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Exponential tracking of adaptive control systems

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Dear editor,

In the last decades, extensive efforts have been focused on the adaptive control problem of uncertain nonlinear systems (e.g., [1–5]). In this study, the problem of exponential tracking control is addressed for a class of nonlinear systems with parametric uncertainty. A design method based on adaptive control, capable of guaranteeing the exponential tracking with zero tracking error, is proposed. In the control design, the following techniques are applied: (a) several technical lemmas which can guarantee the exponential tracking are introduced; (b) two exponential functions with different exponent constants are incorporated into the control law and adaptive law, respectively; (c) all the unknown parameters in the controlled system are lumped together, and only one integrated parameter is adaptively updated. The presented controller has simple structure and the estimated parameter is carefully chosen. To ensure that the control function avoids singularity and remains smooth enough, the hyperbolic tangent function is incorporated into the control law. For stability analysis, the Lyapunov function weighted by an exponential function is appropriately constructed. It is shown that the presented robust adaptive scheme can guarantee the boundedness of all the closed-loop system signals and the convergence of the tracking error to zero exponentially fast. A simulation example is provided to clarify and verify the proposed approach.

Problem formulation. Consider the following nonlinear systems:

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) = bu(t), \tag{1}$$

where $x(t)$ is system state, $u(t)$ is control input, a_i are unknown constants, Y_i are known smooth functions, and control gain b is unknown constant.

The control objective is to design a control law $u(t)$ such that all the closed-loop signals are bounded, while the system state vector $X = [x, \dot{x}, \ldots, x^{(n-1)}]^{\mathrm{T}}$ exponentially tracks a specified desired trajectory $X_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]^{\mathrm{T}}$ with zero tracking error, where $x_d(t)$ is a given reference signal. To this end, we make Assumptions 1 and 2.

Assumption 1. The reference signal $x_d(t)$ and its first n derivatives are known and bounded.

Assumption 2. The sign of uncertain parameter b is known. Here, we assume that $b > 0$.

Preliminary results.

Lemma 1 (Finsler's theorem [6])**.** Let X be an $n \times n$ positive-semidefinite symmetric matrix and let Y be an $n \times n$ symmetric matrix such that $z^{\mathrm{T}} Y z > 0$ for all $z \in \mathbb{R}^n$ satisfying $z \neq 0$, $z^{\mathrm{T}} X z = 0$. Then there exists a constant $\tau > 0$ such that the matrix $Y + \tau X$ is positive definite.

Lemma 2. For any given $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$ and $\sigma \in \mathbb{R}$, a gain matrix K can be chosen such that the inequality

$$(A + BK)^{\mathrm{T}} P + P(A + BK) < -2\sigma P \tag{2}$$

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has a symmetric positive definite solution $P \in \mathbb{R}^{n\times n}$ if and only if there exists a constant $\tau > 0$ such that $P > 0$ satisfies the following inequality:

$$PA + A^{\mathrm{T}}P + 2\sigma P - \tau PBB^{\mathrm{T}}P < 0. \quad (3)$$

The proof of Lemma 2 is provided in Appendix A.

Lemma 3. For given matrixes A, B and constant σ, if there exists a symmetric positive definite matrix S and a positive constant τ such that the following linear matrix inequality (LMI) holds:

$$AS + SA^{\mathrm{T}} + 2\sigma S - \tau BB^{\mathrm{T}} < 0, \quad (4)$$

then we can choose $P = S^{-1}$, $K = -\frac{\tau}{2}B^{\mathrm{T}}P$ such that (2) is satisfied. The proof of Lemma 3 is provided in Appendix B.

Lemma 4 ([7])**.** Let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $\sigma \in \mathbb{R}$. If the pair (A, B) is controllable, then the pair $(A + \sigma I, B)$ is controllable.

Lemma 5. If the pair (A, B) is controllable, and $\sigma > 0$ is a constant, then there exists a constant matrix K and a symmetric positive definite matrix P such that the inequality (2) holds. The proof of Lemma 5 is provided in Appendix C.

Remark 1. In Lemma 2, a sufficient and necessary condition is provided to judge the existence of the pair (P, K) satisfying the inequality (2). To be more important, in Lemma 5, the condition (3) can be transformed into the form of LMIs: (4), $S > 0$ and $\tau > 0$, and the pair (P, K) is clearly specified based on the solutions of LMIs.

Remark 2. Compared with Lemma 2, in Lemma 5 the pair (A, B) is restricted to be controllable. In this case, the pair (P, K) satisfying (2) does exists. On the other hand, it follows from Lemma 2 that the existence of (P, K) indicates that the pair (P, τ) with constant $\tau > 0$ satisfies (3). Therefore, when (A, B) is controllable, the pair (P, K) can be derived in two ways. One way is based on the proof process of Lemma 5, while the other way is to solve LMIs according to Lemma 3.

Lemma 6 ([7])**.** Let $g, V : [0, \infty) \mapsto \mathbb{R}$. Then $\dot{V} \leqslant -\sigma V + g$, $\forall t \geqslant 0$, implies that $V(t) \leqslant \exp(-\sigma t)V(0) + \int_0^t \exp[-\sigma(t-\tau)]g(\tau)\mathrm{d}\tau$, $\forall t \geqslant 0$ for any finite constant σ.

Lemma 7. If, in Lemma 6, $\sigma > 0$, $g(t)$ is chosen as $g(t) = l\exp(-\lambda t)$, where l, λ are positive constants, and $\lambda > \sigma$, then $V(t) \leqslant (V(0) + \frac{l}{\lambda-\sigma})\exp(-\sigma t)$, $\forall t \geqslant 0$. The proof of Lemma 7 is provided in Appendix D.

Lemma 8 ([8])**.** For any $\epsilon > 0$ and any $\eta \in \mathbb{R}$, the inequality $0 \leqslant |\eta| - \eta\tanh(\frac{\eta}{\epsilon}) \leqslant \kappa\epsilon$ holds, where κ is a constant satisfying $\kappa = \mathrm{e}^{-(\kappa+1)}$, that is, $\kappa = 0.2785$.

Adaptive control design and analysis. Let the tracking error $e = X - X_d$, which together with (1) is governed by

$$\dot{e} = Ae + B\left[-\sum_{i=1}^{r} a_iY_i(X) + bu(t) - x_d^{(n)}\right], \quad (5)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \quad (6)$$

Owing to the structure of A, B, it is clear that the pair (A, B) is controllable. From the analysis in Remark 2, there are two ways to obtain the pair (P, K) so that the inequality (2) is satisfied for a given constant $\sigma > 0$. For clarity, we give Algorithms 1 and 2.

Algorithm 1

Step 1. Set σ to be a positive value and Q to be a positive definite matrix;
Step 2. Choose a constant matrix K such that $A+\sigma I+BK$ is stable;
Step 3. Solve the Lyapunov equation (C1) for P.

Algorithm 2

Step 1. Set σ to be a positive value;
Step 2. Solve the LMIs: (4), $S > 0$, $\tau > 0$ to obtain the pair (S, τ);
Step 3. Let $P = S^{-1}$, $K = -\frac{\tau}{2}B^{\mathrm{T}}P$.

Next, we define

$$f(X,t) = \sum_{i=1}^{r}\sqrt{|Y_i(X)|^2 + h_i} + \sqrt{\|e(t)\|^2 + h} + 1, \quad (7)$$

$$\theta = \max\left\{|a_1|, \ldots, |a_r|, \|K\|, \sup_{t\geqslant 0}\left|x_d^{(n)}\right|\right\},$$

$$\theta^* = \frac{\theta}{b}, \quad (8)$$

where h, h_i, $i = 1, 2, \ldots, r$, are positive design constants, and $\|\cdot\|$ denotes the Euclidean norm of a vector. Then, we propose the following control law and parameter update law:

$$u(t) = \frac{-e^{\mathrm{T}}PB\hat{\theta}^2(t)f^2}{e^{\mathrm{T}}PB\tanh[l^{-1}e^{\mathrm{T}}PB\exp(2\lambda t)]\hat{\theta}f + l\exp(-2\lambda t)}, \quad (9)$$

$$\dot{\hat{\theta}} = \gamma\exp(2\sigma t)|e^{\mathrm{T}}PB|f, \quad (10)$$

where f denotes $f(X,t)$, l, λ are positive design constants, $\gamma > 0$ is adaptive gain, λ satisfies $\lambda > \sigma$, $\hat{\theta}(t)$ is the estimate of θ^*, and $\hat{\theta}(0) \geqslant 0$. With the above design scheme, we can obtain the result on the system stability as stated below.

Theorem 1. Consider the closed-loop system consisting of system (1), control law (9) and adaptive law (10) based on Assumptions 1–2. Then, all the closed-loop signals remain bounded, and the tracking error converges to zero exponentially with the rate of not less than σ. The proof of Theorem 1 is provided in Appendix E.

Simulation study. Consider the uncertain nonlinear system studied in [9]

$$\ddot{x} = a_1 \frac{1-\exp(-x)}{1+\exp(-x)} - a_2(\dot{x}^2 + 2x)\sin(\dot{x}) - 0.5 a_3 x \sin(3t) + bu. \quad (11)$$

The plant parameters are given as $a_1 = a_2 = a_3 = b = 1$, which are assumed to be unknown. The initial condition is set to be $x(0) = -2.5$, $\dot{x}(0) = 3.5$. The reference signal is $x_d = 2.5\sin(t)$. From (6) and (7), it follows that

$$e = [e_1, e_2]^{\mathrm{T}}, \quad e_1 = x_1 - x_d, \quad e_2 = x_2 - \dot{x}_d,$$
$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$f(x_1, x_2, t) = \sqrt{\left[\frac{1-\exp(-x_1)}{1+\exp(-x_1)}\right]^2 + h_1} + \sqrt{[(x_2^2 + 2x_1)\sin(x_2)]^2 + h_2} + \sqrt{[0.5x_1 \sin(3t)]^2 + h_3} + \sqrt{\|e\|^2 + h} + 1,$$

where $x_1 = x$, $x_2 = \dot{x}$, and h_1, h_2, h_3, h are selected as $h_1 = h_2 = h_3 = h = 1$. Then, we apply the proposed control scheme to this example. Using Algorithm 2 with $\sigma = 0.01$, we have

$$S = \begin{bmatrix} 1.4183 & -0.4886 \\ -0.4886 & 0.4825 \end{bmatrix}, \quad \tau = 1.4327,$$
$$P = \begin{bmatrix} 1.0828 & 1.0965 \\ 1.0965 & 3.1832 \end{bmatrix}, \quad K = [-0.7855 - 2.2803].$$

According to Theorem 1, the control law and adaptive law can be derived. The design parameters are chosen as $\gamma = 4$, $l = 100$, $\lambda = 0.1$, $\hat{\theta}(0) = 0$. The simulation results are shown in Figure F1 (see Appendix F). From Figure F1(b), we can see that the tracking error converges to zero rapidly. At the same time, the boundedness of control signal u is shown in Figure F1(d). The boundedness of other signals including plant state x and parameter estimate $\hat{\theta}$ is revealed in Figure F1(a) and (c), respectively.

Conclusion. The problem of adaptive tracking control for a class of uncertain nonlinear systems has been considered. We have proposed a class of adaptive controllers for tracking of dynamical signals. We have shown that by employing the presented adaptive tracking controller, the tracking error can be guaranteed to decrease to zero exponentially. In view of simple control structure, the least number of the estimated parameter and exact output tracking, the proposed controller may be applied to the practical control problems in order to obtain better control effects.

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Supporting information Appendixes A–F. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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