducing the latency of SCL decoder is an important issue.

A low complexity SCL decoder is introduced in [3], where SC is performed to decode information bit instead of SCL if the polarized channel is evaluated to be reliable. Authors in [4] propose a low latency SCL decoder through multi-bit decision. Fast simplified SCL decoder to reduce path splitting at rate 1 node [5] is proposed in [6]. In [7], simplified decoding algorithms for some kinds of constituent nodes such as rate 1 node and single-parity check node are proposed and approximate parameter configurations are given to further reduce the latency of SCL decoder. Authors in [6, 7] propose that to accurately decode rate 1 nodes, certain number of path splitting is indispensable.

Fast path metric sorting is an alternative approach to reduce the latency of SCL decoder. Two kinds of path sorting, quick select (QS) and simplified bitonic sorter (SBT), are proposed in [8]. The complexity of QS relies on exchanging rounds and it is suitable for $L \leqslant 8$. The complexity of SBT is $O(L\log_2 L)$ and it is suitable for $L > 8$. In existing studies the sorting algorithms are exact, i.e., L smallest metrics [9] among $2L$ candidates are selected and their latency can be further reduced.

In this study, a latency-reduced SCL decoder is introduced. A suboptimal path sorting scheme is proposed to further reduce the number of comparators required for path sorting. Improved decoding schemes are proposed for first consecutive rate 0 (FCR0) node and most significant rate 1 (MSR1) node, which reduces path splitting in existing studies [6] with no BLER degradation. The suboptimal path metric sorting only requires L comparators, which is less than existing schemes [8]. Simulation results show that the BLER loss due to suboptimal sorting is negligible. The proof of lemmas in this study is in the supplemental material. We first define FCR0 and MSR1 nodes and then give the improved decoding algorithms of them. For simplicity, in rest of this study $\mathcal{F}$ and $\mathcal{M}$ are used to denote FCR0 and MSR1, respectively.

Definition of $\mathcal{F}$ and $\mathcal{M}$. The source bit sequence before encoding is $\boldsymbol{u}_1^N = (u_1, \ldots, u_k, u_{k+1}, \ldots, u_j, u_{j+1}, \ldots, u_N)$. $(u_1, \ldots, u_k)$ is all frozen but u_{k+1} is an information bit, the bit cluster $(u_1, \ldots, u_k)$ is defined as FCR0 node. Assume that $(u_{j+1}, \ldots, u_N)$ is information bit but u_j is frozen.

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Let integer $\gamma = N - 2^{\lfloor \log_2(N-j) \rfloor}$, where $\lfloor z \rfloor$ is the largest integer that does not exceed z. The bit cluster $(u_{\gamma+1}, \ldots, u_N)$ is defined as MSR1 node. More details of $\mathcal{F}$ and $\mathcal{M}$ are given in Appendix A.

Improved decoding of $\mathcal{F}$. When decoding $\mathcal{F}$, there is no information bit so only one decoding path is active. Hence, in $\mathcal{F}$ the process of SCL decoding is the same as SC decoding. Since the bits under rate 0 node are all frozen, there is no need to compute log-likelihood ratio (LLR) on the top of rate 0 nodes [5]. According to the definition, $\mathcal{F}$ may consist of several rate 0 nodes. The LLR on the top of each rate 0 node in $\mathcal{F}$ can be skipped.

Furthermore, in the SCL decoding process of $\mathcal{F}$, only one path is activated. Thus, there is no need to update the value of LLR-based path metric. During decoding $\mathcal{F}$, the metric of the only one active path remains 0, i.e., the initialized value. Hence, LLR calculations of rate 0 nodes and the path metric update during decoding $\mathcal{F}$ are both skipped, which reduces decoding latency.

Note that only the path metric updating of rate 0 nodes in the $\mathcal{F}$ can be skipped. The path metric updating of rate 0 nodes after $\mathcal{F}$ cannot be ignored because path metric increments introduced by frozen bits after $\mathcal{F}$ node have significant impact on the reliability of the surviving paths.

Improved decoding of $\mathcal{M}$. The improved decoding of $\mathcal{M}$ relies on Lemma 1. Some notations are defined as below for a clear elaboration.

$\mathcal{C}$ denotes one binary linear block code and $\boldsymbol{c}_1^N = (c_1, \ldots, c_N) \in \mathcal{C}$. The modulation used to transmit $\boldsymbol{c}_1^N$ is binary-phase shift keying (BPSK). $\boldsymbol{y}_1^N = (y_1, \ldots, y_N)$ denotes the received signal. $\hat{\boldsymbol{c}}_1^N$ denotes the maximum likelihood decision (not decoding) result of $\boldsymbol{y}_1^N$, i.e., if $p(y_i|0) > p(y_i|1)$, then set $\hat{c}_i = 0$ and if $p(y_i|0) < p(y_i|1)$, then set $\hat{c}_i = 1$.

Lemma 1. If $\hat{\boldsymbol{c}}_1^N$ is a code word in $\mathcal{C}$, then $\hat{\boldsymbol{c}}_1^N \in \mathcal{C}$ is the result of maximum likelihood decoding using received signal $\boldsymbol{y}_1^N$ (Appendix B).

Since the number of bits that $\mathcal{M}$ includes is power of 2, these bits can be seen as a sub-polar code with length $|\mathcal{M}|$, where $|\mathcal{M}|$ represents the number of bits in $\mathcal{M}$. This sub-polar code has code rate 1. The maximum likelihood decision result $\boldsymbol{v} = (v_1, \ldots, v_{|\mathcal{M}|})$ is obtained by making decision of the LLRs on the top of $\mathcal{M}$. Since $\mathcal{M}$ represents a sub-polar code with length $|\mathcal{M}|$, we have the sub-source bit sequence $\boldsymbol{w}$ as follows:

$$\boldsymbol{w} = (w_1, \ldots, w_{|\mathcal{M}|}) = \boldsymbol{v}\boldsymbol{G}_{|\mathcal{M}|}^{-1} = \boldsymbol{v}\boldsymbol{G}_{|\mathcal{M}|}, \qquad (1)$$

where $\boldsymbol{G}_{|\mathcal{M}|} = \boldsymbol{B}_{|\mathcal{M}|} F^{\otimes \log_2 |\mathcal{M}|}$ is the generator matrix of this sub-polar code.

By the definition of $\mathcal{M}$, all bits in $\boldsymbol{w}$ are information bits, so $\boldsymbol{w}$ is always a legal source bit sequence and thus $\boldsymbol{v}$ is always a legal code word. According to Lemma 1, $\boldsymbol{v}$ is the result of maximum likelihood decoding using LLRs on the top of $\mathcal{M}$. Thereby, the decoding of $\mathcal{M}$ can be finished in one step by making decision of LLRs on the top of $\mathcal{M}$. After $\boldsymbol{w}$ is obtained, connect $\boldsymbol{w}$ to the previous decoding sequence $\hat{\boldsymbol{u}}_1^{N-|\mathcal{M}|}$ and subsequently we have the decoding result $(\hat{\boldsymbol{u}}_1^{N-|\mathcal{M}|}, \boldsymbol{w})$. Since there are L candidate paths when we start to decode $\mathcal{M}$, after decoding of $\mathcal{M}$ we also get L candidate paths. Eq. (12) in [6] is adopted to update the L path metrics. After L surviving paths are associated with respective metrics, the path with smallest metric is selected as the final decoding output.

Suboptimal path metric sorting algorithm. In sequential decoder, metric of surviving path is calculated to judge whether the current decoding result is correct. If the current decoding result is incorrect, this path metric will become more and more unreliable as the sequential decoding proceeds and the error will be detected with probability one. Similarly, it can be inferred that in SCL decoding, the incorrect surviving paths with large LLR-based metric tend to be more unreliable than the correct one, which indicates that the correct probability of the paths with large LLR-based metric is very small. Even if these unreliable paths are not accurately sorted, the BLER performance loss will be negligible as long as most reliable paths (paths with small metric) are preserved. This gives inspiration to a suboptimal sorting with lower latency.

Though observation of SCL decoding process, the two paths with minimum and second minimum metric tend to be the correct path with high probability (more than 90%, especially in high SNR region). The corresponding simulation results are given in Appendix C. This implies that if paths with minimum and second minimum metrics are preserved, then it is very likely that the correct path is survived. Therefore, it is unnecessary to exactly select paths with L smallest metrics. The proposed suboptimal sorting algorithm is described below. Note that the proposed sorting still preserves L paths in every sorting procedure.

When decoding the information bit u_i, L surviving paths split into $2L$ candidate paths. The matrix that saves the $2L$ values of path metric is

$$\boldsymbol{P} = \begin{pmatrix} \mathrm{PM}_1 & \cdots & \mathrm{PM}_i & \cdots & \mathrm{PM}_j & \cdots & \mathrm{PM}_L \\ \mathrm{PM}_1^+ & \cdots & \mathrm{PM}_i^+ & \cdots & \mathrm{PM}_j^+ & \cdots & \mathrm{PM}_L^+ \end{pmatrix}, \qquad (2)$$

where PM_i^+ denotes $\mathrm{PM}_i^+ \geqslant \mathrm{PM}_i$. The partial order between PM_i^+ and PM_i is obtained as in [9].

The proposed sorting scheme compares PM_i with PM_{i+1}^+ and select the path corresponding to the smaller metric as surviving path, i.e., if $\mathrm{PM}_i \leqslant \mathrm{PM}_{i+1}^+$, then the candidate path with metric PM_i is preserved and the candidate path with

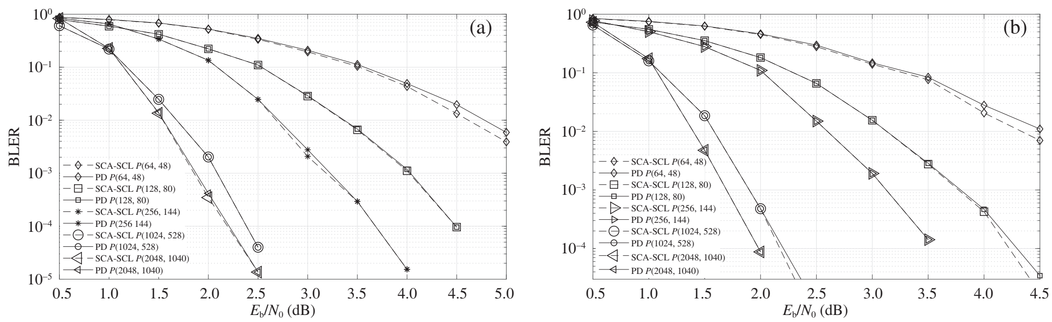

Figure 1 BLER performance comparison 1. (a) List size $L = 8$; (b) list size $L = 16$.

metric PM^+_{i+1} is discarded. PM_L should be compared with PM^+_1. This sorting procedure needs L times of comparisons and as a result, L survival paths are selected.

Lemma 2. Through the proposed sorting algorithm, the two paths corresponding to the minimum and second minimum metrics are preserved (Appendix D).

In the proposed sorting scheme, the number of comparisons in one sorting process is L and the number of required comparators is also L, which is irrelevant to the code length or SNR. The decoding latency is analyzed in Appendix E.

Simulation results. $P(64, 48)$, $P(128, 80)$, $P(256,\ 144)$, $P(1024, 528)$ and $P(2048, 1040)$ are simulated, where $P(N, K)$ denotes polar codes with length N and K information bits. The modulation is BPSK and the channel is AWGNC. $P(1024, 528)$ and $P(2048, 1040)$ are constructed through Bhattacharyya parameter assuming underlying channels are BECs with erasure probability 0.32, while remaining polar codes are constructed with the same method except erasure probability being 0.1. The list sizes of CA-SCL decoder are 8 and 16. All those polar codes are concatenated with the 16 bit CRC $g(x) = x^{16} + x^{15} + x^2 + 1$.

Comparisons between standard CA-SCL (SCA-SCL) and the proposed decoder (PD) are shown in Figure 1. Except $P(64, 48)$ and $P(128, 80)$ suffering a little BLER loss when SNR is high (more than 4 dB), under all remaining polar code configurations, the BLER between SCA-SCL and the PD are almost overlapped for various configurations. The BLER gap of short and moderate polar code in high SNR region may result from that the rate of short and moderate polar code tends to be influenced by the length of outer CRC code. More simulation results are provided in Appendix F.

Conclusion. We propose a latency-reduced SCL decoder. The decoding latency is reduced by two approaches: the improved decoding for FCR0/MSR1 node and the suboptimal sorting algorithm. The improved decoding of FCR0 and MSR1 node is exact, which will not lead to BLER performance loss. Simulation results demonstrate that the BLER performance loss due to the suboptimal sorting algorithm is negligible.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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