SCIENCE CHINA
Information Sciences

• LETTER •

January 2019, Vol. 62 019203:1–019203:3
https://doi.org/10.1007/s11432-018-9542-3

Backstepping control of a quadrotor unmanned aerial vehicle based on multi-rate sampling

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Received 14 May 2018/Revised 22 June 2018/Accepted 16 July 2018/Published online 4 December 2018

Citation Wang F K, Chen W S, Dai H, et al. Backstepping control of a quadrotor unmanned aerial vehicle based on multi-rate sampling. Sci China Inf Sci, 2019, 62(1): 019203, https://doi.org/10.1007/s11432-018-9542-3

Dear editor,

An unmanned aerial vehicle (UAV) is a portable power unit that uses aerodynamic forces for carrying loads. A UAV does not have a human pilot on board; it is controlled remotely or programmed to fly autonomously. In recent years, quadrotor UAVs are being increasingly used in military and civilian areas, such as ground reconnaissance, disaster relief, and agricultural investigations. Therefore, a number of studies are being performed on quadrotors for UAVs. The quadrotor UAV is an under-actuated system with six degrees-of-freedom and four inputs. The quadrotor UAV system is difficult to design because it has characteristics such as multi-variability, nonlinearity, strong coupling, and interference sensitivity. However, a few studies have addressed the flight-control problems of the quadrotor UAV by employing different methods. In [1], a discrete-time sliding-mode controller was applied to control the position and attitude of the quadrotor UAV.

The backstepping control technique has good control effects in nonlinear, strong coupling, time-varying systems such as UAVs, and this technique has a certain anti-interference capability [2]. The backstepping method is widely used in nonlinear, continuous-time systems, and in discrete-time systems. In [3], a discrete-time backstepping controller based on helicopter nonlinear discrete equations was designed to track the predetermined position and the yaw trajectories. However, in practical applications, the controller is implemented only by the computer that handles the digital signal. Therefore, we need to study the backstepping control in the sampled-data control systems.

This study proposes a backstepping method based on multi-rate sampling to solve the actual flight-control problem of quadrotor UAVs. Multi-rate sampling control takes advantage of the cascade structure of strict feedback dynamics, which is particularly suitable for backstepping. The sampled-data control method used in the current mainstream digital controller for the continuous system is emulation, which is carried out by directly discretizing the continuous-time controller [4]. Though it is attractive for its simplicity, the method does not perform well in practical applications because the required sampling period needs to be small enough. Compared with the emulation control, the multi-rate sampling control proposed in this letter performs multiple calculations on the controller during each sampling period of the computer to achieve multiple control of the system in each sampling period and improve the control accuracy. At the same time, the proposed sampling control can ensure that the multi-rate sampling controller can stabilize the system when the computer has a large sampling period; this can help achieve the expected performance in practical applications.

Dynamic model of quadrotor. The following assumptions about the UAV system are necessary in

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the modeling process.

Assumption 1. The UAV body is rigid and has completely uniform symmetry.

Assumption 2. The propeller of a UAV is a rigid body without considering its structure and the elastic deformation.

Assumption 3. The center of mass and the body-fixed frame origin are assumed to coincide.

As per air conventions, the absolute position and attitude of the rotor are represented by (x, y, z) and the Euler angle (ϕ, θ, ψ), respectively. The attitude angles are known as the roll angle, the pitch angle, and the yaw angle.

The equations of dynamics can be derived from the force and moment balance as follows:

$$\begin{cases} \ddot{x} = (C_\phi S_\theta C_\psi + S_\phi S\psi)\dfrac{\sum_{i=1}^4 T_i}{m}, \\ \ddot{y} = (C_\phi S_\theta S_\psi - S_\phi C\psi)\dfrac{\sum_{i=1}^4 T_i}{m}, \\ \ddot{z} = -g + (C_\phi C_\theta)\dfrac{\sum_{i=1}^4 T_i}{m}, \\ \ddot{\phi} = \dot{\theta}\dot{\psi}\left(\dfrac{I_y - I_z}{I_x}\right) + \dfrac{k_1 l}{I_x}(\Omega_4^2 - \Omega_2^2) + \dot{\theta}\Omega_r\dfrac{J_r}{I_x}, \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\left(\dfrac{I_z - I_x}{I_y}\right) + \dfrac{k_1 l}{I_y}(\Omega_3^2 - \Omega_1^2) + \dot{\phi}\Omega_r\dfrac{J_r}{I_y}, \\ \ddot{\psi} = \dot{\theta}\dot{\phi}\left(\dfrac{I_x - I_y}{I_z}\right) + \dfrac{k_2}{I_z}(\Omega_4^2 + \Omega_2^2 - \Omega_1^2 - \Omega_3^2), \end{cases} \tag{1}$$

where m and $I_T = \mathrm{diag}[I_x, I_y, I_z]$ represent the mass and the total inertia matrix of the quadrotor, respectively; l is the distance between the center of the quadrotor and the center of the rotor; Ω_i is the speed of the ith rotor $(i = 1, 2, 3, 4)$; and $\Omega_r = \Omega_4 + \Omega_2 - \Omega_3 - \Omega_1$. J_r is the propeller inertia coefficient, and k_1 and k_2 are proportionality coefficients. $\sum_{i=1}^4 T_i$ is the thrust produced by the four rotors, which equals $k_1(\Omega_1^2 + +\Omega_2^2 + \Omega_3^2 + \Omega_4^2)$.

Backstepping controller design. Model (1) is rewritten in the state-space form as

$$\dot{X} = f(X, U), \tag{2}$$

where the state vector X and the input vector U are defined as follows:

$$\begin{aligned} X &= \left[x_1, x_2, \ldots, x_{12} \right]^{\mathrm{T}} \\ &= \left[x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi} \right]^{\mathrm{T}}, \end{aligned} \tag{3}$$

$$U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} k_1(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ k_1(\Omega_4^2 - \Omega_2^2) \\ k_1(\Omega_3^2 - \Omega_1^2) \\ k_1(\Omega_4^2 + \Omega_2^2 - \Omega_3^2 - \Omega_1^2) \end{bmatrix}. \tag{4}$$

The complete system state equation is given as follows:

$$\begin{cases} \dot{x_1} = x_2, \\ \dot{x_2} = u_x \dfrac{U_1}{m}, \\ \dot{x_3} = x_4, \\ \dot{x_4} = u_y \dfrac{U_1}{m}, \\ \dot{x_5} = x_6, \\ \dot{x_6} = (\cos x_7 \cos x_9)\dfrac{U_1}{m} - g, \\ \dot{x_7} = x_8, \\ \dot{x_8} = x_{10}x_{12}\alpha_1 + \beta_1 U_2 + x_{10}\alpha_2\Omega_r, \\ \dot{x_9} = x_{10}, \\ \dot{x_{10}} = x_8 x_{12}\alpha_3 + \beta_2 U_3 + x_8\alpha_4\Omega_r, \\ \dot{x_{11}} = x_{12}, \\ \dot{x_{12}} = x_{10}x_8\alpha_5 + \beta_3 U_4, \end{cases} \tag{5}$$

where $\alpha_1 = (I_y - I_x)/I_z$, $\beta_1 = l/I_x$, $\alpha_2 = J_r/I_x$, $\beta_2 = l/I_y$, $\alpha_3 = (I_z - I_x)/I_y$, $\beta_3 = 1/I_z$, $\alpha_4 = -J_r/I_y$, $\alpha_5 = (I_x - I_y)/I_z$, $u_x = (\cos x_7 \sin x_9 \cos x_{11} + \sin x_7 \sin x_{11})$, $u_y = (\cos x_7 \sin x_9 \sin x_{11} - \sin x_7 \cos x_{11})$.

Therefore, the control quantity of UAV can be obtained by the backstepping method as follows: $U_1 = \frac{m}{\cos x_7 \cos x_9}(\ddot{x}_{5d} + e_5 + g + \chi_5(e_6 - \chi_5 e_5) + \tau_6 e_6)$, $U_2 = \frac{1}{\beta_1}(\ddot{x}_{7d} + e_7 + \chi_7(e_8 - \chi_7 e_7) - x_{10}x_{12}\alpha_1 - x_{10}\alpha_2\Omega_r + \tau_8 e_8)$, $U_3 = \frac{1}{\beta_2}(\ddot{x}_{9d} + e_9 + \chi_9(e_{10} - \chi_9 e_9) - x_8 x_{12}\alpha_3 - x_8\alpha_4\Omega_r + \tau_{10}e_{10})$, $U_4 = \frac{1}{\beta_3}(\ddot{x}_{11d} + e_{11} + \chi_{11}(e_{12} - \chi_{11}e_{11}) - x_8 x_{10}\alpha_5 + \tau_{12}e_{12})$, where χ_i and τ_i are positive design constants; e_i are the tracking errors.

Multi-rate sampling controller design. It is difficult to apply the continuous-time controller to the practical systems; therefore, we apply the multi-rate sampling method proposed in [5] to the quadrotor UAV system.

Theorem 1. In (5), the proposed single-rate digital feedback controller of the form

$$\widehat{U}_{\lambda k} = U_{\lambda k}^{\delta} = U_{\lambda k0} + \sum_{i \geqslant 1} \frac{\delta^i}{(i+1)!} U_{\lambda ki} \tag{6}$$

makes the closed-loop system (5) globally asymptotically stable, with $\lambda = 1, 2, 3, 4$ and $k \geqslant 0$, where δ and k represent the sampling period and the sampling time, respectively.

Theorem 2. The proposed double-rate digital feedback controller of the form

$$\widetilde{U}_{\lambda k} = U_{\lambda ik}^{\overline{\delta}} = U_{\lambda ik}^{0} + \sum_{j \geqslant 1} \frac{\overline{\delta}^j}{(j+1)!} U_{\lambda ik}^{j}, \quad i = 1, 2 \tag{7}$$

makes the closed-loop system (5) globally asymptotically stable with $\overline{\delta} = \delta/2$ and $\lambda = 1, 2, 3, 4$;

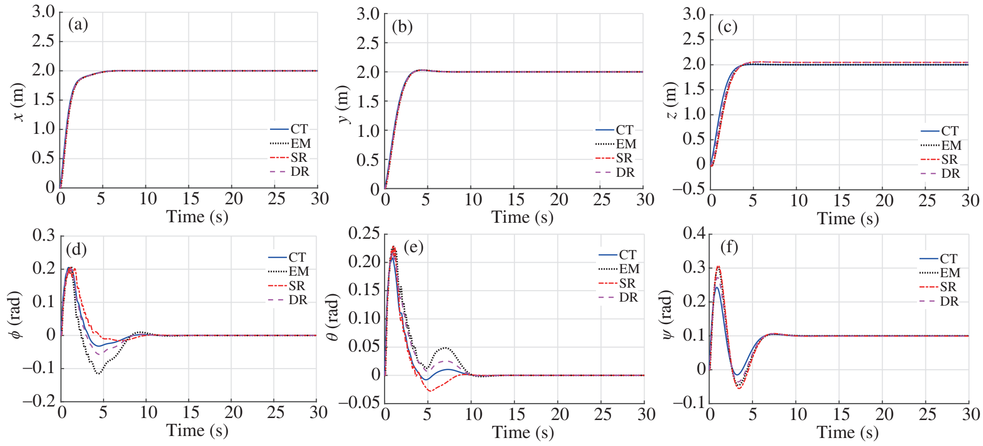

Figure 1 (Color online) State curves of CT, EM, SR, and DR systems at the sampling period $\delta = 0.01$. CT, EM, SR, and DR represent the state curves of the UAV under the continuous controller, emulation controller, single-rate controller, and double-rate controller, respectively. (a) Curves of position x; (b) Curves of position y; (c) Curves of position z; (d) Curves of attitude angle ϕ; (e) Curves of attitude angle θ; (f) Curves of attitude angle ψ.

$i = 1, 2$ represents U_i in the first half and the second half of a sampling period, respectively.

Remark 1. The multi-rate sampling of m orders refers to the actuation of controlling the variable m-times in each time interval. The evolution of the discrete-time dynamic description (5) in the sampling time $t = k$ defines the equivalent sampling data model. Its form is δ-parameterized map $F^{\delta}(\cdot, u_k)$, which is expanded by the following exponential Lie series:

$$x_{k+1} = F^{\overline{\delta}}(\cdots(F^{\overline{\delta}}(x_k, U_{1k}), \cdots), U_{mk})$$
$$= \mathrm{e}^{\overline{\delta}(f+U_{1k}g)} \circ \cdots \circ \mathrm{e}^{\overline{\delta}(f+U_{mk}g)} x_k.$$

In the double-rate sampling controller, $m = 2$. The proof is given in [5] and omitted here. Compared with the single-rate sampling controller, the multi-rate sampling controller adds more control inputs in one sampling period, which improves the sampling precision.

Comparison and verification. Figure 1 shows the state curves of the UAV under the continuous controller, emulation controller, single-rate controller and double-rate controller. From the curves, we can see that the UAV's flight status under the double-rate controller is closer to the continuous-time controller. With increase in the sampling period, the emulation and single-rate controllers gradually lose their effect; however, the double-rate controller can still keep the UAV flying steadily (see Figure A4 in Appendix A).

Conclusion. In this study, we designed multi-rate sampling controllers to realize the flight stability of the quadrotor UAV. Our simulation results proved that the proposed controllers can stabilize the entire system and drive the quadrotor to the desired trajectory. Compared with the traditional controllers, our proposed backstepping controllers based on multi-rate sampling can increase the sampling period of the system. This also means that increasing the sampling period will reduce the hardware requirements when the computer controls the UAV. In future, we could investigate how to extend these results to the case of aperiodic sampled-data systems.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61503292, 61673308, 61673014), Natural Science Foundation of Shaanxi Province (Grant Nos. 2018JM6079, 2018JM6026), Fundamental Research Funds for the Central Universities (Grant No. JB181305), and Innovation Fund of Xidian University.

Supporting information Appendix A. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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