SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

February 2019, Vol. 62 022203:1–022203:12
https://doi.org/10.1007/s11432-017-9417-0

Hybrid event- and time-triggered control for double-integrator heterogeneous networks

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Received 8 November 2017/Revised 5 February 2018/Accepted 2 April 2018/Published online 28 December 2018

Abstract This paper investigates the state consensus for double-integrator networks under heterogeneous interaction topologies. For double-integrator networks, the setting of heterogeneous topologies means that position and velocity information flows are modeled by two different graphs. The corresponding protocol proposed in this paper is based on edge-event-triggered control. The events based on position information are irrelevant to velocity information and independent of the events based on velocity information. And for different edges, the corresponding events are activated independently of each other. Once an event occurs, the agents connected by the associated edge will sample their corresponding relative state information and update their corresponding controllers. Furthermore, under the presented event-triggering rules, the state consensus of double-integrator networks can be achieved by designing appropriate parameters. In addition, the proposed protocol with the event-triggering rules can effectively improve the system performance and avoid the occurrence of Zeno behaviors. Finally, a simulation example is worked out to verify the theoretical analysis.

Keywords consensus, sampled-data control, multi-agent systems, edge-event-triggered control, heterogeneous networks

Citation Duan G P, Xiao F, Wang L. Hybrid event- and time-triggered control for double-integrator heterogeneous networks. Sci China Inf Sci, 2019, 62(2): 022203, https://doi.org/10.1007/s11432-017-9417-0

1 Introduction

Distributed control of networked systems has drawn great attention of researchers in various fields owing to its wide applications [1–20]. Therein, consensus problems of multi-agent systems are of great interest, and they aim to drive all agents to reach a common state with limited and unreliable information transmission by designing appropriate control laws. The typical consensus model under directed networks was given by Olfati-Saber and Murray in [21], where they discussed the consensus problems under directed fixed networks, directed switching networks and undirected fixed networks with communication time-delays, respectively.

Many existing studies were centred on multi-agent systems with multiple first-order integrators. In practical engineering applications, the behavior of some agents needs to be modeled by second-order dynamics [22]

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t), \end{cases} \qquad i \in \underline{n} = \{1, 2, \ldots, n\}, \tag{1}$$

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where $x_i(t)$, $v_i(t)$ are the position and velocity of agent i at time t, respectively, and $u_i(t)$ is the control input. In practice, it is hard to guarantee continuous communication and uninterrupted controller update for agents. Periodic sampled-data models characterize the process of interaction among agents intermittently [23, 24]. Compared with the typical periodic sampled-data control, event-triggered control is a possible alternative with respect to reducing unnecessary data sampling and redundant controller update [25–35]. In event-triggered control, an important task is to design event-triggering schemes. As a topic in event-triggered control, the edge-event-triggered control of multi-agent systems was introduced in [27], where edge events were defined independently for communication links and when the events of some edge are activated, the pair of adjacent agents connected by this edge sample the corresponding state data and update their controllers. In [27], the authors solved the average consensus problem for single-integrator networks based on hybrid event-time triggered control. The combination of time-triggered control and event-triggered control effectively eliminates Zeno behaviors. Furthermore, Cao et al. [32] investigated the average consensus of multiple double-integrator networks based on edge-event-triggered control, and proposed a corresponding protocol as well as event-detecting rules.

In this paper, we focus on the edge-event-based sampled-data consensus of multiple double-integrator systems over heterogeneous topologies. In [26, 36, 37], the centralized event-triggered control was investigated. It relaxes the requirement of continuous controller update. However, it needs global information to evaluate the triggering condition and requires all agents to update their controllers simultaneously. In this paper, we consider the decentralized event-triggered control, in which each agent detects the event-triggering condition only based on the states of its neighbors and its own. And all agents update their control signals asynchronously. In the event-triggered control studied in [25, 26, 31, 33, 36, 37], when an event occurs, the corresponding agent and all its neighbors will update all their controllers accordingly. Different from that mechanism, we consider a novel mechanism based on edge-event-triggered control, which designs events for the edges of the communication graph rather than nodes. At event times, only a pair of adjacent agents exchange information. In the implementation of event-triggered control, an important issue is to avoid Zeno behaviors. However, many existing results were difficult to achieve the goal based on continuous monitoring. To overcome the difficulty, we adopt a hybrid time- and event-triggered mechanism to get rid of the Zeno behaviors. In engineering applications, an agent could be equipped with multiple sensors. And it is not practical to assume that all these sensors work synchronously. Therefore, different types of senors for position measurement and for velocity measurement may lie in different networks, which motivates us to investigate the multi-agent systems with heterogenous topologies. The system we investigate is more general than the double-integrator networks with homogeneous topologies [32]. Theoretically, the convergence analysis of system (1) over heterogeneous networks is more challenging than that over homogeneous networks. In homogeneous networks [32], when some events of some edge occur, the corresponding agents connected by this edge sample both their relative position information and relative velocity information, and update position information and velocity information in their controllers, which leads to energy waste to some extent. However, in the case of heterogeneous networks, position-based edge events and velocity-based edge events occur independently over different graphs. Owing to the independence, when events of some edge occur, the agents jointed by this edge only need to sample the corresponding state information (position information or velocity information) and update corresponding information in their controllers. For instance, at some event-detecting time, if some events over position communication graph are activated, the corresponding agents sample their relative position information and update their position information in controllers. In such a case, the agents do not need to sample their relative velocity information as required in [32], which reduces communication costs and controller-updating costs. We propose a protocol for heterogeneous networks and provide the corresponding event-detecting rules based on edge-event-triggered control, which can effectively reduce communication costs and controller-updating costs. Furthermore, we give a sufficient condition for state consensus which is verified by Lyapunov methods.

The outline of this paper is as follows. In Section 2, we introduce some basic definitions in algebraic graph theory and propose the protocol; in Section 3, the event-detecting rules are presented, and the consensus problem is solved by Lyapunov methods; Section 4 gives a simulation example to demonstrate the effectiveness of the result; finally, we make a brief conclusion of this paper and discuss possible future studies in Section 5.

Notation. $\mathbb{R}$, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the sets of real numbers, n-dimensional real column vectors and all $m\times n$ real matrices, respectively. I and $\mathbf{0}$ are the $n\times n$ identity matrix and a zero vector or matrix with compatible dimensions, respectively. The set $\{1,2,3,\ldots,n\}$ is denoted by $\underline{n}$. In addition, $\|A\|$ represents the (induced) 2-norm of matrix or vector A and $|a|$ is the absolute value of number a. $\lambda_2(A)$ denotes the second smallest eigenvalue of real symmetric matrix A. For matrices A, B, $A>B$ means $A-B$ is positive definite.

2 Preliminaries

Consider the multi-agent system (1) modeled by double-integrator network, which consists of n agents. In this paper, we investigate the consensus of system (1) in the setting that position information and velocity information is transmitted over different networks. Assume that there are m_p communication channels in the position communication network and m_v communication channels in the velocity communication network. The two networks are modeled by graph G_p and G_v, respectively. In G_p and G_v, each vertex represents an agent and each edge represents a communication channel between agents. $G_p=\{\mathcal{V},E_p,A_p\}$ consists of a vertex set $\mathcal{V}=\{\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_n\}$, an edge set $E_p=\{e_1^p,e_2^p,\ldots,e_{m_p}^p\}\subseteq\mathcal{V}\times\mathcal{V}$, and an adjacency matrix $A_p=[a_{ij}^p]\in\mathbb{R}^{n\times n}$. $a_{ij}^p>0$ if there is an edge $(\mathcal{V}_j,\mathcal{V}_i)$ in G_p from $\mathcal{V}_j$ to $\mathcal{V}_i$, otherwise $a_{ij}^p=0$. Moreover, a_{ij}^p is called the weight of edge $(\mathcal{V}_j,\mathcal{V}_i)$. Analogously, $G_v=\{\mathcal{V},E_v,A_v\}$ consists of the vertex set $\mathcal{V}$, an edge set $E_v=\{e_1^v,e_2^v,\ldots,e_{m_v}^v\}\subseteq\mathcal{V}\times\mathcal{V}$, and an adjacency matrix $A_v=[a_{ij}^v]\in\mathbb{R}^{n\times n}$. In G_p (resp. G_v), $\mathcal{V}_j$ is called a neighbor of $\mathcal{V}_i$, if $a_{ij}^p>0$ (resp. $a_{ij}^v>0$). And the set of all neighbors of $\mathcal{V}_i$ in G_p (resp. G_v) is denoted by N_i^p (resp. N_i^v). In addition, define edge weight matrix W_p of G_p (resp. W_v of G_v) as $W_p=\mathrm{diag}(w_1^p,w_2^p,\ldots,w_{m_p}^p)$ (resp. $W_v=\mathrm{diag}(w_1^v,w_2^v,\ldots,w_{m_v}^v)$), where w_q^p (resp. w_q^v) is the weight of edge e_q^p, $q\in\underline{m_p}$ (resp. e_q^v, $q\in\underline{m_v}$), equal to a_{ij}^p with $e_q^p=(\mathcal{V}_j,\mathcal{V}_i)$ (resp. a_{ij}^v with $e_q^v=(\mathcal{V}_j,\mathcal{V}_i)$).

For undirected graph G_p, assign an arbitrary orientation to each edge, and define the incidence matrix $D_p=[d_{ij}^p]\in\mathbb{R}^{n\times m_p}$ as

$$d_{ij}^p=\begin{cases}-1, & \text{if } \mathcal{V}_i \text{ is the tail of the } j\text{-th oriented edge in } G_p,\\ 1, & \text{if } \mathcal{V}_i \text{ is the head of the } j\text{-th oriented edge in } G_p,\\ 0, & \text{otherwise.}\end{cases}$$

Define the Laplacian matrix $L_p=[l_{ij}^p]\in\mathbb{R}^{n\times n}$ of graph G_p as

$$l_{ij}^p=\begin{cases}-a_{ij}^p, & \text{if } i\neq j,\\ \displaystyle\sum_{k\in N_i^p}a_{ik}^p, & \text{if } i=j.\end{cases}$$

Apparently, $L_p=D_pW_pD_p^{\mathrm{T}}$ holds for undirected graph G_p. Similarly, define the incidence matrix $D_v=[d_{ij}^v]\in\mathbb{R}^{n\times m_v}$ and the Laplacian matrix $L_v=[l_{ij}^v]\in\mathbb{R}^{n\times n}$ of graph G_v, and if G_v is an undirected graph, $L_v=D_vW_vD_v^{\mathrm{T}}$ holds as well.

Let t_k, $k=0,1,2,\ldots$, denote the event-detecting times of all edges in G_p and G_v with $t_{k+1}=t_k+h$, where $h>0$ is the event-detecting period. At these times, each agent checks the edge-event-triggering conditions shared with its corresponding neighbor, and decides whether to sample the relative data and to update its controller accordingly. The agents, connected by different edges, follow the aforementioned procedure independently of each other. Furthermore, let $t_{k^{p(ij)}(t)}$ or $t_{k^{p(q)}(t)}$ (resp. $t_{k^{v(ij)}(t)}$ or $t_{k^{v(q)}(t)}$) denote the recent event time before or at time t for edge $e_q^p=(\mathcal{V}_i,\mathcal{V}_j)$ (resp. $e_q^v=(\mathcal{V}_i,\mathcal{V}_j)$). Mathematically, $k^{p(q)}(t)=\max\{k|t_k\leqslant t,\ \text{an edge event of } e_q^p \text{ in } G_p \text{ occurs at } t_k\}, q\in\underline{m_p}$, and $k^{v(q)}(t)=\max\{k|t_k\leqslant t,\ \text{an edge event of } e_q^v \text{ in } G_v \text{ occurs at } t_k\}$, $q\in\underline{m_v}$.

Specifically, we adopt the following protocol:

$$u_i(t)=\sum_{j\in N_i^p}a_{ij}^p(x_j(t_{k^{p(ij)}(t)})-x_i(t_{k^{p(ij)}(t)}))+k\sum_{j\in N_i^v}a_{ij}^v(v_j(t_{k^{v(ij)}(t)})-v_i(t_{k^{v(ij)}(t)})),\tag{2}$$

where k is to be designed later. Under protocol (2), we consider the state consensus of system (1), that is, for any initial states $x_i(0)$, $v_i(0)$, $i \in \underline{n}$, $x_i(t) \to \frac{1}{n}\sum_{j=1}^n x_j(t)$, $v_i(t) \to \frac{1}{n}\sum_{j=1}^n v_j(t)$, $i \in \underline{n}$ as $t \to \infty$.

3 Consensus based on edge-event-triggered control

For convenience, we introduce variables $\delta_i(t) = x_i(t) - \frac{1}{n}\sum_{j=1}^n x_j(t)$, $\sigma_i(t) = v_i(t) - \frac{1}{n}\sum_{j=1}^n v_j(t)$, and it is clear that system (1) achieves the state consensus if $\delta_i(t) \to 0$, $\sigma_i(t) \to 0$, $i \in \underline{n}$. We consider the following Lyapunov function candidate

$$V(t) = \left[\delta^{\mathrm{T}}(t)\ \sigma^{\mathrm{T}}(t)\right] \begin{bmatrix} \beta L_p + \dfrac{k\alpha\lambda_2(L_v)}{2}I & \dfrac{\alpha}{2}I \\ \dfrac{\alpha}{2}I & \beta I \end{bmatrix} \begin{bmatrix} \delta(t) \\ \sigma(t) \end{bmatrix}, \tag{3}$$

where parameters α, β are to be designed, $\delta(t) = [\delta_1(t)\ \delta_2(t)\ \cdots\ \delta_n(t)]^{\mathrm{T}}$, and $\sigma(t) = [\sigma_1(t)\ \sigma_2(t)\ \cdots\ \sigma_n(t)]^{\mathrm{T}}$.

Lemma 1. Assume that G_p and G_v are both connected. The Lyapunov function (3) is positive definite if $\beta > \frac{\alpha}{2k\lambda_2(L_v)} > 0$. Moreover, $V(t) = 0$ if and only if $\delta(t) = \sigma(t) = 0$, which implies system (1) achieves the state consensus.

Proof. Let

$$P = \begin{bmatrix} \beta L_p + \dfrac{k\alpha\lambda_2(L_v)}{2}I & \dfrac{\alpha}{2}I \\ \dfrac{\alpha}{2}I & \beta I \end{bmatrix}.$$

Obviously, $\beta L_p + \frac{k\alpha\lambda_2(L_v)}{2}I$ is invertible. Define

$$Q = \begin{bmatrix} I & -\dfrac{\alpha}{2}\left(\beta L_p + \dfrac{k\alpha\lambda_2(L_v)}{2}I\right)^{-1} \\ \mathbf{0} & I \end{bmatrix}.$$

Then, we have that

$$Q^{\mathrm{T}}PQ = \begin{bmatrix} \beta L_p + \dfrac{k\alpha\lambda_2(L_v)}{2}I & \mathbf{0} \\ \mathbf{0} & \beta I - \dfrac{\alpha^2}{4}\left(\beta L_p + \dfrac{k\alpha\lambda_2(L_v)}{2}I\right)^{-1} \end{bmatrix}.$$

Because L_p is a real symmetric matrix, it is easy to get that $\beta L_p + \frac{k\alpha\lambda_2(L_v)}{2}I$ is positive definite for any $\alpha, \beta > 0$. Because L_p is a symmetric real matrix and positive semi-definite, there exists an invertible matrix Φ such that $L_p = \Phi^{-1}\Lambda\Phi$, where Λ is a diagonal matrix with diagonal elements being the eigenvalues of L_p. Therefore, we have that

$$\beta I - \frac{\alpha^2}{4}\left(\beta L_p + \frac{k\alpha\lambda_2(L_v)}{2}I\right)^{-1} = \Phi^{-1}\left(\beta I - \frac{\alpha^2}{4}\left(\beta\Lambda + \frac{k\alpha\lambda_2(L_v)}{2}I\right)^{-1}\right)\Phi,$$

where $\beta I - \frac{\alpha^2}{4}(\beta\Lambda + \frac{k\alpha\lambda_2(L_v)}{2}I)^{-1}$ is a diagonal matrix. Because $\beta > 0$ and Λ is a diagonal matrix with nonnegative diagonal elements, $\beta\Lambda + \frac{k\alpha\lambda_2(L_v)}{2}I > \frac{k\alpha\lambda_2(L_v)}{2}I$ and $(\frac{k\alpha\lambda_2(L_v)}{2}I)^{-1} > (\beta\Lambda + \frac{k\alpha\lambda_2(L_v)}{2}I)^{-1}$ hold. Therefore, $\beta > \frac{\alpha}{2k\lambda_2(L_v)} > 0$, which is equivalent to $\beta I > \frac{\alpha^2}{4}(\frac{2}{k\alpha\lambda_2(L_v)}I)$ and can guarantee $\beta I > \frac{\alpha^2}{4}(\beta\Lambda + \frac{k\alpha\lambda_2(L_v)}{2}I)^{-1}$. In conclusion, $\beta > \frac{\alpha}{2k\lambda_2(L_v)} > 0$ can guarantee the positive definiteness of $\beta L_p + \frac{k\alpha\lambda_2(L_v)}{2}I$ and $\beta I - \frac{\alpha^2}{4}(\beta L_p + \frac{k\alpha\lambda_2(L_v)}{2}I)^{-1}$. Furthermore, P is positive definite, and $V(t) = 0$ if and only if $[\delta^{\mathrm{T}}\ \sigma^{\mathrm{T}}]^{\mathrm{T}} = \mathbf{0}$.

Afterwards, we introduce variables $y(t) = D_p^{\mathrm{T}}x(t)$ and $z(t) = D_v^{\mathrm{T}}v(t)$, where $x(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^{\mathrm{T}}$, $v(t) = [v_1(t)\ v_2(t)\ \cdots\ v_n(t)]^{\mathrm{T}}$, $y(t) = [y_1(t)\ y_2(t)\ \cdots\ y_{m_p}(t)]^{\mathrm{T}} \in \mathbb{R}^{m_p}$, $z(t) = [z_1(t)\ z_2(t)\ \cdots\ z_{m_v}(t)]^{\mathrm{T}} \in \mathbb{R}^{m_v}$. From the definitions of D_p, D_v, for any $y_{q_1}(t)$, $q_1 \in \underline{m_p}$, there exist agents i_1 and j_1, such that $y_{q_1}(t) = x_{i_1}(t) - x_{j_1}(t)$ with $d^p_{i_1q_1} = 1$ and $d^p_{j_1q_1} = -1$, which denotes the relative position information of agents i_1 and j_1 connected by $e^p_{q_1}$ in G_p at time t. Similarly, for any $z_{q_2}(t)$, $q_2 \in \underline{m_v}$, there exist agents i_2 and j_2, such that $z_{q_2}(t) = v_{i_2}(t) - v_{j_2}(t)$ with $d^v_{i_2q_2} = 1$ and $d^v_{j_2q_2} = -1$, which denotes the relative velocity information of agents i_2 and j_2 connected by $e^v_{q_2}$ in G_v at time t. Moreover, we introduce two vectors as follows: $\hat{y}(t) = [\hat{y}_1(t)\ \hat{y}_2(t)\ \cdots\ \hat{y}_{m_p}(t)]^{\rm T}$ and $\hat{z}(t) = [\hat{z}_1(t)\ \hat{z}_2(t)\ \cdots\ \hat{z}_{m_v}(t)]^{\rm T}$ with $\hat{y}_{q_1}(t) = y_{q_1}(t_{k^{p(q_1)}(t)})$, $q_1 \in \underline{m_p}$ and $\hat{z}_{q_2}(t) = z_{q_2}(t_{k^{v(q_2)}(t)})$, $q_2 \in \underline{m_v}$. According to the definitions of $t_{k^{p(q_1)}(t)}$ and $t_{k^{v(q_2)}(t)}$, it is easy to understand that $\hat{y}_{q_1}(t)$ represents the sampled relative position information between agents i_1 and j_1 at the recent event time before or at time t for edge $e^p_{q_1} = (\mathcal{V}_{i_1}, \mathcal{V}_{j_1})$ in graph G_p. $\hat{z}_{q_2}(t)$ represents the sampled relative velocity information between agents i_2 and j_2 at the recent event time before or at time t for edge $e^v_{q_2} = (\mathcal{V}_{i_2}, \mathcal{V}_{j_2})$ of graph G_v. Furthermore, we can derive the compact form of system (1) with protocol (2) as follows:

$$\begin{cases} \dot{x} = v, \\ \dot{v} = -D_pW_p\hat{y}(t) - kD_vW_v\hat{z}(t). \end{cases} \tag{4}$$

Let $\overline{v}(t) = \frac{1}{n}\sum_{i=1}^n v_i(t)$, and similar to Lemma 2 of [32], it is easy to get that $\overline{v}(t)$ is a constant. And according to the definitions of $\delta(t)$, $\sigma(t)$, we can get the equivalent form of system (4)

$$\begin{bmatrix} \dot{\delta} \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \sigma \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ D_pW_p & kD_vW_v \end{bmatrix} \begin{bmatrix} \hat{y}(t) \\ \hat{z}(t) \end{bmatrix}. \tag{5}$$

Next, we design the event-detecting rules in the heterogeneous networks.

Rule I. If the following inequalities are not all satisfied for the q_1-th edge of G_p at time t_k, the edge event of $e^p_{q_1}$ of G_p occurs:

$$\text{(I-1)}\quad \text{when}\ \hat{y}_{q_1}(t_{k-1}) \geqslant 0, \quad a\hat{y}_{q_1}(t_{k-1}) < y_{q_1}(t_k) < b\hat{y}_{q_1}(t_{k-1});$$

$$\text{(I-2)}\quad \text{when}\ \hat{y}_{q_1}(t_{k-1}) < 0, \quad b\hat{y}_{q_1}(t_{k-1}) < y_{q_1}(t_k) < a\hat{y}_{q_1}(t_{k-1});$$

if the following inequalities are not all satisfied for the q_2-th edge of G_v at time t_k, the edge event of $e^v_{q_2}$ of G_v occurs:

$$\text{(I-3)}\quad \text{when}\ \hat{z}_{q_2}(t_{k-1}) \geqslant 0, \quad c\hat{z}_{q_2}(t_{k-1}) < z_{q_2}(t_k) < d\hat{z}_{q_2}(t_{k-1});$$

$$\text{(I-4)}\quad \text{when}\ \hat{z}_{q_2}(t_{k-1}) < 0, \quad d\hat{z}_{q_2}(t_{k-1}) < z_{q_2}(t_k) < c\hat{z}_{q_2}(t_{k-1}).$$

In the above inequalities, $0 < a, c \leqslant 1$ and $b, d \geqslant 1$.

Rule II. If the following inequalities are not all satisfied for the q_1-th edge with $\hat{y}_{q_1}(t) = x_{i_1}(t_{k^{p(q_1)}(t)}) - x_{j_1}(t_{k^{p(q_1)}(t)})$ of G_p at time t_k, the edge event of $e^p_{q_1}$ of G_p occurs:

$$\text{(II-1)}\quad \text{when}\ \hat{y}_{q_1}(t_{k-1}) \geqslant 0, \quad -\frac{1-a}{2}\hat{y}_{q_1}(t_{k-1}) < x_{i_1}(t_k) - x_{i_1}(t_{k^{p(q_1)}(t_{k-1})}) < \frac{b-1}{2}\hat{y}_{q_1}(t_{k-1});$$

$$\text{(II-2)}\quad \text{when}\ \hat{y}_{q_1}(t_{k-1}) < 0, \quad \frac{b-1}{2}\hat{y}_{q_1}(t_{k-1}) < x_{i_1}(t_k) - x_{i_1}(t_{k^{p(q_1)}(t_{k-1})}) < -\frac{1-a}{2}\hat{y}_{q_1}(t_{k-1});$$

if the following inequalities are not all satisfied for the q_2-th edge with $\hat{z}_{q_2}(t) = v_{i_2}(t_{k^{v(q_2)}(t)}) - v_{j_2}(t_{k^{v(q_2)}(t)})$ of G_v at time t_k, the edge event of $e^v_{q_2}$ of G_v occurs:

$$\text{(II-3)}\quad \text{when}\ \hat{z}_{q_2}(t_{k-1}) \geqslant 0, \quad -\frac{1-c}{2}\hat{z}_{q_2}(t_{k-1}) < v_{i_2}(t_k) - v_{i_2}(t_{k^{v(q_2)}(t_{k-1})}) < \frac{d-1}{2}\hat{z}_{q_2}(t_{k-1});$$

$$\text{(II-4)}\quad \text{when}\ \hat{z}_{q_2}(t_{k-1}) < 0, \quad \frac{d-1}{2}\hat{z}_{q_2}(t_{k-1}) < v_{i_2}(t_k) - v_{i_2}(t_{k^{v(q_2)}(t_{k-1})}) < -\frac{1-c}{2}\hat{z}_{q_2}(t_{k-1}).$$

Before summarizing the conclusion, we introduce two constants

$$M = -a\theta_1\lambda_{\min}(W_p) + \beta\|D_p\|\,\|W_p\|r_1 + (\alpha - \theta_1)\|W_p\|br_1 + \omega_1h\lambda_n^{W_p} + \omega_2h\lambda_n^{d_1} + \omega_3h\lambda_n^{D_1} + \omega_4h\lambda_n^{D_3}, \tag{6}$$

with

$$\begin{cases} \theta_1 = \alpha - \dfrac{k\alpha(\lambda_2(L_v)+\lambda_n(L_v))}{2\lambda_2(L_p)}, \\ \theta_2 = \left(\beta\|D_p\|\,\|W_p\|r_1 + \left(1+\dfrac{h(k+1)}{2}\right)\beta h\lambda_n^{L_p}\right)\dfrac{1}{\lambda_2(L_v)}, \\ \omega_1 = \dfrac{\theta_1}{2} + (b^2+r_1^2)\dfrac{2+h(k+1)}{4}(\alpha-\theta_1), \\ \omega_2 = \dfrac{2\theta_1 h + kh\theta_1}{4} + \left(k\beta - \dfrac{k\alpha}{4} - \dfrac{\alpha}{2\lambda_2(L_v)}\right), \\ \omega_3 = (\alpha-\theta_1)\dfrac{2h+2\beta h+2h^2+h^3+2kh^3}{4}, \\ \omega_4 = \left(\dfrac{k\alpha}{2}+\dfrac{\alpha}{\lambda_2(L_v)}\theta_2\right)(1+h(k+1)), \end{cases}$$

and

$$N = -c\left(2k\beta - k\alpha - \frac{\alpha}{\lambda_2(L_v)}\right)\lambda_{\min}(W_v) + \vartheta_1 r_2\alpha\|W_v\| + \vartheta_2\lambda_n^{W_v} + \vartheta_3 h\lambda_n^{d_2} + \vartheta_4 h\lambda_n^{D_2} + \vartheta_5 h^2\lambda_n^{D_4}, \quad (7)$$

with

$$\begin{cases} \vartheta_1 = \dfrac{k(d+1)}{2} + \dfrac{d}{\lambda_2(L_v)}, \\ \vartheta_2 = \left(\dfrac{k\alpha}{2}+\dfrac{\alpha}{\lambda_2(L_v)}\right)\dfrac{h(1+k)(d^2+r_2^2)}{2} + \theta_1 d^2(1+h+kh), \\ \qquad + \dfrac{hd^2\lambda_n^{L_p}}{\lambda_2(L_v)}\left(\dfrac{\theta_1}{2}+\beta+(\alpha-\theta_1)\left(1+h+\dfrac{h^2+kh^2}{2}\right)\right), \\ \vartheta_3 = \dfrac{kh\theta_1}{4} + \left(\dfrac{1}{2}+k\right)\left(2k\beta - k\alpha - \dfrac{\alpha}{\lambda_2(L_v)}\right) + \dfrac{k\alpha+2k^2\alpha}{4}, \\ \vartheta_4 = (k+kh(k+1))\left(\dfrac{\alpha}{\lambda_2(L_v)}+\dfrac{k\alpha}{2}\right) + \theta_1 k(1+h+kh), \\ \vartheta_5 = (\alpha-\theta_1)k\left(\dfrac{1+h}{2}+\dfrac{h^2(k+2)}{4}\right) + \dfrac{k\beta}{2}, \end{cases}$$

where $r_1 = \max\{1-a, b-1\}$, $r_2 = \max\{1-c, d-1\}$, and $\lambda_n^{L_p}$, $\lambda_n^{L_v}$, $\lambda_n^{W_p}$, $\lambda_n^{W_v}$, $\lambda_n^{d_1}$, $\lambda_n^{d_2}$, $\lambda_n^{D_1}$, $\lambda_n^{D_2}$, $\lambda_n^{D_3}$, $\lambda_n^{D_4}$ denote the largest eigenvalues of matrices L_p, L_v, W_p, W_v, $W_pD_p^{\mathrm{T}}D_pW_p$, $W_vD_v^{\mathrm{T}}D_vW_v$, $W_pD_p^{\mathrm{T}}D_pW_pD_p^{\mathrm{T}}D_pW_p$, $W_vD_v^{\mathrm{T}}D_vW_vD_v^{\mathrm{T}}D_vW_v$, $W_pD_p^{\mathrm{T}}D_vW_vD_v^{\mathrm{T}}D_pW_p$, $W_vD_v^{\mathrm{T}}D_pW_pD_p^{\mathrm{T}}D_vW_v$, respectively.

Theorem 1. Suppose that position topology G_p and velocity topology G_v are both connected. If for given parameter k and the common event-detecting period h, there exist constants α, β, a, b, c, d with $0 < a, c \leqslant 1$, $b, d \geqslant 1$, such that

$$\begin{cases} \alpha - \dfrac{k\alpha(\lambda_2(L_v)+\lambda_n^{L_v})}{2\lambda_2(L_p)} > 0, \\ 2k\beta - k\alpha - \dfrac{\alpha}{\lambda_2(L_v)} > 0, \\ M, N < 0, \end{cases} \quad (8)$$

then multi-agent system (1) achieves the state consensus by protocol (2) under Rule I or II.

Proof. Consider the Lyapunov function (3), and by Lemma 1, we know that matrix P is positive definite. The derivative of $V(t)$ is given by

$$\dot{V}(t) = 2\left[\delta^{\mathrm{T}}\ \sigma^{\mathrm{T}}\right]\begin{bmatrix} \beta L_p + \dfrac{k\alpha\lambda_2(L_v)}{2}I & \dfrac{\alpha}{2}I \\ \dfrac{\alpha}{2}I & \beta I \end{bmatrix}\begin{bmatrix} \dot{\delta} \\ \dot{\sigma} \end{bmatrix}$$

$$= \left[\delta^{\mathrm{T}}\ \sigma^{\mathrm{T}}\right]\begin{bmatrix} \beta L_p + \frac{k\alpha\lambda_2(L_v)}{2}I & \frac{\alpha}{2}I \\ \frac{\alpha}{2}I & \beta I \end{bmatrix}\left(\begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta \\ \sigma \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ D_pW_p & kD_vW_v \end{bmatrix}\begin{bmatrix} \hat{y}(t) \\ \hat{z}(t) \end{bmatrix}\right)$$

$$= 2\beta\delta^{\mathrm{T}}L_p\sigma + k\alpha\lambda_2(L_v)\delta^{\mathrm{T}}\sigma + \alpha\sigma^{\mathrm{T}}\sigma - \alpha y^{\mathrm{T}}W_p\hat{y} - 2\beta\sigma^{\mathrm{T}}D_pW_p\hat{y} - k\alpha\delta^{\mathrm{T}}D_vW_v\hat{z} - 2k\beta z^{\mathrm{T}}W_v\hat{z}.$$

According to the property of undirected graph [38]

$$\min_{\delta\neq 0, 1^{\mathrm{T}}\delta=0}\frac{\delta^{\mathrm{T}}L_p\delta}{\delta^{\mathrm{T}}\delta} = \lambda_2(L_p), \quad \min_{\sigma\neq 0, 1^{\mathrm{T}}\sigma=0}\frac{\sigma^{\mathrm{T}}L_v\sigma}{\sigma^{\mathrm{T}}\sigma} = \lambda_2(L_v).$$

Based on $L_p = D_pW_pD_p^{\mathrm{T}}$, $L_v = D_vW_vD_v^{\mathrm{T}}$, and the inequality $2x^{\mathrm{T}}y \leqslant x^{\mathrm{T}}x + y^{\mathrm{T}}y$, we further get that

$$\begin{aligned} V(t) \leqslant{}& 2\beta\sigma^{\mathrm{T}}D_pW_p(y-\hat{y}) + \frac{k\alpha(\lambda_2(L_v)+\lambda_n^{L_v})}{2\lambda_2(L_p)}y^{\mathrm{T}}W_p(y-\hat{y}) - \left(\alpha - \frac{k\alpha(\lambda_2(L_v)+\lambda_n^{L_v})}{2\lambda_2(L_p)}\right)y^{\mathrm{T}}W_p\hat{y} \\ &+ \left(\frac{k\alpha}{2} + \frac{\alpha}{\lambda_2(L_v)}\right)z^{\mathrm{T}}W_v(z-\hat{z}) - \frac{k\alpha}{2}(z^{\mathrm{T}}-\hat{z}^{\mathrm{T}})W_v\hat{z} - \left(2k\beta - k\alpha - \frac{\alpha}{\lambda_2(L_v)}\right)z^{\mathrm{T}}W_v\hat{z}. \end{aligned} \tag{9}$$

From (4), for any $t \in [t_k, t_{k+1})$, it is easy to get that

$$\begin{cases} v(t) = v(t_k) - h'D_pW_p\hat{y} - kh'D_vW_v\hat{z}, \\ x(t) = x(t) + h'v(t_k) - \frac{h'^2}{2}D_pW_p\hat{y} - k\frac{h'^2}{2}D_vW_v\hat{z}, \end{cases} \tag{10}$$

where $h' = t - t_k$. Furthermore, considering $y(t) = D_p^{\mathrm{T}}x(t)$ and $z(t) = D_v^{\mathrm{T}}v(t)$, we have

$$z(t) = z(t_k) - h'D_v^{\mathrm{T}}D_pW_p\hat{y} - kh'D_v^{\mathrm{T}}D_vW_v\hat{z}, \tag{11}$$

and

$$y(t) = y(t_k) + h'D_p^{\mathrm{T}}v(t_k) - \frac{h'^2}{2}D_p^{\mathrm{T}}D_pW_p\hat{y} - \frac{kh'^2}{2}D_p^{\mathrm{T}}D_vW_v\hat{z}. \tag{12}$$

Equivalently, based on $D_p^{\mathrm{T}}v(t_k) = D_p^{\mathrm{T}}(v(t_k) - \overline{v}) = D_p^{\mathrm{T}}\sigma(t_k)$, (12) yields that

$$y(t) = y(t_k) + h'D_p^{\mathrm{T}}\sigma(t_k) - \frac{h'^2}{2}D_p^{\mathrm{T}}D_pW_p\hat{y} - \frac{kh'^2}{2}D_p^{\mathrm{T}}D_vW_v\hat{z}. \tag{13}$$

Moreover, under Rule I or II, the following inequalities hold [32]:

$$\begin{cases} a\left|\hat{y}_q(t_k)\right| \leqslant \left|y_q(t_k)\right| \leqslant b\left|\hat{y}_q(t_k)\right| \text{ and } y_q(t_k)\hat{y}_q(t_k) > 0,\ q \in \underline{m_p}, \\ c\left|\hat{z}_q(t_k)\right| \leqslant \left|z_q(t_k)\right| \leqslant d\left|\hat{z}_q(t_k)\right| \text{ and } z_q(t_k)\hat{z}_q(t_k) > 0,\ q \in \underline{m_v}. \end{cases} \tag{14}$$

Also, we obtain that

$$\begin{cases} 0 \leqslant \|y(t_k) - \hat{y}(t)\| \leqslant r_1\|\hat{y}(t)\|, \\ 0 \leqslant \|z(t_k) - \hat{z}(t)\| \leqslant r_2\|\hat{z}(t)\|. \end{cases} \tag{15}$$

Then we substitute (11) and (13) into (9). By utilizing the Hölder inequality and the inequality $2x^{\mathrm{T}}y \leqslant x^{\mathrm{T}}x + y^{\mathrm{T}}y$, and by (14) and (15), we can get the following inequalities:

$$\begin{cases} -y(t_k)^{\mathrm{T}}W_p\hat{y} \leqslant -a\lambda_{\min}(W_p)\hat{y}^{\mathrm{T}}\hat{y}, \\ -z(t_k)^{\mathrm{T}}W_v\hat{z} \leqslant -c\lambda_{\min}(W_v)\hat{z}^{\mathrm{T}}\hat{z}, \\ z(t_k)^{\mathrm{T}}(z(t_k) - \hat{z}) \leqslant r_2d\hat{z}^{\mathrm{T}}\hat{z}, \\ y(t_k)^{\mathrm{T}}(y(t_k) - \hat{y}) \leqslant r_1b\hat{y}^{\mathrm{T}}\hat{y}, \\ \hat{z}^{\mathrm{T}}(z(t_k) - \hat{z}) \leqslant r_2\hat{z}^{\mathrm{T}}\hat{z}, \\ \|\sigma\|\,\|y(t_k) - \hat{y}\| \leqslant \frac{r_1}{2}\left(\hat{y}^{\mathrm{T}}\hat{y} + \sigma^{\mathrm{T}}\sigma\right), \\ \sigma^{\mathrm{T}}\sigma \leqslant \frac{1}{\lambda_2(L_v)}z^{\mathrm{T}}W_vz, \\ \sigma(t_k)^{\mathrm{T}}\sigma(t_k) \leqslant \frac{\lambda_n^{W_v}}{\lambda_2(L_v)}d^2\hat{z}^{\mathrm{T}}\hat{z}. \end{cases} \tag{16}$$

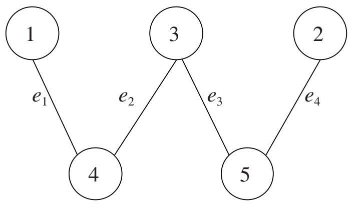

Figure 1 Position topology.

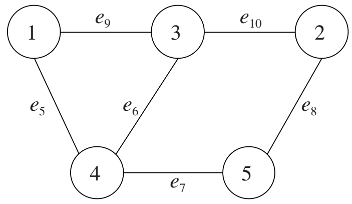

Figure 2 Velocity topology.

In addition, we have

$$\begin{cases} \hat{y}^{\mathrm{T}} W_p \hat{y} \leqslant \lambda_n^{W_p} \hat{y}^{\mathrm{T}} \hat{y}, \\ \hat{z}^{\mathrm{T}} W_v \hat{z} \leqslant \lambda_n^{W_v} \hat{z}^{\mathrm{T}} \hat{z}, \\ \hat{y}^{\mathrm{T}} W_p D_p^{\mathrm{T}} D_p W_p \hat{y} \leqslant \lambda_n^{d_1} \hat{y}^{\mathrm{T}} \hat{y}, \\ \hat{z}^{\mathrm{T}} W_v D_v^{\mathrm{T}} D_v W_v \hat{z} \leqslant \lambda_n^{d_2} \hat{z}^{\mathrm{T}} \hat{z}, \\ \hat{y}^{\mathrm{T}} W_p D_p^{\mathrm{T}} D_p W_p D_p^{\mathrm{T}} D_p W_p \hat{y} \leqslant \lambda_n^{D_1} \hat{y}^{\mathrm{T}} \hat{y}, \\ \hat{z}^{\mathrm{T}} W_v D_v^{\mathrm{T}} D_v W_v D_v^{\mathrm{T}} D_v W_v \hat{z} \leqslant \lambda_n^{D_2} \hat{z}^{\mathrm{T}} \hat{z}, \\ \hat{y}^{\mathrm{T}} W_p D_p^{\mathrm{T}} D_v W_v D_v^{\mathrm{T}} D_p W_p \hat{y} \leqslant \lambda_n^{D_3} \hat{y}^{\mathrm{T}} \hat{y}, \\ \hat{z}^{\mathrm{T}} W_v D_v^{\mathrm{T}} D_p W_p D_p^{\mathrm{T}} D_v W_v \hat{z} \leqslant \lambda_n^{D_4} \hat{z}^{\mathrm{T}} \hat{z}. \end{cases} \tag{17}$$

Therefore,

$$\dot{V} \leqslant M \hat{y}^{\mathrm{T}} \hat{y} + N \hat{z}^{\mathrm{T}} \hat{z} \leqslant 0. \tag{18}$$

Furthermore, $\dot{V}(t) = 0$ suggests that $\hat{y}(t) = 0$, $\hat{z}(t) = 0$, which leads to $\delta(t) = 0$, $\sigma(t) = 0$ by (11), (13), (14), the last inequality in (16), and the definitions of $\delta(t)$ and $\sigma(t)$. Thus, system (1) with protocol (2) achieves the state consensus, that is, $x_i(t) \to \frac{1}{n}\sum_{j=1}^{n} x_j(t)$ and $v_i(t) \to \frac{1}{n}\sum_{j=1}^{n} v_j(t)$, $i \in \underline{n}$ as $t \to \infty$.

Remark 1. Here we show the existence of M, N. In (6) and (7), set $a = b = c = d = 1$. Then $r_1 = r_2 = 0$. Moreover, set $k = k_0$, $\alpha = \alpha_0$ and $\beta = \beta_0$ such that the first and the second inequalities in (8) hold. Then, it is clear that M and N are the functions of h and monotone increasing. Based on this, $M = -\lambda_{\min}(W_p)(\alpha_0 - \frac{k_0\alpha_0(\lambda_2(L_v)+\lambda_n^{L_v})}{2\lambda_2(L_p)}) < 0$ and $N = -\lambda_{\min}(W_v)(2k_0\beta_0 - k_0\alpha_0 - \frac{\alpha_0}{\lambda_2(L_v)}) < 0$ hold with $h = 0$. Therefore, there exists some $h = h_{\max}$ such that $M(h_{\max}) = 0$ and $N(h_{\max}) = 0$. Because of the monotonicity of M and N, there exist appropriate parameters satisfying (8) with any $h \in (0, h_{\max})$.

Remark 2. In this paper, we adopt the hybrid event- and time-triggered mechanism which can effectively avoid the Zeno behaviors. The event-triggered control is based on periodic event detections, that is, every two neighboring event detections are performed discontinuously and separated by a fixed time interval h. Event-triggering conditions are checked at the event-detecting times. Furthermore, the events of each edge occur at some event-detecting times. Therefore, the inter-event time intervals are equal to or greater than h with $h > 0$.

4 Simulation

In this section, in order to verify the correctness of the theoretical result in Section 3, we present a numerical simulation.

Considering system (1) under protocol (2), we suppose that the position communication topology and velocity communication topology are modeled as Figures 1 and 2, respectively, which are both connected. Obviously, system (1) consists of 5 agents, and the position communication graph and velocity communication graph consist of 4 and 6 communication channels, weighted with 1, 1, 1, 1, 1, 3, 2, 2, 5, 5, respectively.

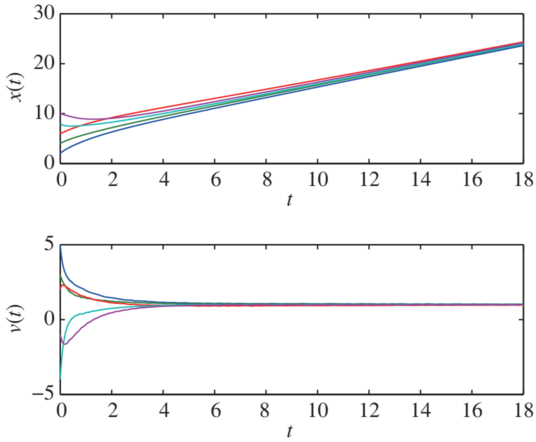

Figure 3 (Color online) State trajectories under Rule I.

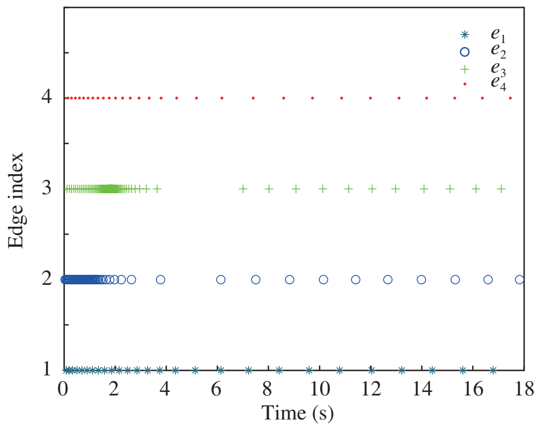

Figure 4 (Color online) Edge-event times in position graph under Rule I.

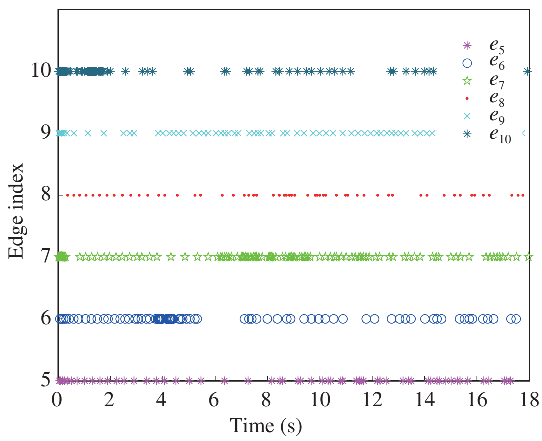

Figure 5 (Color online) Edge-event times in velocity graph under Rule I.

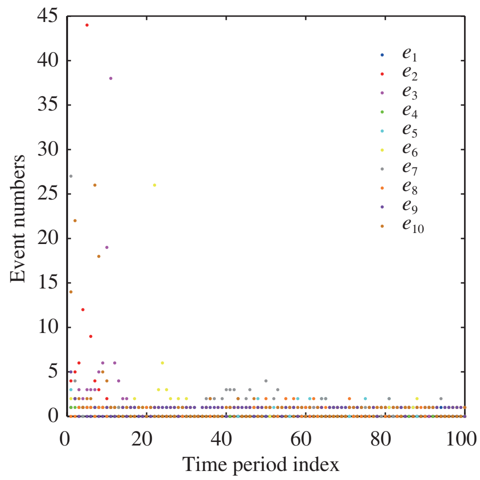

Figure 6 (Color online) Event numbers counted up every 0.18 s under Rule I.

For system (1) with initial states $x(0) = [2\ 4\ 6\ 8\ 10]^{\mathrm{T}}$, $v(0) = [5\ 3\ 2\ \ -4\ \ -1]^{\mathrm{T}}$, we choose the event-detecting period $h = 0.002$, parameter $k = 1$ in protocol (2) and parameters $a = 0.9$, $b = 1.1$, $c = 0.8$, $d = 1.2$ in rules. Simulation results are presented in Figures 3–10. Figures 3 and 7 show that system (1) achieves the state consensus under Rules I and II, respectively. Figures 4 and 5 give the event times under Rule I based on position information in G_p and velocity information in G_v, respectively. About 9000 event detections were performed for each edge. Consequently, there are 27, 104, 114, 29 events in G_p and 57, 100, 126, 56, 73, 137 events in G_v, respectively. Similarly under Rule II, there are 609, 677, 1117, 668 events in G_p and 57, 100, 126, 56, 73, 137 events in G_v, respectively. The corresponding event times are depicted in Figures 8 and 9, respectively. In addition, we count up the event numbers every 0.18 s for each edge, which are shown in Figures 6 and 10 under Rules I and II, respectively. It is clear that by Rules I and II, the event numbers are greatly reduced. Compare Figure 6 with Figure 10, and we can find that event numbers under Rule I appear to be less than those under Rule II. However, from Rule II, we know that at each event-detecting time, the corresponding agent does not require the state information of the corresponding neighbor, so that the system avoids unnecessary information exchanges. By the simulation, we can verify such a conclusion that based on edge-event-triggered control, systems can greatly reduce communication costs and controller-updating costs.

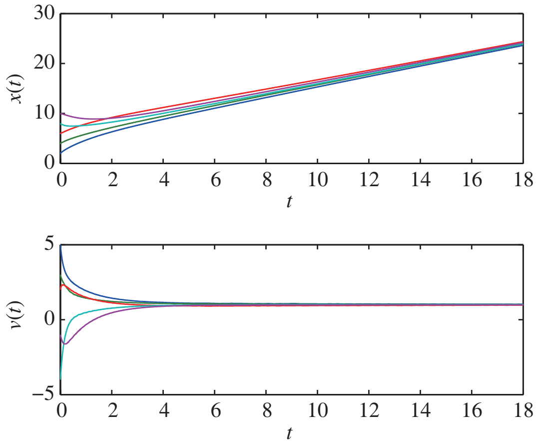

Figure 7 (Color online) State trajectories under Rule II.

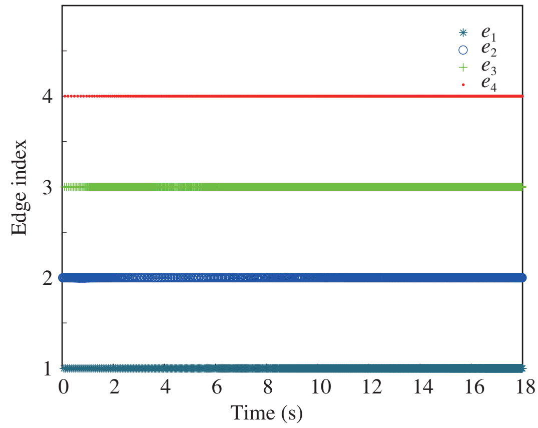

Figure 8 (Color online) Edge-event times in position graph under Rule II.

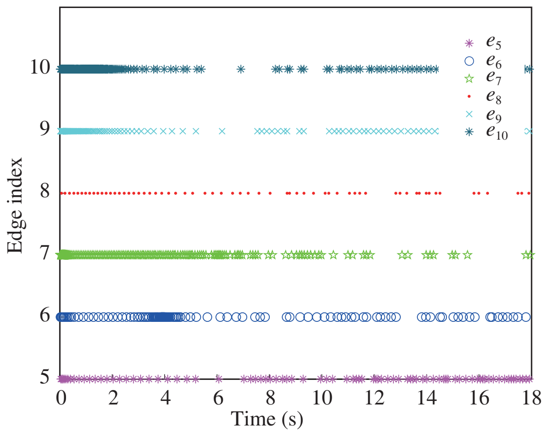

Figure 9 (Color online) Edge-event times in velocity graph under Rule II.

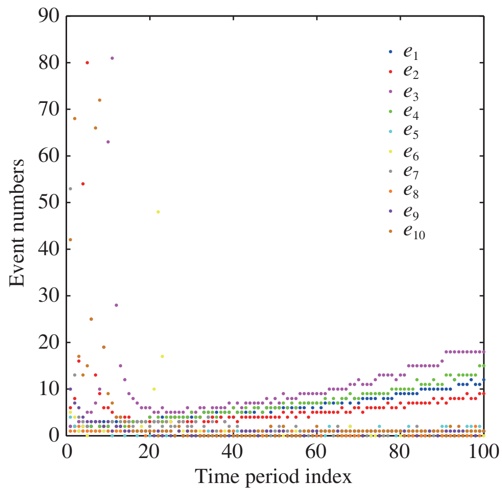

Figure 10 (Color online) Event numbers counted up every 0.18 s under Rule II.

5 Conclusion

In this paper, we solved the state consensus problems of double-integrator multi-agent systems based on edge-event-triggered control under heterogeneous networks. We proposed the corresponding protocol based on periodic event detections and designed the event-detecting rules as well. Furthermore, by Lyapunov methods, we presented sufficient conditions for the state consensus. Moreover, we also gave a simulation example to test the validity of theoretical results and showed the advantages of reduced controller-updating costs and communication costs. Our future work will focus on the design of event-triggering conditions to improve the system performance and on the asynchronous consensus of double-integrator systems under heterogeneous networks with or without time delays and the edge-event-triggered control of heterogeneous multi-agent systems [39].

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61751301, 61533001, 61873074).

References

1 Cheng D Z, He F H, Qi H S, et al. Modeling, analysis and control of networked evolutionary games. IEEE Trans Autom Control, 2015, 60: 2402–2415

2 Cheng D Z, Qi H S. Controllability and observability of Boolean control networks. Automatica, 2009, 45: 1659–1667

3 Zhao Y, Cheng D Z. On controllability and stabilizability of probabilistic Boolean control networks. Sci China Inf Sci, 2014, 57: 012202

4 Zhu W, Cheng D Z. Leader-following consensus of second-order agents with multiple time-varying delays. Automatica, 2010, 46: 1994–1999

5 Lu Z H, Zhang L, Wang L. Controllability analysis of multi-agent systems with switching topology over finite fields. Sci China Inf Sci, 2019, 62: 012201

6 Lin X, Zheng Y S, Wang L. Consensus of switched multi-agent systems with random networks. Int J Control, 2017, 90: 1113–1122

7 Li A, Cornelius S P, Liu Y Y, et al. The fundamental advantages of temporal networks. Science, 2017, 358: 1042–1046

8 Wang L, Shi H, Chu T G, et al. Aggregation of foraging swarms. In: Proceedings of the 17th Australian Joint Conference on Advances in Artificial Intelligence, Cairns, 2004. 766–777

9 Wang L, Xiao F. A new approach to consensus problems in discrete-time multiagent systems with time-delays. Sci China Ser F-Inf Sci, 2007, 50: 625–635

10 Zhao B, Guan Y Q, Wang L. Non-fragility of multi-agent controllability. Sci China Inf Sci, 2018, 61: 052202

11 Adler J L, Satapathy G, Manikonda V, et al. A multi-agent approach to cooperative traffic management and route guidance. Transport Res Part B-Meth, 2005, 39: 297–318

12 Guan Y Q, Wang L. Structural controllability of multi-agent systems with absolute protocol under fixed and switching topologies. Sci China Inf Sci, 2017, 60: 092203

13 Wang L, Jiang F C, Xie G M, et al. Controllability of multi-agent systems based on agreement protocols. Sci China Ser F-Inf Sci, 2009, 52: 2074–2088

14 Dimarogonas D V, Tsiotras P, Kyriakopoulos K J. Leader-follower cooperative attitude control of multiple rigid bodies. Syst Control Lett, 2009, 58: 429–435

15 Wang L, Xiao F. Finite-time consensus problems for networks of dynamic agents. IEEE Trans Autom Control, 2010, 55: 950–955

16 Olfati-Saber R. Flocking for multi-agent dynamic systems: algorithms and theory. IEEE Trans Autom Control, 2006, 51: 401–420

17 Xiao F, Shi Y, Ren W. Robustness analysis of asynchronous sampled-data multiagent networks with time-varying delays. IEEE Trans Autom Control, 2018, 63: 2145–2152

18 Xiao F, Chen T W, Gao H J. Consensus in time-delayed multi-agent systems with quantized dwell times. Syst Control Lett, 2017, 104: 59–65

19 Xiao F, Chen T W. Adaptive consensus in leader-following networks of heterogeneous linear systems. IEEE Trans Control Network Syst, 2018, 5: 1169–1176

20 Jing G S, Zheng Y S, Wang L. Consensus of multiagent systems with distance-dependent communication networks. IEEE Trans Neural Netw Learn Syst, 2017, 28: 2712–2726

21 Olfati-Saber R, Murray R M. Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans Autom Control, 2004, 49: 1520–1533

22 Ren W. On consensus algorithms for double-integrator dynamics. IEEE Trans Autom Control, 2008, 53: 1503–1509

23 Dibaji S M, Ishii H. Resilient consensus of double-integrator multi-agent systems. In: Proceedings of the American Control Conference, Portland, 2014. 5139–5144

24 Gao Y P, Wang L. Sampled-data based consensus of continuous-time multi-agent systems with time-varying topology. IEEE Trans Autom Control, 2011, 56: 1226–1231

25 Borgers D P, Heemels W P M H. Event-separation properties of event-triggered control systems. IEEE Trans Autom Control, 2014, 59: 2644–2656

26 Dimarogonas D V, Frazzoli E, Johansson K H. Distributed event-triggered control for multi-agent systems. IEEE Trans Autom Control, 2012, 57: 1291–1297

27 Xiao F, Meng X Y, Chen T W. Average sampled-data consensus driven by edge events. In: Proceedings of the 31st Chinese Control Conference, Hefei, 2012. 6239–6244

28 Xiao F, Chen T W, Gao H J. Synchronous hybrid event- and time-driven consensus in multiagent networks with time delays. IEEE Trans Cybern, 2016, 46: 1165–1174

29 Xiao F, Meng X Y, Chen T W. Sampled-data consensus in switching networks of integrators based on edge events. Int J Control, 2015, 88: 391–402

30 Xiao F, Chen T W. Sampled-data consensus in multi-agent systems with asynchronous hybrid event-time driven interactions. Syst Control Lett, 2016, 89: 24–34

31 Seyboth G S, Dimarogonas D V, Johansson K H. Event-based broadcasting for multi-agent average consensus. Automatica, 2013, 49: 245–252

32 Cao M T, Xiao F, Wang L. Event-based second-order consensus control for multi-agent systems via synchronous periodic event detection. IEEE Trans Autom Control, 2015, 60: 2452–2457

33 Åström K J, Bernhardsson B. Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In: Proceedings of the 41st IEEE Conference on Decision and Control, Las Vegas, 2002. 2011–2016

34 Duan G P, Xiao F, Wang L. Asynchronous periodic edge-event triggered control for double-integrator networks with communication time delays. IEEE Trans Cybern, 2018, 48: 675–688

35 Zhang Z Q, Hao F, Zhang L, et al. Consensus of linear multi-agent systems via event-triggered control. Int J Control, 2014, 87: 1243–1251

36 Dimarogonas D V, Frazzoli E. Distributed event-triggered control strategies for multi-agent systems. In: Proceedings of the 47th Annual Allerton Conference on Communication, Control, and Computing, Monticello, 2009. 906–910

37 Kharisov E, Wang X F, Hovakimyan N. Distributed event-triggered consensus algorithm for uncertain multi-agent systems. In: Proceedings of AIAA Guidance, Navigation, and Control Conference, Toronto, 2010

38 Godsil C, Royle G. Algebraic Graph Theory. Berlin: Springer-Verlag, 2001

39 Zheng Y S, Ma J Y, Wang L. Consensus of hybrid multi-agent systems. IEEE Trans Neural Netw Learn Syst, 2018, 29: 1359–1365