SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

November 2018, Vol. 61 112208:1–112208:8
https://doi.org/10.1007/s11432-018-9453-x

Leader-following consensus of linear discrete-time multi-agent systems subject to jointly connected switching networks

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Received 6 March 2018/Accepted 3 May 2018/Published online 15 October 2018

Abstract In this paper, we further study the leader-following consensus problem for a class of linear discrete-time multi-agent systems subject to jointly connected switching digraphs. We first establish a stability result for a class of linear switched systems under a more relaxed assumption than those in the literature. Then, we apply this stability result to obtain the solution to our problem, which contains previous results as special cases. Finally, we apply our result to an example that cannot be handled by any existing result.

Keywords multi-agent system, discrete-time consensus, jointly connected digraphs

Citation Liu J W, Huang J. Leader-following consensus of linear discrete-time multi-agent systems subject to jointly connected switching networks. Sci China Inf Sci, 2018, 61(11): 112208, https://doi.org/10.1007/s11432-018-9453-x

1 Introduction

The cooperative control of multi-agent systems is to design distributed control laws, which utilizes the local feedback information only, to achieve global objectives such as consensus, formation, tracking and disturbance rejection. For continuous-time multi-agent systems, various cooperative control problems have been extensively studied since 2000 [1–6]. A comprehensive coverage of the research on the multi-agent control systems can be found in the very recent survey paper [7]. The cooperative control of discrete-time multi-agent systems has also received extensive attention. In Tuna [8], the leaderless consensus problem was addressed for neutrally stable linear multi-agent systems subject to static networks. You and Xie [9] further investigated the leaderless consensus problem for more general linear systems using modified Riccati inequality design method. Hengster-Movric et al. [10] and Liu and Huang [11] studied leader-following consensus problem under static networks, respectively. The two consensus problems subject to switching networks were studied in a few papers [12–14] for neutrally stable linear multi-agent systems. More specifically, the result of Su and Huang [14] considered the jointly connected switching and undirected graphs, which was extended to some class of jointly connected switching and directed graphs in [12]. In Lee et al. [13], the authors further established the conditions for exponential consensus. However, the research for the discrete-time multi-agent systems still lags far behind the study of the continuous-time multi-agent systems.

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In this paper, we consider the leader-following consensus problem for neutrally stable discrete-time multi-agent systems subject to jointly connected switching networks. We first establish a stability result for a class of linear switched systems, which was previously studied in Su and Huang [14] and Huang [12]. However, our result in this paper has relaxed the conditions in Su and Huang [14] and Huang [12] so that it applies to a larger class of linear switched systems. This stability results leads to an improvement of the leader-following consensus results given in [12, 14]. To illustrate the effectiveness of our approach, we apply our approach to one leader-following consensus problem whose digraph is jointly connected. It is noted that this example cannot be solved by any existing approach.

Notation. Let $\mathbb{Z}^+$ denote the set of nonnegative integers. Let $\sigma(A)$ denote the spectrum of A. Let $C(c, r)$ denote the open disk on the complex plane centered at c with radius r. Let $\otimes$ denote the Kronecker product of matrices. Let $||\cdot||$ denote the Euclidean norm of a vector. Let $A^{\frac{1}{2}}$ denote the square root of a positive semi-definite matrix A. $A \in \mathbb{R}^{n\times n}$ is orthogonal if $A^{\mathrm{T}}A = I_n$. A is stochastic if A is a square matrix of nonnegative entries and the sums of each row in A are equal to 1.

The remainder of this paper is organized as follows. In Section 2, we present the leader-following consensus problem together with some assumptions. In Section 3, we establish a stability result for a class of switched systems, which in turn leads to the solvability of the problem. In Section 4, we illustrate our approach by an example. The conclusion is given in Section 5. Finally, concepts of digraphs and switching signals are provided in Appendixes A and B.

2 Problem formulation

Consider a group of linear discrete-time systems

$$x_i(k+1) = Ax_i(k) + Bu_i(k),\ i = 1, \ldots, N, \tag{1}$$

where, for $i = 1, \ldots, N$, $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ are the state and control input of i-th subsystem of (1). Without loss of generality, it is assumed that B is of full column rank. In addition, there is an n-dimensional vector sequence $\{x_0(k)\}$ generated as follows:

$$x_0(k+1) = Ax_0(k). \tag{2}$$

The system composed of (1) and (2) can be viewed as a multi-agent system with (1) as the follower system and (2) as the leader system. To describe the time-varying communication among different subsystems of (1) and (2), we define a switching digraph $\bar{\mathcal{G}}_\sigma(\bar{\mathcal{V}}, \bar{\mathcal{E}}_\sigma)$ with a switching signal $\sigma : \mathbb{Z}^+ \to \mathcal{P}$ and $\mathcal{P} = \{1, \ldots, M\}$. Here $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ with node 0 representing (2) and node i representing the i-th subsystem of (1) for $i = 1, \ldots, N$, $\bar{\mathcal{E}}_p \subset \{(i, j) \mid i, j \in \bar{\mathcal{V}}, i \neq j, i \neq 0\}$, $p \in \mathcal{P}$, and $(j, i) \in \bar{\mathcal{E}}_{\sigma(k)}$ if and only if the control input $u_i(k)$ can make use of the relative state measurement $x_j(k) - x_i(k)$ at time k. For each $p \in \mathcal{P}$, let $\bar{\mathcal{A}}_p$ be the adjacency matrix of $\bar{\mathcal{G}}_p(\bar{\mathcal{V}}, \bar{\mathcal{E}}_p)$, and let $a_{ij}(p)$, $i, j = 0, 1, \ldots, N$, be the (i, j)-th element of $\bar{\mathcal{A}}_p$.

We consider a class of distributed static state feedback control laws of the following form:

$$u_i(k) = \mu K \sum_{j=0}^{N} a_{ij}(\sigma(k))(x_j(k) - x_i(k)), \quad i = 1, \ldots, N, \tag{3}$$

where μ is a positive constant and $K \in \mathbb{R}^{m\times n}$ is called the state feedback gain.

We now describe our problem as follows.

Discrete-time leader-following consensus problem (DLCP). Given the follower system (1), the leader system (2) and the switching digraph $\bar{\mathcal{G}}_\sigma(\bar{\mathcal{V}}, \bar{\mathcal{E}}_\sigma)$, design a distributed state feedback control law of the form (3) such that, for any initial state $x_i(0)$, $i = 0, 1, \ldots, N$, the solution of the closed-loop system (1)–(3) satisfies

$$\lim_{k\to\infty} (x_i(k) - x_0(k)) = 0,\ i = 1, \ldots, N.$$

Denote $\bar{\mathcal{L}}_p$ as the Laplacian matrix of $\bar{\mathcal{G}}_p$, $p \in \mathcal{P}$. Let $e_i = x_i - x_0$ and $e = [e_1^{\mathrm{T}}, \ldots, e_N^{\mathrm{T}}]^{\mathrm{T}}$. Then it can be verified that the error vector $e(k)$ is governed by the following equation:

$$e(k+1) = (I_N \otimes A - \mu \mathcal{H}_{\sigma(k)} \otimes BK)e(k), \tag{4}$$

in which $\mathcal{H}_p$ is the matrix consisting of the last N rows and the last N columns of $\bar{\mathcal{L}}_p$. Thus, the DLCP is solvable if there exist $\mu > 0$ and $K \in \mathbb{R}^{m\times n}$ such that the system (4) is asymptotically stable.

Let us now list some standard assumptions as follows:

Assumption 1. The matrix A is orthogonal.

Assumption 2. The pair (A, B) is controllable.

Assumption 3. There is an infinite subsequence $\{k_j\}_{j=0}^{\infty}$ of $\mathbb{Z}^+$, such that $\{k_{j+1} - k_j\}$ is bounded and for any $j \in \mathbb{Z}^+$, $\cup_{\tau=k_j}^{k_{j+1}-1} \bar{\mathcal{G}}_{\sigma(\tau)}$ is connected.

Remark 1. Because, for any neutral stable matrix A, there exists a nonsingular real matrix T such that TAT^{-1} is orthogonal, by Remark 2.2 of [14], the first assumption can be relaxed to the marginal stability of A. On the other hand, under Assumption 1, the eigenvalues of A are on the unit circle. Thus, under Assumption 1, the controllability of (A, B) is equivalent to the stabilizability of (A, B). The third assumption is the mildest assumption on switching digraphs as it allows the digraph to be disconnected at every time instant [15, 16].

3 Main result

Let us start with considering the stability property of the following linear discrete-time switched system:

$$\xi(k+1) = (I_N \otimes A - \mu \mathcal{F}_{\sigma(k)} \otimes BB^{\mathrm{T}}A)\xi(k), \tag{5}$$

where the pair (A, B) satisfies Assumptions 1 and 2, $\sigma : \mathbb{Z}^+ \to \mathcal{P}$ is a switching signal with $\mathcal{P} = \{1, \ldots, M\}$, $\mu \in \mathbb{R}$ and $\mathcal{F}_p \in \mathbb{R}^{N\times N}$ for all $p \in \mathcal{P}$. To ensure the existence of μ such that system (5) is asymptotically stable, we introduce Assumptions 4–6.

Assumption 4. The switching signal σ has dwell-time d_0, where d_0 is the controllability index of the pair (A, B).

Assumption 5. There exist a positive real number $\hat{\mu}$ and a positive definite matrix $P \in \mathbb{R}^{N\times N}$ such that

$$\mathcal{F}_p^{\mathrm{T}} P + P\mathcal{F}_p - \hat{\mu}\mathcal{F}_p^{\mathrm{T}} P \mathcal{F}_p \geqslant 0 \tag{6}$$

for all $p \in \mathcal{P}$.

Assumption 6. There is an infinite subsequence $\{k_j\}_{j=0}^{\infty}$ of $\mathbb{Z}^+$ such that $\{k_{j+1} - k_j\}$ is bounded and for any $j \in \mathbb{Z}^+$, the matrix $\sum_{\tau=k_j}^{k_{j+1}-1} \mathcal{F}_{\sigma(\tau)}$ is nonsingular.

Remark 2. It is clear that Assumption 5 includes Assumption 3.2 of [12] as a special case, which requires for each $p \in \mathcal{P}$, the eigenvalues of $\mathcal{F}_p$ have nonnegative real parts and (6) holds with $P = I_N$ and some $\hat{\mu} > 0$. It is noted that $(I_N - \hat{\mu}\mathcal{F}_p)^{\mathrm{T}} P (I_N - \hat{\mu}\mathcal{F}_p) - P = \hat{\mu}(\hat{\mu}\mathcal{F}_p^{\mathrm{T}} P \mathcal{F}_p - \mathcal{F}_p^{\mathrm{T}} P - P\mathcal{F}_p)$. If (6) is satisfied, then the matrix $I_N - \hat{\mu}\mathcal{F}_p$ is marginally stable for every $p \in \mathcal{P}$. As a result, for each $p \in \mathcal{P}$, the eigenvalues of $\hat{\mu}\mathcal{F}_p$ lie in $\bar{C}(1, 1)$, which implies the eigenvalues of $\mathcal{F}_p$ have nonnegative real parts due to the fact that $\hat{\mu} > 0$.

To derive the asymptotic stability result of system (5), we need Lemma 1 which extends Lemma 3.1 in [12].

Lemma 1. Suppose Assumption 5 holds. Then, given any positive semi-definite nonzero matrix $Q \in \mathbb{R}^{q\times q}$,

$$\mathcal{F}_p^{\mathrm{T}} P \otimes Q + P\mathcal{F}_p \otimes Q - \frac{\hat{\mu}}{\lambda_{\max}(Q)} \mathcal{F}_p^{\mathrm{T}} P \mathcal{F}_p \otimes Q^2 \geqslant 0$$

holds for any $p \in \mathcal{P}$, where $\lambda_{\max}(Q)$ is the largest eigenvalue of Q.

Proof. In view of (6) and the inequality $\lambda_{\max}(Q)\cdot I_q \geqslant Q$, for any $p\in\mathcal{P}$, we obtain

$$(\mathcal{F}_p^{\mathrm{T}}P+P\mathcal{F}_p)\otimes Q \geqslant \hat{\mu}\mathcal{F}_p^{\mathrm{T}}P\mathcal{F}_p\otimes Q \geqslant \frac{\hat{\mu}}{\lambda_{\max}(Q)}\mathcal{F}_p^{\mathrm{T}}P\mathcal{F}_p\otimes Q^2.$$

Lemma 2. Suppose Assumptions 4 and 5 hold. Then, for any $\mu\in(0,\frac{\hat{\mu}}{\lambda_{\max}})$ with $\lambda_{\max}$ being the largest eigenvalue of $A^{\mathrm{T}}BB^{\mathrm{T}}A$, the solutions of system (5) satisfy

$$\lim_{k\to\infty}(\mathcal{F}_{\sigma(k)}\otimes I_n)\xi(k)=0. \tag{7}$$

Moreover, if Assumption 6 also holds, then system (5) is asymptotically stable.

Proof. Denote $Q=A^{\mathrm{T}}BB^{\mathrm{T}}A$. Consider the Lyapunov function candidate $V(\xi(k))=\xi^{\mathrm{T}}(k)(P\otimes I_n)\xi(k)$, where $P>0$ is given in Assumption 5. The difference of V along the trajectories of system (5) satisfies

$$\begin{aligned}\Delta V(k)&=V(\xi(k+1))-V(\xi(k))\\&=\xi^{\mathrm{T}}(k)\left[\left(I_N\otimes A^{\mathrm{T}}-\mu\mathcal{F}_{\sigma(k)}^{\mathrm{T}}\otimes A^{\mathrm{T}}BB^{\mathrm{T}}\right)(P\otimes I_n)\left(I_N\otimes A-\mu\mathcal{F}_{\sigma(k)}\otimes BB^{\mathrm{T}}A\right)-(P\otimes I_n)\right]\xi(k)\\&=\mu\xi^{\mathrm{T}}(k)\left[\left(-\mathcal{F}_{\sigma(k)}^{\mathrm{T}}P-P\mathcal{F}_{\sigma(k)}\right)\otimes Q+\mu\mathcal{F}_{\sigma(k)}^{\mathrm{T}}P\mathcal{F}_{\sigma(k)}\otimes Q^2\right]\xi(k).\end{aligned}$$

By Lemma 1, we have

$$\left(\mathcal{F}_{\sigma(k)}^{\mathrm{T}}P+P\mathcal{F}_{\sigma(k)}\right)\otimes Q-\mu\mathcal{F}_{\sigma(k)}^{\mathrm{T}}P\mathcal{F}_{\sigma(k)}\otimes Q^2\geqslant\left(\frac{\hat{\mu}}{\lambda_{\max}}-\mu\right)\mathcal{F}_{\sigma(k)}^{\mathrm{T}}P\mathcal{F}_{\sigma(k)}\otimes Q^2.$$

Thus,

$$\begin{aligned}\Delta V(k)&\leqslant-\left(\frac{\hat{\mu}}{\lambda_{\max}}-\mu\right)\mu\xi^{\mathrm{T}}(k)\left(\mathcal{F}_{\sigma(k)}^{\mathrm{T}}P\mathcal{F}_{\sigma(k)}\otimes Q^2\right)\xi(k)\\&=-\left(\frac{\hat{\mu}}{\lambda_{\max}}-\mu\right)\mu\left\|\left(P^{\frac{1}{2}}\mathcal{F}_{\sigma(k)}\otimes Q\right)\xi(k)\right\|^2\\&\leqslant 0\end{aligned}$$

for all $0<\mu<\frac{\hat{\mu}}{\lambda_{\max}}$. This shows system (5) is (uniformly) stable and $V(\xi(k))$ is monotonically nonincreasing, which, given $V(\xi(k))$ is bounded from below by zero, in turn implies $V(\xi(k))$ converges. Hence, $\lim_{k\to\infty}\Delta V(k)=0$, which yields $\lim_{k\to\infty}[P^{\frac{1}{2}}\mathcal{F}_{\sigma(k)}\otimes Q]\xi(k)=0$ because $0<\mu<\frac{\hat{\mu}}{\lambda_{\max}}$. As a result,

$$\lim_{k\to\infty}\left(\mathcal{F}_{\sigma(k)}\otimes B^{\mathrm{T}}A\right)\xi(k)=0 \tag{8}$$

since A is nonsingular and B is of full column rank.

If $d_0=1$, then B is nonsingular, and hence (8) leads to (7). Now suppose $d_0>1$. As the proof of Lemma 3.3 in [12], under Assumption 4, there is a subsequence $\{t_j\}$ of $\{k\}$ such that $d_0\leqslant t_{j+1}-t_j<2d_0$ and $\sigma(t_j)=\sigma(t_j+1)=\cdots=\sigma(t_{j+1}-1)$. Moreover, Eq. (8) and the controllability of (A,B) imply $\lim_{j\to\infty}(\mathcal{F}_{\sigma(t_j)}\otimes I_n)\xi(t_j)=0$. Hence, Eq. (7) is guaranteed by the boundedness of $\{t_{j+1}-t_j\}$ and the finiteness of $\mathcal{P}$. Then, it follows from the proof of Lemma 3.1 in [14] that, $\lim_{k\to\infty}(\mathcal{F}_{\sigma(k+s)}\otimes I_n)\xi(k)=0$ for any $s\in\mathbb{Z}^+$. Thus, $\lim_{j\to\infty}(\sum_{\tau=k_j}^{k_{j+1}-1}\mathcal{F}_{\sigma(\tau)}\otimes I_n)\xi(k_j)=0$, which, together with Assumption 6 and the finiteness of $\mathcal{P}$, implies $\lim_{j\to\infty}\xi(k_j)=0$. Finally, $\lim_{k\to\infty}\xi(k)=0$ due to the fact that $V(\xi(k))$ is monotonically nonincreasing.

Remark 3. The stability property of the switched system (5) was studied in [12, 14], respectively. In particular, it was shown in [12] that, under Assumptions 4–6 with $P=I_N$, system (5) is asymptotically stable for any $\mu\in(0,\frac{\hat{\mu}}{\lambda_{\max}})$. Later, when we apply this result to deal with the leader-following consensus problem, it can be seen that our result include some interesting cases which violate Assumption 3.2 of [12]. Therefore, Lemma 2 extends Lemma 3.3 of [12] in the sense that it accommodates a more general class of switching matrices $\mathcal{F}_{\sigma(k)}$.

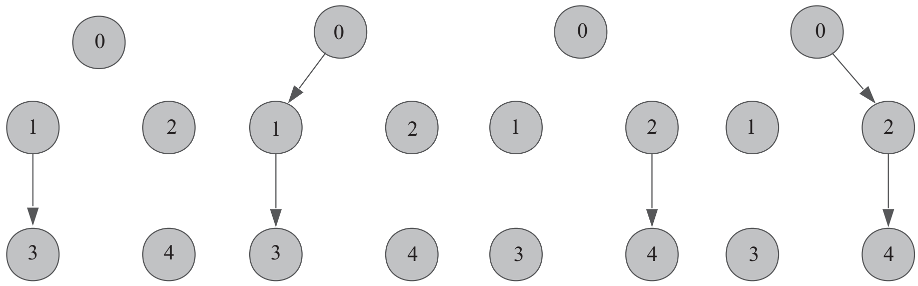

Figure 1 The digraphs $\bar{\mathcal{G}}_1$, $\bar{\mathcal{G}}_2$, $\bar{\mathcal{G}}_3$, $\bar{\mathcal{G}}_4$.

Remark 4. From the proof of Lemma 2, one can see that Assumption 5 guarantees $V(\xi(k)) = \xi^{\mathrm{T}}(k)(P \otimes I_n)\xi(k)$ is a common weak quadratic Lyapunov function [17] for the switched system (5).

Recall from Remark 3.3 of [12] that Assumption 3 implies that the matrix $\sum_{\tau=k_j}^{k_{j+1}-1} \mathcal{H}_{\sigma(\tau)}$ is nonsingular. Using this fact together with Lemma 2 gives the solvability of the DLCP as follows.

Theorem 1. Suppose Assumptions 1–5 with $\mathcal{F}_p = \mathcal{H}_p$ are satisfied. Then, the DLCP is solved by the control law of the form (3) with $K = B^{\mathrm{T}}A$ and any $\mu \in (0, \frac{\hat{\mu}}{\lambda_{\max}})$, where $\lambda_{\max}$ is the largest eigenvalue of $A^{\mathrm{T}}BB^{\mathrm{T}}A$.

Remark 5. Theorem 1 not only extends Theorem 4.2 of [14] where the switching graph is assumed to be undirected, but also includes Theorem 3.1 of [12] as a special case because it can handle a larger class of jointly connected switching graphs than those in [12]. In Section 4, we will provide an example which cannot be handled by the results in [12, 14].

4 An example

Consider four discrete-time controlled harmonic oscillators

$$x_i(k+1) = \begin{bmatrix} \cos(T) & \sin(T) \\ -\sin(T) & \cos(T) \end{bmatrix} x_i(k) + \begin{bmatrix} 1-\cos(T) \\ \sin(T) \end{bmatrix} u_i(k), \quad i = 1,2,3,4,$$

where $T > 0$ is some positive constant. The leader system of the above follower system is a discrete-time uncontrolled harmonic oscillator

$$x_0(k+1) = \begin{bmatrix} \cos(T) & \sin(T) \\ -\sin(T) & \cos(T) \end{bmatrix} x_0(k).$$

The switching graph $\bar{\mathcal{G}}_\sigma$ among the leader and the four followers is dictated by the following switching signal:

$$\sigma(k) = \begin{cases} 1, & 0 \text{ or } 1 \equiv k \pmod 8, \\ 2, & 2 \text{ or } 3 \equiv k \pmod 8, \\ 3, & 4 \text{ or } 5 \equiv k \pmod 8, \\ 4, & 6 \text{ or } 7 \equiv k \pmod 8. \end{cases}$$

The four digraphs $\bar{\mathcal{G}}_i$, $i = 1,2,3,4$, are shown in Figure 1.

First, note that Assumption 1 holds with controllability index 2 if $T \notin \pi\mathbb{Z}^+$, and Assumption 2 is clearly satisfied. Also, it can be verified that Assumption 3 with $k_j = 8j$ and Assumption 4 with $\tau_\sigma = 2$ are satisfied. To verify Assumption 5, let the Laplacian matrices of the four digraphs $\bar{\mathcal{G}}_i$, $i = 1,2,3,4$, be as follows:

$$\bar{\mathcal{L}}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \bar{\mathcal{L}}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \bar{\mathcal{L}}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}, \quad \bar{\mathcal{L}}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

Correspondingly, we have

$$\mathcal{H}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{H}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{H}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \quad \mathcal{H}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

It can be verified that $\mathcal{H}_p^{\mathrm{T}} P_0 + P_0 \mathcal{H}_p \geqslant \mathcal{H}_p^{\mathrm{T}} P_0 \mathcal{H}_p$ is satisfied with

$$P_0 = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

Thus, Assumption 5 holds with $\mathcal{F}_p = \mathcal{H}_p$, $\hat{\mu} = 1$ and $P = P_0$. Therefore, all assumptions in Theorem 1 are satisfied.

Let $T = 0.1$. In this case, the largest eigenvalue of $A^{\mathrm{T}} B B^{\mathrm{T}} A$ equals 0.01. By Theorem 1, we can choose

$$K = B^{\mathrm{T}} A = [-0.005 \; 0.0998],$$
$$\mu = 80 \in (0, 100)$$

in the distributed control law (3).

The performance of this control law is evaluated by computer simulation. Figures 2 and 3 show the simulation results with the initial values being $x_0(0) = -2.0999$, $x_1(0) = -0.6160$, $x_2(0) = 1.6617$, $x_3(0) = -3.6688$, $x_4(0) = -2.2612$, $v_0(0) = -1.7499$, $v_1(0) = -1.8481$, $v_2(0) = 0.8308$, $v_3(0) = 4.5860$, $v_4(0) = 0.7537$, which are randomly chosen. It can be seen that the tracking errors approach zero as k tends to infinity.

Interestingly, it is noted that the matrices $\mathcal{H}_1^{\mathrm{T}} + \mathcal{H}_1$ and $\mathcal{H}_3^{\mathrm{T}} + \mathcal{H}_3$ are not positive semidefinite, which implies that Assumption 3.2 of [12] with $\mathcal{F}_p$ replaced by $\mathcal{H}_p$ does not hold. As a result, one cannot invoke Theorem 3.1 of [12] to handle this example.

5 Conclusion

In this paper, we have studied the leader-following problem for neutrally stable linear discrete-time multi-agent systems over jointly connected digraphs. Our control law is derived from a stability result of a class of switched systems. We have also presented an example which cannot be handled by any existing approaches. One future work will focus on further relaxing Assumption 5 so that we can handle more general class of systems.

Acknowledgements This work was supported by Research Grants Council of the Hong Kong Special Administration Region (Grant No. 14200617).

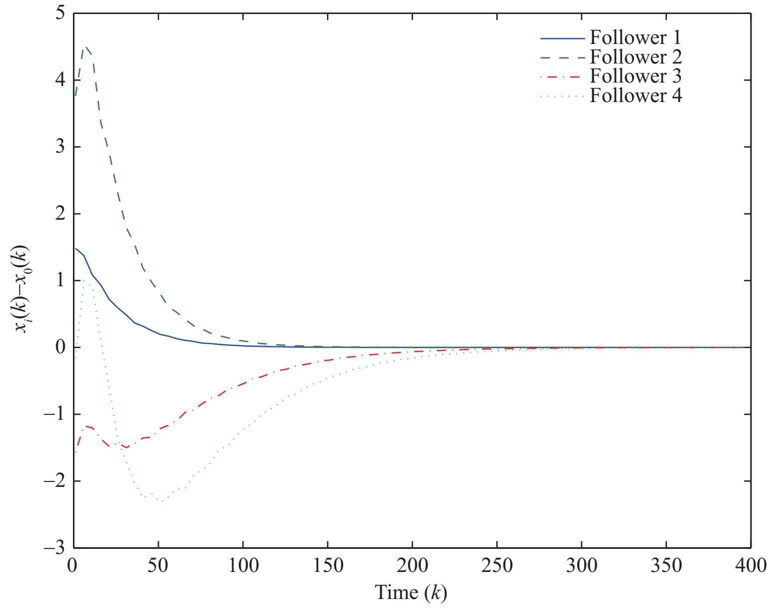

Figure 2 (Color online) The first component of tracking errors of followers.

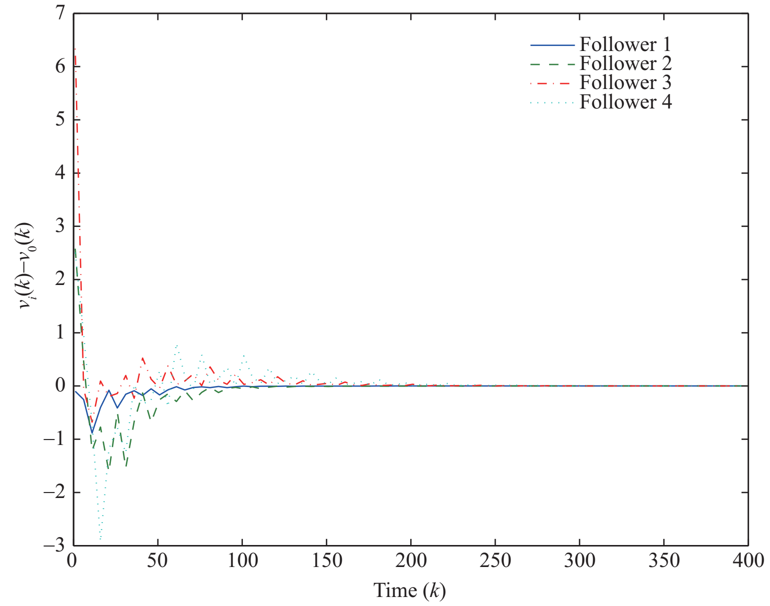

Figure 3 (Color online) The second component of tracking errors of followers.

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Appendix A Digraph

A digraph $\mathcal{G}(\mathcal{V},\mathcal{E})$ consists of a finite node set $\mathcal{V}=\{1,\ldots,M\}$ and an edge set $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$. A digraph $\mathcal{G}_s(\mathcal{V}_s,\mathcal{E}_s)$ is called a subgraph of $\mathcal{G}(\mathcal{V},\mathcal{E})$ if $\mathcal{V}_s\subset\mathcal{V}$ and $\mathcal{E}_s\subset\mathcal{E}$. Given any two nodes $i,j\in\mathcal{V}$ of a digraph $\mathcal{G}(\mathcal{V},\mathcal{E})$, node j is said to be reachable from node i if there exists a sequence of edges $\{(i_k,i_{k+1})\}_{k=0}^{l-1}\subset\mathcal{E}$ such that $i_0=i$ and $i_l=j$. A digraph $\mathcal{G}$ is said to be connected if there exists a node $i_0\in\mathcal{V}$ such that any other node in $\mathcal{V}$ is reachable from i_0. A weighted adjacency matrix of a digraph $\mathcal{G}(\mathcal{V},\mathcal{E})$ is a nonnegative matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{M\times M}$, where $a_{ij}>0$ if $(j,i)\in\mathcal{E}$ and $a_{ij}=0$ otherwise. The Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L}=\mathcal{D}-\mathcal{A}$, where $\mathcal{D}=\mathrm{diag}(d_1,\ldots,d_M)$ is the degree matrix of $\mathcal{G}$ with $d_i=\sum_{j\neq i}a_{ij}$.

Appendix B Switching signal

Let $\sigma:\mathbb{Z}^+\to\mathcal{P}$ with $\mathcal{P}=\{1,2,\ldots,\rho\}$ denote a switching signal in the sense that there exists a subsequence $\{l_j\}_{j=0}^{\infty}$ of $\mathbb{Z}^+$, called switching instants, such that $\sigma(l_j)=\sigma(l_j+1)=\cdots=\sigma(l_{j+1}-1)$. If there exists some positive integer τ such that the switching instants satisfy $l_{j+1}-l_j\geqslant\tau$ for all $j\in\mathbb{Z}^+$, then σ is said to have dwell-time τ.