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# Two novel robust adaptive parameter estimation methods with prescribed performance and relaxed PE condition

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Dear editor,
In many industrial applications, the systems under study are often subject to parametric uncertainties. These uncertainties, which may arise from aging, hardware damages and environmental conditions, can degrade the system's performance [1]. To overcome these uncertainties, the problem of parameter estimation has been of considerable interest. Classical parameter estimation methods were reported, including least square and recursive least square, gradient descent method, projection method, and improvements such as $e$-modification, $\sigma$-modification, deadzone-modification, Kalman filter-based modification.

In most of the existing parameter estimation methods, the convergence rate is not always well characterized. To this end, finite time (FT) parameter estimation scheme was proposed and then developed by many researchers [2,3]. Furthermore, parameter constraints arise in many fields such as computer vision, blending operations and chemical engineering processes. To this end, prescribed performance function (PPF)-based error construction method for parameter estimation was proposed [4].

From the existing literature, it is noted that the persistence of excitation (PE) condition is a limitation of the existing FT parameter estimation methods. We propose two robust adaptive parameter estimation methods for a class of linearly parameterized nonlinear systems with the attempt to relax the PE condition and guarantee the prescribed performance. And then it is further developed by sliding mode technique, which obtains the FT convergence. The main advantage of the proposed methods is that the PE condition is replaced with a less restrictive rank condition via concurrent learning technique. This condition is only a restriction on the recorded data. Particularly it applies only to data that has been specifically selected and recorded, rather than the current data and even the future behavior of the reference/command. Consequently, it is conducive to online monitoring. Furthermore, the design of the external dither signal and the corresponding remove mechanism design are both avoided. Therefore, the proposed parameter estimation is more convenient for practical implementation.

*Methodology.* Consider a linearly parameterized nonlinear system [2, 3]

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) + \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t), t)\boldsymbol{\theta}, \quad (1)$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the system state, $\boldsymbol{u}(t) \in \mathbb{R}^m$ is the control input, $\boldsymbol{f}(\cdot) \in \mathbb{R}^n$ and $\boldsymbol{g}(\cdot) \in \mathbb{R}^{n\times p}$ are both known continuous vector/matrix functions.

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$\boldsymbol{\theta} \in \mathbb{R}^p$ is the vector of unknown constant parameters to be estimated.

To implement the design procedure, a series of filtered variables are introduced firstly as follows:

$$\begin{cases} k\dot{\boldsymbol{x}}_f(t) + \boldsymbol{x}_f(t) = \boldsymbol{x}(t), & \boldsymbol{x}_f(0) = \boldsymbol{0}, \\ k\dot{\boldsymbol{f}}_f(t) + \boldsymbol{f}_f(t) = \boldsymbol{f}(t), & \boldsymbol{f}_f(0) = \boldsymbol{0}, \\ k\dot{\boldsymbol{g}}_f(t) + \boldsymbol{g}_f(t) = \boldsymbol{g}(t), & \boldsymbol{g}_f(0) = \boldsymbol{0}, \end{cases} \tag{2}$$

where $k > 0$ is the filter parameter to be designed. Then the following relationship can be obtained:

$$\dot{\boldsymbol{x}}_f(t) = \frac{\boldsymbol{x}(t) - \boldsymbol{x}_f(t)}{k} = \boldsymbol{f}_f(t) + \boldsymbol{g}_f(t)\boldsymbol{\theta}. \tag{3}$$

The auxiliary filtered and integrated regression matrix $\boldsymbol{G}_{\rm cl}(t) \in \mathbb{R}^{p\times p}$ and vector $\boldsymbol{F}_{\rm cl}(t) \in \mathbb{R}^p$ are defined as follows:

$$\begin{cases} \dot{\boldsymbol{G}}_{\rm cl}(t) = -l\boldsymbol{G}_{\rm cl}(t) + \boldsymbol{g}_f^{\rm T}(t)\boldsymbol{g}_f(t) + \boldsymbol{G}_s, \\ \dot{\boldsymbol{F}}_{\rm cl}(t) = -l\boldsymbol{F}_{\rm cl}(t) + \boldsymbol{g}_f^{\rm T}(t)\boldsymbol{F}_t + \boldsymbol{F}_s, \\ \boldsymbol{G}_s = \displaystyle\sum_{i=1}^{q} \boldsymbol{g}_f^{\rm T}(t_i)\boldsymbol{g}_f(t_i), \\ \boldsymbol{F}_t = \left[\dfrac{\boldsymbol{x}(r) - \boldsymbol{x}_f(r)}{k} - \boldsymbol{f}_f(r)\right], \\ \boldsymbol{F}_s = \displaystyle\sum_{i=1}^{q} \boldsymbol{g}_f^{\rm T}(t_i)\left[\dfrac{\boldsymbol{x}(t_i) - \boldsymbol{x}_f(t_i)}{k} - \boldsymbol{f}_f(t_i)\right], \\ \boldsymbol{G}_{\rm cl}(0) = \boldsymbol{0}, \quad \boldsymbol{F}_{\rm cl}(0) = \boldsymbol{0}, \end{cases} \tag{4}$$

where $l > 0$ is a design freedom. Eq. (4) implies

$$\begin{cases} \boldsymbol{G}_{\rm cl}(t) = \displaystyle\int_0^t {\rm e}^{-l(t-r)}\left[\boldsymbol{g}_f^{\rm T}(r)\boldsymbol{g}_f(r) + \boldsymbol{G}_s\right]{\rm d}r, \\ \boldsymbol{F}_{\rm cl}(t) = \displaystyle\int_0^t {\rm e}^{-l(t-r)}\left\{\boldsymbol{g}_f^{\rm T}(r)\boldsymbol{F}_t + \boldsymbol{F}_s\right\}{\rm d}r, \end{cases} \tag{5}$$

where $\boldsymbol{G}_{\rm cl}(t)$ and $\boldsymbol{F}_{\rm cl}(t)$ is the concurrent learning formulation of the regression matrix and vector. $\boldsymbol{g}_f(t_i)$, $\boldsymbol{x}(t_i)$, $\boldsymbol{x}_f(t_i)$ and $\boldsymbol{f}_f(t_i)$ are the recorded data points at time $t_i$, respectively.

**Condition 1.** There exists an array which has as many linearly independent elements as the dimension of $\boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t), t)$. That is, if $\boldsymbol{H} = [\boldsymbol{g}_1^{\rm T}, \boldsymbol{g}_2^{\rm T}, \ldots, \boldsymbol{g}_q^{\rm T}]$, $q$ is the maximum number of the recorded data points, then ${\rm rank}(\boldsymbol{H}) = p$.

**Lemma 1.** The matrix $\boldsymbol{G}_{\rm cl}$ defined in (5) is positive definite and invertible (i.e., $\lambda_{\min}(\boldsymbol{G}_{\rm cl}) = \sigma > 0$). And $\sigma > 0$ is guaranteed if Condition 1 is met by the data recording algorithm in [5].

The parameter estimation error information is constructed according to (3) and (5):

$$\boldsymbol{E}(t) = \boldsymbol{G}_{\rm cl}(t)\hat{\boldsymbol{\theta}} - \boldsymbol{F}_{\rm cl}(t) = -\boldsymbol{G}_{\rm cl}(t)\tilde{\boldsymbol{\theta}}, \tag{6}$$

where $\tilde{\boldsymbol{\theta}}(t) = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(t)$ is the estimation error.

With the attempt to guarantee the transient and steady performance of the parameter estimation error, a PPF in [6] is introduced as follows:

$$\mu(t) = (\mu_0 - \mu_\infty){\rm e}^{-\alpha t} + \mu_\infty, \tag{7}$$

where $\mu_0 > \mu_\infty > 0$, $\alpha > 0$ are design parameters, $\mu(t) : \mathbb{R}^+ \to \mathbb{R}^+$ is a positive monotonously decreasing function with $\lim_{t\to\infty}\mu(t) = \mu_\infty > 0$, $\mu_\infty$ is the allowable steady-state error.

To obtain the prescribed performance, the following relationship is to be guaranteed:

$$-L_i\mu(t) < \tilde{\theta}_i(t) < U_i\mu(t), \quad i = 1, \ldots, p, \tag{8}$$

where $L_i$ and $U_i$ are design parameters.

The main idea of the PPF-based parameter estimation is to transform the parameter estimation error into a certainly equivalence formulation such that the following strictly monotonically increasing function $T(s_i)$ of the transformed error $s_i \in \mathbb{R}$, $i = 1, \ldots, p$ satisfies

$$\begin{cases} -L_i < T(s_i) < U_i, & \forall s_i \in L_\infty, \\ \displaystyle\lim_{s_i\to-\infty} T(s_i) = -L_i, & \displaystyle\lim_{s_i\to+\infty} T(s_i) = U_i. \end{cases} \tag{9}$$

According to (9), constraint (8) equals to

$$\tilde{\theta}_i(t) = \mu(t)T(s_i), \quad i = 1, \ldots, p. \tag{10}$$

Because $T(s_i)$ is strictly monotonically increasing and $\mu(t) \geqslant \mu_\infty > 0$ holds, the inverse function of $T(s_i)$ is obtained:

$$s_i = T^{-1}\left[\frac{\tilde{\theta}_i(t)}{\mu(t)}\right]. \tag{11}$$

To show the basic idea of the proposed scheme, the function satisfying (9) is introduced:

$$T(s_i) = \frac{U_i{\rm e}^{s_i} - L_i{\rm e}^{-s_i}}{{\rm e}^{s_i} + {\rm e}^{-s_i}}, \quad i = 1, \ldots, p. \tag{12}$$

Then

$$s_i = T^{-1}\left[\frac{\tilde{\theta}_i(t)}{\mu(t)}\right] = \frac{1}{2}\ln\frac{\rho_i(t) + L_i}{U_i - \rho_i(t)}, \tag{13}$$

where $\rho_i(t) = \tilde{\theta}_i(t)/\mu(t)$, $\boldsymbol{\rho} = [\rho_1, \ldots, \rho_p]^{\rm T} \in \mathbb{R}^p$. Thus it is deduced that

$$\begin{aligned} \boldsymbol{S} &= T^{-1}\left[\frac{\tilde{\boldsymbol{\theta}}(t)}{\mu(t)}\right] \\ &= \left[\frac{1}{2}\ln\frac{\rho_1(t) + L_1}{U_1 - \rho_1(t)}, \ldots, \frac{1}{2}\ln\frac{\rho_p(t) + L_p}{U_p - \rho_p(t)}\right]^{\rm T}. \end{aligned} \tag{14}$$

Taking the time derivative of the transformed error (14) yields

$$\dot{\boldsymbol{S}} = \frac{\partial \boldsymbol{S}}{\partial \boldsymbol{\rho}}\dot{\boldsymbol{\rho}} = \boldsymbol{\Phi}\left(\dot{\tilde{\boldsymbol{\theta}}} - \tilde{\boldsymbol{\theta}}\dot{\mu}/\mu\right), \tag{15}$$

where $\boldsymbol{\Phi} = \mathrm{diag}(\varphi_i) \in \mathbb{R}^{p\times p}$ with

$$\varphi_i = \frac{1}{2\mu}\left[\frac{1}{\rho_i + L_i} - \frac{1}{\rho_i - U_i}\right], \quad i = 1, \ldots, p, \quad (16)$$

and then $0 < \varphi_i \leqslant \varphi_{\max} = \frac{L_i+U_i}{\mu_\infty L_i U_i}$.

Based on the deduce above, we design concurrent learning PPF parameter estimation as follows:

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}_1 \boldsymbol{\Phi}^{-1}\boldsymbol{S} - \boldsymbol{G}_{\rm cl}^{-1}\boldsymbol{E}\dot{\mu}/\mu, \quad (17)$$

where $\boldsymbol{\Gamma}_1 = \boldsymbol{\Gamma}_1^{\rm T} > 0$ is the learning rate matrix to be designed.

With the attempt to improve the FT convergence property of (17), it is modified as follows:

$$\dot{\hat{\boldsymbol{\theta}}} = \frac{\boldsymbol{\Gamma}_2 \boldsymbol{\Phi}^{-1}\boldsymbol{S}}{\|\boldsymbol{S}\|} - \boldsymbol{G}_{\rm cl}^{-1}\boldsymbol{E}\dot{\mu}/\mu, \quad (18)$$

where $\boldsymbol{\Gamma}_2 = \boldsymbol{\Gamma}_2^{\rm T} > 0$ is the learning rate matrix to be designed, and $\|\cdot\|$ denotes the Eulerian norm of the corresponding vector/matrix.

To give out a full description of the proposed methods, a detailed block diagram is given in Figure A1.

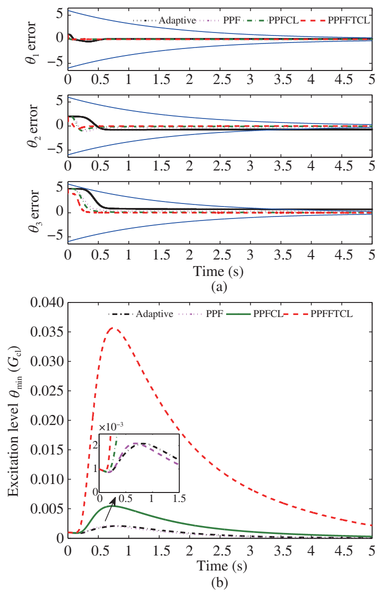


**Figure 1** (Color online) (a) The time history of parameter estimation error; (b) the time history of excitation level.

*Results and discussion.* To validate the effectiveness of the proposed parameter estimation methods, the model in [2] is used. Apply the singular value maximizing data recording algorithm [5] during concurrent learning, the criteria value is set as 0.001 and the maximum number of the recorded data is 8. And the corresponding design parameters of PPF are set to be $\mu_0 = 5$, $\mu_\infty = 0.0001$, $\alpha = 0.6$, $L_i = 1.2$, and $U_i = 1.2$. Then, the simulation results are achieved and shown in Figure 1. Furthermore, the estimation result is shown in Figure A2 for details.

Note that 'True' denotes the true value of the parameter vector to be estimated. 'Adaptive', 'PPF', 'PPFCL', and 'PPFFTCL' denote the simulation results for the parameter estimation laws in [3, 4], (17) and (18), respectively. The parameter estimation errors of PPF-based methods are constrained in the time-diminishing bounds while that of 'Adaptive' is not. Note that the proposed methods gain better convergence compared with the existing literature because of the excitation level improvements brought by concurrent learning. Furthermore, the FT convergence property makes PPFFTCL the fastest one.

*Conclusion.* This study presents two parameter estimation methods for a class of linearly parameterized nonlinear systems suffering from parameter uncertainty and bounded external disturbance. The main features of the proposed methods including: a relaxed PE condition, prescribed performance and finite time convergence.

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**Supporting information** Figures A1 and A2. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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