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Consensus in nonlinear multi-agent systems with nonidentical nodes and sampled-data control

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Abstract This paper primarily discusses the leader-following consensus problem in nonlinear second-order multi-agent systems with nonidentical nodes. Sampled-data-based protocols are applied to reach consensus. Both delay-free and input-delay protocols are proposed. Based on the Lyapunov functional approach and linear matrix inequality (LMI) method, sufficient criteria are obtained to guarantee quasi-consensus for nonlinear heterogeneous multi-agent systems. All the heterogeneous followers can track the leader within a bounded range. Furthermore, the error systems between the leader and each follower eventually converge to a convergence domain that depends on the heterogeneity among the dynamics of the agents. Additionally, leader-following consensus can also be reached as the heterogeneity vanishes. Finally, numerical simulations are provided to illustrate the theoretical results.

Keywords heterogeneous systems, multi-agent systems, quasi-consensus, leader-following consensus, sampled-data control

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1 Introduction

With the rapid development of computer science, the internet and communication technology over the past few decades, multi-agent systems have been extensively researched. Because of broad applications in industry and engineering, the interactive and coordinated control of multi-agent systems has received considerable research attention. For example, people hope to replace large integrated circuit equipment with small devices by employing coordinated control. In general, coordinated control and analysis in networked systems include: synchronization [1–5], containment control [6, 7], flocking [8, 9] and consensus [10–15].

As a typical and basic dynamical behavior in a multi-agent system, consensus means that all the individuals tend toward the same state by employing the local interactions among all the agents. Until now, studies on consensus for multi-agent systems have mainly focused on first-order consensus [16, 17], second-order consensus [18–22], and high-order consensus [23, 24]. Moreover, many studies on consensus

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of multi-agent systems have depended on a precondition that all the agents had identical dynamics, that is, the multi-agent systems were homogeneous. For example, in [13, 20, 25–27], the consensus problems for homogeneous systems were investigated based on the specific topology that a graph had a directed spanning tree or the graph was undirected connected. However, multi-agent systems with nonidentical nodes are more common than homogeneous cases in real situations. Specifically, many factors, including external disturbances, systematic mutations, parameter uncertainties and individual differences, can lead to the heterogeneity of networked systems [1, 5, 28, 29]. Different from consensus in homogeneous multi-agent systems, it is difficult to analyze consensus or synchronization in heterogeneous multi-agent systems or heterogeneous complex networks because heterogeneous agents have different self-dynamics [1,5,17,28].

Generally, it is difficult for nonidentical agents to reach consensus if multi-agent systems are heterogeneous. Therefore, it is necessary to add external controls when multi-agent systems cannot reach consensus under the specific local distributed coupling. Therefore, many useful controls are adopted. For instance, pinning control [12, 25], impulsive control [30, 31] and intermittent control [13, 20, 21, 32] are applied to reach consensus. With the development of communication technology and the demand for low communication costs, sampling control has been widely adopted. By dividing a continuous time period into small sampling intervals, sampling implies that the systems sample the information at each sampling instant and only use the sampled information until new information is sampled. It has been shown that sampled-data control is an effective approach to investigate the consensus problem. Based on sampling control, the consensus problems in multi-agent systems were solved in [33–40]. More specifically, consensus could be reached in multi-agent systems by sampling the current position and velocity data in [33,34], and the current and sampled position data in [35]. Because packet transmissions between multi-agent systems may be lost, the authors studied consensus of linear multi-agent systems with sampled-data and packet losses [36]. In addition to deterministic sampling, stochastic sampling control is also effective at solving the consensus problem in multi-agent systems [37]. If a multi-agent system only uses the sampled information, which in fact is outdated information, is it actually effective at solving the consensus problem for a heterogeneous multi-agent system? It is worth investigating whether consensus can be reached in a heterogeneous multi-agent system via sampling control.

This paper focuses on solving the consensus problem in heterogeneous nonlinear second-order multi-agent systems via sampled-data-based controls. The main contributions are as follows. On one hand, heterogeneous multi-agent systems with nonidentical nonlinear self-dynamics are studied. On the other hand, by employing both delay-free and input-delay sampling controls, quasi-consensus is guaranteed in heterogeneous nonlinear multi-agent systems. By adopting the Lyapunov stability theory and linear matrix inequality (LMI) method, sufficient criteria are obtained for quasi-consensus in heterogeneous nonlinear multi-agent systems via sampled-data-based controls. All the heterogeneous followers can track the leader within a bounded error range. Additionally, the error systems eventually converge to a convergence domain, which depends on the heterogeneity of the dynamics among all the followers and the leader. The upper bound of the quasi-consensus error can be estimated.

The remainder of this paper consists of the following parts. In Section 2, the problem is formulated, and some preliminaries are given. In Section 3, the main results and corresponding proofs are presented. In Section 4, numerical simulations are provided. Conclusion is drawn in Section 5.

2 Problem formulation

2.1 Notation and graph theory

In this paper, $\mathbb{R}$ is the set of real numbers and $\mathbb{N}$ is the set of natural numbers. The symbols $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ represent the sets of vectors with dimension n and $n \times n$ real matrices, respectively. In addition, I_N (0_N) denotes the N-dimensional identity (zero) matrix. Matrix P is positive definite if $P > 0$. $\|\cdot\|$ indicates the Euclidean form, and $\mathrm{diag}(\cdot)$ indicates a diagonal matrix. $*$ represents the symmetric parts of a symmetric matrix.

Let $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ be a weighted digraph, in which $\mathscr{V} = \{v_1, v_2, \ldots, v_N\}$ and $\mathscr{E} \subset \mathscr{V} \times \mathscr{V}$ are the set of nodes and the set of edges, respectively. In addition, directed edge $e_{ij} = (v_i, v_j)$, $i, j \in \{1, 2, \ldots, N\}$ shows that information can be transmitted from node v_i to node v_j. Moreover, $\mathscr{A} = [a_{ij}]_{N\times N}$ represents the adjacency matrix of graph $\mathscr{G}$ with elements a_{ij}. $a_{ii} = 0$, and $a_{ij} > 0$ if and only if $e_{ji} \in \mathscr{E}$. Furthermore, let $L = [l_{ij}]_{N\times N}$ denote the corresponding Laplacian matrix of graph $\mathscr{G}$, where $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ii} = -\sum_{j=1,j\neq i}^{N} l_{ij}$. A graph is undirected if $e_{ji} \in \mathscr{E}$ is equivalent to $e_{ij} \in \mathscr{E}$.

2.2 Problem formulation

Consider a group of nonlinear multi-agent systems with N agents. The i-th agent can be described as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = f_i(t, x_i(t), v_i(t)) + u_i(t), \quad i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ describe the position state vector and velocity state vector of the i-th agent, respectively. $u_i(t)$ represents the input control that needs to be designed. $f_i(t, x_i(t), v_i(t)) : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous and nonlinear function that denotes the self-dynamics of the i-th agent. The agents are assumed to be heterogeneous. In other words, the self-dynamics of the agents are different from each other.

To study consensus for heterogenous multi-agent system (1), this paper introduces a virtual leader $(x_r(t), v_r(t))$ for system (1) as follows:

$$\begin{cases} \dot{x}_r(t) = v_r(t), \\ \dot{v}_r(t) = f_r(t, x_r(t), v_r(t)), \end{cases} \tag{2}$$

where $x_r(t) \in \mathbb{R}^n$ and $v_r(t) \in \mathbb{R}^n$ are the position state vector and velocity state vector, respectively. $f_r(t, x_r(t), v_r(t)) : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is the continuous and nonlinear self-dynamics of the leader.

Assumption 1. There exists a directed path from the leader to each of the other agents.

Remark 1. In fact, Assumption 1 is a necessary condition for achieving consensus in leader-following multi-agent systems [1, 7, 17]. By Assumption 1, the information can be transmitted from the leader to each of the followers through the directed path.

Assumption 2. There exist positive constants $\alpha_i > 0$ and $\beta_i > 0$ such that for any $x, v, y, z \in \mathbb{R}^n$,

$$\|f_i(t, x, v) - f_i(t, y, z)\| \leqslant \alpha_i \|x - y\| + \beta_i \|v - z\|,$$

$i = 1, 2, \ldots, N$.

Assumption 2 shows that the continuous function $f_i(t, x_i(t), v_i(t))$ in system (1) satisfies the Lipschitz conditions.

Assumption 3. There exist nonnegative constants $\delta_i \geqslant 0$ such that

$$\|f_i(t, x_r, v_r) - f_r(t, x_r, v_r)\| \leqslant \delta_i, \quad i = 1, 2, \ldots, N.$$

Remark 2. It is necessary to point out that Assumption 3 holds for many systems that have an equilibrium point, a periodic orbit, or a chaotic attractor. Therefore, the leader agent $(x_r(t), v_r(t))$ is bounded. Because the dynamics f_r and f_i are continuous, Assumption 3 can be easily satisfied.

To save energy and reduce communication cost, the input protocol $u_i(t)$ is designed as a sampled-date-based control:

$$u_i(t) = c\sum_{j=1}^{N} a_{ij}[(x_j(t_k) - x_i(t_k)) + (v_j(t_k) - v_i(t_k))] - cd_i[(x_i(t_k) - x_r(t_k)) + (v_i(t_k) - v_r(t_k))],$$
$$t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \quad i = 1, 2, \ldots, N, \tag{3}$$

where t_k, $k \in \mathbb{N}$, are the sampling moments and satisfy $0 = t_0 < t_1 < \cdots < t_k < \cdots$ and $t_{k+1} - t_k \leqslant h$, with $h > 0$ being the upper bound of the sampling intervals. $d_i > 0$ if there exists an edge (direceted) from the leader to the i-th agent, otherwise $d_i = 0$.

Let $\widehat{x}_i(t) = x_i(t) - x_r(t)$ and $\widehat{v}_i(t) = v_i(t) - v_r(t)$, $i = 1, 2, \ldots, N$. Combining systems (1) and (3) yields the following:

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = c\sum_{j=1}^{N} a_{ij}[(x_j(t_k) - x_i(t_k)) + (v_j(t_k) - v_i(t_k))] - cd_i[\widehat{x}_i(t_k) + \widehat{v}_i(t_k)] + f_i(t, x_i(t), v_i(t)), \end{cases} \tag{4}$$

$t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$, $i = 1, 2, \ldots, N$.

Definition 1 ([4,41])**.** Multi-agent system (1) is said to realize quasi-consensus (or bounded-consensus) with the leader (2) to a bounded set $\mathcal{M}$ if for any initial conditions, the following conditions are satisfied:

$$\lim_{t\to\infty} \mathrm{dist}(\widehat{x}_i(t), \mathcal{M}) = 0 \ \text{ and } \ \lim_{t\to\infty} \mathrm{dist}(\widehat{v}_i(t), \mathcal{M}) = 0,$$

for $i = 1, 2, \ldots, N$, where $\mathrm{dist}(x^*, \mathcal{M})$ stands for the distance from the point x^* to the set $\mathcal{M}$.

Lemma 1 ([42])**.** n and m are positive integers; $\lambda \in (0, 1)$; $R > 0$ is an $n \times n$ matrix; and W_1 and W_2 are two $n \times m$ matrices. Define the function

$$f(\lambda, R) = \frac{1}{\lambda}\xi^{\mathrm{T}} W_1^{\mathrm{T}} R W_1 \xi + \frac{1}{1-\lambda}\xi^{\mathrm{T}} W_2^{\mathrm{T}} R W_2 \xi,$$

for $\xi \in \mathbb{R}^m$. If $\left[\begin{smallmatrix} R & S \\ * & R \end{smallmatrix}\right] > 0$ for an $n \times n$ real matrix S, then

$$\min_{\lambda\in(0,1)} f(\lambda, R) \geqslant \xi^{\mathrm{T}} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R & S \\ * & R \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \xi.$$

For simplicity, this paper only discusses the case of $n = 1$. However, the results of this paper hold for the cases of $n > 1$ by applying Kronecker product $\otimes$.

3 Main results

In this section, some sufficient criteria will be derived to guarantee consensus in second-order nonlinear multi-agent systems (1) and (2).

3.1 Consensus analysis without input delays

The error system of the i-th agent can be rewritten as follows:

$$\begin{cases} \dot{\widehat{x}}_i(t) = \widehat{v}_i(t), \\ \dot{\widehat{v}}_i(t) = \widehat{f}_i(t) - c\sum_{j=1}^{N} l_{ij}[\widehat{x}_j(t_k) + \widehat{v}_j(t_k)] + \widehat{f}_{r_i}(t) - cd_i[\widehat{x}_i(t_k) + \widehat{v}_i(t_k)], \\ t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \end{cases} \tag{5}$$

where $\widehat{f}_i(t) = f_i(t, x_i(t), v_i(t)) - f_i(t, x_r(t), v_r(t))$ and $\widehat{f}_{r_i}(t) = f_i(t, x_r(t), v_r(t)) - f_r(t, x_r(t), v_r(t))$, $i = 1, 2, \ldots, N$.

For simplify, the following symbols are adopted to simplify the formulas:

$$\widehat{x}(t) = \left[\widehat{x}_1^{\mathrm{T}}(t), \widehat{x}_2^{\mathrm{T}}(t), \ldots, \widehat{x}_N^{\mathrm{T}}(t)\right]^{\mathrm{T}}, \quad \widehat{v}(t) = \left[\widehat{v}_1^{\mathrm{T}}(t), \widehat{v}_2^{\mathrm{T}}(t), \ldots, \widehat{v}_N^{\mathrm{T}}(t)\right]^{\mathrm{T}},$$
$$F(t) = \left[\widehat{f}_1^{\mathrm{T}}(t), \widehat{f}_2^{\mathrm{T}}(t), \ldots, \widehat{f}_N^{\mathrm{T}}(t)\right]^{\mathrm{T}}, \quad F_r(t) = \left[\widehat{f}_{r_1}^{\mathrm{T}}(t), \widehat{f}_{r_2}^{\mathrm{T}}(t) \ldots, \widehat{f}_{r_N}^{\mathrm{T}}(t)\right]^{\mathrm{T}}.$$

Define $y(t) = [\widehat{x}^{\mathrm{T}}(t), \widehat{v}^{\mathrm{T}}(t)]^{\mathrm{T}}$. Then, system (5) can be rewritten as the following compact system:

$$\dot{y}(t) = \Omega y(t) + \widetilde{L} y(t - \theta(t)) + \widetilde{F}(t) + \widetilde{F}_r(t), \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \tag{6}$$

where $\theta(t)=t-t_k$, $D=\mathrm{diag}(d_1,d_2,\ldots,d_N)$, $\widetilde{F}(t)=[0_{1\times N},F^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\widetilde{F}_r(t)=[0_{1\times N},F_r^{\mathrm{T}}(t)]^{\mathrm{T}}$,

$$\widetilde{L}=\begin{bmatrix}0_N & 0_N\\ -c(L+D) & -c(L+D)\end{bmatrix} \text{ and } \Omega=\begin{bmatrix}0_N & I_N\\ 0_N & 0_N\end{bmatrix}. \tag{7}$$

Consider the following Lyapunov-Krasovskii functional

$$V(t)=V_1(t)+V_2(t)+V_3(t), \tag{8}$$

where

$$\begin{aligned}V_1(t)&=y^{\mathrm{T}}(t)Py(t),\\ V_2(t)&=\int_{t-h}^{t}\mathrm{e}^{a(s-t)}y^{\mathrm{T}}(s)Qy(s)\mathrm{d}s,\\ V_3(t)&=h\int_{-h}^{0}\int_{t+\sigma}^{t}\mathrm{e}^{a(s-t)}\dot{y}^{\mathrm{T}}(s)R\dot{y}(s)\mathrm{d}s\mathrm{d}\sigma,\end{aligned}$$

with constant $a>0$, $2N\times 2N$ symmetric matrices $P>0$, $Q>0$, and $R>0$.

Similar to [17, 43], Lemma 2 holds.

Lemma 2. There exist constants $a>0$, $b>0$, $2N\times 2N$ symmetric matrices $P>0$, $Q>0$ and $R>0$, such that

$$U(t)\triangleq\dot{V}(t)+aV(t)-b\widetilde{F}_r^{\mathrm{T}}(t)\widetilde{F}_r(t)\leqslant 0, \tag{9}$$

where $\dot{V}(t)$ is the differential of $V(t)$ along the trajectory of (6). Then, error system (6) can exponentially converge to set

$$\mathscr{M}\triangleq\left\{y(t)\in\mathbb{R}^{2N}:y^{\mathrm{T}}(t)Py(t)\leqslant\frac{b}{a}\sum_{i=1}^{N}\delta_i^2\right\}.$$

Proof. In fact, the solution of (9) satisfies

$$V(t)\leqslant\mathrm{e}^{-at}V(0)+b\int_0^t\mathrm{e}^{a(s-t)}\widetilde{F}_r^{\mathrm{T}}(s)\widetilde{F}_r(s)\mathrm{d}s.$$

Combining Assumption 3 and $\widetilde{F}_r^{\mathrm{T}}(t)\widetilde{F}_r(t)=F_r^{\mathrm{T}}(t)F_r(t)$ yields the following:

$$V(t)\leqslant\mathrm{e}^{-at}V(0)+b\sum_{i=1}^{N}\delta_i^2\int_0^t\mathrm{e}^{a(s-t)}\mathrm{d}s.$$

This completes the proof.

Theorem 1. Under Assumptions 1–3, if there exist positive constants $a>0$, $b>0$ and $h>0$, and matrices $P>0$, $Q>0$, $R>0$, S_1, S_2 and S_3 with P, Q, R, S_1, S_2, $S_3\in\mathbb{R}^{2N\times 2N}$, such that $\Phi<0$ and $\Delta>0$, where Φ is a block matrix defined in (10),

$$\Phi=\begin{bmatrix}\Phi_{11} & P+\Omega^{\mathrm{T}}S_1^{\mathrm{T}}-S_2 & \mathrm{e}^{-ah}S_3 & \mathrm{e}^{-ah}R-\mathrm{e}^{-ah}S_3+S_2\widetilde{L} & S_2 & S_2\\ * & h^2R-S_1-S_1^{\mathrm{T}} & 0_{2N} & S_1\widetilde{L} & S_1 & S_1\\ * & * & -\mathrm{e}^{-ah}(Q+R) & \mathrm{e}^{-ah}R-\mathrm{e}^{-ah}S_3^{\mathrm{T}} & 0_{2N} & 0_{2N}\\ * & * & * & -2\mathrm{e}^{-ah}R+\mathrm{e}^{-ah}(S_3+S_3^{\mathrm{T}}) & 0_{2N} & 0_{2N}\\ * & * & * & * & -I_{2N} & 0_{2N}\\ * & * & * & * & * & -bI_{2N}\end{bmatrix}, \tag{10}$$

$$\begin{aligned}&\Phi_{11}=aP+Q-\mathrm{e}^{-ah}R+S_2\Omega+\Omega^{\mathrm{T}}S_2^{\mathrm{T}}+\rho,\\ &\overline{\alpha}=\mathrm{diag}(\alpha_1^2,\alpha_2^2,\ldots,\alpha_N^2),\end{aligned}$$

$$\overline{\beta} = \mathrm{diag}(\beta_1^2, \beta_2^2, \ldots, \beta_N^2),$$
$$\rho = \begin{bmatrix} 2\overline{\alpha} & 0_N \\ 0_N & 2\overline{\beta} \end{bmatrix}, \text{ and } \Delta = \begin{bmatrix} R & S_3 \\ * & R \end{bmatrix}.$$

Then, heterogeneous second-order multi-agent systems (1) and (2) with coordinated protocol (3) can achieve quasi-consensus and $\widehat{x}(t)$, $\widehat{v}(t)$ exponentially converge to the set

$$\mathscr{M}_0 = \left\{ \varepsilon \in \mathbb{R}^N : \|\varepsilon\| \leqslant \sqrt{\frac{b}{a\lambda_{\min}(P)} \sum_{i=1}^{N} \delta_i^2} \right\}.$$

Proof. Considering the derivation of $V(t)$ along the trajectory of (6) and substituting the derivation of $V(t)$ into $U(t)$ yields the following:

$$\begin{aligned} U(t) \leqslant\ & 2y^{\mathrm{T}}(t)P\dot{y}(t) + y^{\mathrm{T}}(t)(aP+Q)y(t) - \mathrm{e}^{-ah}y^{\mathrm{T}}(t-h)Qy(t-h) + h^2\phi(t) \\ & - h\mathrm{e}^{-ah}\int_{t-h}^{t}\phi(s)\mathrm{d}s - b\widetilde{F}_r^{\mathrm{T}}(t)\widetilde{F}_r(t), \end{aligned} \tag{11}$$

where $\phi(s) \triangleq \dot{y}^{\mathrm{T}}(s)R\dot{y}(s)$.

Let

$$\kappa_1(t) = \int_{t-\theta(t)}^{t}\dot{y}(s)\mathrm{d}s, \quad \kappa_2(t) = \int_{t-h}^{t-\theta(t)}\dot{y}(s)\mathrm{d}s.$$

According to Jensen's inequality, one has

$$\begin{aligned} -h\int_{t-h}^{t}\dot{y}^{\mathrm{T}}(s)R\dot{y}(s)\mathrm{d}s =& - h\int_{t-h}^{t-\theta(t)}\dot{y}^{\mathrm{T}}(s)R\dot{y}(s)\mathrm{d}s - h\int_{t-\theta(t)}^{t}\dot{y}^{\mathrm{T}}(s)R\dot{y}(s)\mathrm{d}s \\ \leqslant& -\frac{h}{h-\theta(t)}\kappa_2^{\mathrm{T}}(t)R\kappa_2(t) - \frac{h}{\theta(t)}\kappa_1^{\mathrm{T}}(t)R\kappa_1(t). \end{aligned} \tag{12}$$

Note that

$$\frac{h-\theta(t)}{h} + \frac{\theta(t)}{h} = 1.$$

Therefore, based on Lemma 1, we have the following from $\Delta > 0$ and (12):

$$-h\int_{t-h}^{t}\dot{y}^{\mathrm{T}}(s)R\dot{y}(s)\mathrm{d}s \leqslant -\begin{bmatrix}\kappa_1(t)\\ \kappa_2(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} R & S_3 \\ * & R\end{bmatrix}\begin{bmatrix}\kappa_1(t)\\ \kappa_2(t)\end{bmatrix}. \tag{13}$$

Note that when $\theta(t) = 0$ or $\theta(t) = h$, one has $y(t) - y(t-\theta) = 0$ or $y(t-\theta) - y(t-h) = 0$, respectively. Therefore, (13) still holds.

According to Assumption 2 and $\widetilde{F}^{\mathrm{T}}(t)\widetilde{F}(t) = F^{\mathrm{T}}(t)F(t)$, it is easy to get

$$\widetilde{F}^{\mathrm{T}}(t)\widetilde{F}(t) \leqslant y^{\mathrm{T}}(t)\rho y(t), \tag{14}$$

where $\rho = [\begin{smallmatrix} 2\overline{\alpha} & 0_N \\ 0_N & 2\overline{\beta}\end{smallmatrix}]$, $\overline{\alpha} = \mathrm{diag}(\alpha_1^2, \alpha_2^2, \ldots, \alpha_N^2)$ and $\overline{\beta} = \mathrm{diag}(\beta_1^2, \beta_2^2, \ldots, \beta_N^2)$.

Thus, substituting (13) and (14) into (11) yields

$$\begin{aligned} U(t) \leqslant\ & 2y^{\mathrm{T}}(t)P\dot{y}(t) + y^{\mathrm{T}}(t)(aP+Q)y(t) - \mathrm{e}^{-ah}y^{\mathrm{T}}(t-h)Qy(t-h) + h^2\dot{y}^{\mathrm{T}}(t)R\dot{y}(t) \\ & - \mathrm{e}^{-ah}\kappa_1^{\mathrm{T}}(t)R\kappa_1(t) - \mathrm{e}^{-ah}\kappa_1^{\mathrm{T}}(t)S_3\kappa_2(t) - \mathrm{e}^{-ah}\kappa_2^{\mathrm{T}}(t)S_3^{\mathrm{T}}\kappa_1(t) - \mathrm{e}^{-ah}\kappa_2^{\mathrm{T}}(t)R\kappa_2(t) \\ & - b\widetilde{F}_r^{\mathrm{T}}(t)\widetilde{F}_r(t) - \widetilde{F}^{\mathrm{T}}(t)\widetilde{F}(t) + y^{\mathrm{T}}(t)\rho y(t). \end{aligned} \tag{15}$$

Introducing a new variable $\eta(t) = [y^{\mathrm{T}}(t), \dot{y}^{\mathrm{T}}(t), y^{\mathrm{T}}(t-h),\ y^{\mathrm{T}}(t-\theta(t)), \widetilde{F}^{\mathrm{T}}(t), \widetilde{F}_r^{\mathrm{T}}(t)]^{\mathrm{T}}$, and combining the following equations:

$$-\dot{y}^{\mathrm{T}}(t)S_1\dot{y}(t) + \dot{y}^{\mathrm{T}}(t)S_1\left[\Omega y(t) + \widetilde{L}y(t-\theta(t)) + \widetilde{F}(t) + \widetilde{F}_r(t)\right] = 0,$$

$$
\begin{aligned}
&-\dot{y}^{\mathrm{T}}(t)S_1^{\mathrm{T}}\dot{y}(t)+\left[\Omega y(t)+\widetilde{L}y(t-\theta(t))+\widetilde{F}(t)+\widetilde{F}_r(t)\right]^{\mathrm{T}}S_1^{\mathrm{T}}\dot{y}(t)=0,\\
&-y^{\mathrm{T}}(t)S_2\dot{y}(t)+y^{\mathrm{T}}(t)S_2\left[\Omega y(t)+\widetilde{L}y(t-\theta(t))+\widetilde{F}(t)+\widetilde{F}_r(t)\right]=0,\\
&-\dot{y}^{\mathrm{T}}(t)S_2^{\mathrm{T}}y(t)+\left[\Omega y(t)+\widetilde{L}y(t-\theta(t))+\widetilde{F}(t)+\widetilde{F}_r(t)\right]^{\mathrm{T}}S_2^{\mathrm{T}}y(t)=0,
\end{aligned}
$$

one gets

$$
U(t)\leqslant\eta^{\mathrm{T}}(t)\Phi\eta(t). \tag{16}
$$

According to the conditions of Theorem 1, $\Phi<0$. It follows from Lemma 2 that the norms of consensus errors $\|\widehat{x}(t)\|$ and $\|\widehat{v}(t)\|$ exponentially converge to the domain $\mathscr{M}_0$ under the sampling protocol (3). This completes the proof.

According to Theorem 1, accurate consensus can be reached for the homogeneous leader-following multi-agent systems because $\delta_i\equiv 0$ for all $i=1,2,\ldots,N$.

Corollary 1. Under Assumptions 1 and 2, if there exist positive constants $a>0$, $b>0$ and $h>0$, and matrices $P>0$, $Q>0$, $R>0$, S_1, S_2 and S_3 with P, Q, R, S_1, S_2, $S_3\in\mathbb{R}^{2N}$ such that $\Phi<0$ and $\Delta>0$, then homogeneous nonlinear systems (1) and (2) with protocol (3) can reach consensus when $f_1=f_2=\cdots=f_N=f_r$.

Remark 3. Theorem 1 shows that quasi-consensus in nonlinear second-order multi-agent systems (1) and (2) with sampled-data control can be reached. Moreover, the convergence domain which depends on the heterogeneity of the dynamics of all the agents, is also provided. Corollary 1 shows that nonlinear leader-following second-order multi-agent systems (1) and (2) can reach consensus if the heterogeneity of the dynamics of all the agents vanishes. Consensus may be difficult to reach if there are large differences between the leader and followers.

3.2 Consensus analysis with input delays

Because of the limited rate of communication in a network, time-delay is common and inevitable. If input delays exist, the delayed protocol can be adopted

$$
\begin{aligned}
u_i(t)=&\,c\sum_{j=1}^{N}a_{ij}[(x_j(t_k-\tau)-x_i(t_k-\tau))+(v_j(t_k-\tau)-v_i(t_k-\tau))]\\
&-cd_i[(x_i(t_k-\tau)-x_r(t_k-\tau))+(v_i(t_k-\tau)-v_r(t_k-\tau))],\\
t\in&\,[t_k,t_{k+1}),\quad k\in\mathbb{N},\quad i=1,2,\ldots,N,
\end{aligned} \tag{17}
$$

where τ is an input delay with $0<\tau<h$.

Combining (1) and (2) with protocol (17), the following error systems can be derived:

$$
\begin{cases}
\dot{\widehat{x}}_i(t)=\widehat{v}_i(t),\\
\dot{\widehat{v}}_i(t)=\widehat{f}_i(t)-c\displaystyle\sum_{j=1}^{N}l_{ij}[\widehat{x}_j(t_k-\tau)+\widehat{v}_j(t_k-\tau)]+\widehat{f}_{r_i}(t)-cd_i[\widehat{x}_i(t_k-\tau)+\widehat{v}_i(t_k-\tau)],
\end{cases} \tag{18}
$$

$t\in[t_k,t_{k+1})$, $k\in\mathbb{N}$, $i=1,2,\ldots,N$.

Similar to system (6), system (18) can be rewritten in the following compact form using the delayed input approach:

$$
\dot{y}(t)=\Omega y(t)+\widetilde{L}y(t-\tau-\theta(t))+\widetilde{F}(t)+\widetilde{F}_r(t),\quad t\in[t_k,t_{k+1}),\quad k\in\mathbb{N}. \tag{19}
$$

The initial value of error system (19) is assumed to be

$$
y(t)=y(0),\quad t\in[-\tau,0].
$$

Then, Theorem 2 can be derived.

Theorem 2. Suppose that Assumptions 1–3 hold. If there exist constants $a > 0$, $b > 0$ and $h > 0$, and matrices $P_1 > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, S_1, S_2 and S_3 with P_1, Q_1, Q_2, R_1, R_2, S_1, S_2, $S_3 \in \mathbb{R}^{2N\times 2N}$, such that $\Psi < 0$ and $\Delta_1 > 0$, where Ψ is a block matrix defined in (20),

$$\Psi = \begin{bmatrix} \Psi_{11} & P_1 + \Omega^{\mathrm{T}} S_1^{\mathrm{T}} - S_2 & 0_{2N} & \mathrm{e}^{-a\tau} R_1 & S_2 \widetilde{L} & S_2 & S_2 \\ * & \Psi_{22} & 0_{2N} & 0_{2N} & S_1 \widetilde{L} & S_1 & S_1 \\ * & * & \Psi_{33} & \mathrm{e}^{-2ah} S_3^{\mathrm{T}} & \mathrm{e}^{-2ah} R_2 - \mathrm{e}^{-2ah} S_3^{\mathrm{T}} & 0_{2N} & 0_{2N} \\ * & * & * & \Psi_{44} & \mathrm{e}^{-2ah} R_2 - \mathrm{e}^{-2ah} S_3 & 0_{2N} & 0_{2N} \\ * & * & * & * & -2\mathrm{e}^{-2ah} R_2 + \mathrm{e}^{-2ah}(S_3 + S_3^{\mathrm{T}}) & 0_{2N} & 0_{2N} \\ * & * & * & * & * & -I_{2N} & 0_{2N} \\ * & * & * & * & * & * & -bI_{2N} \end{bmatrix}, \tag{20}$$

$$\begin{aligned} &\Psi_{11} = aP_1 + Q_1 + Q_2 - \mathrm{e}^{-a\tau} R_1 + \rho + S_2\Omega + \Omega^{\mathrm{T}} S_2^{\mathrm{T}}, \\ &\Psi_{22} = \tau^2 R_1 + (2h-\tau)^2 R_2 - S_1 - S_1^{\mathrm{T}}, \\ &\Psi_{33} = -\mathrm{e}^{-2ah}(Q_2 + R_2), \\ &\Psi_{44} = -\mathrm{e}^{-a\tau}(Q_1 + R_1) - \mathrm{e}^{-2ah} R_2, \\ &\Delta_1 = \begin{bmatrix} R_2 & S_3 \\ * & R_2 \end{bmatrix}. \end{aligned}$$

Then, heterogeneous second-order multi-agent systems (1) and (2) under protocol (17) can achieve quasi-consensus and $\widehat{x}(t)$, $\widehat{v}(t)$ exponentially converge to set

$$\mathscr{M}_0' = \left\{ \varepsilon \in \mathbb{R}^N : \|\varepsilon\| \leqslant \sqrt{\frac{b}{a\lambda_{\min}(P_1)} \sum_{i=1}^{N} \delta_i^2} \right\}.$$

Proof. Construct the Lyapunov-Krasovskii functional as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \tag{21}$$

where

$$\begin{aligned} &V_1(t) = y^{\mathrm{T}}(t) P_1 y(t), \\ &V_2(t) = \int_{t-\tau}^{t} \mathrm{e}^{a(s-t)} y^{\mathrm{T}}(s) Q_1 y(s) \mathrm{d}s, \\ &V_3(t) = \int_{t-2h}^{t} \mathrm{e}^{a(s-t)} y^{\mathrm{T}}(s) Q_2 y(s) \mathrm{d}s, \\ &V_4(t) = \tau \int_{-\tau}^{0} \int_{t+\sigma}^{t} \mathrm{e}^{a(s-t)} \dot{y}^{\mathrm{T}}(s) R_1 \dot{y}(s) \mathrm{d}s \mathrm{d}\sigma, \\ &V_5(t) = (2h-\tau) \int_{-2h}^{-\tau} \int_{t+\sigma}^{t} \mathrm{e}^{a(s-t)} \dot{y}^{\mathrm{T}}(s) R_2 \dot{y}(s) \mathrm{d}s \mathrm{d}\sigma. \end{aligned}$$

Therefore, we have

$$\begin{aligned} U(t) \leqslant\ & 2y^{\mathrm{T}}(t) P_1 \dot{y}(t) + y^{\mathrm{T}}(t)(aP_1 + Q_1 + Q_2) y(t) - \mathrm{e}^{-a\tau} y^{\mathrm{T}}(t-\tau) Q_1 y(t-\tau) \\ & - \mathrm{e}^{-2ah} y^{\mathrm{T}}(t-2h) Q_2 y(t-2h) + \tau^2 \dot{y}^{\mathrm{T}}(t) R_1 \dot{y}(t) + (2h-\tau)^2 \dot{y}^{\mathrm{T}}(t) R_2 \dot{y}(t) \\ & - \tau \mathrm{e}^{-a\tau} \int_{t-\tau}^{t} \dot{y}^{\mathrm{T}}(s) R_1 \dot{y}(s) \mathrm{d}s - (2h-\tau)\mathrm{e}^{-2ah} \int_{t-2h}^{t-\tau} \dot{y}^{\mathrm{T}}(s) R_2 \dot{y}(s) \mathrm{d}s - b\widetilde{F}_r^{\mathrm{T}}(t) \widetilde{F}_r(t). \end{aligned} \tag{22}$$

Let

$$\zeta_1(t) = \int_{t-\tau-\theta(t)}^{t-\tau} \dot{y}(s) \mathrm{d}s, \quad \zeta_2(t) = \int_{t-2h}^{t-\tau-\theta(t)} \dot{y}(s) \mathrm{d}s.$$

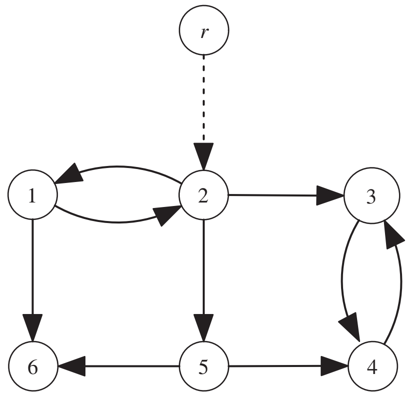

Figure 1 Topology of multi-agent systems.

It follows from Jensen's inequality and Lemma 1 that

$$\begin{aligned}
&-(2h-\tau)\int_{t-2h}^{t-\tau}\dot{y}^{\mathrm{T}}(s)R_2\dot{y}(s)\mathrm{d}s \\
&=-(2h-\tau)\int_{t-2h}^{t-\tau-\theta(t)}\dot{y}^{\mathrm{T}}(s)R_2\dot{y}(s)\mathrm{d}s-(2h-\tau)\int_{t-\tau-\theta(t)}^{t-\tau}\dot{y}^{\mathrm{T}}(s)R_2\dot{y}(s)\mathrm{d}s \\
&\leqslant-\frac{2h-\tau}{2h-\tau-\theta(t)}\zeta_2^{\mathrm{T}}(t)R_2\zeta_2(t)-\frac{2h-\tau}{\theta(t)}\zeta_1^{\mathrm{T}}(t)R_2\zeta_1(t) \\
&\leqslant-\begin{bmatrix}\zeta_1(t)\\ \zeta_2(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}R_2 & S_3\\ * & R_2\end{bmatrix}\begin{bmatrix}\zeta_1(t)\\ \zeta_2(t)\end{bmatrix}.
\end{aligned}$$

Let $\eta_1(t)=[y^{\mathrm{T}}(t),\dot{y}^{\mathrm{T}}(t),y^{\mathrm{T}}(t-2h),y^{\mathrm{T}}(t-\tau),\ y^{\mathrm{T}}(t-\tau-\theta(t)),\widetilde{F}^{\mathrm{T}}(t),\widetilde{F}_r^{\mathrm{T}}(t)]^{\mathrm{T}}$. The remainder of the proof is similar to that of Theorem 1 and thus is omitted here.

For homogeneous leader-following multi-agent systems, Corollary 2 holds.

Corollary 2. Suppose that Assumptions 1 and 2 hold. If there exist constants $a>0$, $b>0$ and $h>0$, and matrices $P_1>0$, $Q_1>0$, $Q_2>0$, $R_1>0$, $R_2>0$, S_1, S_2 and S_3 with P_1, Q_1, Q_2, R_1, R_2, S_1, S_2, $S_3\in\mathbb{R}^{2N\times 2N}$, such that $\Psi<0$ and $\Delta_1>0$, then consensus can be reached in homogeneous nonlinear systems (1) and (2) with protocol (17).

Remark 4. Because the networked systems are heterogeneous and nonlinear, the methods analyzed in [33,35,39] are difficult to apply in this work. In contrast to the conditions that rely on the eigenvalues of Laplacian matrices in [33,35,39], conditions (10) and (20) need to solve LMIs rather than the eigenvalues of Laplacian matrices. Furthermore, conditions (10) and (20) can be solved by MATLAB's LMI Toolbox. On one hand, the LMI method can provide direct criteria for consensus. On the other hand, the LMI method may need mass calculations if the LMI is very large. In the future, decomposition methods will be applied to reduce the variables for some kinds of LMIs [30,36].

4 Numerical simulations

In this section, numerical simulations are shown to verify the theoretical results. Consider a directed graph with six agents. The graph topology is shown in Figure 1, where r represents the leader, and numbers 1 to 6 are the followers.

Example 1. The nonlinear dynamics f_i of the i-th agent is described by Chua's circuit:

$$f_i(t,x_i(t),v_i(t))=A_iv_i(t)+B_ig(v_i(t)), \tag{23}$$

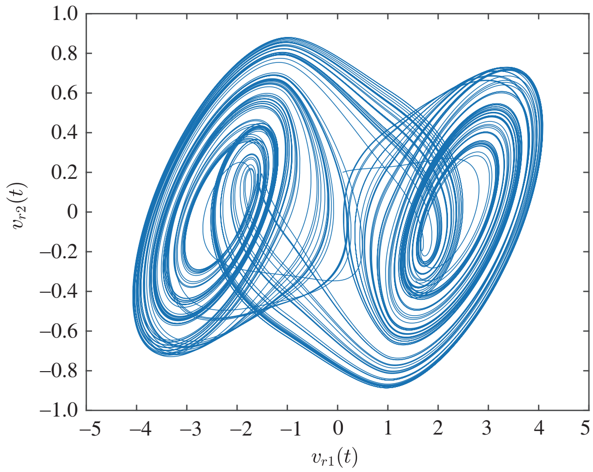

Figure 2 (Color online) Velocity trajectories of the leader.

where $v_i(t) = [v_{i1}(t), v_{i2}(t), v_{i3}(t)]^{\mathrm{T}}$, $g(v_i(t)) = [0.5(|v_{i1}+1| - |v_{i1}-1|), 0, 0]^{\mathrm{T}}$, and

$$A_i = \frac{3}{21+i}\begin{bmatrix} -2.5 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -18 & 0 \end{bmatrix}, \quad B_i = \frac{3}{21+i}\begin{bmatrix} \frac{35}{6} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad i = 1, 2, \ldots, 6. \tag{24}$$

Set the nonlinear dynamics f_r of the leader as

$$f_r(t, x_r(t), v_r(t)) = A_r v_r(t) + B_r g(v_r(t)), \tag{25}$$

where

$$A_r = \begin{bmatrix} -2.5 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -18 & 0 \end{bmatrix}, \quad B_r = \begin{bmatrix} \frac{35}{6} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

As shown in [27], system (25) is chaotic, and the Lipschitz constants of f_r are $\alpha_r = 0$ and $\beta_r = 4.3871$. Hence, Assumptions 2 and 3 hold with $\overline{\alpha} = 0_6$, and

$$\overline{\beta} = \mathrm{diag}\left(\frac{9}{22^2}, \frac{9}{23^2}, \ldots, \frac{9}{27^2}\right)\beta_r^2.$$

Figure 2 shows that system (25) is chaotic.

Coordinated protocol (3) is adopted. Taking $a = 0.1$, $b = 4$ and $c = 18$, and solving the LMIs of Theorem 1, it is possible to obtain an allowed value for sampling interval $h = 0.02$. For simplicity, protocol (3) is designed as periodic sampling, with sampling period $h = 0.02$. Then, the initial values $x_r(0) = [-0.1, -0.2, -0.3]^{\mathrm{T}}$, $v_r(0) = [0.1, 0.2, 0.3]^{\mathrm{T}}$ are selected, along with random initial values for the leader and 6 followers. Figures 3 and 4 depict the evolutions of the position and velocity states, respectively. The dashed lines represents the evolutions of the states of the leader. In Figures 3 and 4, all of the heterogeneous nonlinear follower agents can achieve quasi-consensus with the leader. When $t \in [40, 50]$, the upper bounds of $\|\widehat{x}_i(t)\|$ and $\|\widehat{v}_i(t)\|$ are approximately 1.13 and 2.83, respectively.

If one chooses $A_r = A_1 = \cdots = A_6 = 0.05I_3$ and

$$B_r = B_1 = \cdots = B_6 = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

then consensus in leader-following multi-agent systems (1) and (2) can be reached by employing sampled-data-based control (3) with $h = 0.02$. Figures 5 and 6 show consensus in homogeneous leader-following multi-agent systems.

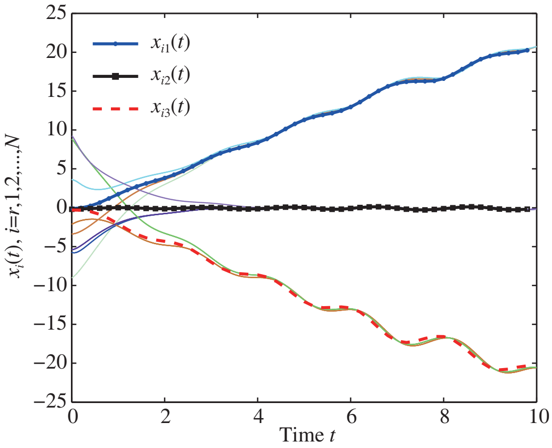

Figure 3 (Color online) Position trajectories of three states of the leader and the followers.

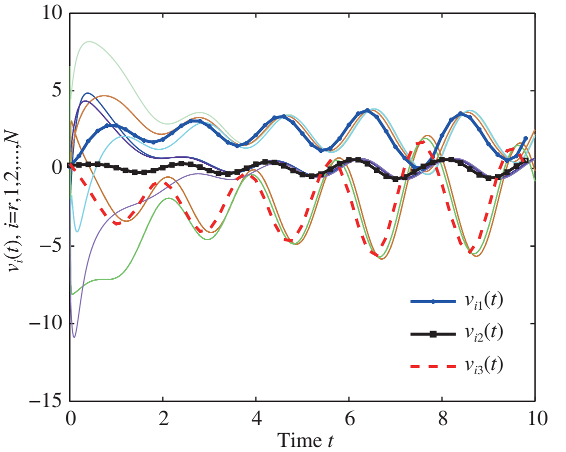

Figure 4 (Color online) Velocity trajectories of three states of the leader and the followers.

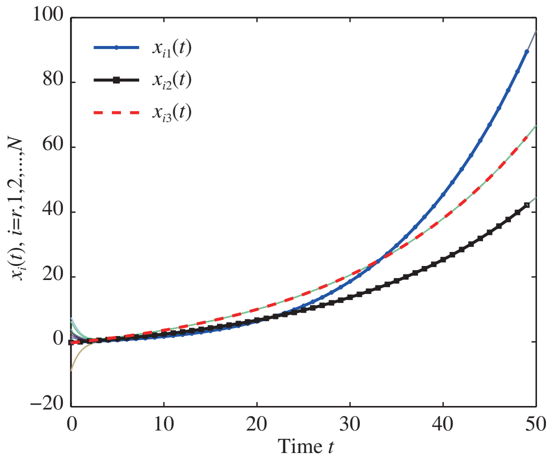

Figure 5 (Color online) Position trajectories of three states of the leader and the followers.

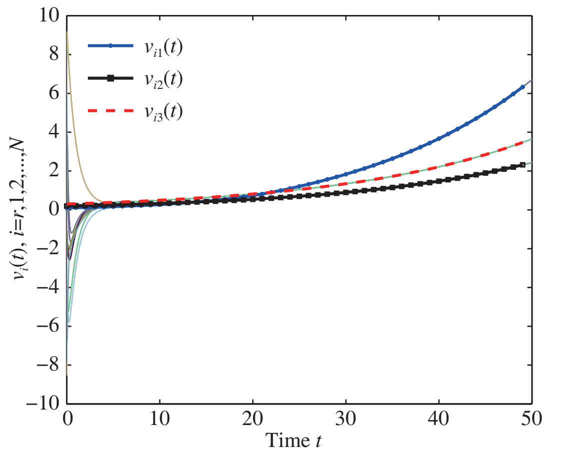

Figure 6 (Color online) Velocity trajectories of three states of the leader and the followers.

Example 2. Considering the same multi-agent systems as Example 1 and taking the same parameters and nonlinear dynamics as Example 1. The consensus protocol (17) is adopted. Let $\tau = 0.005$. Solving the linear matrix inequalities of Theorem 2 makes it possible to obtain an allowed value of sampling interval $h = 0.011$. Similar to Example 1, protocol (17) is designed as periodic sampling with sampling period $h = 0.01$. The initial values $x_r(0) = [-0.1, -0.2, -0.3]^{\rm T}$, $v_r(0) = [0.1, 0.2, 0.3]^{\rm T}$ are selected for the leader, with random initial values for 6 followers. Figures 7 and 8 show the evolutions of the position and velocity states under the delayed protocol (17), respectively. When $t \in [40, 50]$, the upper bounds of $\|\widehat{x}_i(t)\|$ and $\|\widehat{v}_i(t)\|$ are approximately 1.12 and 2.81, respectively.

Remark 5. Protocol (17) considers sampled-data-based control with an input delay. By applying protocol (17), Theorem 2 shows that heterogeneous multi-agent systems (1) and (2) can still reach quasi-consensus for relatively small values of delay τ. When $\tau = 0$, the delay-free results can be obtained from Theorem 2. According to (20), it may be difficult to satisfy $\Psi < 0$ for relatively large values of delay τ. By solving LMI, $\Psi < 0$ will not hold for Example 2 when $\tau \geqslant 0.024$. In addition, in numerical simulations, quasi-consensus will not be reached for Example 2 when $\tau \geqslant 0.025$. Therefore, the implementation of quasi-consensus of multi-agent systems will not be affected for relatively small values of input delay. However, quasi-consensus of multi-agent systems will be destroyed for relatively large values of input delay.

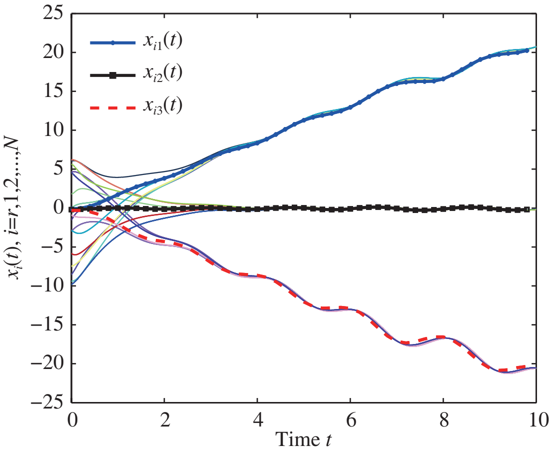

Figure 7 (Color online) Position trajectories of three states of the leader and the followers under delayed protocol.

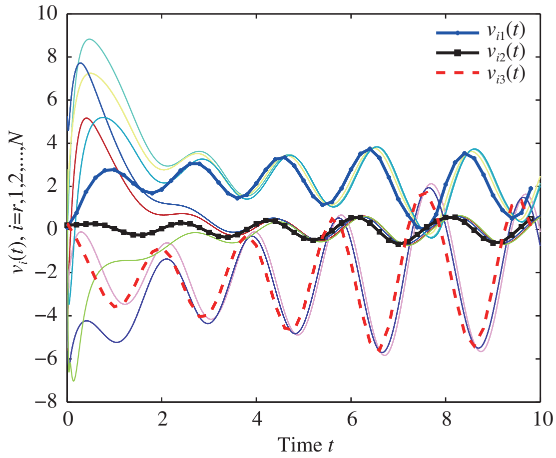

Figure 8 (Color online) Velocity trajectories of three states of the leader and the followers under delayed protocol.

5 Conclusion

This study considered leader-following consensus in heterogeneous second-order multi-agent systems via sampled-data control. Two distributed sampling protocols were proposed depending on the position and velocity information. The delay-free and input-delay cases were both considered. According to the Lyapunov stability theory and LMI method, sufficient criteria were obtained to guarantee quasi-consensus for multi-agent systems. A converged domain, which depended on the heterogeneity of the self-dynamics of all the agents, was estimated. In addition, consensus could also be reached as the heterogeneity of the self-dynamics of all the agents vanished. Based on this study, in the future, attention will be focused on whether heterogeneous multi-agent systems can reach consensus within a finite time under the proper protocols.

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References

1 He W L, Qian F, Lam J, et al. Quasi-synchronization of heterogeneous dynamic networks via distributed impulsive control: error estimation, optimization and design. Automatica, 2015, 62: 249–262

2 Liu X Y, Cao J D, Yu W W, et al. Nonsmooth finite-time synchronization of switched coupled neural networks. IEEE Trans Cybern, 2016, 46: 2360–2371

3 Cao J D, Li R X. Fixed-time synchronization of delayed memristor-based recurrent neural networks. Sci China Inf Sci, 2017, 60: 032201

4 Li Q B, Guo J, Wu Y Y, et al. Global asymptotical bounded synchronization for a class of coupled complex networks with nonidentical nodes. Asian J Control, 2017, 19: 1630–1640

5 Wang Z X, Jiang G P, Yu W W, et al. Synchronization of coupled heterogeneous complex networks. J Franklin Inst, 2017, 354: 4102–4125

6 Ji M, Ferrari-Trecate G, Egerstedt M, et al. Containment control in mobile networks. IEEE Trans Autom Control, 2008, 53: 1972–1975

7 Cao Y C, Ren W, Egerstedt M. Distributed containment control with multiple stationary or dynamic leaders in fixed and switching directed networks. Automatica, 2012, 48: 1586–1597

8 Olfati-Saber R. Flocking for multi-agent dynamic systems: algorithms and theory. IEEE Trans Autom Control, 2006,

51: 401–420
9 Su H S, Wang X F, Lin Z L. Flocking of multi-agents with a virtual leader. IEEE Trans Autom Control, 2009, 54: 293–307
10 Ren W, Beard R W. Consensus seeking in multiagent systems under dynamically changing interaction topologies. IEEE Trans Autom Control, 2005, 50: 655–661
11 Yu W W, Chen G R, Cao M, et al. Delay-induced consensus and quasi-consensus in multi-agent dynamical systems. IEEE Trans Circ Syst I, 2013, 60: 2679–2687
12 Song Q, Cao J D, Yu W W. Second-order leader-following consensus of nonlinear multi-agent systems via pinning control. Syst Control Lett, 2010, 59: 553–562
13 Wen G H, Duan Z S, Yu W W, et al. Consensus in multi-agent systems with communication constraints. Int J Robust Nonlinear Control, 2012, 22: 170–182
14 Wang Z, Cao J D. Quasi-consensus of second-order leader-following multi-agent systems. IET Control Theory Appl, 2012, 6: 545–551
15 Cai H, Huang J. Leader-following adaptive consensus of multiple uncertain rigid spacecraft systems. Sci China Inf Sci, 2016, 59: 010201
16 Jadbabaie A, Lin J, Morse A S. Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans Autom Control, 2003, 48: 988–1001
17 Ding L, Zheng W X. Consensus tracking in heterogeneous nonlinear multi-agent networks with asynchronous sampled-data communication. Syst Control Lett, 2016, 96: 151–157
18 Lin P, Jia Y M. Consensus of second-order discrete-time multi-agent systems with nonuniform time-delays and dynamically changing topologies. Automatica, 2009, 45: 2154–2158
19 Yu W W, Chen G R, Cao M. Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems. Automatica, 2010, 46: 1089–1095
20 Wen G H, Duan Z S, Yu W W, et al. Consensus of second-order multi-agent systems with delayed nonlinear dynamics and intermittent communications. Int J Control, 2013, 86: 322–331
21 Yu Z Y, Jiang H J, Hu C, et al. Consensus of second-order multi-agent systems with delayed nonlinear dynamics and aperiodically intermittent communications. Int J Control, 2017, 90: 909–922
22 Liu X Y, Ho D W C, Cao J D, et al. Discontinuous observers design for finite-time consensus of multiagent systems with external disturbances. IEEE Trans Neural Netw Learn Syst, 2017, 28: 2826–2830
23 Wang Z H, Zhang H S, Fu M Y, et al. Consensus for high-order multi-agent systems with communication delay. Sci China Inf Sci, 2017, 60: 092204
24 Tian Y P, Zhang Y. High-order consensus of heterogeneous multi-agent systems with unknown communication delays. Automatica, 2012, 48: 1205–1212
25 Chen F, Chen Z Q, Xiang L Y, et al. Reaching a consensus via pinning control. Automatica, 2009, 45: 1215–1220
26 Liu X W, Chen T P, Lu W L. Consensus problem in directed networks of multi-agents via nonlinear protocols. Phys Lett A, 2009, 373: 3122–3127
27 Yu W W, Chen G R, Cao M, et al. Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics. IEEE Trans Syst Man Cybern B, 2010, 40: 881–891
28 He W L, Qian F, Han Q L, et al. Synchronization error estimation and controller design for delayed Lur'e systems with parameter mismatches. IEEE Trans Neural Netw Learn Syst, 2012, 23: 1551–1563
29 Su H S, Qiu Y, Wang L. Semi-global output consensus of discrete-time multi-agent systems with input saturation and external disturbances. ISA Trans, 2017, 67: 131–139
30 Zhang W B, Tang Y, Wu X T, et al. Synchronization of nonlinear dynamical networks with heterogeneous impulses. IEEE Trans Circ Syst I, 2014, 61: 1220–1228
31 Hu J Q, Liang J L, Cao J D. Synchronization of hybrid-coupled heterogeneous networks: pinning control and impulsive control schemes. J Franklin Inst, 2014, 351: 2600–2622
32 Wen G H, Duan Z S, Ren W, et al. Distributed consensus of multi-agent systems with general linear node dynamics and intermittent communications. Int J Robust Nonlinear Control, 2014, 24: 2438–2457
33 Yu W W, Zhou L, Yu X H, et al. Consensus in multi-agent systems with second-order dynamics and sampled data. IEEE Trans Ind Inf, 2013, 9: 2137–2146
34 Gao Y P, Wang L. Sampled-data based consensus of continuous-time multi-agent systems with time-varying topology. IEEE Trans Autom Control, 2011, 56: 1226–1231
35 Yu W W, Zheng W X, Chen G R, et al. Second-order consensus in multi-agent dynamical systems with sampled position data. Automatica, 2011, 47: 1496–1503
36 Zhang W B, Tang Y, Huang T W, et al. Sampled-data consensus of linear multi-agent systems with packet losses. IEEE Trans Neural Netw Learn Syst, 2017, 28: 2516–2527

37 He W L, Zhang B, Han Q L, et al. Leader-following consensus of nonlinear multiagent systems with stochastic sampling. IEEE Trans Cybern, 2016, 47: 327–338

38 Wen G H, Duan Z S, Yu W W, et al. Consensus of multi-agent systems with nonlinear dynamics and sampled-data information: a delayed-input approach. Int J Robust Nonlinear Control, 2013, 23: 602–619

39 Huang N, Duan Z S, Chen G R. Some necessary and sufficient conditions for consensus of second-order multi-agent systems with sampled position data. Automatica, 2016, 63: 148–155

40 Wan Y, Cao J D, Alsaedi A, et al. Distributed observer-based stabilization of nonlinear multi-agent systems with sampled-data control. Asian J Control, 2017, 19: 918–928

41 Wang L, Chen S Y, Wang Q G. Eigenvalue based approach to bounded synchronization of asymmetrically coupled networks. Commun Nonlinear Sci Numer Simul, 2015, 22: 769–779

42 Park P G, Ko J W, Jeong C. Reciprocally convex approach to stability of systems with time-varying delays. Automatica, 2011, 47: 235–238

43 Fridman E, Dambrine M. Control under quantization, saturation and delay: an LMI approach. Automatica, 2009, 45: 2258–2264