Adaptive Narrow Band MultiFLIP for Efficient Two-Phase Liquid Simulation

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Outline

- Introduction
- Our Method
  - Narrow band MultiFLIP
  - Variable Coefficient Poisson equation on octrees
- Results
- Conclusion and future work

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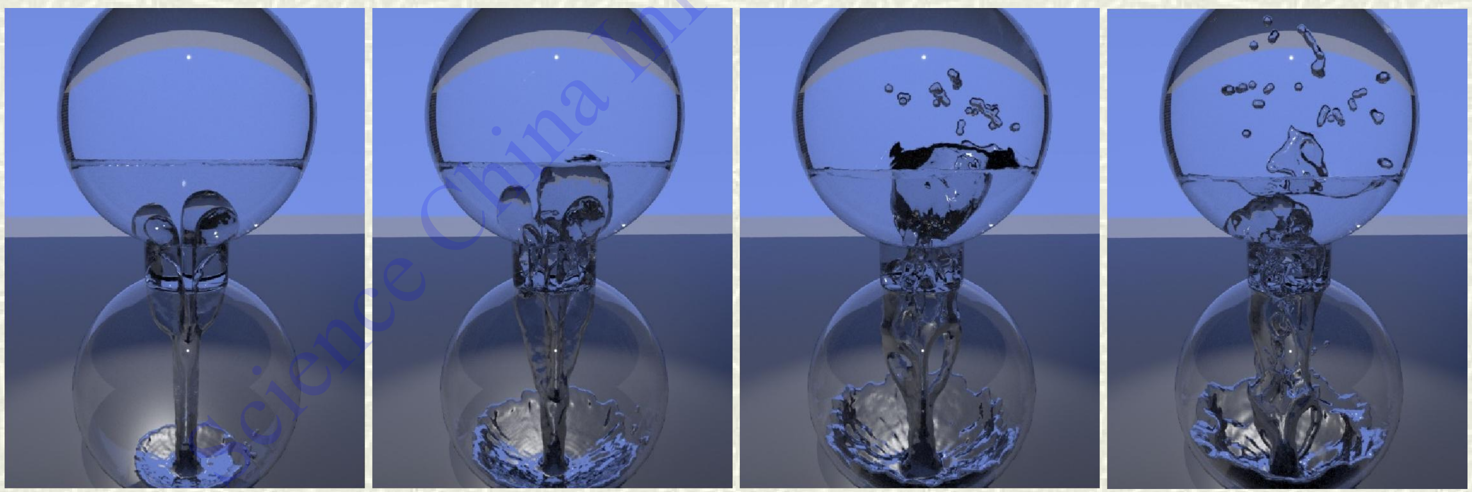
Introduction

- Liquid-gas interactions produce many interesting phenomena such as droplets, bubbles, the "glugging" effect of water pouring, and even the sound of bubbles. which are important in many fluid animation scenarios.



Introduction

- For the sake of computational efficiency, one-phase methods only simulate the liquid phase with the assumption of free-boundary conditions at the interface of gas and liquid.
- In order to simulate two-phase phenomena, Boyd and Bridson proposed the MultiFLIP method, but MultiFLIP has two challenges :
  - Gas and liquid particles are required to fill the gas and liquid volumes;
  - A variable Poisson equation needs to be solved on the entire domain.

Introduction

- We present two techniques to address the two challenges of MultiFLIP.
  - Firstly, only two narrow bands near the embedded interface are used to track the interface. This significantly reduce the number of particles from $8n^3$ to O($8n^2$).
  - Secondly, an octree structure is constructed to adaptively discretize the variable Poisson equation with refined grids near the interface. It is found that the number of the unknown variables can be efficiently reduced from n^3 to O($n^2 logn$).

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Our Method

- Our method is based on the inviscid, incompressible Navier-Stokes equations,

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \frac{1}{\rho} g, \nabla \cdot \mathbf{u} = 0,$$

  where $\mathbf{u}$ is velocity, p is pressure, ρ is density and g is external force such as gravity.

- We solve a variable coefficient Poisson equation,

$$\nabla \cdot \frac{1}{\rho}\nabla p = \nabla \cdot \left(\mathrm{u}_g^* + \mathrm{u}_l^*\right),$$

$$\mathrm{u}_g = \mathrm{u}_g^* - \left(\tfrac{1}{\rho}\nabla p\right)|_g, \mathrm{u}_l = \mathrm{u}_l^* - \left(\tfrac{1}{\rho}\nabla p\right)|_l,$$

  where $\left(\frac{1}{\rho}\nabla p\right)\Big|_g$, $\left(\frac{1}{\rho}\nabla p\right)\Big|_l$ denote the restriction of $\frac{1}{\rho}\nabla p$ to the gas and liquid domains.

Our Method

- Algorithm (MutliFLIP vs. OctNB-MultiFLIP)
  - Advect the gas and liquid particles
  - Track gas-liquid interface
    - [MultiFLIP] Full resolution interface tracking
    - [OctNB-MultiFLIP] Narrow band interface tracking $\Rightarrow \phi$
  - [OctNB-MultiFLIP] Advect $u_g, u_l \Rightarrow u'_g, u'_l$
  - Map gas and liquid particles to grid $\Rightarrow u_g^p$ and u_l^p
  - Update velocity on grid
    - [MultiFLIP] $u_g^* \leftarrow u_g^p, u_l^* \leftarrow u_l^p$
    - [OctNB-MultiFLIP] $u_g^* \leftarrow nbcombine(u_g^p, u'_g, \phi), u_l^* \leftarrow ubcombine(u_l^p, u'_l, \phi)$
  - Bump and resample particles
  - Handle escaped particles
  - Add external forces g
  - Compute the curvature field
  - Project $u_g^*, u_l^* \Rightarrow u_g, u_l$
    - [MultiFLIP] Project on full grid
    - [OctNB-MultiFLIP] Project on adaptive octree
  - Update particle velocity

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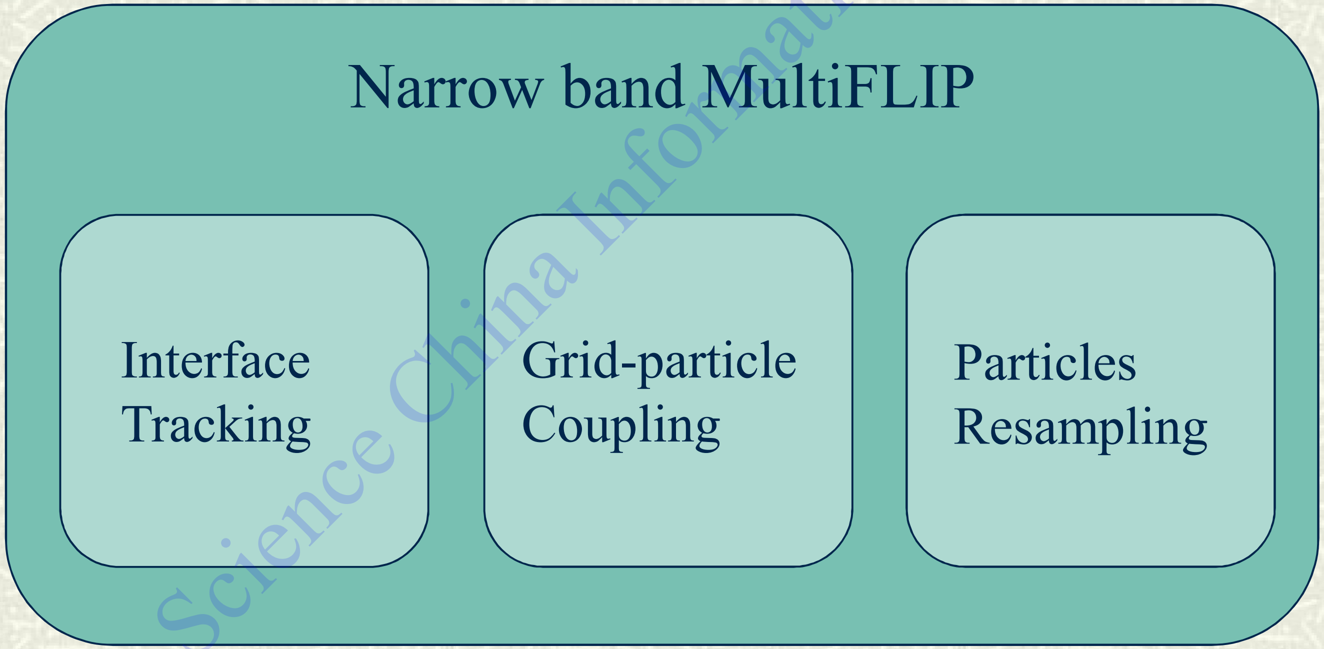
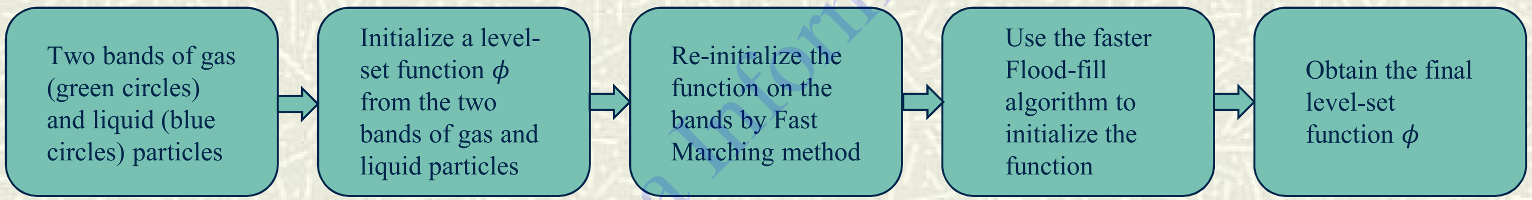
Our Method

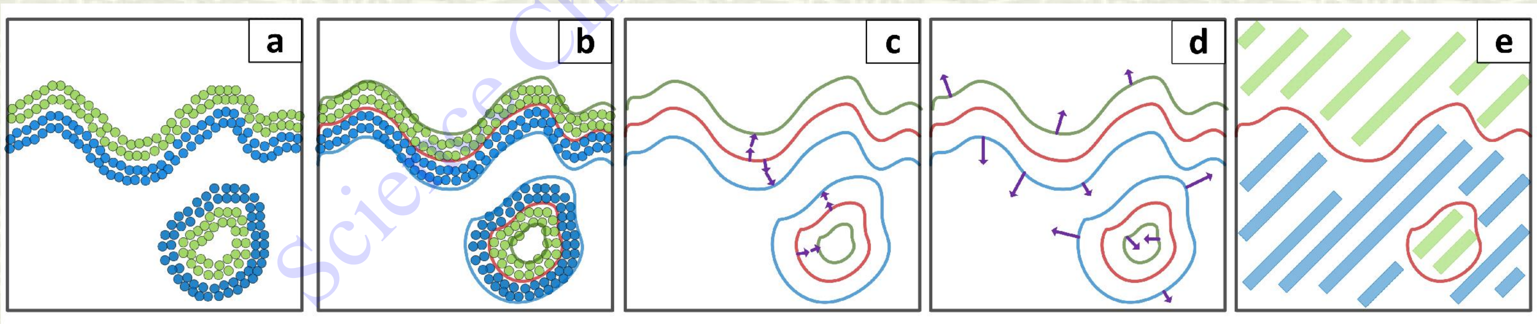
- Narrow band MultiFLIP has three steps:



Our Method

- The figure below illustrates the substeps of our interface tracking.





Our Method

- After advecting the gas and liquid particles, we map them to grid and obtain the gas and liquid velocity: u_g^p and u_l^p.
- We also advect u_g (u_l) on the entire domain by performing a semi-Lagrangian advection step, which yields advected grid velocity u_g' (resp. u_l').
- Finally, we combine the velocities:

$$u_l^* = \begin{cases} u_l^p, if -r \leq \phi \leq 0 \\ u_l', otherwise \end{cases}, u_g^* = \begin{cases} u_g^p, if\ 0 \leq \phi \leq r \\ u_g', otherwise \end{cases},$$

where the distance $r = 2h$ with h being the width of one cell and ϕ is the level-set function obtained in the step of interface tracking.

Our Method

- As the particles being advected, particles would move out or accumulate at the bands, leading to the number of particles in some cell of the bands less (or more) than n (resp. $2n$) (the default $n = 8$).
- To address these issues, we delete the particles outside of the bands or in cells with more than $2n$ particles, and add new particles to cells with less than n particles.

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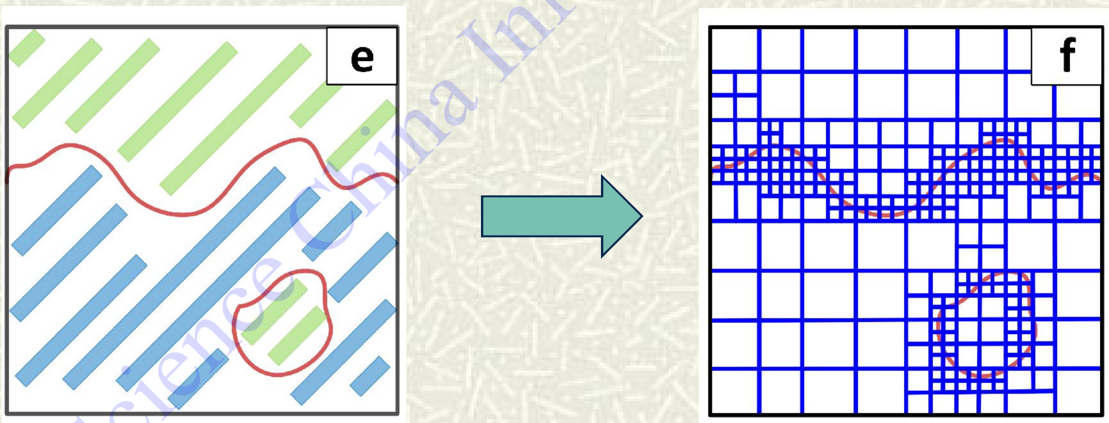
Our Method

- We construct an octree with refined grid cells near the gas-liquid interface according to the constructed level set function.



Our Method

- We discretize the variable coefficient Poisson equation on the octree
- By the divergence theorem, for a cell i, the left term of variable coefficient Poisson equation is discretized as follow

$$V_i \nabla \cdot \frac{1}{\rho} \nabla p = \sum_k A_{i,k} \frac{1}{\rho_{i,k}} \frac{p_i - p_k}{\Delta_{i,k}}$$

where V_i is the volume of i cell ; $A_{i,k}$ and $\Delta_{i,k}$ are the area and distance between i and k cell; p_i and p_k denote the pressure stored in the centers of i cell and its neighbor cells,

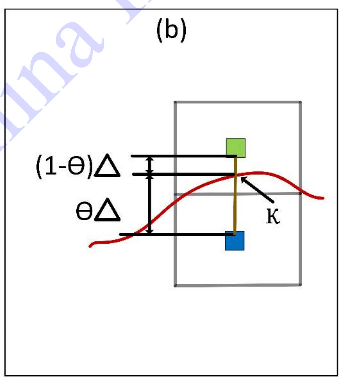
- For the liquid or gas domain, we similarly discretize the divergence of i cell in the right term of variable Poisson equation as follow

$$V_i \nabla \cdot \mathrm{u}^* = \sum_k A_{i,k} (\mathrm{u}^* \cdot \mathrm{n}_{i,k})$$

where $\mathrm{u}^* = \mathrm{u}_g^*$ for the gas or u_l^* for the liquid, and $\mathrm{n}_{i,k}$ is the outer-pointing normal of face $A_{i,k}$. For the domain occupied by both the gas and liquid, we use the same scheme as MultiFlIP to discretize the divergence.

Our Method

- In order to simulate the surface tension, we add $\frac{1}{\rho_{i,k}}\frac{\sigma\kappa}{\Delta_{i,k}^2}$ to the right side of variable coefficient Poisson equation. σ is the surface tension coefficient for the interface and κ is the curvature of the interface. Since κ only has non-zero values on the interface, $\frac{1}{\rho_{i,k}}\frac{\sigma\kappa}{\Delta_{i,k}^2}$ only affects the two cells crossed by the interface with $\rho_{i,k} = \theta\rho_l + (1-\theta)\rho_g$.



- Combining all these terms into one equation, we get a sparse positive semi-definite linear equation, which is solved by PCG.

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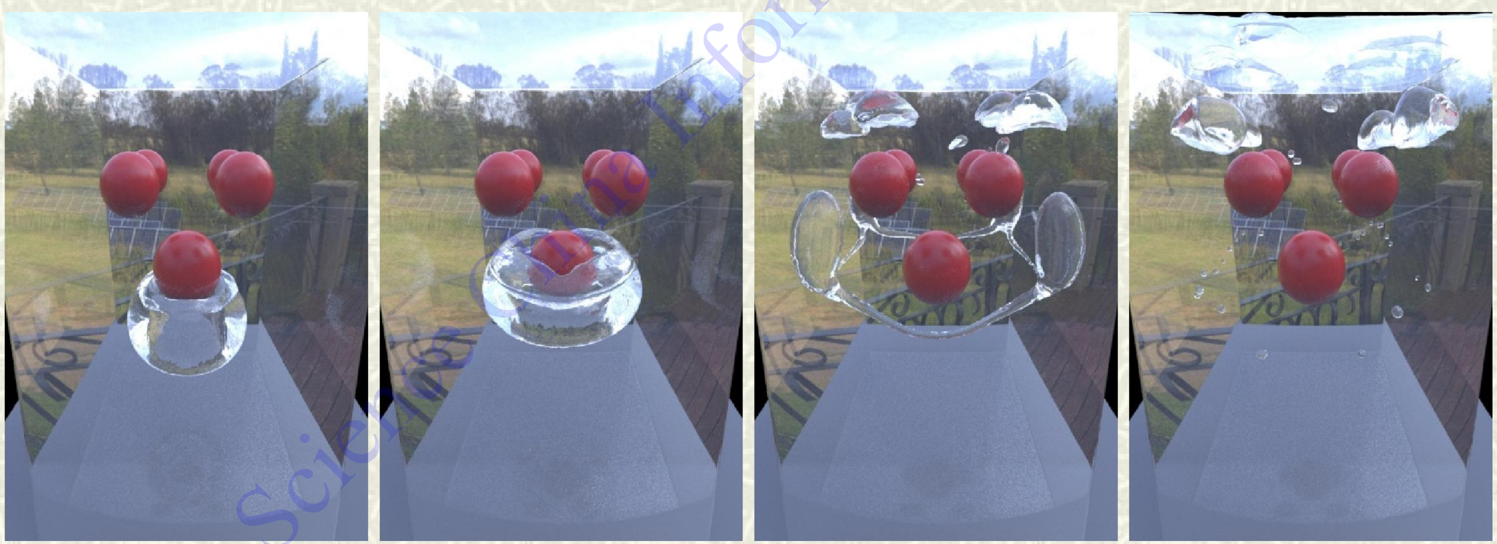
Results

- All experiments run on several different desktop PCs equipped with 3.2GHz/3.4GHz Intel i7-3930K or 3.5GHz E5-1620 processors. We use a weighted average of PIC and FLIP like MultiFLIP with a weight value of 0.95.

Case Name	Resolution	Method	Avg. Particles(Million)			Avg. Time/Timestep(s)			Mem. (MB)
			Gas	Liquid	Total	Projection	Rest	Total	
Tension	128×128×128	MultiFLIP	18.72	1.92	20.65	7.8	28.2	36.0	4932
		Ours	0.47	0.42	0.89	2.3	5.5	7.8	954
Dambreak	128×128×64	MultiFLIP	9.20	2.84	12.04	14.4	21.0	35.4	2648
		Ours	0.38	0.38	0.76	2.6	4.7	7.3	537
Box glugging	64×256×64	MultiFLIP	9.11	1.83	10.90	12.8	19.7	32.5	2426
		Ours	0.47	0.33	0.80	3.4	5.1	8.5	557
Bubble (small)	64×128×64	MultiFLIP	1.44	4.32	5.75	5.4	10.4	15.8	1249
		Ours	0.20	0.25	0.45	1.7	2.9	4.6	282
Bubble	128×256×128	MultiFLIP	-	-	-	-	-	-	-
		Ours	0.69	0.85	1.55	10.8	14.1	24.9	1963
Sandglass	128×256×128	MultiFLIP	-	-	-	-	-	-	-
		Ours	0.75	0.90	1.65	15.6	16.1	31.8	2040

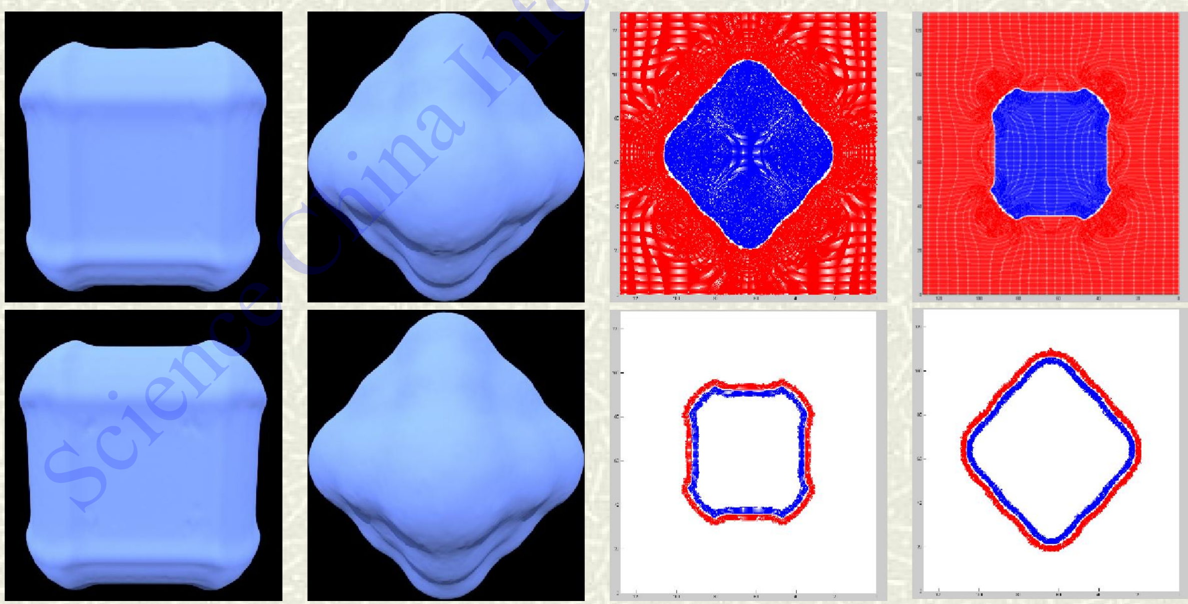
Results

- This figure shows coupling of bubble and obstacle with $\sigma = 2.0$ and an effective resolution of 128×256×128 by OctNB-MultiFLIP. We demonstrate that our method can work well for the coupling of bubble and obstacle.



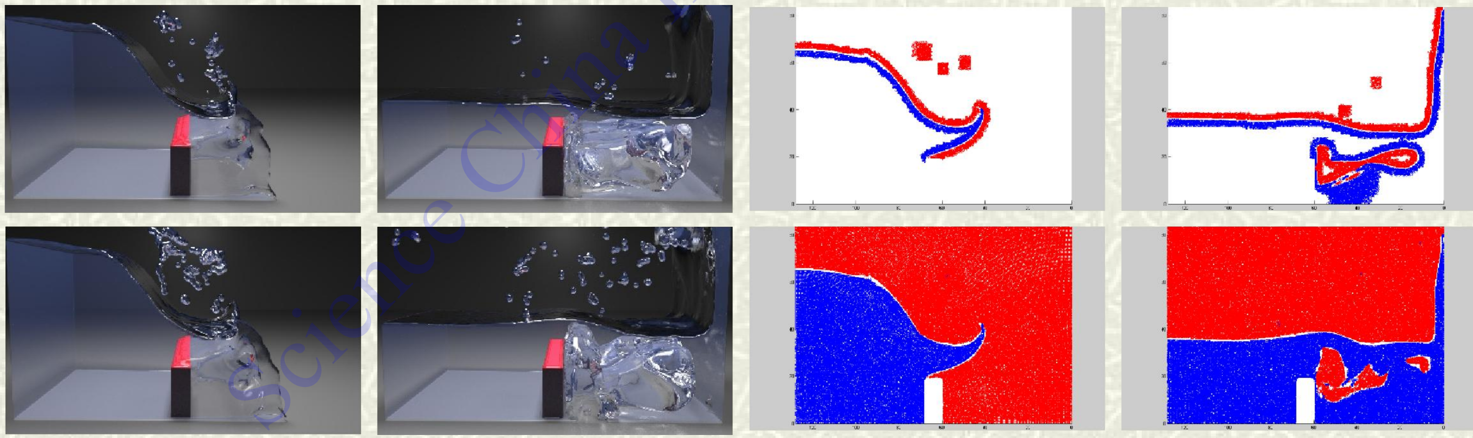
Results

- This figure shows a cube liquid oscillating in zero-gravity due to surface tension. Top row: Frame 18 and 84 by MultiFLIP and the corresponding gas (red color circles) and liquid (blue color circles) particles (Here we show the middle slice along the z-axis). Bottom row: Same sequence of frames by OctNB-MultiFLIP. In this case, our method uses 23 fewer particles (5 fewer memory) than MultiFLIP, and achieves a 4.5 speedup, while producing almost the same result.



Results

- This figure shows dambreak with an obstacle with $\sigma = 2.0$ and an effective resolution of 128×128×64. Column 1 and 3 show Frame 30 and 68 by MultiFLIP. Correspondingly, Column 2 and 4 show the frames by OctNB-MultiFLIP. Our result is similar to MultiFLIP. The difference is that our result exhibits less turbulence. This is reasonable because the velocity on the particle-free regions is tracked by the semi-lagrangian method, resulting in dissipation. Notice that both methods produce bubbles, which can not be reproduced by one-phase solvers like NB-FLIP.



Results

- This figure shows box glugging with $\sigma = 2.0$ and an effective resolution of 64×256×64. Column 1 and 3 show the frames that the first and second bubbles emerge by MultiFLIP. Correspondingly, column 2 and 4 show the frames by OctNB-MultiFLIP. We can see that both method exhibit similar behavior.

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Conclusion and future work

- Conclusion
  - We have introduced two techniques to improve the efficiency of MultiFLIP.
  - Our experiments have showed that the use of two narrow bands to tracking gas-liquid interface can significantly reduce the number of particles while keeping simulation quality.
  - On the other hand, the use of octree structures greatly reduces the unknowns of the variable coefficient Poisson equation, thus speedup the projection step.

Conclusion

- Conclusion
  - In theory, our narrow-band MultiFLIP can drastically reduces the number of particles from 8n^3to O(8n^2), and thus save most of the memory.
  - Our octree-based Poisson solver can reduces the number of unknown variables from n^3to O(n^2logn).
  - By these two techniques, 4 ~ 6x memory reduction and time speedups have been gained over the original MultiFLIP.

Future work

- Future work
  - We consider to use more advanced adaptive Poisson solvers, such as the power diagram method by Aanjaneya.
  - Since the poor parallelism of octree structures, we would also like to explore to use GPU-friendly data structures to discretize the variable Poisson equation like the GVDB by Hoetzlein.
  - Since the simple global volume control mechanism provided by MultiFLIP does not work in some cases, we would like to introduce a more robust volume control method in the future.

Limitations

- Limitations
  - As MultiFLIP, our method also has the stability issue when large time step is used. We would like to address this issue in our future work.
  - Although our method can reduce the memory complexity from 8n^3to O(8n^2) and the number of unknown variables of Poisson equation from n^3 to O(n^2logn), in some worst case, it can degenerate to the full resolution case.

Thank You