A fault tolerant multi-vehicle formation control framework with applications in multi-quadrotor systems

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Research background

- Coordination of multi-vehicle systems (MVSs) has broad applications
- MVSs are severely affected by faults due to the complex flight conditions
- Potential faults could cause performance degradation, and even lead to a chain of failing vehicles, which may cause major catastrophes.
- Effective fault-tolerant control (FTC) capacities are highly required for MVSs

Research background

- Most available literature on FTC are based on centralized architectures. There exist only few contributions for FTC for MVSs.
- In most existing results, MVSs have been often described by ideal models without external disturbance and noise.
- One indispensable assumption is that the adjacent vehicles' full states must be available, which is unrealistic.
- Real experiments on fault tolerant formation of MVSs have been rarely reported.

Innovations

- 1) The FTC problem is investigated for MVSs subject to actuator faults, external disturbance and channel noise.
- 2) A novel observer and fault estimator are developed to design the cooperative FTC scheme.
- 3) The error dynamics and Lyapunov function are designed and analyzed in the framework of Ito stochastic differential equations to guarantee that the formation error system is mean-square asymptotically stable.

Problem formulation

- Consider an MVS of N vehicles, the dynamics of the *i*-th vehicle is

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Ff_i(t) + Ew_i(t), \\ y_i(t) = Cx_i(t) + v_i(t), \end{cases}$$

where $x_i(t)$, $u_i(t)$, $y_i(t)$ and $f_i(t)$ are the state, control input, the measurement and fault. $w_i(t)$ and $v_i(t)$ are the disturbance and measurement noise, respectively.

The desired formation dynamic of the i-th vehicle:

$$\dot{x}_i^r(t) = Ax_i^r(t)$$

Problem formulation

In the current work, the actuator fault is considered. Without loss of generality, it is assumed that $F = [B_{q_1}, B_{q_2}, ..., B_{q_q}]$, where B_{q_j} $(j = 1, 2, .., q)$ is the q_j-th column of matrix B. Therefore, there exists a matrix $\bar{F}$ satisfying that $B\bar{F} = F$. In fact, $\bar{F} \triangleq [\bar{f}]_{i,j}$ is the matrix whose elements are zeros except that $\bar{f}_{q_i,q_i} = 1$ $(i = 1, 2, .., q)$. Therefore, the control signal in affected by actuator faults can be intuitively written as $u_i^f(t) \triangleq u_i(t) + \bar{F} f_i(t)$.

The objective of this paper is to design a proper FTC protocol for the MVS, so that all the vehicles track their desired formation trajectories in mean square and meanwhile maintain a desirable disturbance rejection performance.

Main results

For estimating the state $x_i(t)$, the following observer is designed,

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + \eta(\hat{x}_i(t), t) + F\hat{f}_i(t) \\ \qquad\quad +\bar{L}\big[y_i(t) - \hat{y}_i(t)\big], \\ \hat{y}_i(t) = C\hat{x}_i(t), \end{cases}$$

where $\hat{x}_i(t)$, $\hat{f}_i(t)$ and $\hat{y}_i(t)$ denote the estimation of $x_i(t)$, $f_i(t)$ and $y_i(t)$, respectively; $\bar{L}$ is the observer gain matrix to be designed.

Based on the observer developed above, an adaptive fault estimator is given as follows:

$$\dot{\hat{f}}_i(t) = \Gamma R^T e_{y,i}(t) - \sigma\Gamma\hat{f}_i(t),$$

where it is denoted that $e_{y,i}(t) \triangleq y_i(t) - \hat{y}_i(t)$ is the output estimation error and $e_{f,i}(t) \triangleq f_i(t) - \hat{f}_i(t)$ is the fault estimation error. The weighting matrix $\Gamma = \Gamma^T > 0$ and R and the parameter $\sigma > 0$ are to be determined.

Main results

- The cooperative formation control protocol is designed as

$$u_i(t) = cK \sum_{j \in N_i} a_{ij}\left(1 + \sigma_{ij}\xi_{ij}(t)\right)\left(\hat{x}_j(t) - \hat{x}_i(t) - \Delta_{ij}(t)\right)$$
$$+ cg_i K\left(x_i^r(t) - \hat{x}_i(t)\right) - \bar{F}\hat{f}_i(t),$$

where c and K are the coupling strength and the feedback gain matrix to be determined. $\Delta_{ij}(t) \triangleq x_j^r(t) - x_i^r(t)$ is the desired relative states of vehicle i with respect to its adjacent vehicle j. $\sigma_{ij}\xi_{ij}(t) \in \mathbb{R}$ is the channel noise, $\sigma_{ij} \geq 0$ represents the noise intensity. $\xi_{ij}(t)$ is a random variable for representing the stochastic behavior of channel noise. $\bar{\sigma} \geq 0$ is the maximal noise intensity or estimation of the upper bound of noise intensities. $g_i \in \{0, 1\}$, where $g_i = 1$ means vehicle i gets access to its desired trajectory state $x_i^r(t)$, otherwise $g_i = 0$.

Main results

In the following, $e_i(t) \triangleq x_i(t) - x_i^r(t)$, $e_{x,i}(t) \triangleq x_i(t) - \hat{x}_i(t)$,
Moreover, let $H = L + G$, where $G \triangleq diag\{g_1, g_2, \cdots, g_N\}$
one knows that $0 < \lambda_{min}(\tilde{H})$, where $\tilde{H} = \frac{1}{2}(H + H^T)$.

Definition 1. *For the MVS the fault tolerant formation is reached when the following conditions hold:*

(1) in the absence of disturbance $\lim_{t\to\infty} \sum_{i=1}^{N} \mathbb{E}\|x_i(t) - x_i^r(t)\|^2 = 0$

(2) with the effect of disturbance and fault, under the zero initial condition, the performance variable $\bar{e}_i(t) \triangleq [e_i^T(t), e_{x,i}^T(t), e_{f,i}^T(t)]^T$ *satisfies that* $J = \int_0^\infty \mathbb{E}\|\bar{e}(t)\|^2 - \gamma^2\beta^2 dt \leq 0$, *where* $\beta > 0$ *is the parameter depending on* $\beta_w, \beta_f, \beta_{\dot{f}}$, *which are the upper bounds of the amplitudes of* $w(t)$, $f(t)$ *and* $\dot{f}(t)$, *respectively.* γ *is a positive constants, which can be designed sufficiently small.* $e(t)$, $e_x(t)$ *and* $e_f(t)$ *are, respectively, the compact forms of* $e_i(t)$, $e_{x,i}(t)$ *and* $e_{f,i}(t)$ $(i = 1, 2, \cdots, N)$.

Main results

- **Assumption 1:** Disturbance/noise are independent white Gaussian noises with bounded amplitudes.
- **Assumption 2:** The actuator fault and its derivative are bounded. The multi-vehicle system is strongly connected and balanced.
- Based on the above assumptions and state observers and fault estimators, the main conclusion can be obtained as follows:

Main results

Theorem 1:. The desired formation configuration can be reached for the multi-vehicle system by using the following fault-tolerant control scheme

$$u_i(t) = cK\sum_{j\in N_i} a_{ij}\left(1+\sigma_{ij}\xi_{ij}(t)\right)\left(\hat{x}_j(t)-\hat{x}_i(t)-\Delta_{ij}(t)\right)$$
$$+\, cg_iK\left(x_i^r(t)-\hat{x}_i(t)\right)-\bar{F}\hat{f}_i(t),$$

where the gain matrices K, $\bar{L}$, R, Γ and parameters c and σ satisfy conditions:

$$K = B^TP^{-1},\ R^TC = F^TP^{-1};\quad c \leqslant \frac{\lambda_{min}(\tilde{H})\lambda_{min}(P)}{4\sigma^2\lambda_{max}(B^TB)\lambda_{max}(\tilde{L})};$$

Main results

$$AP + PA^T + 2P^2 + \frac{1}{\gamma_1^2}EE^T + \frac{1}{\gamma_2^2}FF^T - \frac{1}{2}\lambda_{min}(\tilde{H})(I \otimes BB^T) < 0;$$

$$(A - \bar{L}C)P + P(A - \bar{L}C)^T + 2P^2 + \frac{1}{\gamma_3^2}EE^T + c\lambda_{max}(\tilde{H})BB^T < 0;$$

where P is a symmetric positive definite matrix, and γ_1, γ_2, γ_3, γ_4 and γ_5, are positive constants. I is identity matrices with proper dimensions. $\Delta_{ij}(t) \triangleq x_j^r(t) - x_i^r(t)$, $\sigma_{ij}\xi_{ij}(t) \in \mathbb{R}$, and $\sigma_{ij} \geqslant 0$ is the noise intensity. $\xi_{ij}(t)$ is a random variable to show stochastic behavior of channel noises.

Main results

Proof:. Designing the following Lyapunov function:

$$V(t) = e^T(t)(I_N \otimes P^{-1})e(t) + e_x^T(t)(I_N \otimes P^{-1})e_x(t) + e_f^T(t)(I_N \otimes \Gamma^{-1})e_f(t).$$

By using the *Itô* formula as

$$dV(x,t) = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial x^T}F dt + \frac{1}{2}tr(G^T\frac{\partial^2 V}{\partial x \partial x^T}G)dt + \frac{\partial V}{\partial x^T}G dw(t),$$

Then taking the time derivative of $V(t)$ along

$$de_x(t) = \{G_1(e_x(t),t) + [I_N \otimes (A - \bar{L}C)]e_x(t) + (I_N \otimes E)w(t) - (I_N \otimes \bar{L})v(t) + (I_N \otimes F)e_f(t)\}dt,$$

$$de(t) = \{G_2(e(t),t) + [I_N \otimes A - c(H \otimes BK)]e(t) + c(H \otimes BK)e_x(t) + (I_N \otimes E)w(t) + (I_N \otimes F)e_f(t)\}dt + c(I_N \otimes BK)(M - M_x)d\bar{w}(t)$$

Main results

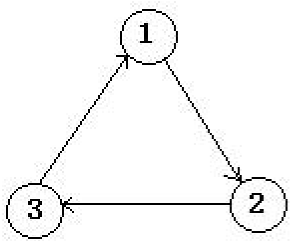
$$de_f(t)=\{\dot{f}(t)-(I_N\otimes\Gamma R^T C)e_x(t)+(I_N\otimes\sigma\Gamma)f(t)-(I_N\otimes\sigma\Gamma)e_f(t)$$
$$-(I_N\otimes\Gamma R^T)v(t)\}dt.$$

and set the performance function

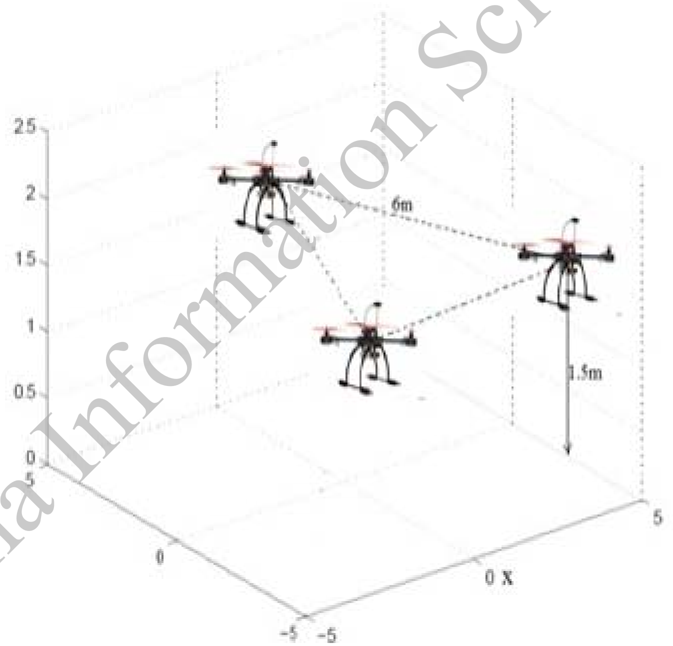
$$J=\int_0^\infty\Big\{\sum_{i=1}^N\mathbb{E}\big[\hat{e}_i^T(t)\big(AP+PA^T+L_\eta^2 I_n+\frac{1}{\gamma_1^2}EE^T+2P^2+\frac{1}{\gamma_2^2}FF^T$$
$$-\frac{1}{2}\lambda_{min}(\tilde{H})(I_N\otimes BB^T)\big)\hat{e}_i(t)\big]+\sum_{i=1}^N\mathbb{E}\big[\hat{e}_{x,i}^T(t)\big((A-\bar{L}C)P$$
$$+P(A-\bar{L}C)^T+2P^2+L_\eta^2 I_n+\frac{1}{\gamma_3^2}EE^T+c\lambda_{max}(\tilde{H})BB^T\big)\hat{e}_{x,i}(t)\big]$$
$$+\sum_{i=1}^N\mathbb{E}\big[e_{f,i}^T(t)\big(\frac{1}{\gamma_4^2}\Gamma^{-1}\Gamma^{-T}+\frac{1}{\gamma_5^2}\sigma^2 I_q-2\sigma I_q+(\gamma_2^2+1)I_q\big)e_{f,i}(t)\big]\Big\}dt$$
$$-\lim_{t\to\infty}m(t)+m(0).$$

we can get the conclusions in the theorem.

Experiment result

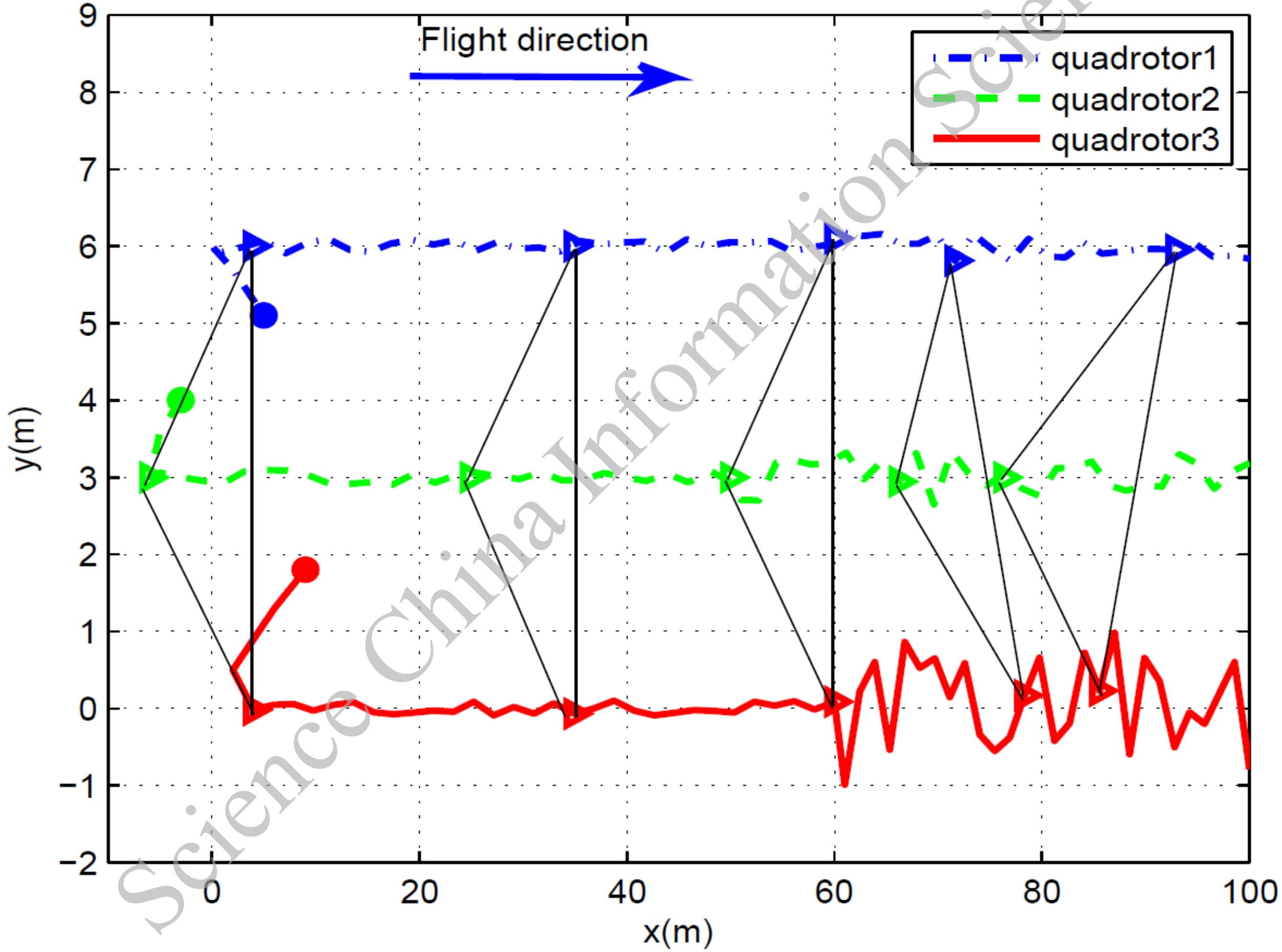


Topology



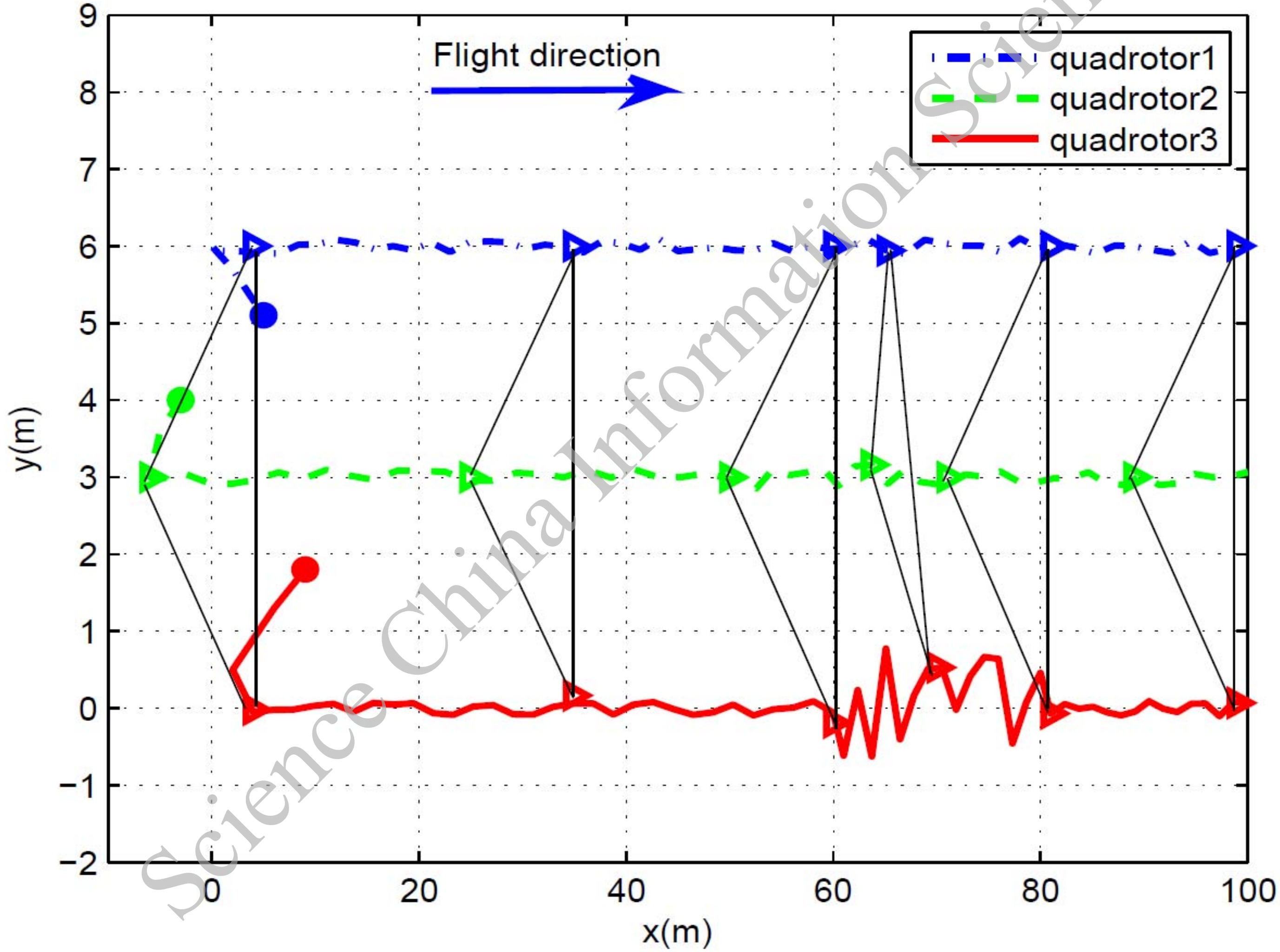
Desired formation conguration

Experiment results



Formation trajectories without FTC

Experiment results



Formation trajectories with FTC

Experiment results

One can observe that the desired formation configuration can be reached under the normal formation control scheme before actuator faults occur. However, the formation configuration has been broken after actuator faults arise in quadrotor 3. The developed cooperative fault-tolerant control scheme can drive all the quadrotors to reach the desired formation configuration in the presence of actuator faults occurring.

Conclusion

In this paper, we investigated the fault-tolerant formation control problem of multi-vehicle systems with actuator faults, disturbance, measurement and channel noises. A novel distributed observer and adaptive fault estimator were proposed to design the fault-tolerant control scheme for driving all vehicles to the desired formation configuration with the prescribed attenuation level.

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