SCIENCE CHINA
Information Sciences

• LETTER •

January 2019, Vol. 62 019201:1–019201:3
https://doi.org/10.1007/s11432-018-9505-6

Minimum input selection of reconfigurable architecture systems for structural controllability

Ting BAI, Shaoyuan LI* & Yuanyuan ZOU

School of Electronic Information and Electrical Engineering, Shanghai Jiao Tong University, Shanghai 200240*, China*

Received 2 May 2018/Accepted 15 June 2018/Published online 7 November 2018

Citation Bai T, Li S Y, Zou Y Y. Minimum input selection of reconfigurable architecture systems for structural controllability. Sci China Inf Sci, 2019, 62(1): 019201, https://doi.org/10.1007/s11432-018-9505-6

Dear editor,

Control configuration selection of complex networks with ensured controllability is of crucial importance for achieving reliable network functionalities [1]. In many scenarios, the exact numerical parameters of system model are not available owing to modeling uncertainties or measurement noise, and the structural controllability is pursued in control configuration design. As first proposed by Lin in [2], a system is said to be structurally controllable if one can seek a numerical realization for the unknown parameters so that the resulting system is controllable in classical sense. More often than not, considering the installation and maintenance cost, it is economic to have full control over the system while minimizing the actuation infrastructures. Therefore, the issue pertaining to minimum input selection (MIS) for structural controllability is of more interest, which can be formally stated as: identifying the minimal subset of actuated states that ensures system's structural controllability.

In recent years, triggered by multifarious demands of switching control targets in complex networks, such as failure isolation, function extension and process improvement, a class of reconfigurable architecture systems with changing network topology has emerged. Motivated by its numerous applications and the significance of optimal inputs allocation, the minimum input selection of reconfigured systems (MIS-RSs) attracts increasing attention [3]. Constrained by existing control configuration, the main challenge to address MIS-RS is how to find the least actuators from a given collection of inputs while achieving structural controllability.

In [4], a distributed method to determine MIS was presented, which benefits the system structural analysis with changing topology. Nevertheless, the constraint on actuated states that are possible for selection is not considered. In existing literature [5], a similar issue called constrained MIS was investigated, where it proved that this problem is NP-hard if each input drives multiple states. Particularly, if a favorable structure is attainable, i.e., the generic-rank condition of structural controllability is certainly satisfied, then the constrained MIS can be reduced to a minimum set covering problem and is polynomially solvable. However, it is a strong hypothesis to network topology and can not be satisfied by a general reconfigured system. In this study, we characterize the optimal solution of MIS-RS and propose a polynomial algorithm to find a minimum input allocation of MIS-RS, which is suitable for reconfigured systems with arbitrary architecture. To this end, the novel procedure is proposed based on the mild assumption that dedicated inputs (i.e., each input actuates at most a single state) are employed, which is the same condition utilized in the classic MIS and is more readily to be satisfied in real scenarios.

* Corresponding author (email: syli@sjtu.edu.cn)

Model and methodology. Now consider a linear reconfigurable architecture system depicted by

$$\dot{x}(t) = A^{\mathrm{re}}x(t) + B^{\mathrm{re}}u(t), \quad (1)$$

where $x, u \in \mathbb{R}^n$ are the state and input vector of reconfigured systems. Let $\bar{A}^{\mathrm{re}} \in \{0,1\}^{n\times n}$ be a binary matrix that encodes the structural pattern of A^{re}. $\bar{B}^{\mathrm{re}} = \mathcal{I}_n^p$ is an $n \times n$ identity matrix with p nonzero diagonal entries. On this basis, the MIS-RS problem satisfying $(\bar{A}^{\mathrm{re}}, \bar{B}^{\mathrm{re}})$ is structurally controllable can be formally posed as $\mathcal{P}$: Given $\bar{A}^{\mathrm{re}} \in \{0,1\}^{n\times n}$, $\bar{B}^{\mathrm{re}} = \mathcal{I}_n^p$, determine

$$\mathcal{J}^* = \arg\min_{\mathcal{J}\subseteq[1:p]} |\mathcal{J}| \quad (2)$$
$$\text{s.t. } (\bar{A}^{\mathrm{re}}, \bar{B}^{\mathrm{re}}_{\mathcal{J}}) \text{ is structurally controllable,}$$

where $\mathcal{J}$ is a subset of indices associated with given inputs. $[1:p]$ is the set $\{1, 2, \ldots, p\}$ and $|\mathcal{J}|$ is the cardinality of $\mathcal{J}$, i.e., the number of elements that $\mathcal{J}$ consists of. $\bar{B}^{\mathrm{re}}_{\mathcal{J}}$ represents the subset of nonzero columns of $\bar{B}^{\mathrm{re}}$ with index in $\mathcal{J}$.

Unless otherwise specified, the graph theoretic notations used in this study can be referenced in [5]. Notably, we use SCC to denote a strongly connected component, which refers to a maximal subgraph of a digraph so that there exists a path between any two vertices of the graph. In addition, non-top linked SCC (NT-SCC) is an SCC that has no incoming edge from another SCC. To proceed, the result used to determine if a system is structurally controllable is provided.

Lemma 1 ([6])**.** The pair $(\bar{A}^{\mathrm{re}}, \bar{B}^{\mathrm{re}})$ is said to be structurally controllable if and only if both the following two conditions hold:

(i) Every NT-SCC of the reconfigured system digraph $\mathcal{D}(\bar{A}^{\mathrm{re}}, \bar{B}^{\mathrm{re}})$ consists of at least one input;

(ii) Any maximum matching of the bipartite graph $\mathcal{B}(\bar{A}^{\mathrm{re}}, \bar{B}^{\mathrm{re}})$ has no right-unmatched vertex.

On this basis, the optimal solution of MIS-RS can be characterized by Theorem 1.

Theorem 1 (Dedicated solution to $\mathcal{P}$)**.** Let r be the number of NT-SCCs of $\mathcal{D}(\bar{A}^{\mathrm{re}})$ and $\mathcal{N}_j^{\mathrm{NT}}$ be the set of states of j-th NT-SCC. Let $\mathcal{R}(M^*_{\bar{A}^{\mathrm{re}},i})$ be the right-unmatched vertex-set of $\mathcal{B}(\bar{A}^{\mathrm{re}})$ with respect to maximum matching $M^*_{\bar{A}^{\mathrm{re}},i}$ and by $d_R = |\mathcal{R}(M^*_{\bar{A}^{\mathrm{re}},i})|$. For any given $\bar{B}^{\mathrm{re}} = \mathcal{I}_n^p$, the number of minimum inputs for MIS-RS is

$$s^* = d_R - \sum_{j=1}^{r} ||\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i}) \cap \mathcal{N}_j^{\mathrm{NT}}|| + r, \quad (3)$$

where $||\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i}) \cap \mathcal{N}_j^{\mathrm{NT}}|| = 1$ if $\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i}) \cap \mathcal{N}_j^{\mathrm{NT}} \neq \emptyset$, otherwise $||\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i}) \cap \mathcal{N}_j^{\mathrm{NT}}|| = 0$. The optimal $\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i})$ is determined by

$$\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i}) = \arg\max \sum_{j=1}^{r} ||\mathcal{R}(M^*_{\bar{A}^{\mathrm{re}},i}) \cap \mathcal{N}_j^{\mathrm{NT}}|| \quad (4)$$
$$\text{s.t. } \mathcal{R}(M^*_{\bar{A}^{\mathrm{re}},i}) \subseteq \mathcal{J}.$$

Proof. First, let η be the number of different maximum matchings of $\mathcal{B}(\bar{A}^{\mathrm{re}})$ and define the new set $\mathcal{X}^{\mathrm{NT}} = \{x_j : x_j = [\mathcal{N}_j^{\mathrm{NT}}, 1], \forall j \in [1:r]\}$, where $[\mathcal{N}_j^{\mathrm{NT}}, 1]$ denotes any one state of $\mathcal{N}_j^{\mathrm{NT}}$. Then according to Lemma 1, the minimum inputs required to achieve condition (i) is r while the minimum actuators needed to achieve condition (ii) is d_R. In consequence, the fewest number of inputs of MIS-RS to ensure structural controllability equals to

$$\begin{aligned} s^* &= |\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i}) \cup \mathcal{X}^{\mathrm{NT}}| \\ &= \min_{i\in[1:\eta]} |\mathcal{R}(M^*_{\bar{A}^{\mathrm{re}},i}) \cup \mathcal{X}^{\mathrm{NT}}| \\ &= d_R - \max_{i\in[1:\eta]} |\mathcal{R}(M^*_{\bar{A}^{\mathrm{re}},i}) \cap \mathcal{X}^{\mathrm{NT}}| + r. \end{aligned} \quad (5)$$

Determined by (4), the optimal $\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i})$ is obtained. Therefore, it derives

$$|\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i}) \cap \mathcal{X}^{\mathrm{NT}}| = \sum_{j=1}^{r} ||\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i}) \cap \mathcal{N}_j^{\mathrm{NT}}||.$$

By substituting this equation to (5), Eq. (3) is proved to be the minimum inputs of MIS-RS.

Owing to the constraint $\mathcal{R}(M^*_{\bar{A}^{\mathrm{re}},i}) \subseteq \mathcal{J}$, the optimal solution of MIS-RS may be suboptimal to MIS, which can be illustrated by an example shown in Figure 1(a)–(d). In light of Theorem 1, the key to solve MIS-RS is how to find $\mathcal{R}^*(M^*_{\bar{A}^{\mathrm{re}},i})$. To do this, Definitions 1–3 are brought in.

Definition 1 (Augmenting path)**.** In $\mathcal{B}(\bar{A}^{\mathrm{re}})$, the augmenting path from a right-unmatched state x_i^- to a right-matched state $x_{[j]}^-$ refers to a path connecting x_i^- and $x_{[j]}^-$ on which the matching edge and the un-matching edge appear alternatively. The augmenting path between x_i^- and $x_{[j]}^-$ is denoted by $\mathrm{aug}(x_i^-, x_{[j]}^-)$.

The augmenting path inversion $\mathrm{aug}^{-1}(x_i^-, x_{[j]}^-)$ can be obtained by turning every matching edge of $\mathrm{aug}(x_i^-, x_{[j]}^-)$ into an un-matching one while changing the un-matching edges into matching ones.

Definition 2 (Complete induced vertex-set)**.** Let $M^*_{\bar{A}^{\mathrm{re}}}$ be any maximum matching of $\mathcal{B}(\bar{A}^{\mathrm{re}})$. $\forall x_i^-(M^*_{\bar{A}^{\mathrm{re}}}) \in \mathcal{R}(M^*_{\bar{A}^{\mathrm{re}}})$, if there exists an augmenting path $\mathrm{aug}(x_i^-(M^*_{\bar{A}^{\mathrm{re}}}), x_{[j]}^-(M^*_{\bar{A}^{\mathrm{re}}}))$, then $x_{[j]}^-(M^*_{\bar{A}^{\mathrm{re}}})$ is an induced vertex of $x_i^-(M^*_{\bar{A}^{\mathrm{re}}})$. The induced vertex-set is denoted by $\mathrm{ind}[x_i^-(M^*_{\bar{A}^{\mathrm{re}}})]$ and complete induced vertex-set is represented as

$$\mathrm{Ind}[x_i^-(M^*_{\bar{A}^{\mathrm{re}}})] = \mathrm{ind}[x_i^-(M^*_{\bar{A}^{\mathrm{re}}})] \cup x_i^-(M^*_{\bar{A}^{\mathrm{re}}}). \quad (6)$$

Definition 3 (Intersecting augmenting path)**.** In $\mathcal{B}(\bar{A}^{\mathrm{re}})$, the path satisfying following conditions is referred to be an intersecting augmenting path: (1) it starts from an induced vertex and ends on

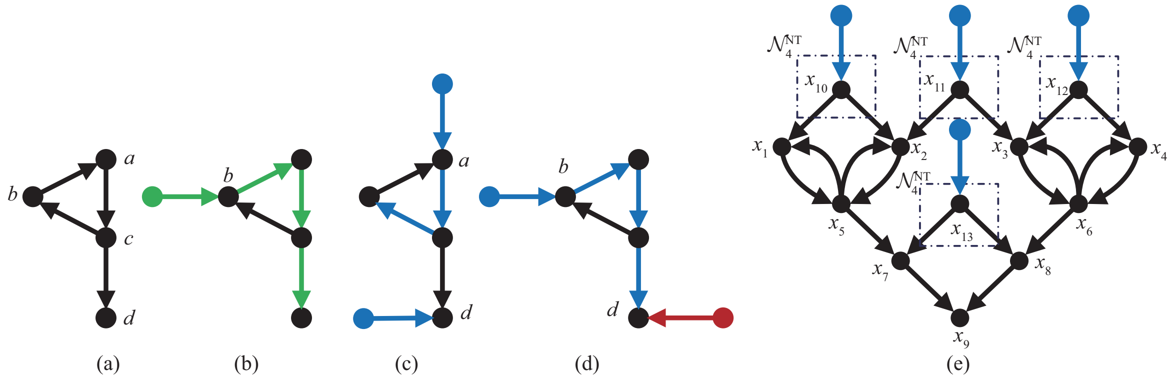

Figure 1 (Color online) (a) Reconfigured state digraph; (b) The MIS of (a) is $\{b\}$; (c) the MIS-RS is $\{a, d\}$ if $\mathcal{J} = \{a, d\}$, which is a suboptimal solution to MIS; (d) the MIS-RS is $\{b\}$ if $\mathcal{J} = \{b, d\}$, which is also an optimal solution to MIS; (e) the example used for algorithm verification.

another induced vertex; (2) it is located on the intersection of different augmenting paths; (3) all the left-matched vertices on it do not connect with an induced vertex. The set of induced vertices on k-th intersecting augmenting path is denoted by $\mathcal{T}_k$.

Then the MIS-RS problem can be solved by Algorithm 1.

Algorithm 1 Solution to the MIS-RS problem

Input: $\bar{A}^{\rm re}$ and $\mathcal{J}$.
Output: The optimal solution $\mathcal{J}^*$ of problem $\mathcal{P}$.
Step1: determine $M^*_{\bar{A}^{\rm re}}$ and $\mathcal{R}(M^*_{\bar{A}^{\rm re}})$ of $\mathcal{B}(\bar{A}^{\rm re})$;
Step2: $\forall x_i^- \in \mathcal{R}(M^*_{\bar{A}^{\rm re}})$, obtain $\mathrm{Ind}[x_i^-(M^*_{\bar{A}^{\rm re}})]$, $\mathcal{T}_k$;
Step3: derive $\mathcal{S}_l = \mathrm{Ind}[x_i^-(M^*_{\bar{A}^{\rm re}})] \cap \mathcal{J}$, $\forall l \in [1:d_R]$;
Step4: find NT-SCCs $\{\mathcal{N}_j^{\rm NT}\}_{j\in[1:r]}$ of $\mathcal{D}(\bar{A}^{\rm re})$;
Step5: In the $\mathcal{B}(\{\mathcal{S}_l\}_{l\in[1:d_R]}, \{\mathcal{N}_j^{\rm NT}\}_{j\in[1:r]}, \mathcal{E}_{\{\mathcal{S}_l\},\{\mathcal{N}_j^{\rm NT}\}})$, $\mathcal{E}_{\{\mathcal{S}_l\},\{\mathcal{N}_j^{\rm NT}\}} = \{(\mathcal{S}_l, \mathcal{N}_j^{\rm NT}) : \mathcal{S}_l \cap \mathcal{N}_j^{\rm NT} \neq \emptyset\}$, find M^*;
Step6:
- $\forall \mathcal{N}_j^{\rm NT}$ matched by $\mathcal{S}_l$, obtain $[\mathcal{S}_l \cap \mathcal{N}_j^{\rm NT}, 1] \in \mathcal{J}_1^*$ satisfying $|\mathcal{J}_1^* \cap \mathcal{T}_k| \leqslant 1$;
- $\forall \mathcal{N}_j^{\rm NT}$ not matched, obtain $[\mathcal{N}_j^{\rm NT} \cap \mathcal{J}, 1] \in \mathcal{J}_2^*$;
- $\forall \mathcal{S}_l$ not matched, obtain $[\mathcal{S}_l, 1] \in \mathcal{J}_3^*$ satisfying $\cap_{l\in[1:d_R]}[\mathcal{S}_l, 1] = \emptyset$, $\mathcal{J}_1^* \cap \mathcal{J}_3^* = \emptyset$, $|\mathcal{J}_1^* \cup \mathcal{J}_3^* \cap \mathcal{T}_k| \leqslant 1$;

Step7: return $\mathcal{J}^* = \mathcal{J}_1^* \cup \mathcal{J}_2^* \cup \mathcal{J}_3^*$.

Because each step of Algorithm 1 is polynomially solvable [7], the MIS-RS issue can be solved by the presented arithmetic in polynomial time.

The established procedure to address MIS-RS is verified through a structural leader-selection problem presented in [5], which is posed as: considering a reconfigured multi-agent system consisting of 13 agents (the system structure is shown in Figure 1(e)). Let part of the agents be equipped with a dedicated input, i.e., $\mathcal{J} = \{x_3, x_5, x_9, x_{10}, x_{11}, x_{12}, x_{13}\}$. In reconfigured systems, the leader-selection problem aims to determine the minimum agents from set $\mathcal{J}$ and employ their inputs to achieve structural controllability.

By Algorithm 1, $\mathcal{R}(M^*_{\bar{A}^{\rm re}}) = \{x_{10}^-, x_{11}^-, x_{12}^-, x_{13}^-\}$ and $\mathcal{T}_k = \emptyset$. For any $x_i^- \in \mathcal{R}(M^*_{\bar{A}^{\rm re}})$, there is $\mathrm{Ind}[x_i^-(M^*_{\bar{A}^{\rm re}})] = \mathcal{S}_l = \{x_i^-\}$, $\forall l \in [1:4]$. The NT-SCCs are shown by dashed boxes in Figure 1(e). Because perfect matching is found between $\{\mathcal{S}_l\}_{l\in[1:4]}$ and $\{\mathcal{N}_j^{\rm NT}\}_{j\in[1:4]}$, $\mathcal{J}_1^* = \{x_{10}, x_{11}, x_{12}, x_{13}\}$, $\mathcal{J}_2^* = \emptyset$ and $\mathcal{J}_3^* = \emptyset$. Thus, $\mathcal{J}^* = \{x_{10}, x_{11}, x_{12}, x_{13}\}$.

Conclusion and future work. We proposed a novel procedure to solve the MIS-RS problem. Under the mild assumption that dedicated input is used, the presented algorithm can select the minimum controls from a given collection of inputs in polynomial time for reconfigured systems with arbitrary network topology. The future research will focus on the minimum input addition problem of reconfigured systems that are not controllable.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61590924, 61773162, 61521063), and Natural Science Foundation of Shanghai (Grant No. 18ZR1420000).

References

1 Liu Y Y, Slotine J J, Barabási A L. Controllability of complex networks. Nature, 2011, 473: 167–173
2 Lin C T. Structural controllability. IEEE Trans Autom Control, 1974, 19: 201–208
3 Guan Y Q, Wang L. Structural controllability of multi-agent systems with absolute protocol under fixed and switching topologies. Sci China Inf Sci, 2017, 60: 092203
4 Mu J B, Li S Y, Wu J. On the structural controllability of distributed systems with local structure changes. Sci China Inf Sci, 2018, 61: 052201
5 Pequito S, Kar S, Aguiar A P. On the complexity of the constrained input selection problem for structural linear systems. Automatica, 2015, 62: 193–199
6 Dion J M, Commault C, van der Woude J. Generic properties and control of linear structured systems: a survey. Automatica, 2003, 39: 1125–1144
7 Assadi S, Khanna S, Li Y, et al. Complexity of the minimum input selection problem for structural controllability. IFAC-Papersonline, 2015, 48: 70–75