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A new method for extending rate equation based VCSEL model with multimode spectral characteristic

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Abstract An accurate VCSEL model is important to evaluate the performance of VCSEL-based high speed short range optical interconnect system. Current VCSEL models show no evidence of the multimode spectral characteristic, which leads to an inaccurate performance estimation of VCSEL-based high speed optical interconnect system especially with large chromatic dispersion. To overcome this problem, we propose a new method to introduce multimode spectral characteristic into current VCSEL models, which extends the spatiotemporal multimode rate equations by calculating the wavelength shift of the VCSEL transverse modes. A VCSEL-multimode fiber (MMF) based short range optical interconnect simulation system is then built using our modified VCSEL model by VPItransmissionMaker. The symbol broadening and the BER performance of 15 Gb/s NRZ signal are tested in both left-tilted differential mode delay (DMD) profile MMF (L-MMF) and right-tilted DMD profile MMF (R-MMF). Simulation results validate our proposed method and indicate that the spectral characteristic is a key component for VCSEL models.

Keywords short-range optical interconnect, vertical cavity surface emitting lasers, VCSEL, multimode rate equations, multimode spectral characteristic, multimode fiber

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1 Introduction

Recently, with the popularity of Internet applications, especially high-definition video applications, there is explosive growth in people's demand for network bandwidth [1]. Meanwhile, due to the increasing demand for high throughput, low latency and energy efficiency, the optical links based on vertical cavity surface emitting lasers (VCSELs) and multimode fiber (MMF) have been widely deployed in data center and the link rate is evolving from 10 to 100 Gb/s because of the exponential increase of the bandwidth requirements [2]. To design system parameters and evaluate system performance, telecom equipment vendors are eager for an effective short range optical interconnect link model, in which the accurate VCSEL model is an essential component.

There are two categories of VCSEL models. One is built on the Maxwell's equations [3], and the other is based on the rate equations [4–11]. The former can accurately analyze the device characteristics of VCSEL. However, it is time-consuming, which is not suitable for system-level simulation. On the other hand, the latter based on rate equations can also describe the time-domain dynamic characteristics of

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VCSEL accurately and consume less computation time. Based on rate equations, Jungo et al. [4] proposed a system-oriented VCSEL model, named VISTAS. In VISTAS, the effects of spatial hole burning (SHB), gain (mode) competition and carrier diffusion losses are modelled and in-depth studied. Perchoux et al. [5] proposed a model derived from the multimode rate equations for oxide-confined VCSEL with the noise contribution adapted to optical links applications. Lenstra et al. [6] proposed a model to describe the deterministic multimode dynamics of a semiconductor laser due to spatial hole burning based on a set of rate equations. In order to analysis small signal response of ultra high speed VCSEL, Hamad et al. [7] expanded the established single-mode laser rate equations towards a multi-mode high speed VCSEL rate-equation model by splitting up the carrier reservoirs. Another VCSEL models have been developed recently, which comprehensively captures the large- and small-signal dynamic response combined with the dc thermal effects on both the electrical and optical behavior [8–10]. However, all these models do not take the spectral characteristic of VCSEL into consideration, thus these VCSEL models do not suitable for system simulation. The VCSEL model included by VPItransmissionMaker is also a single mode VCSEL model, which cannot describe the multimode spectral characteristic of multimode VCSEL and cannot be used to simulate multimode VCSEL based short range optical link. Gholami also proposed a physical model for VCSEL which can be used in 10 Gigabit Ethernet optical communications systems. The static as well as dynamic response, noise, thermal effects and signal coupling to multimode fibers have been investigated in Gholami's model [11]. But the effect of VCSEL spectral characteristic is still ignored. Therefore, Gholami's model is also not accurate enough to describe the VCSEL-based optical link whose speed is more than 10 Gb/s.

Chromatic dispersion affects the optical pulse propagation and brings in inter-symbol interference (ISI) because of the source spectrum broadening. Under linear group delay approximation, the pulse broadening of optical pulse can be expressed as

$$\Delta t = D \cdot \Delta\lambda \cdot z, \tag{1}$$

where the combined effect of light source root mean square (RMS) spectral width $\Delta\lambda$, dispersion coefficient of the source wavelength D and the propagation distance z induce optical pulse broadening Δt. The RMS of a usual oxide-confined VCSEL is nearly 1 nm. The dispersion coefficient at 850 nm is around -85 ps/nm·km. For the short range optical links based on VCSEL and MMF, the communication distance is usually less than 300 m. As a result, the pulse broadening caused by chromatic dispersion may reach 25.5 ps, which cannot be neglected when the system speed is higher than 10 Gb/s (pulse time $<$ 100 ps). For a high-speed ($>$ 10 Gb/s) VCSEL based optical link simulation, it is difficult to evaluate the system performance accurately when the spectral characteristic of VCSEL is not considered correctly.

In this paper, we propose a new method to extend the current multimode rate equations by calculating the wavelength shift of the VCSEL transverse modes. The multimode spectral characteristic of VCSEL can be accurately described. Simulation results validate our proposed modified model and indicate that the spectral characteristic is a key component for high-speed VCSEL links.

2 VCSEL model based on rate equations

The multimode spatiotemporal rate equations [4–11] for modelling VCSEL are listed as

$$\begin{cases} \dfrac{\partial N(\boldsymbol{r},t)}{\partial t} = \dfrac{\eta j(\boldsymbol{r},t)}{eV} - \dfrac{N(\boldsymbol{r},t)}{\tau_{\mathrm{N}}} + D_{\mathrm{N}}\nabla^2 N(\boldsymbol{r},t) - \upsilon_{\mathrm{g}}\displaystyle\sum_{lp} G_{lp}(\boldsymbol{r},t)S_{lp}(t) + F_{\mathrm{N}}(t), \\ \dfrac{\mathrm{d}S_{lp}(t)}{\mathrm{d}t} = \Gamma_{lp}\beta_{lp}\dfrac{\iint N(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r}}{\pi R^2 \tau_{\mathrm{N}}} + \dfrac{\Gamma_{lp}\upsilon_{\mathrm{g}}S_{lp}(t)}{\pi R^2}\displaystyle\iint G_{lp}(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r} - \dfrac{S_{lp}(t)}{\tau_{\mathrm{S}}^{lp}} + F_{\mathrm{S}}(t), \end{cases} \tag{2}$$

where $N(\boldsymbol{r},t)$, $S_{lp}(t)$ and $j(\boldsymbol{r},t)$ are the carrier, photon and injection current density in the active region of VCSEL, respectively. The subscript lp denotes the LP transverse mode. $\boldsymbol{r}$, t are the polar coordinates and time. $F_{\mathrm{N}}(t)$ and $F_{\mathrm{S}}(t)$ are the noise caused by spontaneous emission. $G_{lp}(\boldsymbol{r},t)$ is the model gain.

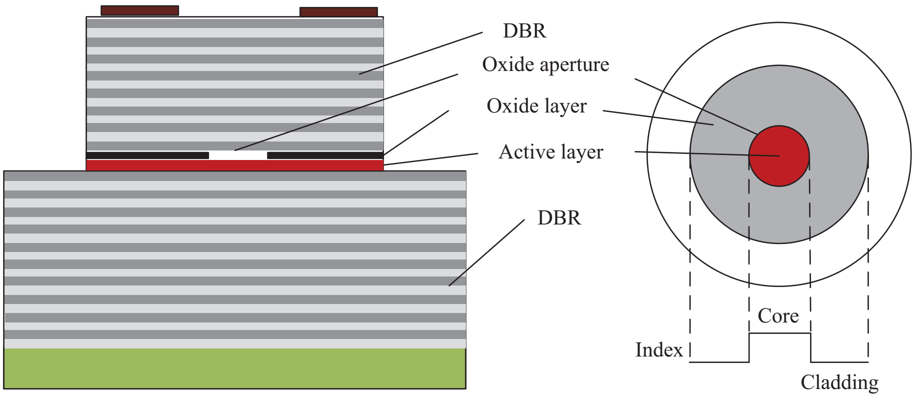

Figure 1 (Color online) VCSEL structure and equivalent step-index optical waveguide.

Based on the equation set (2), we obtain a time domain model, which takes mode competition, carrier diffusion losses and relative intensity noise into consideration.

Laser diode oscillator often has a finite linewidth due to the phase fluctuation of the electric field caused by the spontaneous emission. The phase fluctuation can be characterized as [4]

$$\frac{\mathrm{d}\phi_{lp}(t)}{\mathrm{d}t} = \frac{\alpha}{2}\Gamma_{lp}\upsilon_{\mathrm{g}}a_{\mathrm{N}}[N(t) - N_{\mathrm{th}}], \tag{3}$$

where α is a linewidth enhancement factor, $\phi_{lp}(t)$ is the phase of the output signal of VCSEL, and $a_{\mathrm{N}} = \partial G/\partial N$ is defined as differential gain.

3 Extended VCSEL model with multimode spectral characteristic

In the real short range optical interconnect system, VCSEL is usually a multi-transverse mode laser with RMS spectral width around 1 nm. The wavelength of each transverse mode is different [12, 13]. To evaluate the multimode spectral characteristic of VCSEL, we model the oxide-confined VCSEL as a circularly symmetric step-index optical waveguide to calculate the wavelength of each VCSEL transverse modes. The corresponding principle diagram is shown in Figure 1.

Under the boundary condition for a circularly symmetric step-index optical waveguide with a core radial, we have the eigenvalue equation [14] of wave equation as follows:

$$u\frac{J_{p-1}(u)}{J_p(u)} = -\omega\frac{K_{p-1}(\omega)}{K_p(\omega)}, \tag{4}$$

where u, ω represent the normalized frequencies in core and in the cladding respectively, and J represents first kind of Bessel function and K represents the Hankel function of the first kind. For a given mode, the normalized frequencies u and ω are represented as

$$\begin{aligned} u_{lp} &= r\sqrt{n_{\mathrm{c}}^2k^2 - \beta_{lp}^2} = k\cdot r\sqrt{n_{\mathrm{c}}^2 - n_{\mathrm{eff},lp}^2}, \\ \omega_{lp} &= r\sqrt{\beta_{lp}^2 - n_{\mathrm{cl}}^2k^2} = k\cdot r\sqrt{n_{\mathrm{eff},lp}^2 - n_{\mathrm{cl}}^2}, \end{aligned} \tag{5}$$

where l represents the azimuth mode index and p is the radius mode index. n_{c}, n_{cl} and $n_{\mathrm{eff},lp}$ are the core index, the cladding index and the effective mode index of mode LP. k is wave vector of central wavelength in vacuum. β is the propagation constant in VCSEL cavity.

The wavelength of each transverse mode can be predicted by a simple effective index model [15]

$$\frac{\Delta n_{\mathrm{eff},lp}}{n_{\mathrm{c}}} = \frac{\Delta\lambda}{\lambda_{\mathrm{c}}}, \tag{6}$$

where λ_{c} is the central wavelength. $\Delta n_{\mathrm{eff},lp}$ is the difference between the core index n_{c} and the effective mode index n_{cl}. $\Delta\lambda$ is the wavelength shift with respect to λ_{c}.

Instead of solving $n_{\mathrm{eff},lp}$, we use the phase parameter of step index optical waveguide B to calculate $\Delta n_{\mathrm{eff},lp}$. The phase parameter B of each LP mode is described as

$$B_{lp} = \frac{\omega_{lp}^2}{u_{lp}^2 + \omega_{lp}^2} = \frac{n_{\mathrm{eff},lp}^2 - n_{\mathrm{cl}}^2}{n_{\mathrm{c}}^2 - n_{\mathrm{cl}}^2}. \tag{7}$$

In weakly guided step-index waveguide, there are $n_{\mathrm{c}} + n_{\mathrm{cl}} \approx 2n_{\mathrm{c}}$ and $\Delta n_{\mathrm{eff},lp}^2 \approx 0$. Therefore, $\Delta n_{\mathrm{eff},lp}$ can be represented as

$$\Delta n_{\mathrm{eff},lp} \approx \frac{n_{\mathrm{c}}^2 - n_{\mathrm{cl}}^2}{2n_{\mathrm{c}}}(1 - B_{lp}) + \Delta n_{\mathrm{eff},lp}^2 \approx (n_{\mathrm{c}} - n_{\mathrm{cl}})(1 - B_{lp}). \tag{8}$$

Substituting (8) into (6), we obtain the wavelength shift relative to λ_{c} for each mode $\Delta\lambda_{lp}$ via

$$\Delta\lambda_{lp} = \lambda_{\mathrm{c}} \frac{n_{\mathrm{c}} - n_{\mathrm{cl}}}{n_{\mathrm{c}}}(1 - B_{lp}). \tag{9}$$

Hence, the wavelength of each transverse mode is derived as

$$\lambda_{lp} = \lambda_{\mathrm{c}} - \lambda_{\mathrm{c}} \frac{n_{\mathrm{c}} - n_{\mathrm{cl}}}{n_{\mathrm{c}}}(1 - B_{lp}). \tag{10}$$

According to (2), (3) and (10), the calculation flowchart of the VCSEL output optical signal $E(t)$ is shown in Figure 2. The process of calculating the wavelength of VCSEL transverse modes and integrating the wavelength of each VCSEL mode into traditional rate equations are our main contributions. First, we calculate the mode power P via multimode rate equations. Second, the phase of each transverse mode ϕ_{lp} is calculated by (3). Third, the center wavelength of each transverse mode λ_{lp} is obtained via (10). Finally, the output optical signal $E(t)$ is calculated by

$$E(t) = \sum_{lp} \sqrt{P_{lp}} \exp(-i(w_{lp}t + \phi_{lp})), \tag{11}$$

where the mode frequency $w_{lp} = \lambda_{lp}/c$, and c is the light speed.

4 Results and discussion

Although the current VCSEL models [4–11] are different in form with each other, they are the same in essence. In these VCSEL models, VISTAS [4,16] is a popular VCSEL model based on rate equations which includes the effects of mode competition, carrier diffusion losses and relative intensity noise. Compared with other VCSEL models, VISTAS also contains electrical parasitic effect, thermal effect and a model to calculate the VCSEL-to-fiber coupling, which makes it an excellent time domain 2D VCSEL model. Therefore, we choose VISTAS to integrate with our proposed method. The typical parameters for the VCSEL simulation are listed in Table 1 [16].

The optical spectrum of VISTAS without and with our proposed method are shown in Figure 3. The spectrum shown in Figure 3(a) contains only single wavelength, which is not consistent with the optical spectrum of actual multimode VCSEL used in data centers. The optical spectrum and RMS optical spectrum of the modified VISTAS with our proposed method shows multimode spectral characteristic model in Figures 3(b) and (c). The −20 dB spectral width and the RMS spectral width are 4 nm and 0.9228 nm, respectively.

In order to verify our proposed model on multimode fiber transmission system, a short range optical interconnect system is built using the VPItransmissionMaker V9.5. The system block diagram is shown in Figure 4. The name of VPI modules that used in simulation are also indicated in Figure 4. For comparison, the VISTAS model under two scenarios, with and without our proposed method, are utilized in the simulations. A 15 Gbps OOK signal drives the VCSEL model with 6 mA bias. The modulated optical signal is directly coupled into 300 m OM4 MMF without offset. At the end of MMF, the transmitted signal is received by a PIN photodetector and a TIA with a bandwidth of 22 GHz.

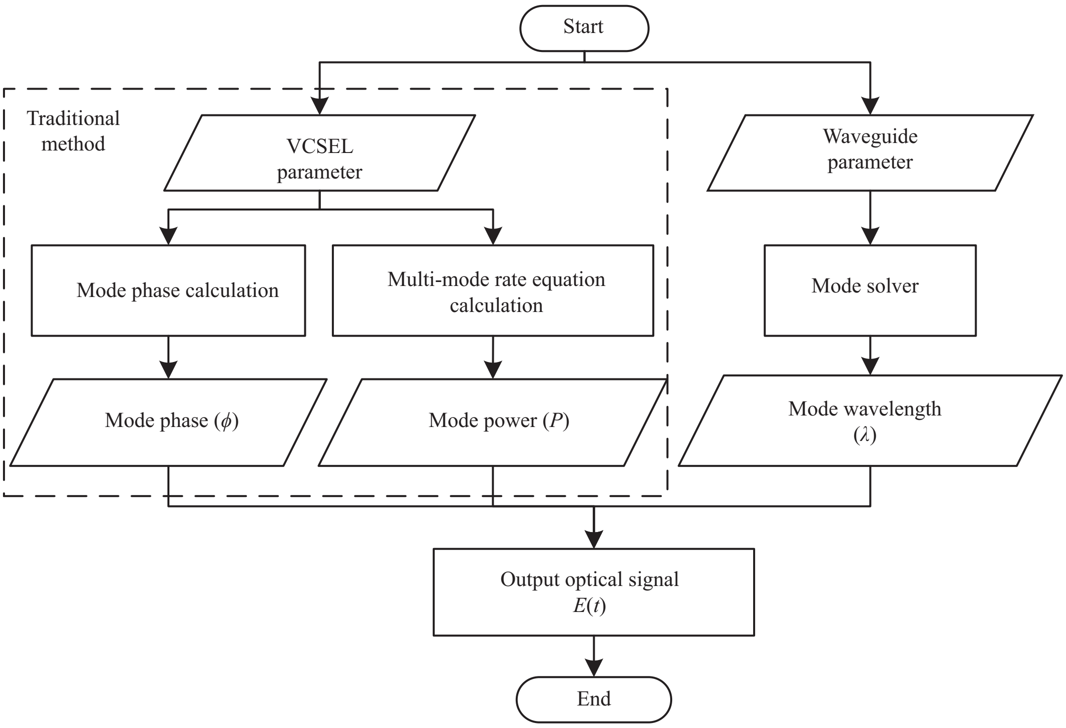

Figure 2 Calculation flowchart of the output optical signal $E(t)$ based on our proposed VCSEL model.

Table 1 Model typical parameters

Symbol	Description	Value
η	Injection efficiency	0.8
e	Elementary charge	1.6E−19 C
V	Active region volume	1.2E−12 m^3
τ_N	Carrier lifetime	5E−10 s
D_N	Diffusion coefficient	0.012 m^2/s
β	Spontaneous recombination coefficient	3E−5
τ_S	Photon lifetime	2E−10 s
α	Linewidth enhancement factor	10
Γ	Optical confinement factor	0.04
υ_g	Group velocity	7.14E+7 m/s
N_{th}	Threshold carrier density	2.9E+18 m^{-3}
n_c	Equivalent core index	3.6
n_{cl}	Equivalent cladding index	3.56
a	Diameter of oxide	3 μm

The eye diagrams of original VISTAS for back-to-back and 300 m OM4 transmission are shown in Figures 5(a) and (b). Because of the narrow spectral width of VISTAS, the symbol width after 300 m propagation is almost the same with the back-to-back case. The eye diagrams of the modified VISTAS are shown in Figures 5(c) and (d). By using our proposed method, the symbol width after 300 m propagation becomes much wider with a closer eye, which is in accordance with (1) because of the larger spectral width. It shows that the spectral characteristic of VCSEL cannot be ignored in VCSEL modelling.

We also measure the BER curve to evaluate the transmission performance of VISTAS with and without our proposed method in Figure 6. Both left-tilted differential mode delay (DMD) profile (L-MMF) and right-tilted DMD profile (R-MMF) are tested. The effective modal bandwidth (EMB) of two MMF types are 7.3 GHz·km. Because of the modal spectral bias, the modal dispersion can be compensated by chromatic dispersion, which results in a better BER performance in L-MMF case [12]. In Figure 6,

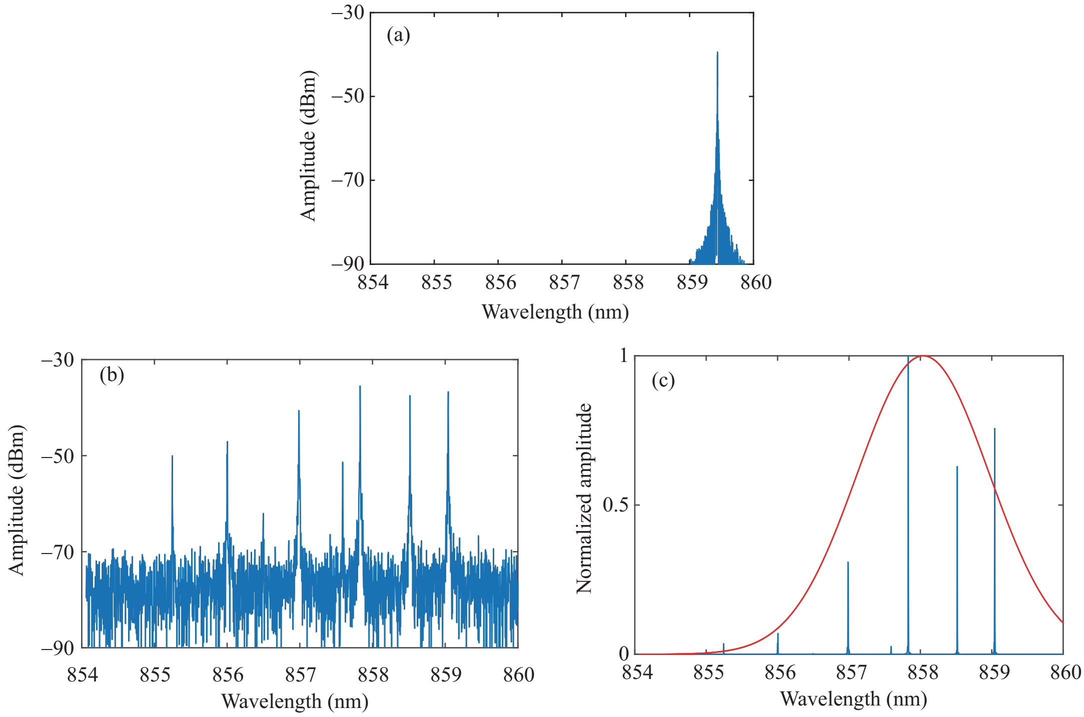

Figure 3 (Color online) Optical spectrum of (a) VISTAS original, (b) VISTAS with our proposed method, and normalized optical spectrum of (c) VISTAS with our proposed method.

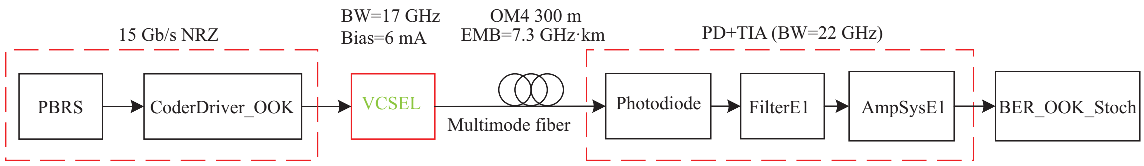

Figure 4 (Color online) Simulation block diagram.

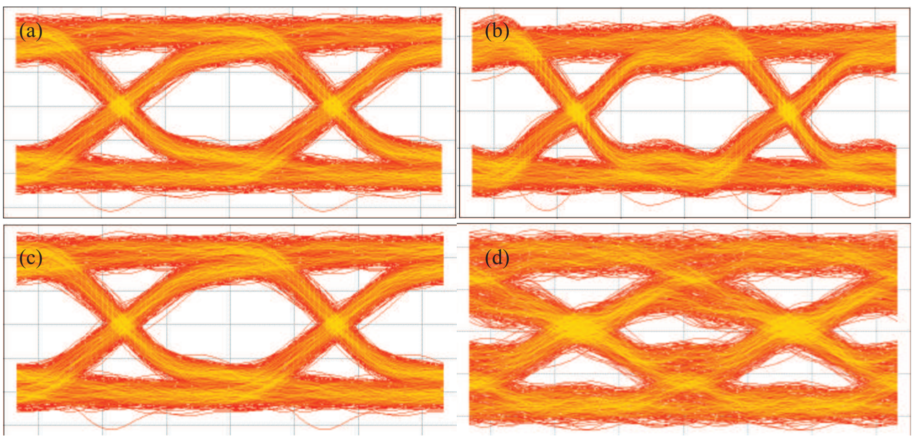

Figure 5 (Color online) Eye diagram of (a) BTB and (b) 300 m propagation of VISTAS without our proposed method, (c) BTB and (d) 300 m propagation of VISTAS with our proposed method.

after applying our modified model, the BER performance of L-MMF transmission is better than the R-MMF one which is consistent with the conclusion. However, for the original VISTAS model, the BER performances in L-MMF and R-MMF are almost the same. This proves that our proposed model is valid and the spectral characteristic is a key component for VCSEL model.

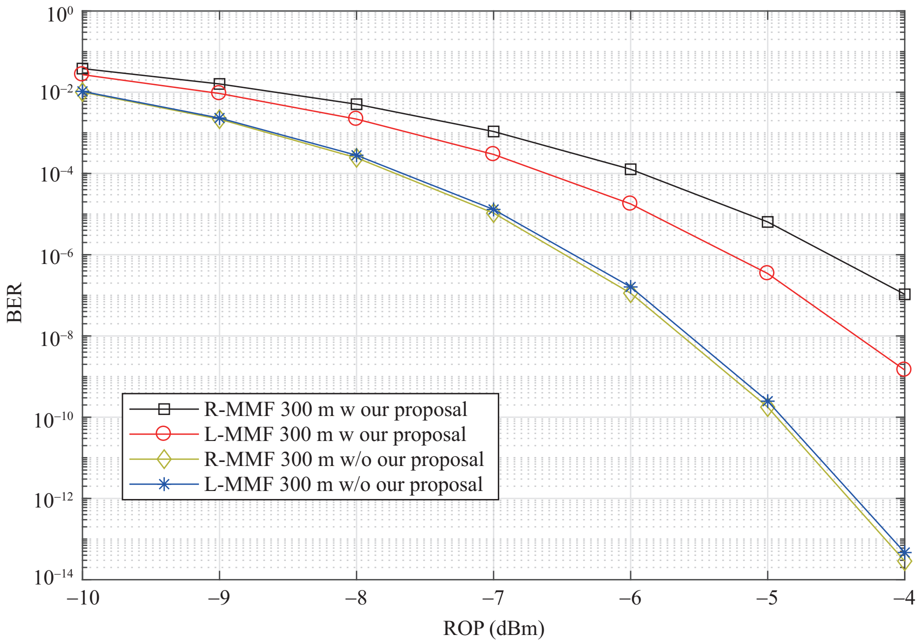

Figure 6 (Color online) BER performance of VISTAS with and without our proposed multimode spectral characteristic model.

5 Conclusion

In this paper, we proposed a new method to introduce multimode spectral characteristic into current VCSEL models, which extends the spatiotemporal multimode rate equations by calculating the wavelength shift of the VCSEL transverse modes. Simulation results show that the VCSEL model that is extended by our proposed method, is able to accurately estimate the effect of chromatic dispersion on the system performance, as well as the performance difference in L-MMF and R-MMF. Our proposed method is able to be integrated with any rate equations based VCSEL model, and the modified VCSEL model is accurately enough to evaluate the performance of high-speed short-range optical interconnect system.

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