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Trajectory optimization for RLV in TAEM phase using adaptive Gauss pseudospectral method

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Dear editor,

With the development of space technology, the trajectory optimization for reusable launch vehicle (RLV) in terminal area energy management (TAEM) phase has aroused increasing attention among researchers because of its nonlinearity and complex path constraints. TAEM phase starts from the terminal entry point (TEP) with an altitude of 30 km and velocity of 3 Ma, and ends at the approach and landing interface (ALI) with an altitude of 3 km and velocity of 0.5 Ma [1].

For trajectory optimization in the TAEM phase, major studies have referred to the geometric trajectory generation scheme. An off-line trajectory planner including ground-track planner and vertical-trajectory planning methodology was developed in [1]. In [2], a reference trajectory of the TAEM phase is generated through iterative solution of a parameter optimization problem in the presence of large variations of initial states. An energy-tube concept was introduced in the terminal area trajectory optimization, and a cross-section of the energy tube is defined by the altitude and velocity information in [3]. An advantage of this method is that it provides sufficient capabilities to deal with off-nominal conditions when energy dissipation is required. Mu et al. [4] designed TAEM trajectory by iterating the motion equations at each node of altitude. Guo et al. [5] proposed a fast algorithm where the Gauss pseudospectral method (GPM) and model predictive control are combined.

In this study, we consider the coupling between the longitudinal motion and the lateral motion of an RLV, path constraints, and terminal constraint to design the 3-DOF optimal TAEM trajectory through the adaptive Gauss pseudospectral method (AGPM). The combination of adaptive scheme and the GPM can improve the accuracy and rapidity of trajectory optimization effectively. Further, the trajectory obtained by our scheme is the basic of the TAEM real-time trajectory.

Problem statement. Based on the 3-DOF translational equations used for trajectory optimization in the TAEM phase mentioned in [5], the objective of TAEM trajectory optimization is to minimize a cost function (1) (In this work, the cost function we chose is to minimize the terminal time) by using the AGPM when considering the constraints involving the heating rate constraint (2), dynamic pressure constraint (3), load constraint (4) and terminal condition (5) (x represents the state mentioned in [6] and t_f is the terminal time).

$$\min J = t_f, \tag{1}$$

$$Q = k_Q \rho^{0.5} V^{3.15} \leqslant Q_{\max}, \tag{2}$$

$$\bar{q}_L \leqslant \bar{q} \leqslant \bar{q}_H, \tag{3}$$

$$n = \sqrt{L^2 + D^2}/(mg) \leqslant n_{\max}, \tag{4}$$

$$x(t_f) = x_f. \tag{5}$$

Gauss pseudospectral method. First, because the GPM is the core algorithm of the TAEM trajectory

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design, a brief review of this numerical method is given in the following. The purpose of the TAEM trajectory optimization problem is to find the control variable (α and σ) to minimize discretized objective function (6) with the constraints.

$$J = \Phi[X(t_0), t_0, X(t_f), t_f] + \int_{t_0}^{t_f} g(X(t), U(t), t)\mathrm{d}t. \tag{6}$$

To express the reentry trajectory optimization problem in time domain $\tau \in [-1, 1]$, $[t_0, t_f]$ needs to go through the following transformation:

$$\tau = \frac{2t}{t_f - t_0} - \frac{t_f + t_0}{t_f - t_0}. \tag{7}$$

Then, the discretization process begins by approximating the states as

$$\begin{aligned} \dot{x}(\tau) \approx \dot{X}(\tau) &= \sum_{i=0}^{N} \dot{L}_i(\tau) x(\tau_i) \\ &= \sum_{i=0}^{N} D_{ki} X(\tau_i), \end{aligned} \tag{8}$$

where $L_i(\tau) = \prod_{j=0, j\neq i}^{N} \frac{\tau_i - \tau_j}{\tau - \tau_j} (i = 0, \ldots, N)$ is the $(N+1)$th degree Lagrange interpolating polynomial and D_{ki} is the element of the $N \times (N+1)$ differentiation matrix described in [6].

The dynamic constraint is transcribed into algebraic constraints, which is expressed as follows:

$$\sum_{i=0}^{N} D_{ki} X(\tau_i) - \frac{t_f - t_0}{2} F(X(\tau_k), U(\tau_k); t_0, t_f) = 0. \tag{9}$$

Moreover, the terminal state X_f is transformed into the following form:

$$\begin{aligned} X(\tau_f) = &\frac{t_f - t_0}{2} \sum \omega_k f(X(\tau_k), U(\tau_k); t_0, t_f) \\ &+ X(\tau_0), \end{aligned} \tag{10}$$

where ω_k is the quadrature weight. Based on the abovementioned transformation, the objective function (6) is rewritten as

$$\begin{aligned} J = &\Phi(X(\tau_0), t_0, X(\tau_f), t_f) \\ &+ \frac{t_f - t_0}{2} \sum_{k=1} N \omega_k g(X(\tau_k), U(\tau_k), \tau_k) \end{aligned} \tag{11}$$

with the boundary conditions and path constraints:

$$\Lambda[X(\tau_0), t_0, X(\tau_f), t_f] = 0, \tag{12}$$

$$C(x_k, u_k, t_0, t_f) \leqslant 0, \quad k = 1, 2, \ldots, N. \tag{13}$$

In brief, the objective is to find the NLP variables

$$\begin{aligned} x_{\mathrm{NLP}} = [&x(\tau_0), \ldots, x(\tau_N), x(\tau_f), \\ &u(\tau_0), \ldots, x(\tau_N), x(\tau_f)] \end{aligned}$$

to minimize (11) under the constraints (9), (10), (12), and (13).

Adaptive discrete strategy. The adaptive discrete strategy is included in the GPM to improve the accuracy and rapidity. Suppose that the flight trajectory has already been divided into S segments. Let $[t_{n-1}, t_n]$ be the Nth interval and suppose that the solution is collocated at N^n Legendre-Gauss (LG) points on this interval. The objective of the adaptive method is to determine if a segment should be divided into more segments or if the number of LG points in a segment should be increased. Define the following midpoint residual vector:

$$\begin{aligned} r(\bar{\tau}_{nk}) = \Bigg| &\sum_{i=0}^{N_n} D_{ki} X(\bar{\tau}_{ni}) \\ &- \frac{t_n - t_{n-1}}{2} F(X(\bar{\tau}_{nk}), U(\bar{\tau}_{nk})) \Bigg| \in \mathbb{R}^n, \end{aligned} \tag{14}$$

where $\bar{\tau}_{nk} = (\tau_{nk} + \tau_{n(k+1)})/2 \quad (k = 1, \ldots, N_s - 1)$, and $X(\bar{\tau}_{nk})$ and $U(\bar{\tau}_{nk})$ are obtained through interpolation. We define the tolerance $\varepsilon_{\max}$. The curvature of the Nth component of the state in mesh interval k is given as

$$\kappa^{(k)}(\tau) = |\ddot{x}_s(\tau)| \cdot \left| \left[1 + \dot{x}s^{(k)}(\tau)^2 \right]^{\frac{-3}{2}} \right|. \tag{15}$$

Further, we define the variable $r^{(k)}$:

$$r^{(k)} = \frac{\kappa_{\max}^{(k)}}{\bar{\kappa}^{(k)}}, \tag{16}$$

where $\bar{\kappa}^{(k)}$ is the mean value of $\kappa^{(k)}$ and $\kappa_{\max}^{(k)}$ is the maximum value of $\kappa^{(k)}$.

When $r^{(k)} < r_{\max}$, the number of LG points in the Nth is updated as follows:

$$\bar{N}_N^* = \bar{N}_N + \mathrm{ceil}[\log_{10}(e_{\max}^k / \varepsilon_{\max})], \tag{17}$$

or the Nth segment is divided into more segments by the following equation:

$$\bar{n}_N^k = \mathrm{ceil}[\log_{10}(e_{\max}^k / \varepsilon_{\max})], \tag{18}$$

where ceil is the operator that rounds to the next highest integer and $e_{\max}^k$ is the maximum tolerance of the equality constraints.

Simulation. To provide the effectiveness of our method, simulation is performed based on the model of X-33. And the simulation results are shown in Figure 1.

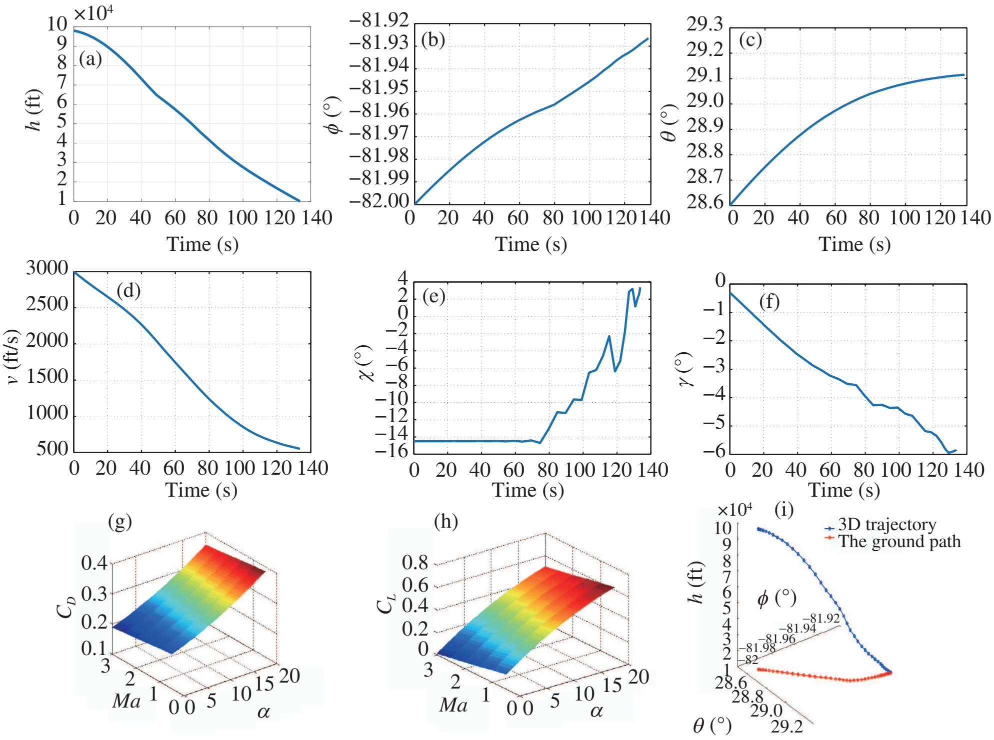

Figure 1 (Color online) Simulation diagrams of TAEM trajectory based on AGPM. (a) Altitude; (b) longitude; (c) latitude; (d) velocity; (e) flight path angle; (f) heading angle; (g) the 3D-surface plots of the drag coefficient; (h) the 3D-surface plots of the lift coefficient; and (i) the 3D path of the generated TAEM trajectory.

Figures 1(a)–(f) show the optimal curves of six state variables. The simulation results show that the generated trajectories change smoothly and satisfy all the path constraints. Because the RLV in the reentry phase is unpowered, the velocity is directly proportional to the energy of the RLV. From the variation of velocity in Figure 1(d), it can be seen that the velocity of the RLV decreases monotonously. Figures 1(g) and (h) are the 3D-surface plots of the drag coefficient and the lift coefficient, respectively. Figure 1(i) shows the ground path and 3D path of the generated TAEM trajectory. From the simulation results, it can be concluded that the AGPM achieves a better entry into the approach and landing interface (ALI).

Conclusion. This study proposed a trajectory optimization scheme for a RLV in a TAEM phase. The AGPM was used to generate a 3D optimal trajectory in the TAEM phase, which is necessary for the safety and reliability of RLV in the TAEM phase. The efficiency of the proposed algorithm was verified through numerical simulation of a typical X-33. Simulation results indicated that the optimal TAEM trajectory designed by the AGPM can deliver the RLV to approach and landing phase safely and reliably.

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