

Control design in the presence of actuator saturation: from individual systems to multi-agent systems

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The importance of taking actuator saturation into consideration in the design of practical control systems has been well recognized. Two facts are clear. Actuator saturation reduces the anticipated closed-loop performance of the closed-loop system, and in a severe situation, may cause loss of closed-loop stability. A saturation avoidance control design incurs conservativeness in achieving closed-loop performance. The analysis and design of control systems in the presence of actuator saturation have been an active research area for dozens of years now and many fundamental results as well as analysis and design tools are available for individual systems. Actuator saturation has also been considered in some early studies on coordinated control of multi-agent systems and has attracted increasing attention as the research field gradually matures. As seen in the recent literature, we can draw on the research results on individual control systems subject to actuator saturation in our new endeavor of coordinated control of multi-agent systems subject to actuator saturation.

Simple examples show that an exponentially unstable linear system cannot be globally stabilized in the presence of actuator saturation. Indeed, it has been established that global stabilization is possible if and only if the system is asymptotically null controllable with bounded controls (ANCBC) [1]. A linear system is ANCBC if it is stabilizable and all its open-loop poles are in the closed left-half plane. Even for systems that are ANCBC, global stabilization in the presence

of actuator saturation in general entails a nonlinear feedback law [2, 3]. Only systems of simple structures, such as single and double integrator systems and neutrally stable systems, can be globally stabilized by linear feedback [4]. For a general ANCBC system, nonlinear feedback laws in the form of nested saturations have been developed that achieve global stabilization [1, 5].

Even though linear feedback is not able to globally stabilize a general ANCBC in the presence of actuator saturation, linear low gain feedback achieves semi-global stabilization [6]. Low gain feedback refers to a family of linear feedback laws where the gain matrix is parameterized in a low gain parameter and designed in such a way that, for any a priori given arbitrarily large bounded set, the low gain parameter can be tuned small enough such that actuator saturation does not occur for any initial condition that belongs to the given set. That is, low gain feedback achieves stabilization with an arbitrarily large bounded domain of attraction. A low gain feedback law can be enhanced by adding to it a high gain feedback component to result in a low-and-high gain feedback law [6], which improves various closed-loop performances without diminishing the domain of attraction that is guaranteed by the original low gain feedback. Both the low gain and high gain parameters can be scheduled as a function of the state. For example, by decreasing the low gain parameter as the state increases, semi-global stabilization can be turned into global stabilization [7]. By fixing a small low

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gain parameter and increasing the high gain parameter as the output approaches the constant reference, the damping ratio is increased from a small value as the output nears its steady state, thus achieving a small rise time that comes with a low damping ratio at the beginning of the transience and eliminating overshoot with the high damping ratio. The resulting feedback law is referred to as the composite nonlinear feedback (CNF) law [8].

When a linear system is exponentially unstable, only local stabilization is possible in the presence of actuator saturation. The control objectives are then to achieve as large a domain of attraction as possible and, within the achieved domain of attraction, to improve performances of different measures. An effective approach to achieving these objectives is to represent the saturated linear feedback in a convex hull of auxiliary linear feedback laws, and based on this representation, formulate and solve the analysis and design problems as constrained optimization problems [9, 10].

Consideration of actuator saturation in the coordinated control of multi-agent systems started early in its study and has followed a similar pattern of and drawn on the results and tools from the development for individual systems, with the additional element of network topologies. It started with simple agent dynamics, including those of single and double integrator dynamics and neutrally stable agent dynamics, under various network topologies [11–17]. For agent dynamics that are ANCBC, global consensus has been achieved by distributed control protocols of nested saturation type [18, 19]. Also for ANCBC agent dynamics, low gain and low-and-high gain feedback design techniques have been widely adopted to arrive at various semi-global consensus results [20–33].

In summary, guided by the facts learned in our study of stabilization problem for individual systems, the study of global and semi-global coordinated control, global consensus in particular, of multi-agent systems, has been focused on agent dynamics that are ANCBC. When the agent dynamics are those of single or double integrators or those of neutrally stable linear systems, distributed linear control protocols have been constructed to achieve global consensus. When the agent dynamics are those of general linear ANCBC systems, nonlinear distributed control protocols of nested saturation type were constructed that achieve global consensus. Low gain and low-and-high gain feedback design techniques have been widely adopted to construct linear distributed control protocols that achieve various forms of semi-global consensus. These results have been developed under various assumptions on the network

topologies. Large room remains for relaxation of some of these assumptions.

In comparison with global and semi-global stabilization of individual systems, global consensus of multi-agent systems, besides requiring the agent dynamics to be ANCBC, involves assumptions on the network topology under which global consensus can be achieved with appropriately designed distributed control laws. An extra layer of complexity is anticipated when studying coordinated control of multi-agent systems where the agent dynamics are exponentially unstable. In this case, only local consensus can be achieved. Simple examples have shown that, in both the continuous-time setting [34] and the discrete-time setting [35], the network topology has strong influence on the size of the achievable domain of consensus, the set of agent initial conditions for which the multi-agent system reaches consensus, and the precise understanding of such influence appears to be complex and requires much attention from our research community.

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