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Asymptotically efficient non-truncated identification for FIR systems with binary-valued outputs

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Dear editor,
Recently, set-valued observations have increasingly emerged in practical fields owing to limited communication resources [1]. Numerous set-valued sensors, such as switching sensors for exhaust gas oxygen and pressure switches have proliferated because of their cost economy [2].

For many finite impulse response (FIR) systems, their outputs cannot be measured accurately. Instead, what can be measured is whether they belong to a certain number of sets. For instance, if the outputs only belong to only two sets, the system is called an FIR system with binary-valued outputs.

The identification of set-valued systems has been studied for more than a decade. Several excellent studies on parameter estimation have been conducted [2–5]. The first comprehensive study on the genre of binary-valued observations was performed by Wang et al. [2]. They investigated their empirical measure method and proved that the estimate can converge to the true parameter. The asymptotic efficiency of this method was studied [6, 7], wherein the inverse of the distributed function was required to be uniformly bounded. To solve this problem, Zhao et al. [8] proposed the truncated empirical measure method and provided a rigorous proof of the asymptotic efficiency. Nevertheless, formulating the truncation was the challenge that Wang and Zhao [9] addressed by proposing a non-truncated empirical measure algorithm for single-parameter systems. However, the efficiency of the algorithm was not analyzed.

We study the convergence and efficiency properties of the non-truncated identification algorithm for FIR systems with binary-valued observations. First, we study the algorithm for single-parameter systems. Using the discrete properties of the binary-valued observations, we prove that it is asymptotically unbiased and asymptotically efficient. Next, we transform the identification problem of multi-parameter systems into multiple identification problems of single-parameter systems. Finally, we construct a non-truncated identification algorithm for multi-parameter systems and prove that the algorithm is asymptotically efficient.

Identification for single-parameter systems. Consider the FIR system with a single parameter and binary-valued outputs

$$\begin{cases} y(k) = \theta + d(k), \\ s(k) = I_{\{y(k)\leqslant C\}}, \quad k = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where θ is an unknown parameter, and $d(k)$ is the disturbance. The output $y(k)$ is measured by a binary-valued sensor with a threshold C represented by the indicator function

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$$s(k) = I_{\{y(k)\leqslant C\}} = \begin{cases} 1, & y(k) \leqslant C, \\ 0, & \text{otherwise.} \end{cases}$$

Here, the objective of identification is to estimate the parameter θ by the binary-valued outputs $s(k)$, $k = 1, \ldots, N$.

Assumption 1. The noises $\{d(k), k = 1, 2, \ldots\}$ are independent and are identically normally distributed random variables with a known distribution function $F(\cdot)$, and the associated density function satisfies $f(x) = \mathrm{d}F(x)/\mathrm{d}x \neq 0$.

Based on the truncated empirical measure method [8], we defined the non-truncated empirical measure method as follows:

$$\psi_N = \frac{1}{N}\sum_{k=1}^{N} s(k),\ \zeta_N = \begin{cases} \frac{1}{2}, & \text{if } \psi_N = 0; \\ \psi_N, & \text{if } 0 < \psi_N < 1; \\ \frac{1}{2}, & \text{if } \psi_N = 1, \end{cases}$$

$$\hat{\theta}_N^1 = C - F^{-1}(\zeta_N), \tag{2}$$

where $\frac{1}{2}$ can be changed into any constant in the interval $(0, 1)$.

Considering the Assumption 1, Theorems 1 and 2 can be proved.

Theorem 1 (Asymptotic unbias)**.** The estimator $\hat{\theta}_N^1$ given by (2) is asymptotically unbiased in the sense that $E\hat{\theta}_N^1 \to \theta$ as $N \to \infty$. Moreover,

$$E(\hat{\theta}_N^1 - \theta) = O\left(\frac{1}{N}\right).$$

Theorem 2 (Asymptotic efficiency)**.** The mean square error of estimator $\hat{\theta}_N^1$ given in (2) is

$$E(\hat{\theta}_N^1 - \theta)^2 = \frac{F(C-\theta)(1-F(C-\theta))}{Nf(C-\theta)^2} + O\left(\frac{1}{N^2}\right).$$

Moreover, estimator $\hat{\theta}_N^1$ is asymptotically efficient in the sense that

$$N\big(E(\hat{\theta}_N^1 - \theta)^2 - \sigma_{\mathrm{CR}}^2(\theta, N)\big) \to 0, \text{ as } N \to \infty,$$

where $\sigma_{\mathrm{CR}}^2(\theta, N) = \frac{F(C-\theta)(1-F(C-\theta))}{Nf(C-\theta)^2}$ is the Cramer-Rao (CR) lower bound.

Identification for multi-parameter systems. Consider the FIR system with multi-parameter and binary-valued outputs

$$\begin{cases} y(k) = \phi^{\mathrm{T}}(k)\theta + d(k), \\ s(k) = I_{\{y(k)\leqslant C\}}, \end{cases} \quad k = 1, 2, \ldots, \tag{3}$$

where $\phi(k) = [u(k-1), u(k-2), \ldots, u(k-n)]^{\mathrm{T}}$ is the input and $\theta = [a_1, a_2, \ldots, a_n]^{\mathrm{T}}$ is the unknown parameter with $n \geqslant 1$. Next, we address the way to estimate parameter θ by the binary-valued outputs $s(k)$, $k = 1, 2, \ldots$.

The main idea is to transform the multi-parameter identification problem to multiple single-parameter identification problems. To this end, we consider Assumption 2.

Assumption 2. The inputs $u(k)$, $k = 1, \ldots, (n+1)N$ are n-periodic with $u(1) = u_1, \ldots, u(n) = u_n$, where $u = [u_n, u_{n-1}, \ldots, u_1]^{\mathrm{T}}$ is full rank.

Definition 1. A vector $v = [v_n, \ldots, v_1]^{\mathrm{T}}$ is called full rank if the circulant matrix generated from v

$$T(v) = \begin{bmatrix} v_n & v_{n-1} & v_{n-2} & \cdots & v_2 & v_1 \\ v_1 & v_n & v_{n-1} & \cdots & v_3 & v_2 \\ v_2 & v_1 & v_n & \cdots & v_4 & v_3 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ v_{n-1} & v_{n-2} & v_{n-3} & \cdots & v_1 & v_n \end{bmatrix}$$

is full rank.

Considering Assumption 2, $\phi(k)$ is n-periodic. Let

$$Y(l) = [y(ln+1), y(ln+2), \ldots, y((l+1)n)]^{\mathrm{T}},$$
$$\Phi(l) = [\phi(ln+1), \phi(ln+2), \ldots, \phi((l+1)n)]^{\mathrm{T}},$$
$$D(l) = [d(ln+1), d(ln+2), \ldots, d((l+1)n)]^{\mathrm{T}},$$

and

$$S(l) = [s(ln+1), s(ln+2), \ldots, s((l+1)n)]^{\mathrm{T}}.$$

Then, the input matrix $\Phi(l) = T(u) \triangleq \Phi$ is a constant matrix with full rank, and the FIR system (3) can be written as

$$\begin{cases} Y(l) = \Phi\theta + D(l), \\ S(l) = \mathbb{I}_{\{Y(l)\leqslant C\}}, \end{cases} \quad l = 1, 2, \ldots, N, \tag{4}$$

where $\mathbb{I}_{\{a\leqslant C\}} = [I_{\{a(1)\leqslant C\}}, I_{\{a(2)\leqslant C\}}, \ldots, I_{\{a(n)\leqslant C\}}]^{\mathrm{T}}$ for any n-dimensional vector $a = [a(1), \ldots, a(n)]^{\mathrm{T}}$.

If we choose $\Phi = I_{n\times n}$, system (4) can be transformed into n single-parameter cases with N observations

$$\begin{cases} y(ln+i) = a_i + d(ln+i), & l = 1, \ldots, N, \\ s(ln+i) = I_{\{y(ln+i)\leqslant C\}}, & i = 1, \ldots, n. \end{cases}$$

The estimates for a_i are given by algorithm (2) for single-parameter systems. The estimate of θ can be written in the following vector form:

$$\Psi_N = [\Psi_N(1), \ldots, \Psi_N(n)]^{\mathrm{T}} = \frac{1}{N}\sum_{l=1}^{N} S(l),$$

$$\Xi_N = \left[\Xi_N(i) = \begin{cases} \frac{1}{2}, & \text{if } \Psi_N(i) = 0; \\ \Psi_N(i), & \text{if } 0 < \Psi_N(i) < 1; \\ \frac{1}{2}, & \text{if } \Psi_N(i) = 1, \end{cases} \right]_{n\times 1},$$

$$\hat{\theta}_N^I = C\mathbb{1}_n - \tilde{F}^{-1}(\Xi_N), \tag{5}$$

where $\mathbb{1}_n$ is an n-dimensional vector with all the elements being 1, and $\tilde{F}^{-1}(\Xi_N) = [F^{-1}(\Xi_N(1)), \ldots, F^{-1}(\Xi_N(n))]^{\mathrm{T}}$.

For a general input matrix Φ, we can estimate $\Phi\theta$ by (5). Then, the estimate for θ is given by

$$\hat{\theta}_N = \Phi^{-1}(C\mathbb{1}_n - \tilde{F}^{-1}(\Xi_N)). \quad (6)$$

If $n = 1$ and $u(i) \equiv 1$, $i = 1, 2, \ldots$, system (4) is a single-parameter system and estimator $\hat{\theta}_N$ in (6) degenerates to estimator $\hat{\theta}_N^1$ in (2). If $\Phi = I_n$, estimator $\hat{\theta}_N$ in (6) will be $\hat{\theta}_N^I$ in (5). Estimator (6) is a more general estimator for FIR systems.

For estimator (6) of the multi-parameter systems, Theorems 3 and 4 can be formulated.

Theorem 3 (Convergence)**.** For a binary-valued FIR system (3), the estimator $\hat{\theta}_N$ given by (6) converges almost certainly to the true value of parameter θ, that is

$$\hat{\theta}_N \to \theta, \quad \text{a.s.} \quad \text{as } N \to \infty.$$

Theorem 4 (Asymptotic efficiency)**.** Considering the Assumptions 1 and 2, the estimator $\hat{\theta}_N$ given by (6) is asymptotically efficient such that

$$N\big(E(\hat{\theta}_N - \theta)(\hat{\theta}_N - \theta)^{\mathrm{T}} - \Sigma^2_{\mathrm{CR}}(\theta, N)\big) \to 0_{n\times n},$$

where $\Sigma^2_{\mathrm{CR}}(\theta, N)$ is the CR lower bound for multi-parameter systems.

Simulations. We consider the multi-parameter system $y(k) = \phi^{\mathrm{T}}(k)\theta + d(k)$ with $\theta = [1, 5, 8]^{\mathrm{T}}$ and $\phi(k) = [u(k-1), u(k-2), u(k-3)]$. Let $u(k)$ be 3-periodic with $u(1) = 3$, $u(2) = 2$, $u(3) = 3$. Then $y(3l+1) = [1, 2, 3]\theta + d(3l+1)$, $y(3l+2) = [3, 1, 2]\theta + d(3l+2)$, $y(3l+3) = [2, 3, 1]\theta + d(3l+3)$, $l = 1, \ldots, N$. Let $Y(l) = [y(3l+1), y(3l+2), y(3l+3)]^{\mathrm{T}}$. We have $Y(l) = \Phi\theta + D(l)$, where $\Phi = \left[\begin{smallmatrix}1 & 2 & 3\\ 3 & 1 & 2\\ 2 & 3 & 1\end{smallmatrix}\right]$, and $D(l) = [d(3l+1), d(3l+2), d(3l+3)]^{\mathrm{T}}$. Consequently, we can get the binary-valued output $S(l) = \mathbb{I}_{\{Y(l)\leqslant C\}}$. Using $S(l)$, $l = 1, \ldots, N$, we can get the estimates of the parameter θ by the algorithm (6). The simulation results are shown in Figure 1, where the estimates can converge to the true value of the parameter.

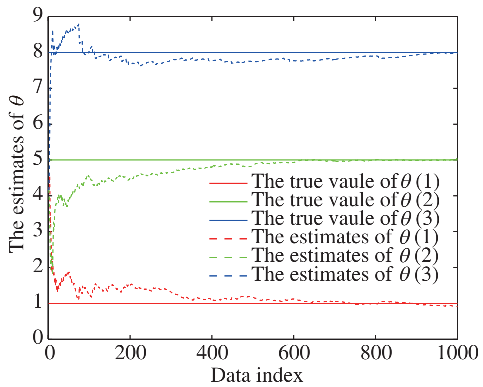

Figure 1 (Color online) The identification for an FIR system.

Conclusion. We studied the identification of FIR systems with binary-valued outputs in both single-parameter and multi-parameter cases. For the single-parameter systems, we proved that the first moment of the estimation error has the same convergence rate $O(1/N)$ as the mean square error, and proved that the estimate is asymptotic efficient. For the multi-parameter systems, we proposed a non-truncated identification algorithm and proved its convergence and asymptotic efficiency. For future research, identification of FIR systems with non-periodic inputs is a key area of interest.

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Supporting information Some definitions, lemmas, proofs of the theorems, and simulations are in Appendices A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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