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Composite anti-disturbance resilient control for Markovian jump nonlinear systems with general uncertain transition rate

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Abstract In this paper, the issue of disturbance observer based resilient control is addressed for Markovian jump nonlinear systems with multiple disturbances and general uncertain transition rates. The disturbances are divided into two parts: one has a bounded H_2 norm, and the other is given by an exogenous system. The general uncertain transition rate matrix is composed of unknown elements and uncertain ones. The uncertain transition rate only has a known approximate range. First, the disturbance described by the exogenous system is estimated by a disturbance observer, and its estimation is used for the controller as feedforward compensation. Subsequently, by using the resilient control method, a composite anti-disturbance resilient controller is constructed to guarantee stochastic stability with $L_2 - L_\infty$ performance of the closed-loop systems. Subsequently, the Lyapunov stability method and linear matrix inequality technique are applied to obtain the controller gain. Finally, an application example is provided to illustrate the effectiveness of the proposed approach.

Keywords composite anti-disturbance control, resilient controller, Markovian jump nonlinear systems, general uncertain transition probabilities, multiple disturbances, $L_2 - L_\infty$ performance

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1 Introduction

Markovian jump systems have attracted increasing attention in the past few decades and many useful results are derived [1–14]. Numerous results are reported for Markovian jump systems with completely known transition rate matrix [1–3]. However, it is difficult to precisely know the transition rate matrix in many actual systems. In recent times, many researchers are devoted to the study of the stability and controller design for Markovian jump systems with partly unknown transition rates [4–10]. In [4–10], the transition rate is either exactly known or completely unknown. In practice, it is difficult to precisely estimate the transition rate. Therefore, a more practical situation is that the transition rate can be completely unknown or its bound is known. This transition rate is referred to as the general uncertain transition rate. Some useful results are provided for Markovian jump systems with general uncertain transition rates [11, 12, 15].

In addition, many kinds of disturbances widely exist in practical system [16–19] and have adverse effects on the control performance of closed-loop system. To enhance the anti-disturbance ability of the system,

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many control methods are proposed to solve the disturbance problems, such as the disturbance-observer-based-control (DOBC) method [20–26], adaptive control scheme [27], and H_∞ control theory [28, 29]. Among them, the DOBC scheme has been used in numerous fields owing to its strong robustness, efficiency, and practicability, e.g., airbreathing hypersonic vehicle [30], missile [31] and spacecraft [32] systems. In the literatures, researchers often consider a single disturbance. Nevertheless, the practical systems always suffer from multiple disturbances, such as internal noise and external disturbance. In [33, 34], the method of composite hierarchical anti-disturbance control is introduced.

Recently, some meaningful results have been presented for Markovian jump systems with multiple disturbances via composite hierarchical anti-disturbance control [2, 3, 35]. However, in the above results, the authors only consider the case where the controller can be realized exactly. In the realization of the controller, the existences of round-off errors in numerical arithmetic, parameter drift, aging of controller devices, finite word length in digital systems, and other factors result in some gain variations of the controller. Therefore, the resilient controller has been applied to improve the control performance in [19, 36, 37]. To the best of our knowledge, for Markovian jump nonlinear systems with multiple disturbances and general uncertain transition rates, the composite anti-disturbance resilient control problem remains open.

In this paper, the composite anti-disturbance resilient control problem is addressed for Markovian jump nonlinear systems with multiple disturbances and uncertain transition rates. For the given systems, the transition rate matrix is assumed to be generally uncertain, i.e., some elements are unknown, whereas for the others, only their approximate ranges are known. The disturbances are divided into two types: one is given by an exogenous system, and the other is assumed to belong to the space $L_2[0,\infty)$. A disturbance observer is established to estimate the disturbance generated by the exogenous system, for which the estimation is introduced to the controller. Subsequently, a composite anti-disturbance resilient control strategy is applied to attenuate and reject the disturbances and ensure stochastic stability with $L_2 - L_\infty$ control performance of the closed-loop systems. Some sufficient conditions for solving the controller gain are provided by using the linear matrix inequalities method and Lyapunov function technique. Finally, an application example is provided to illustrate the effectiveness of the proposed approach. Recently, in [35], a composite anti-disturbance controller is established for Markovian jump nonlinear systems via a disturbance observer approach. We list the main contributions of our study compared with [35], as follows.

- Unlike the assumption in [35], the transition rate matrix is assumed to be generally uncertain, and for some of its elements, only the approximate ranges are known, whereas others are completely unknown.
- In order to deal with the actuator uncertainties, an anti-disturbance resilient controller is designed to enhance the robust stability of the closed-loop systems.
- Considering that parts of the disturbances belong to $L_2[0,\infty)$ space, an energy-to-peak control performance is introduced to reflect the ability of attenuation and rejection of the disturbance.
- Inspired by [1,4], a single link robot arm system is introduced and modeled as a Markovian jump nonlinear systems. For this practical example, we construct a composite anti-disturbance resilient controller and verify the ability of attenuation and rejection of disturbance. Through simulation, we demonstrate the effectiveness of our main results.

2 System model and problem formulation

The probability space $(\Omega, \mathcal{F}, \mathcal{P})$ is fixed, and the Markovian jump nonlinear system is considered:

$$\dot{x}(t) = A(r_t)x(t) + F(r_t)f(x(t),t) + G(r_t)[u(t) + d_1(t)] + H(r_t)d_2(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ represents the control input, and $f(x(t),t) \in \mathbb{R}^q$ denotes the nonlinear vector function. The disturbance $d_1(t) \in \mathbb{R}^m$ is given by an exogenous system in Assumption 1. The other $d_2(t) \in \mathbb{R}^q$ belongs to $L_2(0,\infty)$. Symbols $A(r_t)$, $F(r_t)$, $G(r_t)$ and $H(r_t)$ denote known matrices with appropriate dimensions. The function $\{r_t\}$ has right continuous trajectories for a continuous-time Markovian process, whose values are considered from a finite set $S = \{1, 2, \ldots, N\}$. $\Pi \triangleq \{\pi_{ij}\}$ is the transition rate matrix given by

$$P\{r_{t+h} = j | r_t = i\} = \begin{cases} 1 + \pi_{ii}h + o(h), & \text{if } j = i, \\ \pi_{ij}h + o(h), & \text{if } j \neq i, \end{cases} \tag{2}$$

where $h > 0$, $\lim_{h\to 0}(o(h)/h) = 0$ and π_{ij} denotes the transition rate from mode i at time t to mode j at time $t + h$, $\pi_{ii} = -\sum_{j=1,j\neq i}^{N} \pi_{ij}$ and $\pi_{ij} \geqslant 0$ when $i \neq j$.

Remark 1. In this paper, the Markovian jump nonlinear system (1) is discussed using the general uncertain transition rate matrix. For instance, the transition probability-rate matrix that relates the four operation modes is given as follows:

$$\Pi = \begin{pmatrix} ? & \widehat{\pi}_{12} + \Delta_{12} & \widehat{\pi}_{13} + \Delta_{13} & ? \\ ? & \widehat{\pi}_{22} + \Delta_{22} & ? & \widehat{\pi}_{24} + \Delta_{24} \\ ? & ? & ? & \widehat{\pi}_{34} + \Delta_{34} \\ \widehat{\pi}_{41} + \Delta_{41} & \widehat{\pi}_{42} + \Delta_{42} & ? & ? \end{pmatrix},$$

where "?" represents the unknown element, symbols $\widehat{\pi}_{ij}$ and $\Delta_{ij} \in [-\delta_{ij}, \delta_{ij}]$ $(\delta_{ij} \geqslant 0)$ denote the estimation value and estimation error of the uncertain transition rate π_{ij}, and parameters $\widehat{\pi}_{ij}$ and δ_{ij} are known. For clarity, we define $S = S_i^k + S_i^{\mathrm{uk}}$, $i \in S$, and

$$S_i^{\mathrm{uk}} = \{j \mid \widehat{\pi}_{ij} \text{ is unknown for } j \in S\},$$
$$S_i^k = \{j \mid \widehat{\pi}_{ij} \text{ is known for } j \in S\}.$$

Moreover, if $S_i^k \neq \emptyset$, it is described as $S_i^k = \{l_i^1, l_i^2, \ldots, l_i^m\}$, where l_i^m represents the m-th bound-known element with the index l_i^m in the i-th row of matrix Π.

Therefore, in this study, we aim at designing a composite anti-disturbance controller such that the closed-loop system has stochastic stability with $L_2 - L_\infty$ control performance. Accordingly, some assumptions and lemmas are introduced.

Assumption 1 ([35]). $d_1(t)$ is the disturbance in the control input path, which can be expressed as the following system:

$$\begin{aligned} \dot{w}(t) &= W(r_t)w(t) + M(r_t)d_3(t), \\ d_1(t) &= V(r_t)w(t), \end{aligned} \tag{3}$$

where $W(r_t)$, $M(r_t)$, $V(r_t)$ are known matrices with appropriate dimensions, and $d_3(t) \in \mathbb{R}^l$ belongs to $L_2[0, \infty)$.

Remark 2. In (3), the parameters $W(r_t)$, $M(r_t)$, and $V(r_t)$ are known in advance, but the initial condition of the system (3) is unknown. This indicates that the disturbances cannot be precisely known beforehand. In engineering, many kinds of disturbances can be illustrated using the system (3), for example, unknown constants and harmonics with unknown phase and magnitude [33].

Assumption 2 ([11]). (i) If $S_i^k \neq S$, $i \notin S_i^k$, then $\widehat{\pi}_{ij} + \Delta_{ij} \geqslant 0$ $(\forall j \in S_i^k)$;

(ii) If $S_i^k \neq S$, $i \in S_i^k$, then $\widehat{\pi}_{ij} + \Delta_{ij} \geqslant 0$ $(\forall j \in S_i^k, j \neq i)$, $\widehat{\pi}_{ii} + \Delta_{ii} \leqslant 0$, and $\sum_{j\in S_i^k} \widehat{\pi}_{ij} \leqslant 0$;

(iii) If $S_i^k = S$, then $\widehat{\pi}_{ij} + \Delta_{ij} \geqslant 0$ $(\forall j \in S, j \neq i)$, $\widehat{\pi}_{ii} = -\sum_{j\in S, j\neq i} \widehat{\pi}_{ij}$, and $\delta_{ii} = -\sum_{j\in S, j\neq i} \delta_{ij}$.

Assumption 3. $f(x(t), t)$ is assumed to satisfy

(i) $f(0, t) = 0$;

(ii) $\|f(x_1(t), t) - f(x_2(t), t)\| \leqslant \|U(x_1(t) - x_2(t))\|$, where U is a known constant matrix.

For simplicity, when $r_t = i$, $i \in S$, the matrix $A(r_t)$ is denoted by A_i; the same setting holds for the other matrices, e.q., $F(r_t)$ is denoted by F_i.

Lemma 1 ([11, 38]). Given any matrix Q and any real number ε, the matrix inequality

$$\varepsilon(Q+Q^{\mathrm{T}}) \leqslant \varepsilon^2 P + QP^{-1}Q^{\mathrm{T}}$$

holds for any matrix $P>0$.

Lemma 2 ([39]). D, E and F are known matrices. Suppose $F^{\mathrm{T}}F \leqslant I$, and then

$$DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} \leqslant \varepsilon DD^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E,$$

for any scalar $\varepsilon > 0$.

3 Main results

In this section, the cases of known and unknown nonlinearity function $f(x(t),t)$ are discussed. A disturbance observer is built to estimate the disturbance $d_1(t)$ under the two cases, and by using the L_2-L_∞ resilient control method, the composite anti-disturbance resilient controller is designed to guarantee that the closed-loop systems are stochastically stable with L_2-L_∞ control performance.

3.1 The case of known nonlinearity

The Markovian jump nonlinear system (1) is discussed under Assumptions 1 and 3, and $f(x(t),t)$ is known. We build the disturbance observer as

$$\begin{aligned}
&\widehat{d}_1(t) = V_i\widehat{w}(t),\\
&\widehat{w}(t) = v(t) - \overline{L}x(t),\\
&\dot{v}(t) = (W_i + \overline{L}G_iV_i)(v(t) - \overline{L}x(t)) + \overline{L}(A_ix(t) + F_if(x(t),t) + G_iu(t)),
\end{aligned} \tag{4}$$

where $\overline{L} = L + \Delta L$, ΔL is the variation in observer, and L denotes the observer gain.

Subsequently, the resilient controller can be constructed as follows:

$$u(t) = -\widehat{d}_1(t) + \overline{K}_ix(t), \tag{5}$$

where $\overline{K}_i = K_i + \Delta K_i$, $\widehat{d}_1(t)$ is the estimation of $d_1(t)$, K_i is the controller gain, and ΔK_i denotes the variation in controller gain.

From [36], we assume that $\Delta K_i = N_iB_i(t)D_i$ and $\Delta L = RS(t)T$, where N_i, D_i R and T are known matrices, and $B_i(t)$ and $S(t)$ are uncertain matrices satisfying $B_i^{\mathrm{T}}(t)B_i(t) \leqslant I$, $S^{\mathrm{T}}(t)S(t) \leqslant I$.

Remark 3. The composite resilient controller is divided into two parts: the first part is the negative of the estimation of the disturbance $d_1(t)$, which originates from the disturbance observer (4); the second part is the classical state feedback resilient control law. It is evident that, with the control scheme (5), the disturbances d_1 can be compensated via the first part of the controller, whereas the second plays an important role in guaranteeing the dynamical system stability and satisfying performance requirement.

We define $e_w(t) = w(t) - \widehat{w}(t)$, from (1), (3), (4), and the error dynamics is expressed as

$$\dot{e}_w(t) = (W_i + \bar{L}G_iV_i)e_w(t) + M_id_3(t) + LH_id_2(t). \tag{6}$$

Remark 4. In order to satisfy the performance requirement, the disturbance observer gain is chosen to guarantee the stability of the following systems:

$$\dot{e}_w(t) = (W_i + \bar{L}G_iV_i)e_w(t). \tag{7}$$

The Lyapunov function $V_0(e_w(t),i,t) = e_w^{\mathrm{T}}(t)Pe_w(t)$ is chosen. Let $\mathcal{A}$ be the weak infinitesimal generator of the random process $\{\xi(t), r_t\}$ [40]; thus, we have

$$\mathcal{A}V_0(e_w(t),i,t) = \dot{e}_w^{\mathrm{T}}(t)Pe_w(t) + e_w^{\mathrm{T}}(t)P\dot{e}_w(t) + \sum_{j=1}^{N}\pi_{ij}e_w^{\mathrm{T}}(t)Pe_w(t). \tag{8}$$

Noting that $\sum_{j=1}^{N}\pi_{ij}=0$, $\mathcal{A}V_0(e_w(t),i,t)$ can be written as

$$\begin{aligned}&\mathcal{A}V_0(e_w(t),i,t)\\&=e_w^{\mathrm T}(t)((W_i+\overline{L}G_iV_i)^{\mathrm T}P+P(W_i+\overline{L}G_iV_i))e_w(t)\\&=e_w^{\mathrm T}(t)(W_i^{\mathrm T}P+PW_i+V_i^{\mathrm T}G_i^{\mathrm T}L^{\mathrm T}P+PLG_iV_i+V_i^{\mathrm T}G_i^{\mathrm T}T^{\mathrm T}S^{\mathrm T}(t)R^{\mathrm T}P+PRS(t)TG_iV_i)e_w(t).\end{aligned}$$

Hence, $\mathcal{A}V_0(e_w(t),i,t)<0$ if

$$W_i^{\mathrm T}P+PW_i+V_i^{\mathrm T}G_i^{\mathrm T}L^{\mathrm T}P+PLG_iV_i+V_i^{\mathrm T}G_i^{\mathrm T}T^{\mathrm T}S^{\mathrm T}(t)R^{\mathrm T}P+PRS(t)TG_iV_i<0. \tag{9}$$

According to Lemma 2 and the Schur complement lemma, defining $Y=PL$, there exists a real number $\alpha>0$, such that (9) holds under

$$\begin{pmatrix}W_i^{\mathrm T}P+PW_i+V_i^{\mathrm T}G_i^{\mathrm T}Y^{\mathrm T}+YG_iV_i+\alpha V_i^{\mathrm T}G_i^{\mathrm T}T^{\mathrm T}TG_iV_i & PR\\ * & -\alpha I\end{pmatrix}<0. \tag{10}$$

Thus, the system (7) is stochastically stable under the condition (10), and the disturbance observer gain is $L=P^{-1}Y$.

Remark 5. According to the disturbance estimation error system (7), a method to obtain the observer gain is presented, in which a common Lyapunov function is chosen. This may lead to some conservatism.

Thus, by combining (1) and (5) with (6), we have

$$\begin{aligned}(\Sigma)\quad \dot{\xi}(t)&=\overline{A}_i(t)\xi(t)+\overline{F}_if(\xi(t),t)+\overline{H}_id(t),\\ z(t)&=C_{1i}x(t)+C_{2i}e_w(t)=\overline{C}_i\xi(t),\end{aligned} \tag{11}$$

where $\xi^{\mathrm T}(t)=(x(t),e_w(t))$, $f(\xi(t),t)=f(x(t),t)$, $d^{\mathrm T}(t)=(d_2(t),d_3(t))$ and $z(t)$ is the reference output, with

$$\overline{A}_i=\begin{pmatrix}A_i+G_i\overline{K}_i & G_iV_i\\ 0 & W_i+\bar{L}G_iV_i\end{pmatrix},\quad \overline{H}_i=\begin{pmatrix}H_i & 0\\ \bar{L}H_i & M_i\end{pmatrix},\quad \overline{F}_i=\begin{pmatrix}F_i\\ 0\end{pmatrix},\quad \overline{C}_i=(C_{1i}\ \ C_{2i}).$$

For the system (1) with (3), our task is to design a composite anti-disturbance resilient controller (5), such that the closed-loop system satisfies the following conditions:

(i) The composite closed-loop system in (Σ) is stochastically stable under $d(t)=0$;

(ii) Under zero initial condition, the following inequality holds:

$$\|z(t)\|_{E_\infty}^2<\gamma\|d(t)\|_2^2, \tag{12}$$

where $\|z(t)\|_{E_\infty}^2=\sup_{t>0}E\{z^{\mathrm T}(t)z(t)\}$, the parameter $\gamma>0$ is a prescribed L_2-L_∞ performance, and $d(t)\in L_2[0,\infty)$.

Now, a sufficient condition is presented to guarantee that the system (Σ) is stochastically stable with L_2-L_∞ performance.

Theorem 1. Given the parameters $\gamma>0$ and $\lambda>0$, the composite system (11) is stochastically stable with L_2-L_∞ performance, under the disturbance observer (4) and resilient controller (5), if there exist matrices $P_i>0$, $i\in S$, such that the following inequalities hold:

$$\begin{pmatrix}\Gamma_i & P_i\overline{F_i} & P_i\overline{H_i}\\ * & -\frac{1}{\lambda^2}I & 0\\ * & * & -I\end{pmatrix}<0, \tag{13}$$

$$-P_i+\gamma^{-1}\overline{C}_i^{\mathrm T}\overline{C}_i<0, \tag{14}$$

where

$$\Gamma_i=P_i\overline{A}_i+\overline{A}_i^{\mathrm T}P_i+\sum_{j}^{N}\pi_{ij}P_j+\frac{1}{\lambda^2}\overline{U},\quad \overline{U}=\begin{pmatrix}U^{\mathrm T}U & 0\\ 0 & 0\end{pmatrix}.$$

Proof. A Lyapunov function candidate is defined as

$$V(x(t),i,t)=\xi^{\mathrm{T}}(t)P_i\xi(t)+\frac{1}{\lambda^2}\int_0^t\left(\|Ux(\tau)\|^2-\|f(x(\tau),\tau)\|^2\right)\mathrm{d}\tau, \tag{15}$$

where $P_i>0$, $i\in S$, $\lambda>0$.

Let $\mathcal{A}$ be the weak infinitesimal generator of the random process $\{\xi(t),r_t\}$ [40]. For each $r_t=i$, $i\in S$, we have

$$\begin{aligned}\mathcal{A}V(x(t),i,t)=&\,\xi^{\mathrm{T}}(t)\left(P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_j^N\pi_{ij}P_j\right)\xi(t)+2\xi^{\mathrm{T}}(t)P_i\overline{F}_if(x(t),t)+2\xi^{\mathrm{T}}(t)P_i\overline{H}_id(t)\\&+\frac{1}{\lambda^2}\left(x^{\mathrm{T}}(t)U^{\mathrm{T}}Ux(t)-f^{\mathrm{T}}(x(t),t)f(x(t),t)\right).\end{aligned} \tag{16}$$

Now, to prove the L_2-L_∞ performance of the composite system, we introduce the following performance index:

$$J(T)=E\left\{V(x(T),i,T)-\int_0^{\mathrm{T}}d^{\mathrm{T}}(\tau)d(\tau)\mathrm{d}\tau\right\}. \tag{17}$$

Under the zero initial condition, based on (16) and (17), we have

$$J(T)=E\left\{\int_0^{\mathrm{T}}\mathcal{A}V(x(\tau),i,\tau)-d^{\mathrm{T}}(\tau)d(\tau)\mathrm{d}\tau\right\}=E\left\{\int_0^{\mathrm{T}}\eta^{\mathrm{T}}(\tau)\Theta_i\eta(\tau)\mathrm{d}\tau\right\}, \tag{18}$$

where $\eta(t)=(\ \xi^{\mathrm{T}}(t),\ f^{\mathrm{T}}(x(t),t),\ d^{\mathrm{T}}(t)\ )$, and

$$\Theta_i=\begin{pmatrix}\Gamma_i & P_i\overline{F_i} & P_i\overline{H_i}\\ * & -\frac{1}{\lambda^2}I & 0\\ * & * & -I\end{pmatrix}.$$

From (13) and (17), we have

$$E\{V(x(T),i,T)\}<E\left\{\int_0^{\mathrm{T}}d^{\mathrm{T}}(\tau)d(\tau)\mathrm{d}\tau\right\}. \tag{19}$$

According to (11) and (14), we have

$$\gamma^{-1}\|z(t)\|_{E_\infty}^2=\gamma^{-1}\sup_{t>0}E\{z^{\mathrm{T}}(t)z(t)\}\leqslant E\{V(x(t),i,t)\}. \tag{20}$$

Combining (14) with (20) and letting $T\to\infty$ yields

$$\|z(t)\|_{E_\infty}^2\leqslant E\{\gamma V(x(t),i,t)\}\leqslant\gamma\|d(t)\|_2^2. \tag{21}$$

Hence, the composite system Σ satisfies the L_2-L_∞ performance.

Thus, the stochastic stability of the composite system is confirmed under $d(t)=0$.

Inspired by [29], as Θ_i is a negative definite matrix, there exists a constant $k>0$, and for each $i\in S$, the following equality holds:

$$\begin{pmatrix}P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_j^N\pi_{ij}P_j+\overline{U} & P_i\overline{F}_i\\ * & -\frac{1}{\lambda^2}I\end{pmatrix}<\begin{pmatrix}-kI & 0\\ 0 & 0\end{pmatrix}. \tag{22}$$

Hence, according to Dynkin's formula, we obtain

$$E\left\{\int_0^t\xi^{\mathrm{T}}(s)\xi(s)\mathrm{d}s\right\}\leqslant k^{-1}E\{V(x(0),i,0)\}\leqslant\xi^{\mathrm{T}}(0)M\xi(0), \tag{23}$$

where M is a positive definite matrix and $M\geqslant P_i$, $i\in S$.

Therefore, according to [35], we conclude that the composite system (11) is stochastically stable.

In Theorem 1, a sufficient condition is provided to guarantee the stochastic stability of a Markovian jump nonlinear system with $L_2{-}L_\infty$ performance. Now, considering the case of Markovian jump nonlinear system with general uncertain transition rates, the condition in Theorem 1 is not solvable. Hence, some solvable sufficient conditions are further developed such that the Markovian jump nonlinear system is stochastically stable with $L_2 - L_\infty$ performance.

Theorem 2. Consider the system (1) under Assumptions 1 and 3. Given the parameters $\gamma > 0$ and $\lambda > 0$, the closed-loop system (11) is stochastically stable with $L_2 - L_\infty$ performance, if there exist matrices $P_i > 0$, $J_{ij} > 0$, $V_{ijk} > 0$, $S_{ij} > 0$, $i, j, k \in S$, such that the following inequalities hold:

$$-P_i + \gamma^{-1}\overline{C}_i^{\mathrm{T}}\overline{C}_i < 0. \tag{24}$$

(i) If $i \in S_i^{\mathrm{uk}}$,

$$\begin{pmatrix} \Gamma_{1i} & P_i\overline{F_i} & P_i\overline{H_i} \\ * & -\frac{1}{\lambda^2}I & 0 \\ * & * & -I \end{pmatrix} < 0, \tag{25}$$

$$P_i - P_j \geqslant 0, \quad \forall j \in S_i^{\mathrm{uk}}, \quad j \neq i; \tag{26}$$

(ii) If $i \in S_i^k$ and $\sum_{j\in S_i^{\mathrm{uk}}} \pi_{ij} \neq 0$,

$$\begin{pmatrix} \Gamma_{2i} & P_i\overline{F_i} & P_i\overline{H_i} \\ * & -\frac{1}{\lambda^2}I & 0 \\ * & * & -I \end{pmatrix} < 0, \quad \forall k \in S_i^{\mathrm{uk}}; \tag{27}$$

(iii) If $i \in S_i^k$ and $\sum_{j\in S_i^{\mathrm{uk}}} \pi_{ij} = 0$,

$$\begin{pmatrix} \Gamma_{3i} & P_i\overline{F_i} & P_i\overline{H_i} \\ * & -\frac{1}{\lambda^2}I & 0 \\ * & * & -I \end{pmatrix} < 0, \tag{28}$$

where

$$\Gamma_{1i} = P_i\overline{A}_i + \overline{A}_i^{\mathrm{T}}P_i + \sum_{j\in S_i^k} \widehat{\pi}_{ij}(P_j - P_i) + \frac{1}{\lambda^2}\overline{U} + \sum_{j\in S_i^k}\left[\frac{\delta_{ij}^2}{4}J_{ij} + (P_j - P_i)J_{ij}^{-1}(P_j - P_i)\right],$$

$$\Gamma_{2i} = P_i\overline{A}_i + \overline{A}_i^{\mathrm{T}}P_i + \sum_{j\in S_i^k} \widehat{\pi}_{ij}(P_j - P_k) + \frac{1}{\lambda^2}\overline{U} + \sum_{j\in S_i^k}\left[\frac{\delta_{ij}^2}{4}T_{ijk} + (P_j - P_k)T_{ijk}^{-1}(P_j - P_k)\right],$$

$$\Gamma_{3i} = P_i\overline{A}_i + \overline{A}_i^{\mathrm{T}}P_i + \sum_{i\in S_i^k, j\neq i} \widehat{\pi}_{ij}(P_j - P_i) + \frac{1}{\lambda^2}\overline{U} + \sum_{i\in S_i^k, j\neq i}\left[\frac{\delta_{ij}^2}{4}S_{ij} + (P_j - P_i)S_{ij}^{-1}(P_j - P_i)\right].$$

Proof. First, consider the case where $i \in S_i^{\mathrm{uk}}$. Note that $-\sum_{j\in S_i^{\mathrm{uk}}} \pi_{ij} = \sum_{j\in S_i^k} \pi_{ij}$, $\pi_{ii} \leqslant 0$, $\pi_{ij} \geqslant 0$, $i \neq j$.

Subsequently, using (26), we have

$$\begin{aligned} P_i\overline{A}_i + \overline{A}_i^{\mathrm{T}}P_i + \sum_{j=1}^{N}\pi_{ij}P_j &\leqslant P_i\overline{A}_i + \overline{A}_i^{\mathrm{T}}P_i + \sum_{j\in S_i^k}\pi_{ij}P_j + \sum_{j\in S_i^{\mathrm{uk}}}\pi_{ij}P_i, \\ &= P_i\overline{A}_i + \overline{A}_i^{\mathrm{T}}P_i + \sum_{j\in S_i^k}(\widehat{\pi}_{ij} + \Delta_{ij})(P_j - P_i). \end{aligned} \tag{29}$$

From Lemma 1, we obtain

$$\sum_{j\in S_i^k}\Delta_{ij}(P_j-P_i)\leqslant\sum_{j\in S_i^k}\left[\frac{\delta_{ij}^2}{4}J_{ij}+(P_j-P_i)J_{ij}^{-1}(P_j-P_i)\right]. \tag{30}$$

According to (29) and (30), we derive

$$\begin{aligned}&P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j=1}^{N}\pi_{ij}P_j+\frac{1}{\lambda^2}\overline{U}\\&\leqslant P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j\in S_i^k}\widehat{\pi}_{ij}(P_j-P_i)+\sum_{j\in S_i^k}\left[\frac{\delta_{ij}^2}{4}J_{ij}+(P_j-P_i)J_{ij}^{-1}(P_j-P_i)\right]+\frac{1}{\lambda^2}\overline{U}.\end{aligned} \tag{31}$$

This indicates that the system (11) is stochastically stable with L_2-L_∞ performance if (25) is true. Second, consider the case where $i\in S_i^k$ and $\sum_{j\in S_i^{\mathrm{uk}}}\pi_{ij}\neq 0$. Note that in this case

$$\sum_{j\in S_i^k}\pi_{ij}=-\sum_{j\in S_i^{\mathrm{uk}}}\pi_{ij},\quad \sum_{j\in S_i^k}\pi_{ij}<0,\quad \pi_{ij}\geqslant 0,\quad j\in S_i^{\mathrm{uk}}. \tag{32}$$

After some manipulations, we obtain

$$\begin{aligned}P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j=1}^{N}\pi_{ij}P_j&=P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j\in S_i^k}\pi_{ij}P_j+\sum_{k\in S_i^{\mathrm{uk}}}\pi_{ik}P_k\\&=\frac{\sum_{k\in S_i^{\mathrm{uk}}}\pi_{ik}}{-\sum_{j\in S_i^k}\pi_{ij}}\left(P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j\in S_i^k}\pi_{ij}(P_j-P_k)\right).\end{aligned} \tag{33}$$

From (33), Eq. (13) is equivalent to

$$\frac{\sum_{k\in S_i^{\mathrm{uk}}}\pi_{ik}}{-\sum_{j\in S_i^k}\pi_{ij}}\begin{pmatrix}\Psi_{1i} & P_i\overline{F_i} & P_i\overline{H_i}\\ * & -\frac{1}{\lambda^2}I & 0\\ * & * & -I\end{pmatrix}<0, \tag{34}$$

where $\Psi_{1i}=P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j\in S_i^k}\pi_{ij}(P_j-P_k)+\frac{1}{\lambda^2}\overline{U}$. By using Lemma 1, we obtain

$$\begin{aligned}&P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j\in S_i^k}\pi_{ij}(P_j-P_k)+\frac{1}{\lambda^2}\overline{U}\\&\leqslant P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j\in S_i^k}\widehat{\pi}_{ij}(P_j-P_k)+\frac{1}{\lambda^2}\overline{U}+\sum_{j\in S_i^k}\left[\frac{\delta_{ij}^2}{4}T_{ijk}+(P_j-P_k)T_{ijk}^{-1}(P_j-P_k)\right].\end{aligned} \tag{35}$$

Hence, Eq. (34) holds under (27), and this implies (13).

Third, consider the case where $i\in S_i^k$ and $\sum_{j\in S_i^{\mathrm{uk}}}\pi_{ij}=0$. Observe that $-\pi_{ii}=\sum_{j\in S_i^k j\neq i}\pi_{ij}$. Thus, the following holds:

$$P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j=1}^{N}\pi_{ij}P_j=P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j\in S_i^k,j\neq i}(\widehat{\pi}_{ij}+\Delta_{ij})(P_j-P_i).$$

Applying Lemma 1 to (36) yields

$$P_i\overline{A}_i+\overline{A}_i^{\mathrm{T}}P_i+\sum_{j\in S_i^k,j\neq i}(\widehat{\pi}_{ij}+\Delta_{ij})(P_j-P_i)+\frac{1}{\lambda^2}\overline{U}$$

$$\leqslant P_i\overline{A}_i + \overline{A}_i^{\mathrm{T}} P_i + \sum_{j\in S_i^k, j\neq i} \widehat{\pi}_{ij}(P_j - P_i) + \frac{1}{\lambda^2}\overline{U} + \sum_{j\in S_i^k, j\neq i}\left[\frac{\delta_{ij}^2}{4}S_{ij} + (P_j - P_i)S_{ij}^{-1}(P_j - P_i)\right]. \tag{36}$$

This together with (28) yields (13).

Therefore, according to Theorem 1 and the above analysis, we conclude that the composite system (Σ) is stochastically stable with $L_2 - L_\infty$ performance. The proof is completed.

Here, in order to obtain the gains of the disturbance observer and resilient controller, the linear matrix inequality (LMI) technique is applied to derive the sufficient conditions.

Theorem 3. Consider the system (1) under Assumptions 1 and 3. Given the parameters $\gamma > 0$ and $\lambda > 0$, by designing a disturbance observer and a resilient controller in the forms of (4) and (5), the closed-loop system (Σ) is stochastically stable with $L_2 - L_\infty$ performance, if there exist matrices $Q_i > 0$, $P_{2i} > 0$, $\overline{J}_{1ij} > 0$, $J_{2ij} > 0$, $j \in S_i^k$, $\overline{T}_{1ijk} > 0$, $T_{2ijk} > 0$, $j \in S_i^k$, $k \in S_i^{\mathrm{uk}}$, $\overline{S}_{1ij} > 0$, $S_{2ij} > 0$, $j \in S_i^k$, and X_i, and positive numbers ε_i, $i \in S$, $\beta_1 > 0$, and $\beta_2 > 0$, such that the following inequalities hold:

$$\begin{pmatrix} -P_{2i} & C_{2i}^{\mathrm{T}} \\ * & -\gamma I + C_{1i}Q_iC_{1i}^{\mathrm{T}} \end{pmatrix} < 0. \tag{37}$$

(i) If $i \in S_i^{\mathrm{uk}}$,

$$\begin{pmatrix} \Pi_{11i} & G_iV_i & F_i & H_i & 0 & 0 & 0 & \Xi_{1i} \\ * & \Pi_{22i} & 0 & P_{2i}LH_i & P_{2i}M_i & P_{2i}R & P_{2i}R & \Xi_{2i} \\ * & * & -\frac{1}{\lambda^2}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I+\beta_2H_i^{\mathrm{T}}T^{\mathrm{T}}TH_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\beta_1 I & 0 & 0 \\ * & * & * & * & * & * & -\beta_2 I & 0 \\ * & * & * & * & * & * & * & \Xi_{3i} \end{pmatrix} < 0, \tag{38}$$

$$P_{2j} - P_{2i} \leqslant 0, \quad \forall j \in S_i^{\mathrm{uk}},\ j \neq i, \tag{39}$$

$$-Q_j + Q_i \leqslant 0, \quad \forall j \in S_i^{\mathrm{uk}},\ j \neq i; \tag{40}$$

(ii) If $i \in S_i^k$ and $\sum_{j\in S_i^{\mathrm{uk}}} \pi_{ij} \neq 0$, then, for $\forall k \in S_i^{\mathrm{uk}}$

$$\begin{pmatrix} \Pi'_{11i} & G_iV_i & F_i & H_i & 0 & 0 & 0 & \Xi'_{1i} \\ * & \Pi'_{22i} & 0 & P_{2i}LH_i & P_{2i}M_i & P_{2i}R & P_{2i}R & \Xi'_{2i} \\ * & * & -\frac{1}{\lambda^2}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I+\beta_2H_i^{\mathrm{T}}T^{\mathrm{T}}TH_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\beta_1 I & 0 & 0 \\ * & * & * & * & * & * & -\beta_2 I & 0 \\ * & * & * & * & * & * & * & \Xi'_{3i} \end{pmatrix} < 0, \tag{41}$$

$$-Q_i + Q_k \leqslant 0; \tag{42}$$

(iii) If $i \in S_i^k$ and $\sum_{j\in S_i^{\rm uk}} \pi_{ij}=0$,

$$\begin{pmatrix} \Pi''_{11i} & G_iV_i & F_i & H_i & 0 & 0 & 0 & \Xi''_{1i} \\ * & \Pi''_{22i} & 0 & P_{2i}LH_i & P_{2i}M_i & P_{2i}R & P_{2i}R & \Xi''_{2i} \\ * & * & -\frac{1}{\lambda^2}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I+\beta_2H_i^{\rm T}T^{\rm T}TH_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\beta_1 I & 0 & 0 \\ * & * & * & * & * & * & -\beta_2 I & 0 \\ * & * & * & * & * & * & * & \Xi''_{3i} \end{pmatrix} < 0, \tag{43}$$

where

$$\Xi_{1i}=(Q_iD_i^{\rm T}\ Q_iU^{\rm T}\ \Pi_{18i}\ \Pi_{19i}\ 0\ \Pi_{111i}),\quad \Xi_{3i}={\rm diag}(-\varepsilon_iI\ -\lambda^2I\ \Pi_{88i}\ \Pi_{99i}\ \pi_{1010i}\ \Pi_{1111i}),$$

$$\Xi_{2i}=(0\ 0\ 0\ 0\ \Pi_{210i}\ 0),\quad \Pi_{18i}=(Q_i\ \cdots\ Q_i),\quad \Pi_{19i}=(Q_i-Q_{l_i^1}\ \cdots\ Q_i-Q_{l_i^m}),$$

$$\Pi_{111i}=\left(\frac{\delta_{il_i^1}}{2}Q_i\ \cdots\ \frac{\delta_{il_i^m}}{2}Q_i\right),\quad \Pi_{210i}=\left(P_{2l_i^1}-P_{2i}\ \cdots\ P_{2l_i^m}-P_{2i}\right),$$

$$\Pi_{88i}=-{\rm diag}\left(\widehat{\pi}_{il_i^1}^{-1}Q_{l_i^1}\ \cdots\ \widehat{\pi}_{il_i^m}^{-1}Q_{l_i^m}\right),\quad \Pi_{99i}=-{\rm diag}\left(2Q_{l_i^1}-\overline{J}_{1il_i^1}\ \cdots\ 2Q_{l_i^m}-\overline{J}_{1il_i^m}\right),$$

$$\Pi_{1010i}=-{\rm diag}\big(J_{2il_i^1}\ \cdots\ J_{2il_i^m}\big),\quad \Pi_{1111i}=-{\rm diag}\left(\widehat{\pi}_{il_i^1}^{-1}\overline{J}_{1il_i^1}\ \cdots\ \widehat{\pi}_{il_i^m}^{-1}\overline{J}_{1il_i^m}\right),$$

$$\Xi'_{1i}=(Q_iD_i^{\rm T}\ Q_iU^{\rm T}\ \Pi'_{18i}\ \Pi'_{19i}\ 0\ \Pi'_{111i}),\quad \Xi'_{3i}={\rm diag}(-\varepsilon_iI\ -\lambda^2I\ \Pi'_{88i}\ \Pi'_{99i}\ \pi'_{1010i}\ \Pi'_{1111i}),$$

$$\Xi'_{2i}=(0\ 0\ 0\ 0\ \Pi'_{210i}\ 0),\ \Pi'_{18i}=(Q_i\cdots Q_i\cdots Q_i),\ \Pi'_{19i}=(Q_i-Q_{l_i^1}\cdots Q_i-Q_{l_i^r}\cdots Q_i-Q_{l_i^m})_{l_i^r\neq i},$$

$$\Pi'_{111i}=\left(\frac{\delta_{il_i^1}}{2}Q_i\cdots \frac{\delta_{il_i^r}}{2}Q_i\cdots \frac{\delta_{il_i^m}}{2}Q_i\right)_{l_i^r\neq i},\quad \Pi'_{210i}=(P_{2l_i^1}-P_{2k}\ \cdots\ P_{2l_i^m}-P_{2k}),$$

$$\Pi'_{88i}=-{\rm diag}\left(\widehat{\pi}_{il_i^1}^{-1}Q_{l_i^1}\cdots \widehat{\pi}_{il_i^r}^{-1}Q_{l_i^r}\cdots \widehat{\pi}_{il_i^m}^{-1}Q_{l_i^m}\right)_{l_i^r\neq i},\quad \Pi'_{1010i}=-{\rm diag}\left(T_{2il_i^1k}\ \cdots\ T_{2il_i^mk}\right),$$

$$\Pi'_{1111i}=-{\rm diag}\left(\widehat{\pi}_{il_i^1}^{-1}\overline{T}_{1il_i^1k}\cdots\widehat{\pi}_{il_i^1}^{-1}\overline{T}_{1il_i^1k}\cdots\widehat{\pi}_{il_i^m}^{-1}\overline{T}_{1il_i^mk}\right)_{l_i^r\neq i},\ \Xi''_{1i}=(Q_iD_i^{\rm T}\ Q_iU^{\rm T}\ \Pi''_{18i}\ \Pi''_{19i}\ 0\ \Pi''_{111i}),$$

$$\Xi''_{3i}={\rm diag}(-\varepsilon_iI\ -\lambda^2I\ \Pi''_{88i}\ \Pi''_{99i}\ \Pi''_{1010i}\ \Pi''_{1111i}),\quad \Xi''_{2i}=(0\ 0\ 0\ 0\ \Pi''_{210i}\ 0),\quad \Pi''_{18i}=(Q_i\cdots Q_i\cdots Q_i),$$

$$\Pi''_{19i}=(Q_i-Q_{l_i^1}\cdots Q_i-Q_{l_i^r}\cdots Q_i-Q_{l_i^m})_{l_i^r\neq i},\quad \Pi''_{111i}=\left(\frac{\delta_{il_i^1}}{2}Q_i\cdots\frac{\delta_{il_i^r}}{2}Q_i\cdots\frac{\delta_{il_i^m}}{2}Q_i\right)_{l_i^r\neq i},$$

$$\Pi''_{210i}=(P_{2l_i1}-P_{2i}\cdots P_{2l_ir}-P_{2i}\cdots P_{2l_i^m}-P_{2i})_{l_i^r\neq i},\ \Pi''_{88i}=-{\rm diag}\left(\widehat{\pi}_{il_i^1}^{-1}Q_{l_i^1}\cdots\widehat{\pi}_{il_i^r}^{-1}Q_{l_i^r}\cdots\widehat{\pi}_{il_i^m}^{-1}Q_{l_i^m}\right)_{l_i^r\neq i},$$

$$\Pi''_{1010i}=-{\rm diag}(S_{2il_i^1}\cdots S_{2il_i^r}\cdots S_{2il_i^m})_{l_i^r\neq i},\quad \Pi''_{1111i}=-{\rm diag}\left(\widehat{\pi}_{il_i^1}^{-1}\overline{S}_{1il_i^1}\cdots\widehat{\pi}_{il_i^1}^{-1}\overline{S}_{1il_i^1}\cdots\widehat{\pi}_{il_i^m}^{-1}\overline{S}_{1il_i^m}\right)_{l_i^r\neq i},$$

$$\Pi_{11i}=A_iQ_i+Q_iA_i^{\rm T}+\varepsilon_iG_iN_iN_i^{\rm T}G_i^{\rm T}+G_iX_i+X_i^{\rm T}G_i^{\rm T}-\sum_{j\in S_i^k}\widehat{\pi}_{ij}Q_i,$$

$$\Pi_{22i}=P_{2i}W_{ij}+P_{2i}LG_iV_i+W_i^{\rm T}P_{2i}+V_i^{\rm T}G_i^{\rm T}L^{\rm T}P_{2i}^{\rm T}+\beta_1V_i^{\rm T}G_i^{\rm T}T^{\rm T}TG_iV_i+\sum_{j\in S_i^k}\left(\widehat{\pi}_{ij}P_{2j}-\widehat{\pi}_{ij}P_{2i}+\frac{\delta_{ij}^2}{4}J_{2ij}\right),$$

$$\Pi'_{11i}=A_iQ_i+Q_iA_i^{\rm T}+\varepsilon_iG_iN_iN_i^{\rm T}G_i^{\rm T}+G_iX_i+X_i^{\rm T}G_i^{\rm T}+\widehat{\pi}_{ii}\frac{\delta_{ii}^2}{4}(2Q_i-\overline{T}_{1iik})-\sum_{j\in S_i^k,j\neq i}\widehat{\pi}_{ij}Q_i,$$

$$\Pi'_{22i}=P_{2i}W_{ij}+P_{2i}LG_iV_i+W_i^{\rm T}P_{2i}+V_i^{\rm T}G_i^{\rm T}L^{\rm T}P_{2i}^{\rm T}+\beta_1V_i^{\rm T}G_i^{\rm T}T^{\rm T}TG_iV_i+\sum_{j\in S_i^k}\left(\widehat{\pi}_{ij}P_{2j}-\widehat{\pi}_{ij}P_{2k}+\frac{\delta_{ij}^2}{4}T_{2ijk}\right),$$

$$\Pi''_{11i}=A_iQ_i+Q_iA_i^{\rm T}+\varepsilon_iG_iN_iN_i^{\rm T}G_i^{\rm T}+G_iX_i+X_i^{\rm T}G_i^{\rm T}+\widehat{\pi}_{ii}\frac{\delta_{ii}^2}{4}(2Q_i-\overline{S}_{1ii})-\sum_{j\in S_i^k,j\neq i}\widehat{\pi}_{ij}Q_i,$$

$$\Pi''_{22i}=P_{2i}W_{ij}+P_{2i}LG_iV_i+W_i^{\mathrm{T}}P_{2i}+V_i^{\mathrm{T}}G_i^{\mathrm{T}}L^{\mathrm{T}}P_{2i}^{\mathrm{T}}+\beta_1V_i^{\mathrm{T}}G_i^{\mathrm{T}}T^{\mathrm{T}}TG_iV_i+\sum_{j\in S_i^k,j\neq i}\left(\widehat{\pi}_{ij}P_{2j}-\widehat{\pi}_{ij}P_{2i}+\frac{\delta_{ij}^2}{4}S_{2ij}\right),$$

$$\Pi'_{99i}=-\mathrm{diag}(2Q_{l_i^1}-\overline{T}_{1il_i^1k}\cdots 2Q_{l_i^r}-\overline{T}_{1il_i^rk}\cdots 2Q_{l_i^m}-\overline{T}_{1il_i^mk})_{l_i^r\neq i},$$

$$\Pi''_{99i}=-\mathrm{diag}\big(2Q_{l_i^1}-\overline{S}_{1il_i^1}\cdots 2Q_{l_i^r}-\overline{S}_{1il_i^r}\cdots 2Q_{l_i^m}-\overline{S}_{1il_i^m}\big)_{l_i^r\neq i}.$$

Moreover, the controller gains are given by $K_i=X_iQ_i^{-1}$.

Proof. First, if $i\in S_i^{\mathrm{uk}}$, according to Lemma 1 and letting $\varepsilon=1$, we have $\Pi_{99i}\geqslant-\mathrm{diag}(Q_{l_i^1}\overline{J}_{1il_i^1}^{-1}Q_{l_i^1},\ldots,Q_{l_i^m}\overline{J}_{1il_i^m}^{-1}Q_{l_i^m})$. We define $\Lambda_i=\mathrm{diag}(\,Q_{l_i^1}^{-1},\,\ldots,\,Q_{l_i^m}^{-1}\,)$, $Q_i=P_{1i}^{-1}$, $X_i=K_iQ_i$, $J_{1ij}=\overline{J}_{1ij}^{-1}$. Pre-multiplying and post-multiplying (38) with the term Π_{99i} replaced by the element of $-\mathrm{diag}(Q_{l_i^1}\overline{J}_{1il_i^1}^{-1}Q_{l_i^1},\ldots,Q_{l_i^m}\overline{J}_{1il_i^m}^{-1}Q_{l_i^m})$, simultaneously using $\mathrm{diag}\{\,Q_i^{-1}\;I\;I\;I\;I\;I\;I\;I\;\Lambda_i\;I\;I\,\}$, we have

$$\begin{pmatrix}\overline{\Pi}_{11i}&P_{1i}G_iV_i&P_{1i}F_i&P_{1i}H_i&0&0&0&\overline{\Xi}_{1i}\\ *&\Pi_{22i}&0&P_{2i}LH_i&P_{2i}M_i&P_{2i}R&P_{2i}R&\Xi_{2i}\\ *&*&-\frac{1}{\lambda_i^2}I&0&0&0&0&0\\ *&*&*&-I&0&0&0&0\\ *&*&*&*&-I&0&0&0\\ *&*&*&*&*&-\beta_1I&0&0\\ *&*&*&*&*&*&-\beta_2I&0\\ *&*&*&*&*&*&*&\overline{\Xi}_{3i}\end{pmatrix}<0, \tag{44}$$

where

$$\overline{\Pi}_{11i}=P_{1i}A_i+A_i^{\mathrm{T}}P_{1i}+\varepsilon_iP_{1i}G_iN_iN_i^{\mathrm{T}}G_i^{\mathrm{T}}P_{1i}+P_{1i}G_iK_i+K_i^{\mathrm{T}}G_i^{\mathrm{T}}P_{1i}-\sum_{j\in S_i^k}\widehat{\pi}_{ij}P_{1i},$$

$$\overline{\Xi}_{1i}=(\,D_i^{\mathrm{T}}\;U^{\mathrm{T}}\;\overline{\Pi}_{18i}\;\overline{\Pi}_{19i}\;\;0\;\overline{\Pi}_{111i}\,),\;\;\overline{\Pi}_{18i}=(\,I,\ldots,I\,),\;\;\overline{\Pi}_{19i}=(\,P_{1l_i^1}-P_{1i},\ldots,P_{1l_i^m}-P_{1i}\,),$$

$$\overline{\Xi}_{3i}=\mathrm{diag}(\,-\varepsilon_iI\;-\lambda^2I\;\Pi_{88i}\;\overline{\Pi}_{99i}\;\pi_{1010i}\;\Pi_{1111i}\,),\;\overline{\Pi}_{111i}=(\,I,\ldots,I\,),\;\overline{\Pi}_{99i}=-\mathrm{diag}(\,J_{1il_i^1},\ldots,J_{1il_i^m}\,).$$

According to Lemma 2, we obtain

$$\begin{pmatrix}\Upsilon_{1i}&G_iV_i&F_i&H_i&0\\ *&\Upsilon_{2i}&0&P_{2i}\overline{L}H_i&P_{2i}H_i\\ *&*&-\frac{1}{\lambda_i^2}I&0&0\\ *&*&*&-I&0\\ *&*&*&*&-I\end{pmatrix}<0, \tag{45}$$

where

$$\begin{aligned}\Upsilon_{1i}={}&P_{1i}A_i+A_i^{\mathrm{T}}P_{1i}+P_{1i}G_iK_i+K_i^{\mathrm{T}}G_i^{\mathrm{T}}P_{1i}+P_{1i}G_iN_iB_i(t)D_i+D_i^{\mathrm{T}}B_i^{\mathrm{T}}(t)N_i^{\mathrm{T}}P_{1i}+\frac{1}{\lambda^2}U_i^{\mathrm{T}}U_i\\ &+\sum_{j\in S_i^k}(\widehat{\pi}_{ij}P_{1j}-\widehat{\pi}_{ij}P_{1i})+\sum_{j\in S_i^k}\left(\frac{\delta_{ij}^2}{4}J_{1ij}+(P_{1j}-P_{1i})J_{1ij}^{-1}(P_{1j}-P_{1i})\right),\\ \Upsilon_{2i}={}&P_{2i}W_{ij}+P_{2i}\overline{L}G_iV_i+W_i^{\mathrm{T}}P_{2i}+V_i^{\mathrm{T}}G_i^{\mathrm{T}}\overline{L}^{\mathrm{T}}P_{2i}+\sum_{j\in S_i^k}(\widehat{\pi}_{ij}P_{2j}-\widehat{\pi}_{ij}P_{2i})\\ &+\sum_{j\in S_i^k}\left(\frac{\delta_{ij}^2}{4}J_{2ij}+(P_{2j}-P_{2i})J_{2ij}^{-1}(P_{2j}-P_{2i})\right).\end{aligned}$$

Let $P_i = \left(\begin{smallmatrix} P_{1i} & 0 \\ 0 & P_{2i} \end{smallmatrix}\right)$ and $J_{ij} = \left(\begin{smallmatrix} J_{1ij} & 0 \\ 0 & J_{2ij} \end{smallmatrix}\right)$. We obtain (25) and (27) from (39) and (40).

Subsequently, if $i \in S_i^k$ and $\sum_{j\in S_i^{\rm uk}} \pi_{ij} \neq 0$, or $i \in S_i^k$ and $\sum_{j\in S_i^{\rm uk}} \pi_{ij} = 0$, following similar steps as the case (i), we can derive (27) from (41) and (42), and (28) from (43). Thus, by using Schur complement, (24) could be deduced if (37) holds. Hence, according to Theorem 2, we conclude that the composite system (11) is stochastically stable with $L_2 - L_\infty$ performance.

3.2 The case of unknown nonlinearity

In this subsection, we suppose that Assumptions 1 and 3 hold, and the nonlinear function $f(x(t), t)$ is unknown; thus the disturbance observer is designed as [35]

$$\begin{aligned} &\widehat{d}_1(t) = V_i\widehat{w}(t), \\ &\widehat{w}(t) = v(t) - \bar{L}x(t), \\ &\dot{v}(t) = (W_i + \bar{L}G_iV_i)(v(t) - \bar{L}x(t)) + \bar{L}(A_ix(t) + G_iu(t)). \end{aligned} \tag{46}$$

The composite resilient controller is constructed in the same form as (5). We define $e_w(t) = w(t) - \widehat{w}(t)$, and subsequently, we express the error system as

$$\dot{e}_w(t) = (W_i + \bar{L}G_iV_i)e_w(t) + M_id_3(t) + LH_id_2(t). \tag{47}$$

Combining (1) and (5) with (47), the composite system can be expressed as

$$\begin{aligned} (\Sigma') \quad \dot{\xi}(t) &= \overline{A}_i(t)\xi(t) + \overline{F}_if(\xi(t), t) + \overline{H}_id(t), \\ z(t) &= C_{1i}x(t) + C_{2i}e_w(t) = \overline{C}_i\xi(t), \end{aligned} \tag{48}$$

where $\xi(t) = (x^{\rm T}(t), e_w^{\rm T}(t))^{\rm T}$, $d(t) = (d_2^{\rm T}(t), d_3^{\rm T}(t))^{\rm T}$, $f_i(\xi(t), t) = f(x(t), t)$ and $z(t)$ is reference output, with

$$\overline{A}_i = \begin{pmatrix} A_i + G_i\overline{K}_i & G_iV_i \\ 0 & W_i + \bar{L}G_iV_i \end{pmatrix}, \quad \overline{H}_i = \begin{pmatrix} H_i & 0 \\ \bar{L}H_i & M_i \end{pmatrix}, \quad \overline{F}_i = \begin{pmatrix} F_i \\ \bar{L}F_i \end{pmatrix}, \quad \overline{C}_i = [C_{1i}, C_{2i}].$$

Now, we present a sufficient condition by using the LMI technique, such that the augmented system in (Σ') satisfies the conditions (i) and (ii).

By comparing the system matrices in (11) and (48) and following similar arguments for Theorem 3, we can directly obtain Corollary 1.

Corollary 1. Consider system (1) under Assumptions 1 and 3. Given the parameters $\gamma > 0$ and $\lambda > 0$, by designing a disturbance observer and a resilient controller in the forms of (4) and (5), the closed-loop system (Σ) is stochastically stable with $L_2 - L_\infty$ performance, if there exist matrices $Q_i > 0$, $P_{2i} > 0$, $\overline{J}_{1ij} > 0$, $J_{2ij} > 0$, $j \in S_i^k$, $\overline{T}_{1ijk} > 0$, $T_{2ijk} > 0$, $j \in S_i^k$, $k \in S_i^{\rm uk}$, $\overline{S}_{1ij} > 0$, $S_{2ij} > 0$, $j \in S_i^k$ and X_i, and positive numbers ε_i, $i \in S$, such that the following inequalities hold:

$$\begin{pmatrix} -P_{2i} & C_{2i}^{\rm T} \\ * & -\gamma I + C_{1i}Q_iC_{1i}^{\rm T} \end{pmatrix} < 0. \tag{49}$$

(i) If $i \in S_i^{\rm uk}$,

$$\begin{pmatrix} \Pi_{11i} & G_iV_i & F_i & H_i & 0 & 0 & 0 & \Xi_{1i} \\ * & \Pi_{22i} & P_{2i}LF_i & P_{2i}LH_i & P_{2i}M_i & P_{2i}R & P_{2i}R & \Xi_{2i} \\ * & * & -\frac{1}{\lambda^2}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I+\beta_2H_i^{\rm T}T^{\rm T}TH_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\beta_1I & 0 & 0 \\ * & * & * & * & * & * & -\beta_2I & 0 \\ * & * & * & * & * & * & * & \Xi_{3i} \end{pmatrix} < 0, \tag{50}$$

$$P_{2j} - P_{2i} \leqslant 0, \quad \forall j \in S_i^{\rm uk}, \quad j \neq i, \tag{51}$$

$$-Q_j + Q_i \leqslant 0, \quad \forall j \in S_i^{\rm uk}, \quad j \neq i; \tag{52}$$

(ii) If $i \in S_i^k$ and $\sum_{j\in S_i^{\rm uk}} \pi_{ij} \neq 0$, then, for $\forall k \in S_i^{\rm uk}$

$$\begin{pmatrix} \Pi'_{11i} & G_iV_i & F_i & H_i & 0 & 0 & 0 & \Xi'_{1i} \\ * & \Pi'_{22i} & P_{2i}LF_i & P_{2i}LH_i & P_{2i}M_i & P_{2i}R & P_{2i}R & \Xi'_{2i} \\ * & * & -\frac{1}{\lambda^2}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I+\beta_2H_i^{\rm T}T^{\rm T}TH_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\beta_1I & 0 & 0 \\ * & * & * & * & * & * & -\beta_2I & 0 \\ * & * & * & * & * & * & * & \Xi'_{3i} \end{pmatrix} < 0, \tag{53}$$

$$-Q_i + Q_k \leqslant 0; \tag{54}$$

(iii) If $i \in S_i^k$ and $\sum_{j\in S_i^{\rm uk}} \pi_{ij} = 0$,

$$\begin{pmatrix} \Pi''_{11i} & G_iV_i & F_i & H_i & 0 & 0 & 0 & \Xi''_{1i} \\ * & \Pi''_{22i} & P_{2i}LF_i & P_{2i}LH_i & P_{2i}M_i & P_{2i}R & P_{2i}R & \Xi''_{2i} \\ * & * & -\frac{1}{\lambda^2}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I+\beta_2H_i^{\rm T}T^{\rm T}TH_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -\beta_1I & 0 & 0 \\ * & * & * & * & * & * & -\beta_2I & 0 \\ * & * & * & * & * & * & * & \Xi''_{3i} \end{pmatrix} < 0. \tag{55}$$

The corresponding matrices are given in Theorem 3. Moreover, the controller gains are given by $K_i = X_iQ_i^{-1}$.

4 Application example

In this section, we demonstrate the effectiveness of the proposed control method using a single link robot arm system inspired by [1,4,41],

$$\ddot{\alpha}(t) = -\frac{mgl}{J}\sin(\alpha(t)) - \frac{D}{J}\dot{\alpha}(t) + \frac{1}{J}[u(t) + d_1(t)] + \frac{1}{J}d_2(t), \tag{56}$$

where α is the angle position of the arm, $u(t)$ denotes the control input, and $d_2(t)$ represents the external disturbance. The disturbance $d_1(t)$ can be described by the exogenous system (3). Parameters l, D, g, m, and J denote the arm length, viscous friction, gravity acceleration, payload mass, and inertia moment, respectively. The values of some important parameters are given as $D = 1$, $l = 0.1$, and $g = 9.8$.

Remark 6. When the single link robot arm functions under different environmental conditions and with changing payload, it can be modeled as a Markovian jump system [1,4].

Let $x_1(t) = \alpha(t)$ and $x_2(t) = \dot{\alpha}(t)$; the reference output is chosen as $z^{\rm T}(t) = (x_1(t), x_2(t))$. Thus, the single link robot arm system (56) is modeled as a Markovian jump system with four modes

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 0 & -\frac{D}{J_i} \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ -\frac{m_igl}{J_i} \end{pmatrix} \sin(x_1(t)) + \begin{pmatrix} 0 \\ J_i \end{pmatrix} [u(t) + d_1(t)] + \begin{pmatrix} 0 \\ J_i \end{pmatrix} d_2(t), \tag{57}$$

$$z(t) = (x_1(t)\ x_2(t))^{\mathrm{T}}. \tag{58}$$

Mode 1:

$$J_1 = 3.3,\ m_1 = 0.4,\ V_1 = (2.0\ 1.8),\ N_1 = 0.9,\ W_1 = \begin{pmatrix} 0 & -2.0 \\ 2.0 & 0 \end{pmatrix},\ M_1 = \begin{pmatrix} -0.8 \\ 0.9 \end{pmatrix},\ D_1 = (-0.4\ 1.0).$$

Mode 2:

$$J_2 = 2.1,\ m_2 = 0.9,\ V_2 = (1.9\ -0.5),\ N_2 = 2.0,\ W_2 = \begin{pmatrix} 0 & 2.0 \\ -2.0 & 0 \end{pmatrix},\ M_2 = \begin{pmatrix} 2.8 \\ -1.7 \end{pmatrix},\ D_2 = (-0.2\ 1.3).$$

Mode 3:

$$J_3 = 1,\ m_3 = 0.7,\ V_3 = (-1.4\ -0),\ N_3 = 1.0,\ W_3 = \begin{pmatrix} 0 & -1.0 \\ 1.0 & 0 \end{pmatrix},\ M_3 = \begin{pmatrix} 1.0 \\ -1.1 \end{pmatrix},\ D_3 = (0.3\ 0.9).$$

Mode 4:

$$J_4 = 3,\ m_4 = 0.4,\ V_4 = (1.0\ -0.2),\ N_4 = 2,\ W_4 = \begin{pmatrix} 0 & 1.0 \\ -1.0 & 0 \end{pmatrix},\ M_4 = \begin{pmatrix} 1.3 \\ 0.5 \end{pmatrix},\ D_4 = (1.1\ 1.2).$$

The transition rate matrix Π is given as

$$\Pi = \begin{pmatrix} -0.7 + \Delta_{11} & 0.1 + \Delta_{12} & ? & ? \\ 0.3 + \Delta_{21} & ? & ? & 0.2 + \Delta_{24} \\ ? & 0.4 + \Delta_{32} & ? & 0.2 + \Delta_{34} \\ 0.2 + \Delta_{41} & 0.3 + \Delta_{42} & 0.3 + \Delta_{43} & -0.8 + \Delta_{44} \end{pmatrix},$$

and $\delta_{11} = 0.05$, $\delta_{12} = 0.02$, $\delta_{21} = 0.01$, $\delta_{24} = 0.02$, $\delta_{32} = 0.02$, $\delta_{34} = 0.02$, $\delta_{41} = 0.01$, $\delta_{42} = 0.02$, $\delta_{43} = 0.01$, $\delta_{44} = 0.03$. The uncertain matrices $B_i(t)$ are chosen as $B_i(t) = \sin(t)$ and $S(t)$ is chosen as $S(t) = \cos(t)$, $i \in 1, 2, 3, 4$. The external disturbance is $d_2(t) = \mathrm{e}^{-t}\sin(t)$ and uncertain variation is $d_3(t) = \mathrm{e}^{-t}\sin(t)$.

Thus, we confirm that the composite anti-disturbance resilient control methods for the nonlinear functions are known and unknown.

4.1 The case of known nonlinear function

Based on (6), we obtain $L = \left(\begin{smallmatrix} 0 & -2.4731 \\ 0 & -3.0237 \end{smallmatrix}\right)$, and from Theorem 3, we obtain

$$K_1 = (\,-3.4647\ -0.5687\,),\quad K_2 = (\,-4.6521\ -1.2711\,),$$
$$K_3 = (\,-10.1173\ -2.4719\,),\quad K_4 = (\,-3.7536\ -0.8171\,).$$

The simulation results are shown in Figure 1. In Figure 1(a), under the designed composite anti-disturbance resilient controller, the closed-loop system shows good control performance when suffering from the above multiple disturbances. In Figure 1(b), the control input is shown. The switching signal is presented in Figure 1(c). We can observe from Figure 1(d) that the disturbance $d_1(t)$ is efficiently estimated using a disturbance observer.

4.2 The case of unknown nonlinear function

According to (47), we obtain $L' = \left(\begin{smallmatrix} 0 & -2.2247 \\ 0 & -1.8199 \end{smallmatrix}\right)$. By using Corollary 1, we obtain

$$K_1' = (\,-4.4407\ -1.4418\,),\quad K_2' = (\,-4.1981\ -1.5704\,),$$
$$K_3' = (\,-7.5673\ -2.8038\,),\quad K_4' = (\,-5.3893\ -1.7894\,).$$

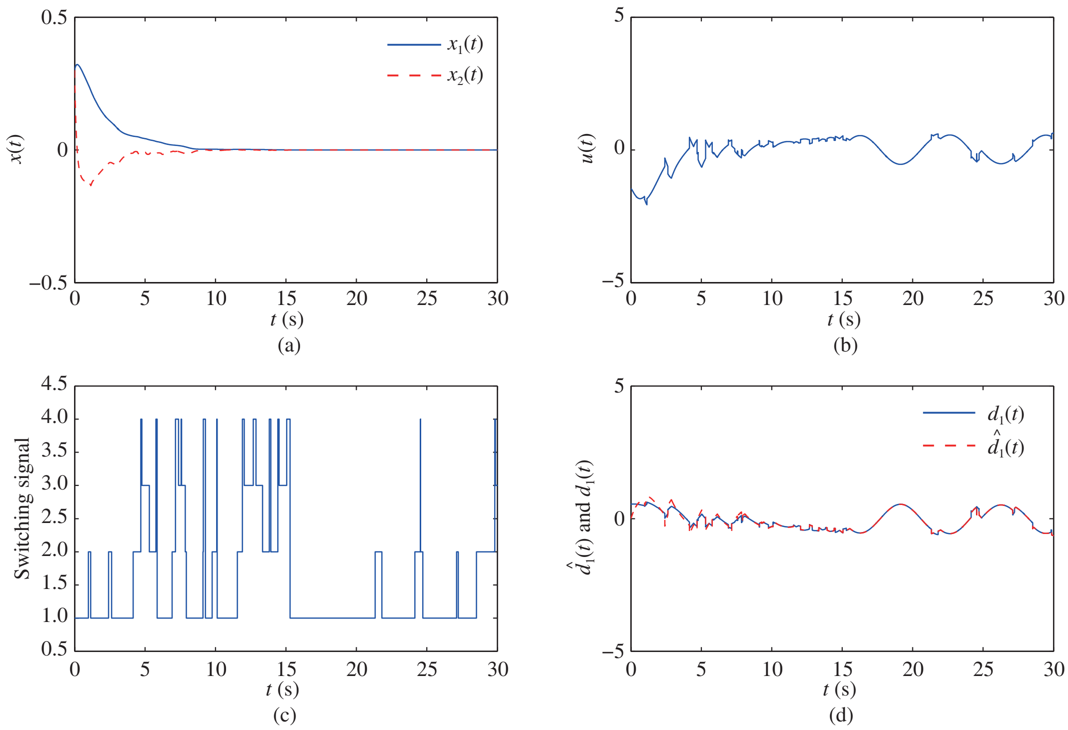

Figure 1 (Color online) Curves of simulation trajectories with known nonlinearity function. (a) System states; (b) control input; (c) switching signal; (d) disturbances and disturbance estimation.

The corresponding simulation results are presented in Figure 2. The state trajectories of the closed-loop system are given in Figure 2(a), which demonstrates that the composite anti-disturbance resilient control method is feasible in spite of existing multiple disturbances. The control input is shown in Figure 2(b). The switching signal is shown in Figure 2(c). In Figure 2(d), the exogenous disturbances $d_1(t)$ is estimated effectively using the proposed disturbance observer.

4.3 Comparison between the proposed method and the conventional L_2–L_∞ control scheme

In order to show the superiority of the proposed method, we compare the proposed algorithm with the conventional L_2-L_∞ control scheme. From Figure 3, we can observe that, in contrast to the conventional L_2-L_∞ control, our approach can achieve smaller overshoots and better rejection and attenuation of multiple disturbances.

Remark 7. In this study, the single link robot arm system (56) is modelled as a Markovian jump nonlinear system. Compared with the existing literatures, the disturbance of the single link robot arm system (56) has been divided into two kinds, which promotes the anti-disturbance ability of the system. Further, the transition rate matrix is generally uncertain, which makes the model suitable to describe a wide range of practical systems. Using the designed composite anti-disturbance resilient controller, we achieve a satisfactory attenuation level.

Remark 8. The system parameters of a single link robot arm are chosen according to [1,4]. With the aid of the MATLAB software, the controller gains are solved via LMI Toolbox. The SIMULINK simulation model of the single link robot arm is established, which is based on the S-function. In addition, the simulation results are presented.

5 Conclusion

We have investigated the composite anti-disturbance resilient control problem for Markovian jump nonlinear systems with general uncertain transition rates and multiple disturbances. Two cases of disturbances

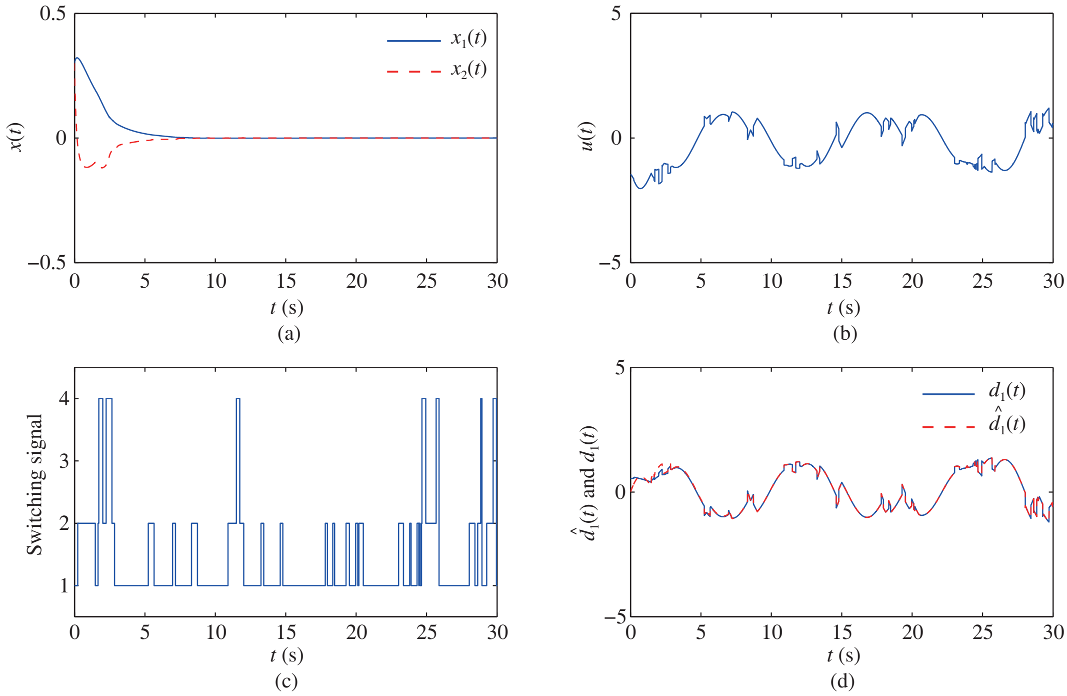

Figure 2 (Color online) Curves of simulation trajectories with unknown nonlinearity function. (a) System states; (b) control input; (c) switching signal; (d) disturbances and disturbance estimation.

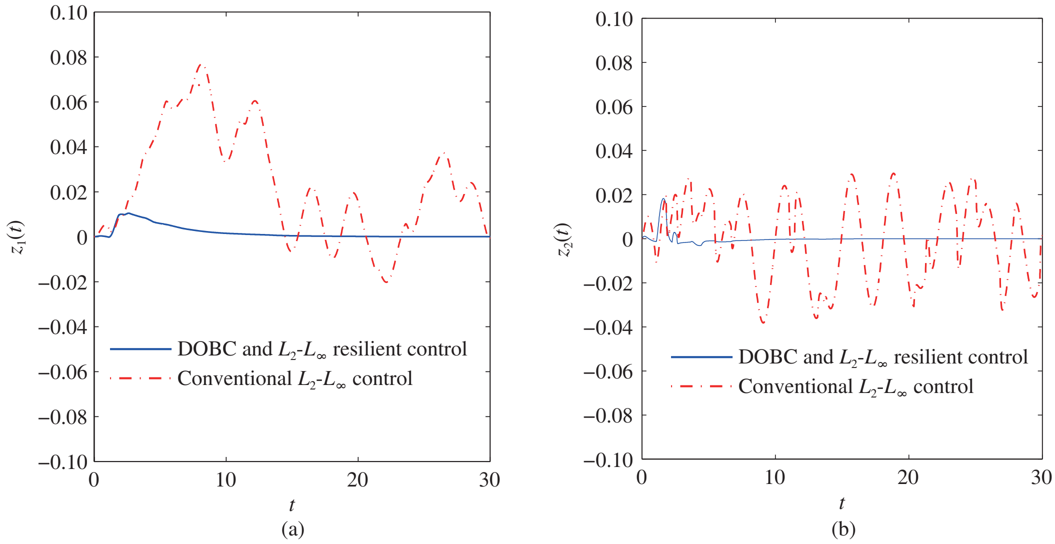

Figure 3 (Color online) Curves of $z(t)$. (a) Composite anti-disturbance resilient control; (b) the conventional control.

are fully considered and some sufficient conditions have been established for verifying the stochastic stability while ensuring the $L_2 - L_\infty$ performance of the closed-loop system. A composite anti-disturbance resilient controller has been designed by solving a set of convex optimization conditions. All the conditions have been transformed into the form of linear matrix inequalities for the sake of calculation. Finally, we have modeled the single link robot arm system as a Markovian jump nonlinear system and studied its composite anti-disturbance resilient control problem, which verifies the effectiveness of the main algorithm in this paper.

In the future, in order to obtain less conservative results, the uncertain transition rate of polytoptic type will be considered [10, 15] and the composite anti-disturbance controller will be designed for Markovian jump systems.

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