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New reachability trees for analyzing unbounded Petri nets with semilinear reachability sets

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Dear editor,

Analysis of reachability sets is of fundamental importance for a Petri net (PN). A reachability set can be represented by a reachability tree (RT), which is a powerful tool for intuitively checking the properties of PNs. Thus, properly construction of an RT is critical. Actually, constructing RTs to exactly characterize reachability sets for unbounded PNs is challenging because such reachability sets are infinite. Over the past 50 years, much effort has been devoted to the finite representation of RTs for PNs with infinite reachability sets. Karp et al. [1] proposed a finite RT (FRT) in which a special symbol, ω, is introduced to denote an infinite component of a marking. It has been proven that an FRT can determine properties such as boundedness and safeness. Unfortunately, the introduction of the symbol ω in an FRT causes information loss. To avoid this problem, Wang et al. [2] proposed a modified reachability tree (MRT) in which the expression $k\omega_n+q$ rather than ω is adopted to denote infinite components of a marking. Additionally, they proved that an MRT is a finite tree capable of determining properties such as reachability, deadlock-freedom, and liveness. Counterexamples in [3] indicate that the marking set represented by an MRT is not necessarily equivalent to a reachability set. For ω-independent unbounded nets, Wang et al. [4] constructed a new modified reachability tree (NMRT) to exactly represent the reachability set of a net and thus correctly determines deadlocks and liveness [5]. Enlightened by the previous work here we proposed a new reachability tree (NRT) for a more general class of unbounded PNs. This study's contributions include the following: (1) a modified definition of ω-numbers is proposed, by which independent and dependent ω-markings can be differentiated; (2) an NRT is proposed for a class of unbounded PNs that exactly characterizes the reachable marking set of a PN; and (3) the role of an NRT in determining deadlocks for unbounded nets is verified.

The basic concepts of PNs and related notions of ω-numbers as well as ω-markings [2–4] are reviewed in Appendix A. We introduce modified ω-numbers and ω-markings as well as their notions and results for NRT. The related proofs are provided in Appendix B. The sets of integers, nonnegative integers, and positive integers are denoted by $\mathbb{Z}$, $\mathbb{N}$, and $\mathbb{N}^+$, respectively.

Definition 1. A subset of integer S is called an ω-number if $\exists q \in \mathbb{Z}$, $k_1, k_2, \ldots, k_m \in \mathbb{N}$ and $k_1+k_2+\cdots+k_m \neq 0$ such that $S = \boldsymbol{\omega}(k_1^{(1)}, k_2^{(2)}, \ldots, k_m^{(m)}; q) \equiv k_1\omega^{(1)} + k_2\omega^{(2)} + \cdots + k_m\omega^{(m)} + q \equiv \{i^{(1)}k_1 + i^{(2)}k_2 + \cdots + i^{(m)}k_m + q | i^{(1)}, i^{(2)}, \ldots, i^{(m)} \in \mathbb{N}\}$, where $m \in \mathbb{N}^+$.

$\boldsymbol{\omega}(k_1^{(1)}, k_2^{(2)}, \ldots, k_m^{(m)}; q)$ or $k_1\omega^{(1)}+k_2\omega^{(2)}+\cdots+k_m\omega^{(m)}+q$ is called a canonical ω-number, where $\omega^{(j)}$ is called an ω element with superscript j, k_j

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the base related to $\omega^{(j)}$, $j \in \{1, 2, \ldots, m\}$ and q the starting value. Moreover, an ω-number has z dimension if it contains z non-zero bases.

Let $S = \boldsymbol{\omega}(k_1^{(1)}, k_2^{(2)}, \ldots, k_m^{(m)}; q)$, $S_1 = \boldsymbol{\omega}(k_{11}^{(1)}, k_{12}^{(2)}, \ldots, k_{1m}^{(m)}; q_1)$, and $S_2 = \boldsymbol{\omega}(k_{21}^{(1)}, k_{22}^{(2)}, \ldots, k_{2m}^{(m)}; q_2)$ be three ω-numbers.

Definition 2. S is called a simple ω-number if its dimension is one; otherwise, it is a compound ω-number.

Definition 3. S_1 and S_2 are ω-numbers with the same form if $k_{1i} = k_{2i}$, $\forall i \in \{1, 2, \ldots, m\}$.

Definition 4. Let $a \in \mathbb{Z}$. $S + a = \boldsymbol{\omega}(k_1^{(1)}, k_2^{(2)}, \ldots, k_m^{(m)}; q + a)$.

Definition 5. $S_1 + S_2 = \boldsymbol{\omega}((k_{11} + k_{12})^{(1)}, (k_{12} + k_{22})^{(2)}, \ldots, (k_{1m} + k_{2m})^{(m)}; q_1 + q_2)$.

Definition 6. Let S_1 and S_2 be two ω-numbers with the same form (i.e., $k_{1i} = k_{2i}$, $\forall i \in \{1, 2, \ldots, m\}$). $S_1 \geqslant S_2 (S_1 > S_2)$ if $q_1 \geqslant q_2 (q_1 > q_2)$.

Property 1. Let S_1 and S_2 be two ω-numbers with the same form. $S_1 \subseteq S_2$ iff $q_1 - q_2 = c_1 k_1 + c_2 k_2 + \cdots + c_m k_m$, $c_1, c_2, \ldots, c_m \in \mathbb{N}$.

Definition 7. We say S_1 is independent of S_2 if $k_{1i} \cdot k_{2i} = 0$, $\forall i \in \{1, 2, \ldots, m\}$.

Based on the modified definition of ω-numbers, the ω-vector (resp. ω-marking) exactly represents only one ordinary vector set (resp. ordinary marking set). Let $\mu = (S_1, S_2, \ldots, S_n)$ be a vector, where $\forall x \in \{1, 2, \ldots, n\}$, $S_x = \{i^{(1)} k_{x1} + i^{(2)} k_{x2} + \cdots + i^{(m)} k_{xm} + q_x | i^{(1)}, i^{(2)}, \ldots, i^{(m)} \in \mathbb{N}\}$, in which $q_x \in \mathbb{Z}$, $k_{xy} \in \mathbb{N}$, $\forall y \in \{1, 2, \ldots, m\}$. Note that S_x is an integer (i.e., $S_x = q_x$) if $k_{xy} = 0$, $\forall y \in \{1, 2, \ldots, m\}$; otherwise, it is an ω-number.

Remark 1. An ω-vector (resp. ω-marking) defined in this study is essentially a linear set [6].

Let $\mu = (S_1, S_2, \ldots, S_n)$, $\mu_1 = (S_{11}, S_{12}, \ldots, S_{1n})$, and $\mu_2 = (S_{21}, S_{22}, \ldots, S_{2n})$ be three ω-vectors.

Definition 8. μ is an independent ω-vector if $\forall i, j \in \{1, 2, \ldots, n\}$, and $i \neq j$, S_i and S_j are independent of each other; otherwise, μ is a dependent ω-vector.

Property 2. We have $\mu = \Delta$ iff μ is an independent ω-vector, where $\Delta = \{(a_1, a_2, \ldots, a_n) | a_g \in S_g$ (or $a_g = S_g$ if S_g is an integer), $\forall g \in \{1, 2, \ldots, n\}\}$.

Definition 9. μ_1 and μ_2 are ω-vectors with the same form if S_{1x} and S_{2x} are ω-numbers with the same form or are both integers, $\forall x \in \{1, 2, \ldots, n\}$.

Definition 10. Let μ_1 and μ_2 be two ω-vectors with the same form. $\mu_2 \geqslant \mu_1$ if $S_{2i} \geqslant S_{1i}$, $\forall i \in \{1, 2, \ldots, n\}$. Note that $\mu_2 > \mu_1$ is defined as $\mu_2 \geqslant \mu_1$, but $\mu_2 \neq \mu_1$.

Property 3. Let μ_1 and μ_2 be two independent ω-vectors. $\mu_1 \subseteq \mu_2$ iff $S_{1i} \subseteq S_{2i}$ or $S_{1i} = S_{2i}$ or $S_{1i} \in S_{2i}$, $\forall i \in \{1, 2, \ldots, n\}$.

Property 4. Let μ_1 and μ_2 be two ω-vectors with the same form. $\mu_1 \subseteq \mu_2$ iff $C_{1\times m} = (c_1, c_2, \ldots, c_m) \in \mathbb{N}^m$ exists such that $\forall x \in \{1, 2, \ldots, n\}$:

(1) $q_x - q'_x = C_{1\times m} \cdot (k_{x1}, k_{x2}, \ldots, k_{xm})^{\mathrm{T}}$ if $S_{1x} = \boldsymbol{\omega}(k_{x1}^{(1)}, k_{x2}^{(2)}, \ldots, k_{xm}^{(m)}; q_x)$ and $S_{2x} = \boldsymbol{\omega}(k_{x1}^{(1)}, k_{x2}^{(2)}, \ldots, k_{xm}^{(m)}; q'_x)$ are ω-numbers with the same form;

(2) $S_{1x} = S_{2x}$ if S_{1x} and S_{2x} are both integers.

Based on the modified definitions of ω-numbers and ω-vectors as well as their related notions, the construction algorithm (Algorithm 1) of an NRT for unbounded PNs is developed. In Algorithm 1, the next-state function $\delta(\mu, t)$ is repeatedly called to compute the marking that results from firing t once at the current marking μ. The detailed computation of $\delta(\mu, t)$ proceeds as in [4]. In Step 8 of Algorithm 1, determining which ω-marking is the bigger one and whether an inclusion relation exists between two ω-markings can be accomplished by means of Definition 10 and Properties 3 or 4, respectively. In addition, four types of nodes are used to construct an NRT (i.e., terminal, duplicate, ω-duplicate, and common nodes [4]). We note that an independent ω-marking can be easily distinguished from a dependent one in an NRT.

Definition 11. Let (N, μ_0) be an unbounded PN. (N, μ_0) is said to be an ω-independent net if its NRT does not contain any dependent ω-marking. Otherwise, it is said to be an ω-dependent net.

In what follows, the following assumption is made for unbounded PNs.

Assumption 1. Finite ω-numbers with different superscripts are introduced when Algorithm 1 is applied.

Under Assumption 1, the finiteness of NRTs is guaranteed, and NRTs can be used to analyze the reachability and determine whether unbounded PNs contain deadlocks, which are shown by Theorems 1–3.

Theorem 1 (Finiteness)**.** The NRT of an unbounded PN is finite.

Theorem 2 (Reachability)**.** The NRT of an unbounded PN consists of only but all reachable markings from its initial marking.

Before presenting Theorem 3, we explain that a full conditional node is a node in an NRT with all its direct successors linked by dotted arcs.

Theorem 3 (Deadlock checking)**.** An unbounded PN has deadlocks iff its NRT contains terminal or full conditional nodes.

Algorithm 1 Construction of an NRT

Input: An unbounded net (N, μ_0);
Output: An NRT of (N, μ_0);
Let x_0 be the root node of the tree and μ_0 the marking of node x_0;
$\Xi := \{x_0\}$ and label x_0 as a new node;
while there exists a new node x in Ξ **do**
  Label x as an old node and let μ_x be the marking of node x;
  for each $t \in T$ **do**
    if t is enabled or conditionally enabled at μ_x **then**
      Compute the next-state $\delta(\mu_x, t)$ and create a new node z in the NRT;
      if there exists a node y on the path from the root node to x such that $\delta(\mu_x, t) > \mu_y$ and $\delta(\mu_x, t) \nsubseteq \mu_y$ **then**
        if $\delta(\mu_x, t)$ is an ordinary marking **then**
          $j = 1$;
        else
          $j = 1 + g$, where g is the maximal dimension of all the ω-numbers in $\delta(\mu_x, t)$;
        end if
        for each $p \in P$ **do**
          if $\delta(\mu_x, t)(p) > \mu_y(p)$ **then**
            $\mu_z(p) := \delta(\mu_x, t)(p) + k\omega^{(j)}$, where $k = \delta(\mu_x, t)(p) - \mu_y(p)$;
          else
            $\mu_z(p) := \delta(\mu_x, t)(p)$;
          end if
        end for
      else
        $\mu_z := \delta(\mu_x, t)$;
      end if
    end if
    if t is enabled at μ_x **then**
      Add a solid arc t from x to z; /* t is enabled at μ_x */
    else
      Add a dotted arc t from x to z; /* t is conditionally enabled at μ_x */
    end if
    if z is a terminal node, ω-duplicate node, or duplicate node **then**
      Let node z be an old node;
    else
      Let node z be a new node;
    end if
    $\Xi := \Xi \cup \{z\}$;
  end for
end while

Remark 2. We note that a finite NRT can be constructed only for a class of PNs whose reachability sets are semilinear, where a semilinear set is a finite union of linear sets [6]. We can see that each node in an NRT corresponds to an ω-marking that is actually a linear set. Therefore, an NRT with finite nodes can only characterize a reachability set that is a semilinear set. It is worth noting that whether a given PN has a semilinear reachability set is decidable, which has been proven independently by Hauschildt [7] and Lambert [8]. In addition, a wide variety of subclasses of PNs enjoy semilinear reachability sets. Indeed, Ref. [9] proved that persistent PNs, weakly persistent PNs, almost persistent PNs, sinkless PNs, almost sinkless PNs, and cyclic PNs all have semilinear reachability sets. Moreover, we note that the NRT overcomes the drawback of the MRT (i.e., the set of markings represented by the nodes of the MRT covers the set of reachable markings, but it is not necessarily equal to that set) and breaks the limitations of the NMRT that is applicable to ω-independent unbounded nets only.

Conclusion. This study proposed an NRT that provides more useful structural information than does FRT, MRT, and NMRT. Moreover, for unbounded PNs with semilinear reachability sets, a finite NRT was successfully constructed to characterize precisely their infinite reachability sets. Based on the finite NRT, we can correctly determine whether an unbounded net contains a deadlock.

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Supporting information Appendixes A and B. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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