

Fully distributed consensus of second-order multi-agent systems using adaptive event-based control

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Dear editor,

In recent years, the distributed consensus of multi-agent systems (MASs) has attracted compelling attention because of its potential applications in many fields such as sensor networks, unmanned aerial vehicle (UAV) formation flying, and multiple signals tracking. The central task in the distributed consensus of MASs is to design distributed control protocols to achieve the consensus using only local information.

Autonomous agents are often equipped with a simple embedded digital micro-processor, that usually has limited on-board resources, limited computing power, and limited communication and actuation capabilities. Owing to its advantages in conserving communication resources and reducing the communication burden, event-based control has attracted much attention in the control community. Many studies have focused on the event-based consensus of various MASs following the work of [1], in recent years. Event-based consensus for single-integrator MASs and double-integrator MASs were studied in [2]. Some studies on event-based consensus for MASs with general linear dynamics were presented in [3]. However, the control

protocols in [2, 3] are not fully distributed as they require the global information of underlying communication graphs to design the triggering functions and control protocols. This requirement is very difficult to meet with regard to implementation for MASs with a large number of agents.

On the other hand, distributed adaptive consensus strategies were designed to overcome the limitation of requiring global information. In [4], an adaptive controller was proposed for the consensus problem of linear MASs with directed communication graphs. In [5], distributed adaptive strategies were presented to address the containment problem of heterogeneous linear MASs, in which the global information is not needed. In [6], the distributed output consensus tracking problem was considered, in which each agent only required the local information. However, it should be pointed out that the adaptive protocols in [4–6] rely on continuous communication between neighboring agents. This often results in unnecessary communication and energy overheads.

More recently, distributed adaptive event-based control has been developed for consensus of MASs [7–9]. In [7], some adaptive controllers were

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proposed for the consensus of first-order MASs. Two distributed event-triggering conditions were presented to determine the event time instants. In [8], an adaptive algorithm was presented, in which global information was not required for determining the parameters of the triggering function. In [9], by combining an adaptive control scheme and an event-based control law, a novel protocol was developed to achieve the leader-following consensus of MASs. To the best of our knowledge, few studies on the consensus of MASs with distributed adaptive event-based controllers have been conducted.

Motivated by the above discussion, a fully distributed adaptive event-based control scheme has been developed for second-order MASs, and presented in this study. The contributions of this work can be summarized as follows.

(1) A number of novel triggering functions and triggering conditions are proposed. The global information/parameters of overall systems are not required. Hence, the adaptive event-based control protocols are fully distributed.

(2) The proposed event-based protocol does not need an agent to communicate with neighbors continuously. In fact, agents only communicate with their neighbors at the instants when they are triggered.

(3) The closed-loop system does not exhibit Zeno-behavior.

Mathematical preliminaries. Consider the second-order MASs described by

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t), \end{cases} \quad i = 1, \dots, N, \quad (1)$$

where $x_i(t)$, $v_i(t)$, and $u_i(t) \in \mathbb{R}$ denote the position, velocity, and control input of agent i , respectively.

The communication among N agents is presented by a graph $G = (\tilde{V}, E, A)$, in which $\tilde{V} = \{1, \dots, N\}$ denotes the set of nodes, $E \subseteq \tilde{V} \times \tilde{V}$ is the set of edges, and $A = (a_{ij})_{N \times N}$ is defined as $a_{ii} = 0$, $a_{ij} > 0$, if $(i, j) \in E$ and $a_{ij} = 0$ otherwise. $N_i = \{j \in V | (i, j) \in E, i \neq j\}$ is the set of agents adjacent to the i -th agent. The Laplacian matrix is denoted as $L = D - A$, where $D = \text{diag}\{d_1, \dots, d_N\}$ and $d_i = \sum_{j=1}^N a_{ij}$. It is clear that $L1_N = 0$, where $1_N = (1, \dots, 1)^T$.

Definition 1. Second-order MASs (1) are said to reach consensus under a designed adaptive event-based controller $u_i(t)$, if there exist $\xi(t) \in \mathbb{R}, \zeta \in \mathbb{R}$ such that

$$\begin{cases} \lim_{t \rightarrow \infty} |x_i(t) - \xi(t)| = 0, \\ \lim_{t \rightarrow \infty} |v_i(t) - \zeta| = 0 \end{cases}$$

holds for $i \in \{1, \dots, N\}$ and any initial conditions.

To discuss the consensus of MASs (1), the following distributed adaptive event-based protocol is proposed for $t \in [t_k^i, t_{k+1}^i)$, $k = 1, 2, \dots$,

$$u_i(t) = -\alpha x_i(t) - c_i^2(t)q_{vi}(t_k^i), \quad (2)$$

$$\dot{c}_i(t) = -c_i(t)q_{vi}^2(t_k^i) - \frac{1}{2}\gamma c_i(t), \quad (3)$$

where $\alpha \geq 1$, $\gamma > 0$, $c_i(t)$ is the time-varying coupling weight for agent i , $q_{vi}(t) = \sum_{i=1}^N a_{ij}(v_i(t) - v_j(t))$, and the triggering time instants t_k^i for agent i is defined by

$$t_{k+1}^i = \inf\{t : t > t_k^i, f_i(t) \geq 0\} \quad (4)$$

with

$$f_i(t) = 2|e_{vi}(t)| - |q_{vi}(t_k^i)| - \beta e^{-\frac{1}{2}\gamma(t-t_k^i)} \quad (5)$$

being the triggering function for $\beta > 0$ and $e_{vi}(t) = q_{vi}(t_k^i) - q_{vi}(t)$. Therefore, $e_{vi}(t)$ is reset to 0 at t_k^i . We assume that $t_0^i = t_0$ for agent i and $0 < c_i(t_0) \leq \psi$, where t_0 is the initial time and $\psi \in \mathbb{R}^+$.

Definition 2. There is no Zeno-behavior for the closed-loop system in (1) if $\inf_k \{t_{k+1}^i - t_k^i\} > 0, \forall i$.

Main results. A number of sufficient conditions for the consensus for MASs (1) with the controllers (2)–(4) are presented while excluding the Zeno-behavior.

Theorem 1. Consider a second-order MAS (1) with a distributed adaptive event-based protocols (2) and (3). The triggering time instants for each agent are defined by (4). If the communication topology is connected, then for any $\alpha \geq 1$, $\beta > 0$ and $\gamma > 0$, the consensus for MASs (1) can be achieved.

Remark 1. From Theorem 1, it should be noted that no global information is used to determine the parameters β and γ in the triggering function. This means that the triggering time instants t_k^i are independent of the global information unlike those in [2, 3]. Moreover, it can be seen from the triggering function (5) that the determination of the triggering time instants t_k^i only relies on the information of the velocities.

The next result shows that the closed-loop system does not exhibit Zeno-behaviour.

Theorem 2. The Zeno-behavior of MASs (1) with the controllers (2)–(4) can be avoided using the same conditions as Theorem 1.

Remark 2. From the triggering function (5), in order to compute $|e_{vi}(t)|$, continuous communication between neighboring agents is still required. We provide a more conservative triggering function to overcome this issue.

For $t \in [t_k^i, t_{k+1}^i)$, the triggering function is defined as follows:

$$\widehat{f}_i(t) = \frac{2h_{kk'}^{ij}}{\alpha} \left(e^{\alpha(t-t_k^i)} - 1 \right) - |q_{vi}(t_k^i)| - \beta e^{-\frac{1}{2}\gamma(t-t_k^i)}, \quad (6)$$

where $h_{kk'}^{ij} = \alpha|q_{xi}(t_k^i)| + |q_{vi}(t_k^i)| + \psi^2 \sum_{j=1}^N a_{ij} (|q_{vi}(t_k^i)| + |q_{vj}(t_{k'}^j)|)$ and $q_{xi}(t) = \sum_{j=1}^N a_{ij}(x_i(t) - x_j(t))$.

Then, the triggering time instants t_k^i for agent i is determined by

$$t_{k+1}^i = \inf \left\{ t > t_k^i : \widehat{f}_i(t) > 0 \right\}. \quad (7)$$

Theorem 3. Consider the second-order MAS in (1) with a distributed adaptive event-based protocols (2) and (3). The triggering time instants for each agent are defined by (7). If the communication topology is connected, then for any $\alpha \geq 1$, $\beta > 0$ and $\gamma > 0$, the consensus for the system can be achieved. Furthermore, the Zeno-behavior of the closed-loop system does not occur.

Remark 3. Theorem 3 shows that agents do not need to communicate continuously with their neighbors. In other words, the proposed protocol in this study saves more energy than those in [4–6], all of which rely on continuous communication. However, the values of $x_i(t_k^i)$ and its neighbors' information $x_j(t_k^i)$, $q_{vj}(t_{k'}^j)$ are required to determine t_{k+1}^i . This shows that the information of its neighbors' neighbors are needed.

The proof of Theorems 1–3 are included in Appendixes A–C, respectively. The proposed results are illustrated by a numerical simulation, shown in Appendix D.

Conclusion. The consensus problem of second-order MASs via distributed adaptive event-based protocol was studied in this study. Using the proposed triggering function and triggering condition (which uses only local information of the underlying system), a number of sufficient conditions for reaching the consensus are presented. It is shown that not only can the consensus be achieved using only local information, but also neighbors do not have to communicate continuously. Furthermore, Zeno-behavior does not occur for the closed-loop system. The adaptive event-based consensus with general linear MASs will be studied in the future.

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Supporting information Appendixes A–D. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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