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• **Supplementary File** •

An advanced operating environment for mathematics education resources

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Appendix A Cascading-Free Data Structure

For supporting the full requirements of mathematics teaching and learning, the operating environment needs to absorb special features of various software in teaching and learning, such as dealing with dynamic geometry system, word processing, multimedia editing and e-book production. The dynamic geometry is the main part. According to the principle of dynamic geometry [1], there is a close relationship between objects created by users, such as points, lines and circles. For example, take any three points A, B, C as vertexes and connect them to three line segments, a triangle is thus created. Point A, B, C are basic objects, named as the first-class objects, and line segment AB, BC, CA are the second-class objects. Specifically, line segment AB is the child object of point A, B, and point A, B is the parent object of line segment AB, and so on. If we continue to make perpendicular bisector of each edge, the perpendicular bisectors are the third-class object, and intersection point of each two perpendicular bisectors is the forth-class object. There will form the fifth-class object if make a circumcircle of $\triangle ABC$ taking the intersection point as circle center, shown in Figure A1(a). When we drag or delete any object, we must consider the impact on other objects associated with the object. If we delete the Point C, the graph would be like Figure A1(b) according to their relationship. The inheritance relationship of these graphs is shown in Figure A2. In these systems, the relationships can easily become highly complicated when the number of objects included increases or the intent behind making a drawing is ambiguous.

In fact, all objects, e.g. points, rectangles, curves, text boxes, fonts, and colors, can be represented as some kind of visual data. Once a new object is created, the OE provides a way of data visualization (metaphors data), such as creating points A, B for displaying coordinates $(a, b), (c, d)$, creating a segment line AB related to the data (a, b, c, d), creating text for displaying variable values. The data may be independent variables with a large value range, or random variables based on the setting conditions, or values obtained by measuring geometric objects or evaluating expressions. They may be introduced by the previous operations, or they may be new variables which are introduced through new operations. The data used to characterize objects is a part of object attributes, which can be a variable or an expression of some variables. When changing the variables values, all related objects will be updated accordingly, thus all objects are directly associated with corresponding variables. To support this feature, two lists are built into the system for data management, including a data list and another object list as shown Figure A3. If a newly created object is associated with new variables, these variables will be added into the data list and considered as parameters of the new object.

There are two ways to make the dynamic process happen. The first is changing the values of variables directly by dragging the sliders corresponding to the variables or clicking the animated buttons corresponding to the variables. With the changing of the values of variables, the measured values and the values of the expressions related to these variables are updated successively, which for the concerned objects generate dynamic visual effects. The second is dragging an object to drive the value's changes of related variables. Dragging a free point can usually cause the changing of the values of two variables, and dragging a semi-free point (the point on a line or curve) can cause the changing of the value of a variable, then the followings are similar as the ones described above.

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Figure A1 (a) Original graphs; (b) After deleted the point C.

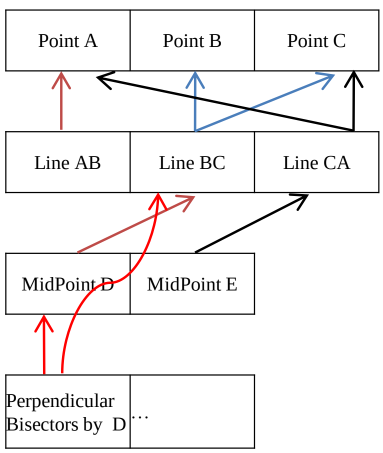

Figure A2 Inheritance relationship.

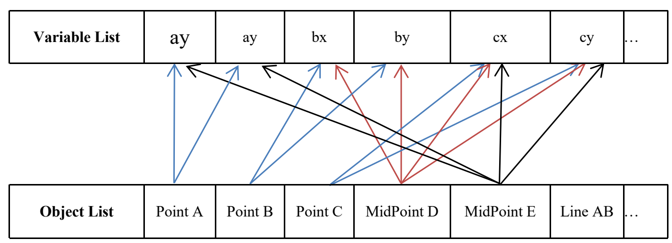

Figure A3 New structure.

Appendix B Intelligent Drawing Technology

In current DGS, users need to frequently click on toolbar buttons or menu items to draw basic geometric graphs, points, lines, and circles. This method not only makes the drawing inefficiency but also increases users' operation burden and visual overhead. To address this problem, we propose an algorithm of intelligent drawing that is context aware. The algorithm flow chart is shown in Figure B1. The pseudo-code of this algorithm can be presented as Algorithm B1.

Algorithm B1 Pseudo-code of intelligent drawing algorithm

Input: matching model: $M(M_1, M_2, \cdots, M_n)$; current operations: *oper*;
Output: drawing tip;
get current graph context $G(G_1, G_2, \cdots, G_k)$;
filter M by current operation *oper*, get sub-matching mode $MS(M_{s_1}, M_{s_2}, \cdots, M_{s_k}), 1 \leqslant k < n$;
for M_i in $(M_{s_1}, M_{s_2}, \cdots, M_{s_k})$ **do**
    get $(G_{j_1}, G_{j_2}, \cdots, G_{j_m})$ from G, $j \in [1, u]$, according to the construction condition of M_i;
    for $a = 1$ to u **do**
        if $D(oper, G_{j_1}, G_{j_2}, \cdots, G_{j_m}) < M_i.d$ **then**
            return $M_i.tip$;
        end if
    end for
end for

First, according to the priority and construction conditions of each kind of geometric graphs, we establish the matching model M of graphs and choose a corresponding threshold d. Second, when users draw by operations *oper*, e.g. clicks, double click, and drags, we get current all geometric graphs G in document and the sub-matching model MS through filtering the matching model M by current operations *oper*. Third, traverse MS, according to the construction condition of each M_i of MS, get $(G_{j_1}, G_{j_2}, \cdots, G_{j_m})$ from G, $j \in [1, u]$, then calculate each threshold D of $(oper, G_{j_1}, G_{j_2}, \cdots, G_{j_m})$, if D is less then the threshold d of M_i, then return the recommendation graph tips. With this algorithm, users can accurately and efficiently draw more than twenty kinds of geometric graphs including introducing middle points, foot points, parallel lines, and equilateral triangles, through mouse operations alone and without any need to click on toolbar buttons or menu items.

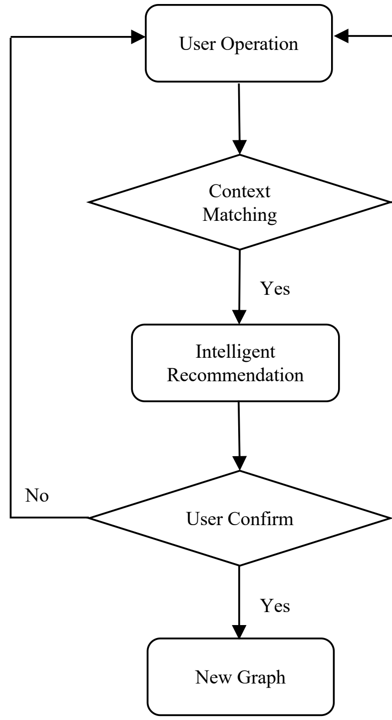

Figure B1 The flow chart of intelligent drawing algorithm

Such an operational design aims to closely mimic is the traditional paradigm of drawing on a conventional blackboard in chalks.

Appendix C Introduction of SSP

Based on the proposed technical framework and key technologies, we developed an operating environment for elementary mathematics education resources, Super Sketchpad(SSP). Teachers and students can do almost everything for teaching and learning mathematics by using SSP. The functions of SSP include geometric graphs and function curves drawing, geometric transformations, texts and formulas, numerical and symbolic computations, geometric information measuring, geometric theorems proofing, courseware demonstration, programming and so on.

Appendix C.1 The functions of SSP

(1) Text and Formulas

This includes inputting and editing text, numbers, symbols, formulas and expressions. With the data-driven system design, we realize the dynamic data functions. The data in text or formulas can be varied according to the graphs or variables changing. As shown in Figure C1, the data of questions and answers are dynamic. When changing the graphs with buttons, all the data of questions and answers changes correspondingly.

In SSP, we make that the formulas can be used for numerical and symbolic computation directly. So, the formulas in text can reflect the actual meaning rather than the form, which makes the education resources more vivid, abundant and convenient. There are a variety of tools to inserted the formulas into the text, but these formulas are usually not allowed to be computed.

(2) Drawing Dynamic Graphs

Drawing includes plane and 3D geometry, function curves and conic curves, space curves, explicit or implicit equations of surfaces, interpolated curves, tracks of a point, line and circle, graphical iterations, statistical diagrams and so on. In SSP, there are five methods, Intelligent Pen, toolbar icons, menus, text commands and procedures, for users to draw graphs.

Intelligent Pen is a specific function based on context-aware technology. With Intelligent Pen, users can draw more than 20 kinds of geometric graphics only with the mouse according to the automatic tips, e.g. free point, foot point, middle point, line, parallel line, tangent line, circle and so on. The Intelligent Pen not only makes SSP easy to use, but also greatly improves users' efficiency. Most importance is that this is fit to traditional habit.

The text commands and procedures drawing are based on knowledge enrichment technology. With the functions, it becomes easy or possible to draw some complex graphs.

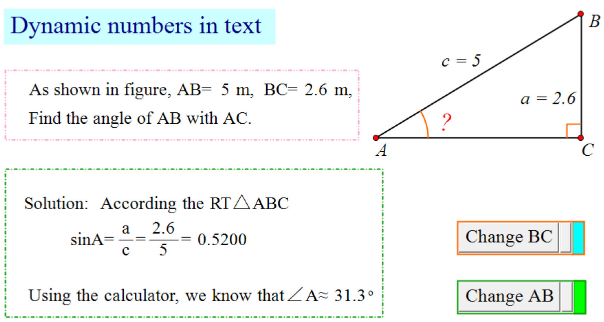

Figure C1 Text and formulas.

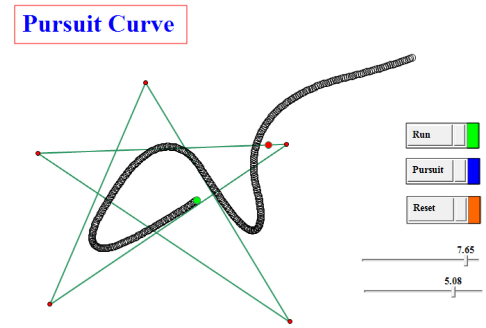

Figure C2 Pursuit curve.

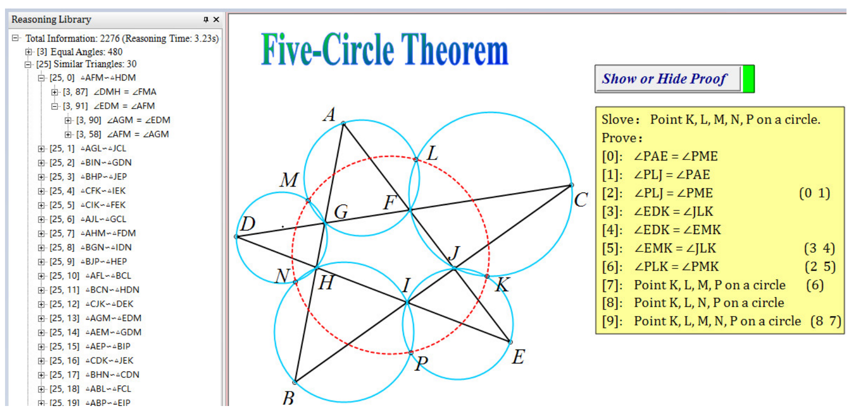

Figure C3 The automated reasoning of the Five-Circle Theorem.

(3) Programming

SSP provides a simple interpretive language for programming. The syntax is similar to language C. With the programming environment, users can realize the related algorithms in secondary school mathematics education. The procedures drawing is realized by the text command drawing and programming environment.

(4) Dynamic Measure

Users can get the dynamic data at any time by measuring geometric objects or expressions, and these data is shown in dynamic texts. Users can measure various geometric quantities, including coordinate, length, angle, area, slope and so on, the equations of lines or conic curves, the numeric values of expressions, and random variables. The function of measuring expressions can be used in a virtual experiment or constructing new objects. The pursuit curve in Figure C2 is formed by using the location data of the one being pursued to adjust the direction of the pursuer.

(5) Geometry Automated Reasoning

Based on our automated reasoning technology [2–6], Geometry automated reasoning of SSP can solve most elementary geometric problems, generates a readable proof, and discover all information contained in the graphs. The proper application of reasoning, not only can reduce the mechanized labor of teachers and students, improve learning interest and leave more time for creative work, but also provide more information to explore new ideas. In Figure C3 is about the five-circle theorem. After performing automated reasoning based on geometric graphs, the generated readable proof of the theorem is shown in the text box. At the same time, the reasoning system generates a total of 2276 pieces of information with the detailed derivation process, including 30 pieces of information about similar triangles, 480 pieces of information about equal angels. Teachers, students, book editors and question bank developers can extract a variety of materials for exercises from equipped Reasoning Library.

(6) Numerical and Symbolic Computation

SSP has powerful computing functions, including large numerical computation, polynomial computation, factorization, derivative, integral and so on. Figure C4 shows the square wave superimposed on the sine wave, which is realized by the computation of trigonometric series.

(7) Resources Demonstrations

In SSP, the development environment is the demonstration environment. Hence, users can edit the resources directly according to the teaching design when demonstrating the resources, this is like the traditional chalk teaching. For example, they can make an important point or curve, measure an important data, from which users can understand not only a knowledge but its generating process.

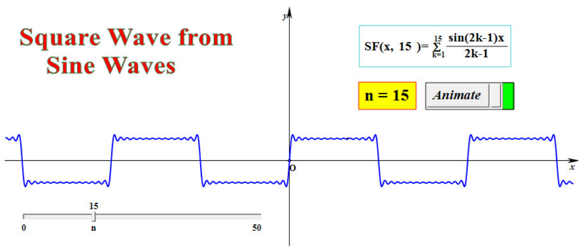

Figure C4 Square wave from sine wave.

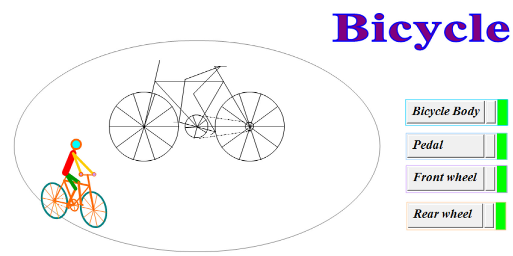

Figure C5 A bicycle by affine transformation.

Q1：Can SSP enhance your understanding of knowledge?		
Answer	**Results**	**Percent**
Yes	31	65.96%
Not sure	12	25.53%
No	4	8.51%
Total	47	

Q2：Do you like that your teacher teaches with SSP?		
Answer	**Results**	**Percent**
Very Like	17	36.17%
Like	28	59.57%
Not Like	2	4.26%
Very Not Like	0	0.00%
Total	47	

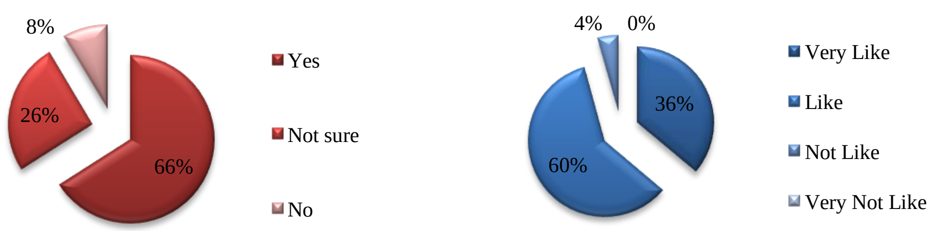

Figure C6 A Survey about SSP.

(8) Geometric Transformations

Geometric transformations includes translation, reflection, rotation, affine transformation, etc. Geometric transformations are not only functional operations, but also mathematical knowledge. Using geometric transformations properly not only helps to strengthen the interaction of the teaching process, but also enables students to understand the production of the resources, which also has deeper educational value. For example, Figure C5 is the dynamic effects of the bicycle made by the affine transformation. These dynamic effects can stimulate the interest of students, make them think and have a deeper understanding of the relevant knowledge.

Appendix C.2 Practical Evaluation

Because it is easy to use and powerful functionalities, SSP is deployed in over 10,000 schools for elementary mathematics education and receives highly positive assessment. A large number of mathematics education researches based on SSP have been done by teachers and researchers [7–20]. Almost all of these researches show the effectiveness and efficiency of SSP in elementary mathematics education.

In [9], XU summarizes the positive educational values of SSP from both educational theories and teaching practices, thinks SSP is the excellent platform for implementing the variation theory, obtaining mathematics experiences, mathematical modeling and so on. In [10], YANG proves the effectiveness of the integration of SSP and function teaching in senior school as three aspects, (1) improving the students' score, (2) improving the students' learning interest, (3) improving the teaching efficiency of the class. In [12], FENG studies the application of SSP in the primary mathematics teaching. In [14], after a lot of teaching practice, CHEN concludes that teaching cone curves with SSP can effectively improve the students' self-learning and self-exploration ability, and students use SSP for mathematical inquiry experiments can improve the students' research ability. Figure C6 is the result of CHEN's survey to his students, it shows 95.72% students like SSP and 65.95% students think SSP can enhance their understanding of knowledge. In [16], DENG studies how to optimize analytical geometry teaching with the support of SSP. In [17], ZHENG studies the application of SSP in mathematical inquiry teaching. In [18], QIAO et al study the cultivation of students' intuitive imagination based on SSP.

Table D1 The functional comparison

Item	SSP	GSP
Dynamic Geometry	strong	strong
Geometric Transformations	strong	strong
Locus, Trace and Animation	strong	strong
Geometric Iteration	weak	strong
Curves with Parameters	strong	weak
Dynamic Measure	strong	weak
Statistic Graph	strong	weak
Visualize Drawing	strong	weak
Numeric Computation	strong	weak
Text and Formulas	strong	weak
Demonstration	strong	weak
Extensibility	strong	weak
Procedure Drawing	strong	none
Symbolic Computation	strong	none
Programming	strong	none
Geometry Automated Reasoning	strong	none
Trigonometric Reasoning	strong	none

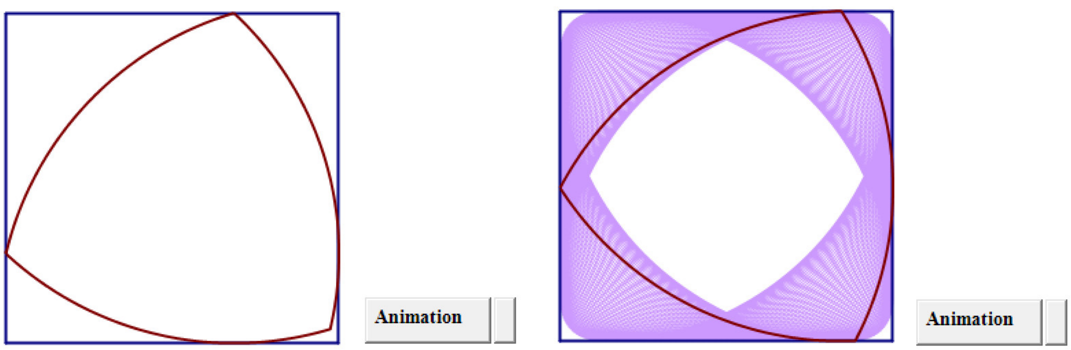

Figure D1 Square Hole Drilling (a) initial graph; (b) run a round.

Appendix D Experiments

(1) Experiment 1

As reported in Table D1, SSP is equipped with a set of more powerful functions than Geometer's Sketchpad (GSP) [21], the latter of which is the most famous operating environment for mathematics education resources. Thus, SSP comprehensively and effectively supports the needs of teaching and learning elementary mathematics.

(2) Experiment 2

Based on the programming environment and API of SSP, we can easily extend the functions of SSP by defining custom function packages. For example, ZHANG et al defined an extend package named "FBM" [22] included 198 functions. FBM can draw special graphs, solve equations, and calculate the value of a determinant. Equipped with the extension by FBM, SSP is then empowered to draw the Square Hole Drilling with the extended command "fkz()" provided by FBM. Figure D1 shows an example created by the above extension.

(3) Experiment 3

Based on the Usability Testing and Evaluation Model for Educational Software (UTEM-ES) [23, 24], we evaluate the operation, efficiency and effectiveness of SSP by the switching count (SC) of menu, toolbar and drawing area, the operating amount (OA) and the operating simplicity ratio (OSR), and comparing with GSP.

UTEM-ES introduces the following metrics for calculating the operating amount (OA) and the operating simplicity ratio (OSR):

- The category count of operating events: Tev;
- The category count of operating objects: Tob;
- The count of operating events: Cev;
- The count of operating objects: Cob;
- The operating amount: OA, $OA = \frac{EvA+ObA}{2}$, where a smaller value means a better quality;
- The average count of operations: Peo, $Peo = \frac{Cev}{Cob}$;

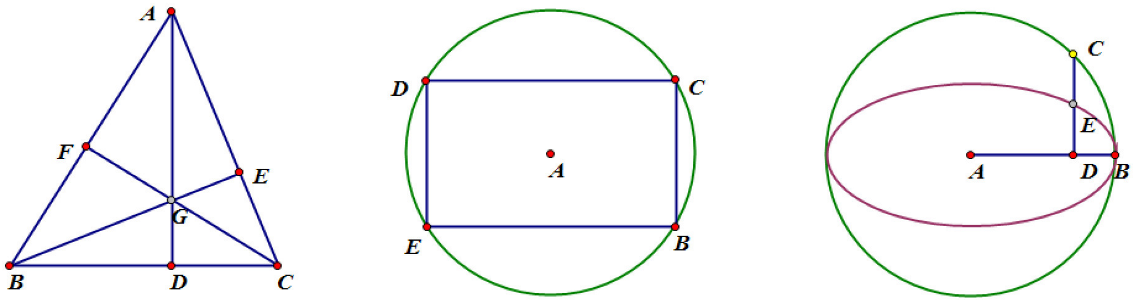

Figure D2 Test cases (a) case 1; (b) case 2; (c) case 3.

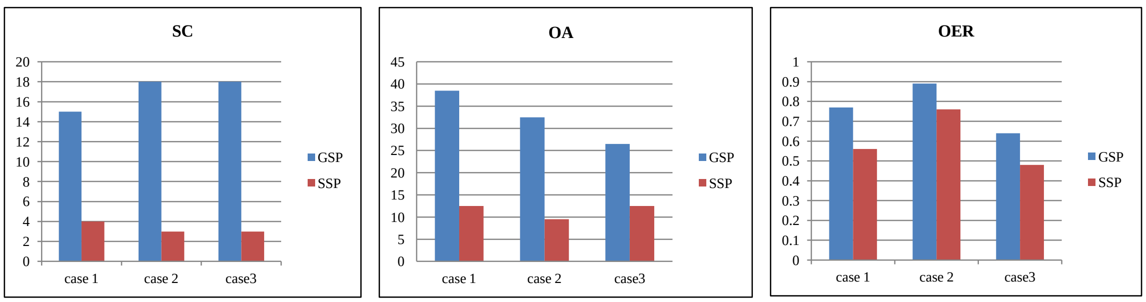

Figure D3 The experimental results.

- The maximum count of operating events: Mcev;
- The maximum count of operating objects: Mcev;
- The easiness rate of operating events: ERev, $ERev = \frac{Mcev}{Cev} \times \frac{1}{Tev}$;
- The easiness rate of operating objects: ERob, $ERob = \frac{Mcob}{Cob} \times \frac{1}{Tob}$;
- The operating easiness ratio: OER, $OER = ERev + Peo \times ERob$, where a smaller value means a better quality;
- The switching count of menu, toolbar and drawing area: SC, where a smaller value means a better quality;

We take three mathematics teaching examples as the test cases, as shown in Figure D2.

Table D2 reports metrics collected of the drawing process of case 1 of Figure D2 using GSP.

Table D3 reports metrics collected of the drawing process of case 1 of Figure D2 using SSP.

According to the drawing process of table , we can obtain the UTEM-ES data of case 1, as shown in Table D4.

Similarity, we can obtain the UTEM-ES data of case 2 and case 3, as reported in Table D5 and Table D6.

The experimental results of the three cases show $SC(GSP) > SC(SSP), OA(GSP) > OA(SSP), OER(GSP) > OER(SSP)$, and the average switching count SC of SSP is about 1/5 of GSP, the average operating amount OA is about SSP is 1/3 of GSP, the average operating easiness ratio OER of SSP is about 72% of GSP.

All experimental results above collectively demonstrate the user-friendliness, domain support for mathematic education, extensibility, and efficiency of SSP in comparison with the state-of-the-art.

References

1 Kortenkamp U. Foundations of Dynamic Geometry. PhD thesis. Swiss Federal Institute of Technology Zurich, 1999

2 Zhang J Z, Gao X S, Zhou X Q. Geometric invariant methods of geometric theorem proving. Beijing: the Science Publishing Company, 2015

3 Zhang J Z, Zhang C J, Zheng H, et al. The realization and application of symbolic computation modules in SGARP. Journal of Computer Research and Development, 2014,51(6):1341-1351

4 Zou Y, Zhang J Z. Automated generation of readable proofs for constructive geometry statements with the mass point method. Proceedings of the 8th International Workshop on Automated Deduction in Geometry (ADG 2010). Berlin: Springer, 2010: 221-258

5 Zhang J Z, Gao X S, Chou S C. The geometry information search system by forward reasoning. Chinese Journal of Computers, 1996,19(10):722-727

6 Zhang J Z, Yang L, Gao X S, Chou S C. Automated generation of readable proofs in geometry. Chinese Journal of Computers, 1995,18(5):380-394

7 Ji L. The switch functions of the symbolic function in the mathematics education technology. Journal of Tibet University, 2012,(2):92-97

8 Wang P Y. Research about application and innovation of mathematics education technology. Nanning: Guangxi Education Publishing House, 2010

Table D2 The drawing process of case 1 using GSP

Step	Aim	Operations	Operating Events	Events Count	Operating Objects	Objects Count
1	select ”Line Tool”	click toolbar button	click	1	toolbar	1
2	draw a segment line	click and drag	click, drag	2	drawing area	1
3	draw a segment line	click and drag	click, drag	2	drawing area	1
4	draw a segment line	click and drag	click, drag	2	drawing area	1
5	select “Line Tool”	click toolbar button	click	1	toolbar	1
6	select the point and line	click the point and line	click	2	drawing area	1
7	draw a perpendicular line	click menu “draw - perpendicular”	click	2	menu	1
8	select the point and line	click the point and line	click	2	drawing area	1
9	draw a perpendicular line	click menu “draw - perpendicular”	click	2	menu	1
10	select the point and line	click the point and line	click	2	drawing area	1
11	draw a perpendicular line	click menu “draw - perpendicular”	click	2	menu	1
12	draw foot points	click the intersection point of two lines	click	3	drawing area	3
13	select three perpendicular lines	click perpendicular lines	click	3	drawing area	3
14	hide perpendicular lines	click menu “view - hide”	click	2	menu	1
15	select “Line Tool”	click toolbar button	click	1	toolbar	1
16	draw segment line	click and drag	click, drag	2	drawing area	1
17	draw segment line	click and drag	click, drag	2	drawing area	1
18	draw segment line	click and drag	click, drag	2	drawing area	1
19	select “Select Tool”	click toolbar button	click	1	toolbar	1
20	draw the orthocenter	click the intersection point of two perpendicular segment lines	click	1	drawing area	1
21	select “Text Tool”	click toolbar button	click	1	toolbar	1
22	display points name	click all points	click	7	drawing area	7

Table D3 The drawing process of case 1 using SSP

Step	Aim	Operations	Operating Events	Events Count	Operating Objects	Objects Count
1	select ”Intelligent Pen”	click toolbar button	click	1	toolbar	1
2	draw a segment line	click and drag	click, drag	2	drawing area	1
3	draw a segment line	click and drag	click, drag	2	drawing area	1
4	draw a segment line	click and drag	click, drag	2	drawing area	1
5	draw a perpendicular segment line and the foot point	click and drag	click, drag	2	drawing area, tips	2
6	draw a perpendicular segment line and the foot point	click and drag	click, drag	2	drawing area, tips	2
7	draw a perpendicular segment line and the foot point	click and drag	click, drag	2	drawing area, tips	2
8	draw the orthocenter	click the intersection point of two perpendicular segment lines	click	1	drawing area	1

Table D4 UTEM-ES data of case 1

	SC	Tev	Tob	Cev	Cob	**OA**	Peo	Mcev	Mcob	ERev	ERob	**OER**
GSP	**15**	2	3	45	32	**38.5**	1.41	39	23	0.43	0.24	**0.77**
SSP	**1**	2	3	14	11	**12.5**	1.27	8	7	0.29	0.21	**0.56**

Table D5 UTEM-ES data of case 2

	SC	Tev	Tob	Cev	Cob	**OA**	Peo	Mcev	Mcob	ERev	ERob	**OER**
GSP	**18**	2	3	37	28	**32.5**	1.31	39	23	0.53	0.27	**0.89**
SSP	**1**	2	3	11	8	**9.5**	1.38	8	7	0.36	0.29	**0.76**

Table D6 UTEM-ES data of case 3

	SC	Tev	Tob	Cev	Cob	**OA**	Peo	Mcev	Mcob	ERev	ERob	**OER**
GSP	**18**	2	3	28	25	**26.5**	1.12	25	13	0.45	0.17	**0.64**
SSP	**1**	2	3	14	11	**12.5**	1.27	8	5	0.29	0.16	**0.48**

9 Xu Z T. The educational value of the super sketchpad and its application in teaching. Beijing: the Science Publishing Company, 2015
10 Yang L. Effectiveness research on the integration of super sketchpad and function teaching in senior shool. Haikou: Hainan Normal University, 2012
11 Xu Z T, Zhang J Z. Learn how to think in a virtual environment created by Super Sketchpad. Shuxue Tongxun, 2014,(9):58-59
12 Feng X W, Feng C W. exploration of primary mathematics teaching based on Super Sketchpad. China Educational Technology and Equipment, 2014,(23):25-26
13 Yan Y H. Improve deaf students geometry problem proof ability by using Super Sketchpad. Learning and Research in Mathematics, 2015,(2):35-35
14 Chen Q H. Action research on super sketchpad application in secondary school teaching of cone curve. Wuhan: Central China Normal University, 2015
15 Su Y H. Research on primary mathematics teaching practice based on Super Sketchpad. New Curriculum, 2015,(12):494-495
16 Deng C. How to optimize analytical geometry teaching with the support of Super Sketchpad. 2016,(3):8-11
17 Zheng Y. The application of super sketchpad in mathematical inquiry teaching. Teaching and Learning of New Curriculum, 2016,(6):13-14
18 Qiao Q, Gao L, Pang Z Y, et al. On the cultivation of studentsintuitive imagination in super sketchpad. Journal of Guizhou Education University, 2016,32(9):68-72
19 Xiao H Z, Zhang C J, Zhu L. Research on cultivating students' ability of mathematical inquiry using Super Sketchpad. Maths Teaching In Middle School, 2016,(10):27-29
20 Li W L. Develop geometric intuitive literacy by Super Sketchpad. Education Science Forum, 2016,(23):45-46
21 The Geometer's Sketchpad: http://www.dynamicgeometry.com
22 FBM: http://ssp.gzhu.edu.cn
23 Fang H G, Zhang J Z. Research on evaluation of educational software usability. E-education Research, 2008,(2):63-67
24 Squires D, Jenny P. Usability and learning: evaluating the potential of educational software. Computers & Education 1996,(27)1:15-22