**SCIENCE CHINA**
Information Sciences

• **Supplementary File** •

# A Distributed Consensus Filter for Sensor Networks with Heavy-tailed Measurement Noise

Peng DONG*, Zhongliang JING*, Kai SHEN & Minzhe LI

*School of Aeronautics and Astronautics, Shanghai Jiao Tong University, Shanghai,* 200240*, China*

## Appendix A

The target dynamic consists of the state $x = [p_x, \dot{p}_x, p_y, \dot{p}_y]^T$ which can be modeled by Eq. (1) according to

$$F_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, Q_k = G_k \Delta G_k^T, \tag{A1}$$

where $\Delta = \mathrm{diag}([w_x^2, w_y^2])$, $w_x^2 = w_y^2 = 0.1$, sample time $T = 1\mathrm{s}$ and

$$G_k = \begin{bmatrix} T^2/2 & T & 0 & 0 \\ 0 & 0 & T^2/2 & T \end{bmatrix}^T, \tag{A2}$$

The target trajectory (see Figure A1) is generated by the above model with the following true initial state

$$x_0 = [2600\mathrm{m}, 20\mathrm{m/s}, 3800\mathrm{m}, 10\mathrm{m/s}]^{\mathrm{T}}.$$

In the simulations, initial states for filters are chosen randomly from $\mathrm{N}(x_0, P_0)$ in each turn, where

$$P_0 = \mathrm{diag}([50^2\mathrm{m}^2, 5^2\mathrm{m}^2/\mathrm{s}^2, 50^2\mathrm{m}^2, 5^2\mathrm{m}^2/\mathrm{s}^2]).$$

There are 20 sensor nodes in the sensor network of which the graphical topology representation is shown in Figure A1. The measurement model is given by Eq. (2) and

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{A3}$$

Suppose we have a nominal measurement noise variance $R = \mathrm{diag}([(10\mathrm{m})^2, (10\mathrm{m})^2])$, then the heavy-tailed measurement noise of sensor node $i$ is generated by a mixture of Gaussian according to

$$r_k^i \sim \begin{cases} \mathrm{N}(0, R), & \text{with probability } 1 - p_o \\ \mathrm{N}(0, 100R), & \text{with probability } p_o \end{cases}, \tag{A4}$$

where $p_o = 0.1$ is the probability of the measurement outlier. It is wildly used to evaluate the performance of Student-$t$ based filters.

The consensus step is $L = 3$ and $\Lambda_k^i = R^{-1}$. The consensus weights of sensor nodes are set to $\pi^{i,j} = 1/|\mathcal{N}^i|$ if $j \in \mathcal{N}^i$ and $\pi^{i,j} = 0$ if $j \notin \mathcal{N}^i$. The DOFs of the proposed filter are set to $\nu^i = 15$. The fixed iteration step of VB is set to $M = 4$ and it is enough to achieve convergence.

The proposed method is summarized in Table A1. Simulations results are given in Figures A2-A4 and Table A2.

* Corresponding author (email: dongpengkty@sjtu.edu.cn, zljing@sjtu.edu.cn)

**Table A1** The proposed DVBSCI algorithm

**Update:**

For each node $i \in \mathcal{N}$, set $\bar{\lambda}_k^{i,0} = 1$, given $\Omega^i_{k|k-1}$ and $q^i_{k|k-1}$, run VB iteration:

**for** $q = 1$ **to** $M$

(1) Obtain the local posterior information pair:

$\bar{L}_k^{i,q} = \bar{\lambda}_k^{i,q-1} \Lambda_k^i$

$\Omega_{k,0}^{i,q} = \Omega^i_{k|k-1} + \left(H_k^i\right)^T \bar{L}_k^i H_k^i$

$q_{k,0}^{i,q} = q^i_{k|k-1} + \left(H_k^i\right)^T \bar{L}_k^i z_k^i$

(2) Consensus:

**for** $l = 1$ **to** $L$

$q_{k,l}^{i,q} = \sum_{j \in \mathcal{N}^i} \pi^{i,j} q_{k,l-1}^{j,q}$, $\Omega_{k,l}^{i,q} = \sum_{j \in \mathcal{N}^i} \pi^{i,j} \Omega_{k,l-1}^{j,q}$

**end for**

set $q_k^{i,q} = q_{k,L}^{i,q}$ and $\Omega_k^{i,q} = \Omega_{k,L}^{i,q}$

(3) Evaluate sufficient statistic of measurement distribution:

$\hat{x}_k^{i,q} = \Omega_k^{i,q} q_k^{i,q}$, $P_k^{i,q} = (\Omega_k^{i,q})^{-1}$

$\hat{z}_k^{i,q} = H_k^i \hat{x}_k^{i,q}$

$D_k^{i,q} = H_k^i P_k^{i,q} (H_k^i)^T + (z_k^i - \hat{z}_k^{i,q})(z_k^i - \hat{z}_k^{i,q})^T$

$\bar{\gamma}_k^{i,q} = \mathrm{tr}(D_k^{i,q} \Lambda_k^i)$

$a_k^{i,q} = \frac{\nu^i + d}{2}$, $b_k^{i,q} = \frac{\nu^i + \bar{\gamma}_k^{i,q}}{2}$

$\bar{\lambda}_k^{i,q} = \frac{a_k^{i,q}}{b_k^{i,q}}$

**end for**

Set $\hat{x}_k^i = \hat{x}_k^{i,M}$, $P_k^i = P_k^{i,M}$, $\Omega_k^i = \Omega_k^{i,M}$, $q_k^i = q_k^{i,M}$.

**Prediction:**

For each node $i \in \mathcal{N}$, compute $q^i_{k+1|k}$, $\Omega^i_{k+1|k}$ via Eq. (14) and Eq. (15).

**Table A2** RMSE of position and velocity for different $p_o$

| $p_o$ | DCI filter | | DVBSCI filter | |
|---|---|---|---|---|
| | Position (m) | Velocity (m/s) | Position (m) | Velocity (m/s) |
| 0 | 3.8471 | 0.9669 | 4.0887 | 1.0202 |
| 0.1 | 7.5076 | 1.2899 | 4.3926 | 1.0517 |
| 0.2 | 9.7272 | 1.5345 | 4.6203 | 1.0849 |
| 0.3 | 11.3421 | 1.7150 | 4.9282 | 1.1272 |
| 0.4 | 12.9377 | 1.9119 | 5.3105 | 1.1777 |

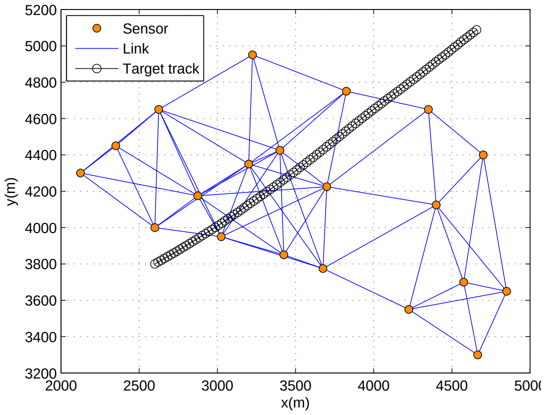


**Figure A1** Topology of the distributed sensor network and track of target.

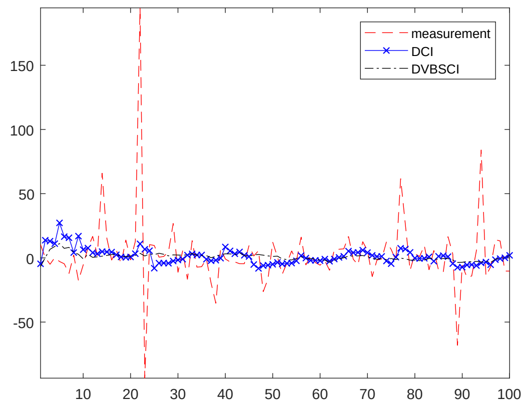


**Figure A2** Position errors of x-axis in a typical run for sensor node 1.

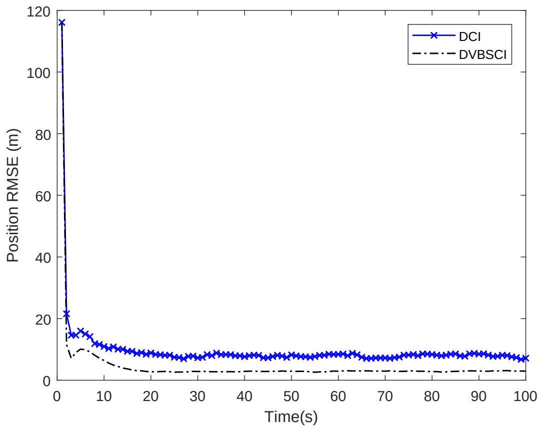


**Figure A3** RMSE of position over time.

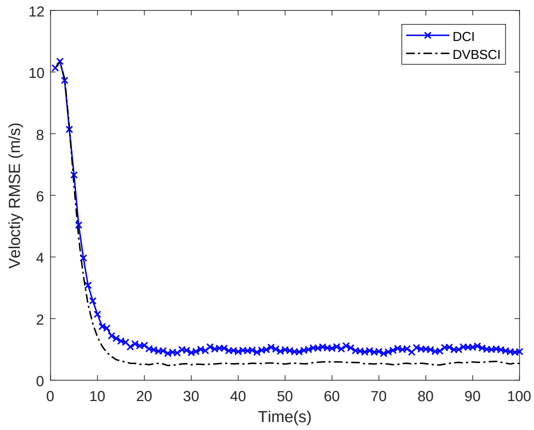


**Figure A4** RMSE of velocity over time.