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Stability criteria for stochastic singular systems with time-varying delays and uncertain parameters

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Dear editor,

It is well known that singular systems can better describe physical systems and they are widely used in chemical processes, microelectronic circuits, economic systems, and network control systems [1]. We can effectively model singular systems as stochastic singular systems when the structure of singular systems is unexpectedly altered by the environment. Several meaningful contributions were reported in [2–4].

The stability problem in stochastic singular systems with time delays has recently attracted significant research interest. In particular, considerable attention has been focused on research concerning delay-dependent stability because the stability criteria in stochastic singular systems are less conservative. There are many meaningful results recently about this topic have been obtained [5–7]. To our knowledge, solutions to the problems associated with stochastic stability for uncertain continuous singular systems with random process and time-varying delays still do not exist.

This study proposes new delay-dependent stability criteria for a class of stochastic singular systems with time-varying delays and uncertain parameters. To reduce the conservatism, we construct the appropriate Lyapunov-Krasovskii functionals, and then utilize the free-weighting-matrix approach and linear matrix inequality (LMI) technique based on an auxiliary vector function. The new delay-dependent stability criteria are derived to ensure the considered system is regular, impulse-free, and stochastically stable in the mean square.

Problem statement. Consider the stochastic singular system defined in a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$,

$$\begin{aligned} E\mathrm{d}x(t) &= ((A+\Delta A(t))x(t) + (A_d+\Delta A_d(t)) \\ &\quad \times x(t-d(t)))\mathrm{d}t + (J+\Delta J(t))x(t)\mathrm{d}\omega(t), \\ x(t) &= \phi(t), \quad t\in[-d_0, 0]\,, \end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, the matrix $E\in\mathbb{R}^{n\times n}$ maybe singular, and we assume $\mathrm{rank}(E) = r \leqslant n$. A, A_d, J are known constant matrices with appropriate dimensions. $\omega(t)$ is a one-dimensional standard Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathcal{P})$, which satisfies $\mathscr{E}\{\mathrm{d}\omega(t)\} = 0$, $\mathscr{E}\{\mathrm{d}\omega^2(t)\} = \mathrm{d}t$. $\Delta A(t)$, $\Delta A_d(t)$ and $\Delta J(t)$ are uncertainties in system matrices of the form

$$[\Delta A(t)\ \Delta A_d(t)\ \Delta J(t)] = MF_1(t)\,[N_A\ N_d\ N_J], \tag{2}$$

where M, N_A, N_d, and N_J are known real constant matrices. The time-varying nonlinear function $F_1(t)$ satisfies $F_1^{\mathrm{T}}(t)F_1(t)\leqslant I$. $\varphi(t)$ is the initial condition that relates to the time-varying delay $d(t)$, satisfying for all $t\geqslant 0$, $0\leqslant d(t)\leqslant d_0$, $\dot{d}(t)\leqslant \bar{d}\leqslant 1$, where d_0 and $\bar{d}$ are scalars.

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Assumption 1. $\mathrm{rank}([E\ \ J + MF_1(t)N_J]) = \mathrm{rank}(E)$.

Main results. An auxiliary vector function $\eta(t)$ is defined such that

$$\eta(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - d(t)). \quad (3)$$

Using (1), we can obtain the following integral equality:

$$Ex(t) - Ex(t - d(t)) = \int_{t-d(t)}^{t} \eta(s)\mathrm{d}s + \int_{t-d(t)}^{t} (J + \Delta J(t))x(s)\mathrm{d}\omega(s). \quad (4)$$

Theorem 1. For a scalar $\bar{d} > 0$, the system (1) is regular, impulse-free, and stochastically stable in the mean square if there exist matrices P, $Q > 0$, $Q_1 > 0$, $Z > 0$, $\hat{M}$, $\hat{N}$ and real numbers $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$ such that

$$E^{\mathrm{T}}P = P^{\mathrm{T}}E \geqslant 0, \quad (5)$$

$$E^{\mathrm{T}}P = E^{\mathrm{T}}Q_1E, \quad (6)$$

$$\Pi = \begin{bmatrix} \Phi_1 & \Phi_2 \\ * & \Phi_3 \end{bmatrix} < 0, \quad (7)$$

where $\Phi_3 = \mathrm{diag}\{\Lambda_6, \Lambda_7\}$,

$$\Phi_1 = \begin{bmatrix} \Lambda_1 & \Lambda_2 \\ * & \Lambda_3 \end{bmatrix}, \ \Phi_2 = \begin{bmatrix} \Lambda_4 & \Lambda_5 \\ 0 & 0 \end{bmatrix}, \ \Lambda_3 = \begin{bmatrix} -d_0^2 Z & 0 \\ * & -Z \end{bmatrix},$$

$$\Lambda_1 = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0 & E^{\mathrm{T}}\hat{M} \\ 0 & E^{\mathrm{T}}\hat{N} \end{bmatrix},$$

$$\Lambda_4 = \begin{bmatrix} J^{\mathrm{T}}(E^{+})^{\mathrm{T}}E^{\mathrm{T}} & P^{\mathrm{T}}M \\ 0 & 0 \end{bmatrix}, \ \Lambda_5 = \begin{bmatrix} P^{\mathrm{T}}M & N_J^{\mathrm{T}} \\ 0 & 0 \end{bmatrix},$$

$$\Lambda_6 = \mathrm{diag}\{\Theta_{31}, -\varepsilon_1 I\}, \ \Lambda_7 = \mathrm{diag}\{-\varepsilon_3 I, -\varepsilon_2 I\},$$

$$\Theta_{11} = A^{\mathrm{T}}P + P^{\mathrm{T}}A + \psi_{11},$$

$$\Theta_{12} = P^{\mathrm{T}}A_d + E^{\mathrm{T}}(\hat{M} - \hat{N}^{\mathrm{T}})E,$$

$$\psi_{11} = Q - E^{\mathrm{T}}(\hat{M} + \hat{M}^{\mathrm{T}})E + \varepsilon_1 N_A^{\mathrm{T}}N_A,$$

$$\Theta_{22} = -(1 - \bar{d})Q + E^{\mathrm{T}}(\hat{N} + \hat{N}^{\mathrm{T}})E + \varepsilon_3 N_d^{\mathrm{T}}N_d,$$

$$\Theta_{31} = -\hat{Q} + \varepsilon_2 EE^{+}MM^{\mathrm{T}}(E^{+})^{\mathrm{T}}E^{\mathrm{T}},$$

$$\hat{Q} = Q_1^{-1}.$$

Proof. First, we prove the system (1) is regular and impulse-free. Under Assumption 1, if $\mathrm{rank}(E) = r$, there are nonsingular matrices U and V such that

$$UEV = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad UAV = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

$$UA_dV = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix}, \ U^{-\mathrm{T}}PV = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}. \quad (8)$$

From (5), it follows that

$$V^{\mathrm{T}}E^{\mathrm{T}}U^{\mathrm{T}}U^{-\mathrm{T}}PV = V^{\mathrm{T}}P^{\mathrm{T}}U^{-1}UEV \geqslant 0.$$

Then, we have $P_{11} = P_{11}^{\mathrm{T}}$, $P_{12} = 0$.

From (7), it can be implied that

$$A^{\mathrm{T}}P + P^{\mathrm{T}}A - E^{\mathrm{T}}(\hat{M} + \hat{M}^{\mathrm{T}})E < 0. \quad (9)$$

Thus,

$$V^{\mathrm{T}}A^{\mathrm{T}}U^{\mathrm{T}}U^{-\mathrm{T}}PV + V^{\mathrm{T}}P^{\mathrm{T}}U^{-1}UAV - V^{\mathrm{T}}E^{\mathrm{T}}U^{\mathrm{T}}U^{-\mathrm{T}}(\hat{M} + \hat{M}^{\mathrm{T}})U^{-1}UEV < 0, \quad (10)$$

that is

$$\begin{bmatrix} \otimes & \tilde{\otimes} \\ * & (A_{22}^{\mathrm{T}}P_{22} + P_{22}^{\mathrm{T}}A_{22}) \end{bmatrix} < 0. \quad (11)$$

Because $\otimes$ and $\tilde{\otimes}$ are irrelevant to the results of the following discussion, the real expression of these two variables are omitted here. According to the expression (11), it is easy to see that $A_{22}^{\mathrm{T}}P_{22} + P_{22}^{\mathrm{T}}A_{22} < 0$, which implies that A_{22} is nonsingular. Thus, the pair (E, A) is regular and impulse-free.

In addition, from the expression (7), we have

$$\Lambda_1 < 0. \quad (12)$$

Pre- and post-multiplying (12) by $[I\ \ I]$ and its transpose, we can easily obtain

$$(A^{\mathrm{T}} + A_d^{\mathrm{T}})P + P^{\mathrm{T}}(A + A_d) + \bar{d}Q < 0.$$

Using a similar approach as mentioned above, we have

$$\begin{bmatrix} \star & \tilde{\star} \\ * & (A_{22}^{\mathrm{T}} + A_{d22}^{\mathrm{T}})P_{22} + P_{22}^{\mathrm{T}}(A_{22} + A_{d22}) \end{bmatrix} < 0. \quad (13)$$

Because $\star$ and $\tilde{\star}$ are irrelevant to the results of the following discussion, the real expression of these two variables have been excluded. From (13), it can be easily seen that $(A_{22}^{\mathrm{T}} + A_{d22}^{\mathrm{T}})P_{22} + P_{22}^{\mathrm{T}}(A_{22} + A_{d22}) < 0$, which implies that the pair $(E, A + A_d)$ is regular and impulse-free. Therefore, the system (1) is regular and impulse-free for any time-varying delay $d(t)$ satisfying $0 \leqslant d(t) \leqslant d_0$.

A candidate Lyapunov-Krasovskii functional is then constructed as follows:

$$V(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t)), \quad (14)$$

where $V_1(x(t)) = x^{\mathrm{T}}(t)E^{\mathrm{T}}Px(t)$,

$$V_2(x(t)) = \int_{t-d(t)}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s,$$

$$V_3(x(t)) = d_0 \int_{-d_0}^{0} \int_{t+\theta}^{t} \eta^{\mathrm{T}}(s)Z\eta(s)\mathrm{d}s\mathrm{d}\theta.$$

The stochastic derivative of $V(x(t))$ along the trajectory of the system (1) can be obtained as follows:

$$\begin{aligned}\mathrm{d}V(x(t)) &= \mathscr{L}V(x(t))\mathrm{d}t \\ &\quad + 2x^{\mathrm{T}}(t)P^{\mathrm{T}}(J+\Delta J)x(t)\mathrm{d}\omega(t), \quad (15)\end{aligned}$$

where

$$\mathscr{L}V(x(t)) = \mathscr{L}V_1(x(t)) + \mathscr{L}V_2(x(t)) + \mathscr{L}V_3(x(t)).$$

Based on Proposition 2.1 in [8] for $\mathscr{L}V_1(x(t))$, using (6) and $\hat{Q} = Q_1^{-1}$, there exist real numbers $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$ such that

$$\begin{aligned}\mathscr{L}V_1(x(t)) \leqslant\ & x^{\mathrm{T}}(t)(P^{\mathrm{T}}A + A^{\mathrm{T}}P + \varepsilon_1^{-1}P^{\mathrm{T}}MM^{\mathrm{T}}P \\ & + \varepsilon_1 N_A^{\mathrm{T}}N_A + (EE^+J)^{\mathrm{T}}(\hat{Q} - \varepsilon_2 EE^+ \\ & \times MM^{\mathrm{T}}(E^+)^{\mathrm{T}}E^{\mathrm{T}})^{-1}EE^+J \\ & + \varepsilon_2^{-1}N_J^{\mathrm{T}}N_J + \varepsilon_3^{-1}P^{\mathrm{T}}MM^{\mathrm{T}}P)x(t) \\ & + x^{\mathrm{T}}(t)(P^{\mathrm{T}}A_d + A_d^{\mathrm{T}}P)x(t-d(t)) \\ & + x^{\mathrm{T}}(t-d(t))\varepsilon_3 N_d^{\mathrm{T}}N_d x(t-d(t)); \\ \mathscr{L}V_2(x(t)) \leqslant\ & x^{\mathrm{T}}(t)Qx(t) \\ & - (1-\bar{d})x^{\mathrm{T}}(t-d(t))Qx(t-d(t)); \\ \mathscr{L}V_3(x(t)) \leqslant\ & d_0^2\eta^{\mathrm{T}}(t)Z\eta(t) \\ & - \int_{t-d(t)}^{t}\eta^{\mathrm{T}}(s)\mathrm{d}sZ\int_{t-d(t)}^{t}\eta(s)\mathrm{d}s.\end{aligned}$$

From (4), for any matrices $\hat{M}$, $\hat{N}$, we have

$$\begin{aligned}0 &= 2[x^{\mathrm{T}}(t)E^{\mathrm{T}}\hat{M} + x^{\mathrm{T}}(t-d(t))E_e^{\mathrm{T}}\hat{N}] \\ &\quad \cdot \left[\int_{t-d(t)}^{t}\eta(s)\mathrm{d}s + \int_{t-d(t)}^{t}(J+\Delta J)x(s)\mathrm{d}\omega(s)\right. \\ &\quad \left. - Ex(t) + Ex(t-d(t))\right]. \quad (16)\end{aligned}$$

Furthermore, from the expressions (15) and (16), we have

$$\begin{aligned}\mathrm{d}V(x(t)) &= \mathcal{L}\tilde{V}(x(t))\mathrm{d}t \\ &\quad + 2x^{\mathrm{T}}(t)P^{\mathrm{T}}(J+\Delta J)x(t)\mathrm{d}\omega(t) \\ &\quad + 2[x^{\mathrm{T}}(t)E^{\mathrm{T}}\hat{M} + x^{\mathrm{T}}(t-d(t))E^{\mathrm{T}}\hat{N}] \\ &\quad \times \int_{t-d(t)}^{t}(J+\Delta J)x(s)\mathrm{d}\omega(s),\end{aligned}$$

where $\mathcal{L}\tilde{V}(x(t)) \leqslant \xi^{\mathrm{T}}(t)\Theta\xi(t)$, and

$$\xi^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-d(t))\ \eta^{\mathrm{T}}(t)\ \int_{t-d(t)}^{t}\eta^{\mathrm{T}}(s)\mathrm{d}s\right],$$

$$\Theta = \begin{bmatrix}\tilde{\Theta}_{11} & \Theta_{12} & 0 & E^{\mathrm{T}}\hat{M} \\ * & \Theta_{22} & 0 & E^{\mathrm{T}}\hat{N} \\ * & * & -d_0^2 Z & 0 \\ * & * & * & -Z\end{bmatrix},$$

$$\begin{aligned}\tilde{\Theta}_{11} &= A^{\mathrm{T}}P + P^{\mathrm{T}}A + \psi_{11} + \varepsilon_1^{-1}P^{\mathrm{T}}MM^{\mathrm{T}}P \\ &\quad + \varepsilon_3^{-1}P^{\mathrm{T}}MM^{\mathrm{T}}P + (EE^+J)^{\mathrm{T}}(\hat{Q} - \varepsilon_2 EE^+ \\ &\quad \times MM^{\mathrm{T}}(E^+)^{\mathrm{T}}E^{\mathrm{T}})^{-1}EE^+J + \varepsilon_2^{-1}N_J^{\mathrm{T}}N_J.\end{aligned}$$

For the condition (7), based on the Schur complement lemma, we have $\Theta < 0$. Thus,

$$\begin{aligned}&\mathscr{E}\{\mathcal{L}V(x(t))\} \leqslant \mathscr{E}\{\mathcal{L}\tilde{V}(x(t))\} \\ &\quad \leqslant -\lambda_{\max}(\Theta)\|\xi(t)\|^2 \leqslant -\lambda_{\max}(\Theta)\|x(t)\|^2.\end{aligned}$$

Therefore, system (1) is stochastically stable in the mean square. This completes the proof.

Conclusion. We discussed the stochastic stability problem of stochastic singular systems with time-varying delays and uncertain parameters, and a new stochastic stability solution was proposed. The results of our proposed solution can be further extended to stochastic singular nonlinear systems with time-varying delays.

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