SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

September 2018, Vol. 61 092208:1–092208:15
https://doi.org/10.1007/s11432-017-9256-3

Distributed filtering for time-varying networked systems with sensor gain degradation and energy constraint: a centralized finite-time communication protocol scheme

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Received 29 July 2017/Accepted 22 September 2017/Published online 10 August 2018

Abstract This paper focuses on the distributed filtering problem for a class of time-varying networked systems with sensor gain degradation and energy constrained communication protocol. To satisfy the requirement of power consumption and reduce the schedule computing complexity, centralized cyclic finite-time communication strategy optimization is taken into account. The networked system considered in this paper consists of spatially distributed sensors linked to their neighbor sensors, where each sensor node suffers from different gain degradation, and the transmission decisions of all the communication channels obey the centralized transmission schedule strategy identically. First, we present scattered communication action based on single-sensor transmission modeling with an energy constraint. Subsequently, an optimal communication protocol considering expected average error covariance is derived between the target system and each sensor node over the distributed sensor systems, based on a centralized finite-time scheme. Finally, by transforming the overall estimation error covariance of the systems at each sampling time into quadratic form, a conditionally unbiased least-square recursive distributed filtering technique over the networked system is designed at each sensor node. The system stability condition under such an optimal schedule is also accomplished with bounded covariance. A numerical example is provided to demonstrate the utility and effectiveness of the distributed filtering technique using the proposed optimized energy constrained communication protocol.

Keywords networked systems, distributed filtering, communication protocol, centralized finite-time schedule, energy constraint, sensor gain degradation

Citation Zhao Y, He X, Zhou D H. Distributed filtering for time-varying networked systems with sensor gain degradation and energy constraint: a centralized finite-time communication protocol scheme. Sci China Inf Sci, 2018, 61(9): 092208, https://doi.org/10.1007/s11432-017-9256-3

1 Introduction

In recent years, spurred by widespread application in various fields including battlefield surveillance, environment monitoring, intelligent transportation, and health care, massive research efforts have been devoted to distributed estimation based on networked systems in information fusion, control, and industrial process monitoring. The distributed sensor system is one type of networked system composed of spatially distributed sensor nodes which have the capability to measure certain parameters of interest,

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such as temperature, humidity, angle, position, velocity of vehicle and vehicle emission [1–3]. These sensor nodes are able to cooperate and coordinate to implement global tasks by communicating with their neighbors. Although, the networked system possesses the advantages of convenience for implementing remote operation and monitoring, low cost, high flexibility, and reliability with each sensor node capable of wireless communication and intelligence for signal processing, there are some particular types of characteristics which can be collectively referred to as network constraints: energy constraint [4], packet dropout [5], uncertainty observation [6], and multiple noise interference [7], compared with the traditional single sensor system. Considering the above constraints, distributed estimation is among the more fundamental and significant tasks that a networked system needs to accomplish. This task has been investigated in recent years and is among the potential and practical subjects remaining to be further studied in the following years.

In many practical applications, communication and computing capabilities of sensor nodes are limited due to various design and implementation considerations such as small battery volume providing limited power supply. Specifically, the energy for data collection and transmission is constrained, and sensor nodes may not transmit data continuously. It would be expensive to replace existing battery systems with long-lasting batteries or to replace existing sensors with harvesting sensors in many applications. Therefore, proper sensor scheduling is still important and is often subject to communication power consumption and battery lifetime for sensor networked systems. Various studies have investigated scheduling techniques [8–20]. The effective energy-efficient methodologies can be mainly categorized into three types. (1) Stochastic scheduling [8–10]: each sensor node sends data to a remote estimator according to a given probability at each time step, and an optimal probability distribution is obtained by solving an optimization problem. (2) Deterministic scheduling [11–15]: the sensor node activation sequence is fixed and indicates the moment in which to send data. In [11], the design and performance evaluation of a communication protocol were presented for distributed sensor systems with dynamic and limited energy. (3) Event-triggered scheduling [16–20]: these schedules utilize real-time measurements from selected sensors which are able to trigger an event.

Regarding the network constraint concerning the sensor nodes, this is a common phenomenon that has critical impact on the distributed estimation. Similarly, the network uncertainty constraint is a class of phenomenon that frequently occurs, and has a significant impact on the estimation performance and stability of the network. Moreover, the network uncertainty further deteriorates the accuracy of the overall sensor measurement and distributed estimation. So different researches have considered and studied the network uncertainty constraints [21–29]. From our point of view, the network uncertainty constraint can be mainly classified into three categories. (1) Wireless channel uncertainty [21, 22]: measurements are transmitted over fading channels. (2) Sensing uncertainty [23, 24]: an incomplete specified sensing process is modeled at each sensor. (3) Sensor gain degradation [25–29]: the sensor gains degrade in random fashion due to various networked systems or environments, namely intermittent sensor outages/faults, sensor aging, sensor packet dropout, sensor saturation, sensor dead zones or transmission congestions, which are particularly true for real-world systems under variable working conditions. Examples involve thermal sensors for detection of moving ground vehicles [27], oceanic platform mounted sonar arrays for acoustic signals [28], and pressure sensors for internet-based three-tank system leakage detection [29].

So far, distributed filtering of networked sensor systems has been a heavily-researched topic, and several studies have tackled this challenging problem [26,30–39]. For example, the consensus strategy was applied in distributed Kalman filters [30–35] to obtain the consensus matrix and the update gain. This allows the nodes in the sensor network to track the average of the sensor measurements based on consensus filters. Optimal distributed filters were proposed to minimize the filtering error covariance [26, 36, 37]. In these optimal filters, the parameters of the filter were adjusted at each step to achieve the MMSE[1)] of the estimation error based on the received signals. In [38, 39], the distributed fusion filtering problem was addressed for a sensor network, where the outputs were perturbed by additive or multiplicative correlated noise in the measurement process. We can note that, among most of the reported results, the

1) MMSE is the abbreviated form of minimum mean square error.

target system and sensor nodes have been limited to time-invariant systems where the circumstances of sensor gain degradation have not been considered.

Based on the literature mentioned above, we focus on transmission schedule coping with constrained energy and the distributed filter design for communication protocol optimization. However, in many cases, it is of practical importance to assume that the energy for data collection and transmission is constrained for networked sensor systems supplied with cyclic rechargeable power. Except for time-varying systems where sensor gain degradation is considered, taking a centralized cyclic finite-time communication protocol into account with an energy constraint has been investigated by only a few studies, to the best of the authors' knowledge.

In this paper, we focus on distributed filtering for a class of time-varying networked systems with energy constrained, centralized finite-time communication protocol and sensor gain degradation, which is motivated by the restrictions of practical communication conditions. Particularly, we consider a process between the target system and each sensor node, where each intelligent sensor node receives a transmission data packet of the target system's state and correspondingly re-estimates the target system state. Adopting distributed filtering, we can achieve state estimation at each time step and the sensor network can be changeable because the network is informed of a change in state (e.g., sensor losses) which is suitable for practical applications without a fusion center. Employing the centralized cyclic finite-time schedule strategy, the sensor networked system can, on the one hand, reduce the burden of computing complexity, and lessen the energy consumption caused by necessary computing, on the other.

To the best of our knowledge, this is the first study on a centralized cyclic finite-time communication protocol for a time-varying networked system distributed filter design with sensor gain degradation. Main contributions of this paper are summarized as follows: (1) The problem of distributed filter design with sensor gain degradation for a networked time-varying networked system is formulated while the centralized finite-time communication strategy optimization with an energy constraint is taken into account. (2) The packet received is modeled with a certain probability when launching the communication action which can portray the network packet dropout phenomenon and the scheme of centralized cyclic finite-time communication protocol with distributed filtering design for a networked time-varying system is adopted to control the cyclic power supply. (3) The optimal communication protocol in a networked system is studied and the stability is analyzed under the proposed schedule.

The remainder of the paper is organized as follows. In Section 2, the system model and communication model are formulated respectively and some preliminary details are discussed. In Section 3, the optimal communication protocol is designed and the corresponding cost is also calculated. In Section 4, the distributed filter with sensor gain degradation under the optimal centralized cyclic finite-time communication protocol scheme is designed, and the stability based on the optimal schedule is also studied. In Section 5, a numerical example is shown to illustrate and clarify the results. Finally, Section 6 concludes the paper.

Notations. $\mathbb{R}^n$ stands for the n-dimensional Euclidean space. $\mathrm{Tr}(\cdot)$ is the trace of the square matrix. $\|\cdot\|$ is the Euclidean norm of the vector. $(\cdot)$ stands for the transposition of the matrix. $\mathrm{E}\{\cdot\}$ stands for the expectation of the stochastic variable. I is the identity matrix with compatible dimension. $\mathrm{diag}_n\{\cdot\}$ and $\mathrm{vec}_n(\cdot)$ stands for a block-diagonal matrix and a row vector, respectively.

2 Problem formulation and preliminaries

2.1 System model

Consider the following class of time-varying discrete-time target system:

$$x(k+1) = A(k)x(k) + w(k), \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state with $n_x \in \mathbb{Z}^+$; $A(k)$ is the known matrix with appropriate dimensions; $w(k) \in \mathbb{R}^{n_x}$ is the additive white noise with mean 0 and covariance $S(k)$. $\mathrm{E}\{x(0)\}$ and $\mathrm{E}\{x(0)x^{\mathrm{T}}(0)\}$ are assumed to be known.

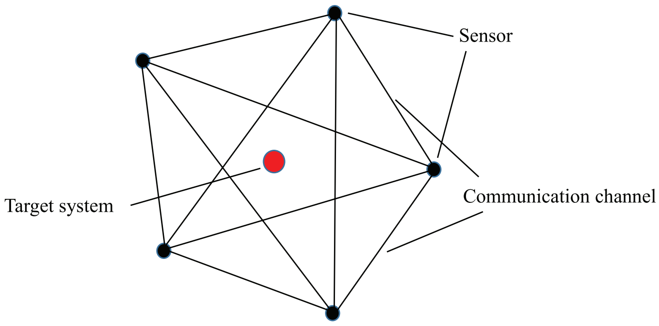

Figure 1 (Color online) Schematic architecture of the networked sensor system [40].

For each sensor node i $(i = 1, 2, \dots, n)$, the measurement of the i-th sensor before communication is described respectively by

$$y_i(k) = \lambda_i(k)C_i(k)x(k) + v_i(k), \tag{2}$$

where $\lambda_i(k)$, indicating the generic sensor gain degradation in the i-th node, is the random variable distributed on the interval $[a_i, b_i]$ $(0 \leqslant a_i \leqslant b_i \leqslant 1)$ with mean $m_i(k)$ and variance $l_i(k)$, which are both scalars assumed to be known when designing the proposed distributed filtering systems. $y_i(k) \in \mathbb{R}^{n_y}$ is the sensor measurement of the i-th node with $n_y \in \mathbb{Z}^+$; $C_i(k)$ is a known matrix with appropriate dimensions for all $i = 1, 2, \dots, n$; $v_i(k) \in \mathbb{R}^{n_y}$ is the additive white noise of the i-th node with mean 0 and variance $V_i(k)$.

We model a networked sensor system whose topology is given by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{D})$ with the set of nodes $\mathcal{V} = \{1, 2, \dots, n\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a switched adjacency matrix $\mathcal{D} = [d_{ij}]$ with adjacency elements d_{ij} representing whether there exists physical communication channel between node i and node j (1 denotes there is communication while 0 denotes none). Figure 1 directly demonstrates the schematic architecture of the networked system. We assume $d_{ii} = 1$ for all $i \in \mathcal{V}$. The set of all the neighbors of node i plus the node itself are denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : d_{ij} \in \{0, 1\}\}$. Moreover, $\gamma(k)$ denotes the centralized communication decision variable at time k showing communication action or not (1 denotes communication action while 0 denotes not).

Denote $\hat{x}_i(k) \in \mathbb{R}^{n_x}$ as the state estimate of the target from the i-th node. The key point to design the distributed filters for the networked sensor system is to integrate the information available for the filter on sensor node i on the one hand from the sensor i itself, and on the other from its neighbors [41]. Every filter out of the class is equivalent and uses the same amount of the information. Thus here, the filter to be designed is of the following structure for the sensor node i:

$$\hat{x}_i(k+1) = A(k)\hat{x}_i(k) + \sum_{j \in \mathcal{N}_i} H_{ij}(k)a_{ij}(k)[y_j(k) - m_j(k)C_j(k)\hat{x}_j(k)], \tag{3}$$

$$a_{ij}(k) = d_{ij}\gamma(k), \tag{4}$$

where $a_{ij}(k)$ can be considered as an adjacency element of generalized changeable topology, the matrix $H_{ij}(k)$ is the parameter needed to be set, and $\gamma(k)$ is the communication decision variable at time k, whose possible values are 0 and 1. The initial value of the state estimate of the target from sensor node i is $\hat{x}_i(0) = \mathrm{E}\{x(0)\}$ for all $1 \leqslant i \leqslant n$. The proposed structures (3) and (4) represent how the sensor nodes communicate with their neighbors via $\mathcal{N}_i$ so as to not only satisfy the communication condition, but also guarantee unbiased and optimized estimation performance under the specific restriction.

2.2 Communication model

Let $\gamma(k) = 1$ or 0 be the communication decision variable at time k, and γ be the schedule that defines $\gamma(k)$ at each time k. When the communication generator launches the transmission, the sensor data packet will be received with the probability α. Denote $\theta(k) = 1$ or 0 as the indicator representing whether the data packet is received or not. Assume that $\theta(k)$ is an independently and identically distributed Bernoulli random variable with $\mathrm{E}(\theta(k)) = \alpha$ for each k. The consequence of communication action $\gamma(k) = 1$ is

$$\theta(\gamma(k)) = \begin{cases} 0, & \text{with probability } 1-\alpha, \\ 1, & \text{with probability } \alpha. \end{cases} \tag{5}$$

If the communication generator does not take action and remains dormant at time k, i.e., $\gamma(k) = 0$, the data packet cannot be received by the distributed estimator, i.e., $\theta(\gamma(k)) = 0$.

Specifically, consider a finite horizon T as one period where T is a positive integer. Denote $\gamma = (\gamma(1), \gamma(2), \ldots, \gamma(T))$ as the communication schedule on the horizon in a certain period.

The networked sensor system is supplied with cyclic rechargeable energy whose period is T, the communication generator of the networked system can only launch n transmissions in one period of time T from $k = 1$ to $k = T$. Thus, we formulate the power constraint as

$$\|\gamma\|_0 = n, \quad n < T. \tag{6}$$

Definition 1. Define $J(\gamma)$ as the average expected estimation error covariance matrix for a given communication schedule γ in one period of the cyclic finite-time communication protocol, i.e.,

$$J(\gamma) = \frac{1}{T}\sum_{k=1}^{T} \mathrm{E}[P(\gamma(k))]. \tag{7}$$

In this paper, we aim to solve Problem 1 from the viewpoint of the communication generator.

Problem 1. Solving the optimization problem with a limitation factor as below:

$$\min_{\gamma\in\Gamma} \ \mathrm{Tr}[J(\gamma)], \tag{8}$$

$$\text{s.t.} \ \ \|\gamma\|_0 = n, \tag{9}$$

where γ is the communication schedule on a certain finite-time period horizon T and $\Gamma = \{\gamma|\gamma(k) \in \{0, 1\}, \forall k \in \{1, 2, \ldots, T\}\}$ is the communication schedule space.

Remark 1. If the expected estimation error covariance matrix is a non-negative definite matrix, then according to the Kalman filtering algorithm, the trace of the matrix is a non-negative real number. Thus the error covariance matrix represents the accuracy of the estimation, so it is reasonable to measure the quality of distributed filtering with the summation of the trace value of each sampling time.

3 Optimal communication schedule analysis

3.1 State estimation under communication scheduling

In this subsection, we present lemmas applicable to the estimation error for one sensor node system under the communication schedule.

Considering that noise signals are relatively small compared to the estimated state and the sensors have sufficient computational capability [15], the dynamic state is eliminated because of the fast process. The system uses a local Kalman filter to obtain $\hat{x}^s(k)$ and sends $\hat{x}^s(k)$ to the remote estimator from each sensor node. It can be deduced directly that the optimal filtering state estimate and error covariance at the remote estimator are computed as

$$\begin{aligned} &(\hat{x}(k+1), P(k+1)) \\ &= \begin{cases} (A\hat{x}(k), AP(k)A' + \Sigma_w), & \text{if } \gamma(k+1) = 0,\ \theta(k+1) = 0, \\ (\hat{x}^s(k+1), P^s(k+1)), & \text{if } \gamma(k+1) = 0,\ \theta(k+1) = 1 \text{ or solo } \gamma(k+1) = 1. \end{cases} \end{aligned} \tag{10}$$

For $P^s(0) \geqslant 0$, $P^s(k)$ converges to $\bar{P}$ exponentially on condition that the sensors have sufficient computation, so without much loss of generality, we assume that the Kalman filter converges to its steady-state value at the remote estimator. Then $P(k+1)$ is simply given by

$$P(k+1) = \begin{cases} AP(k)A' + \Sigma_w, & \text{if } \gamma(k+1) = 0,\ \theta(k+1) = 0, \\ \bar{P}, & \text{if } \gamma(k+1) = 0,\ \theta(k+1) = 1 \text{ or solo } \gamma(k+1) = 1. \end{cases} \tag{11}$$

For the sake of brevity and simplicity, we assume that $P(0) = \bar{P}$ and $\bar{P} \approx 0$. It is straightforward to show that

$$\mathrm{E}[P(k+1)|P(k) = 0] = \begin{cases} \alpha\Sigma_w, & \text{if } \gamma(k+1) = 0, \\ 0, & \text{otherwise,} \end{cases} \tag{12}$$

and

$$\mathrm{E}[P(k+1)|P(k) \neq 0] = \begin{cases} \alpha(AP(k)A' + \Sigma_w), & \text{if } \gamma(k+1) = 0, \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Lemma 1 shows how the consecutive dormancy action (non-communication action) schedule affects the expected error covariance.

Lemma 1. For a given consecutive dormancy action schedule time interval $[s+1, s+t]$, i.e., $\gamma(s) = 1, \gamma(s+1) = \gamma(s+2) = \cdots = \gamma(s+t) = 0$, $\gamma(s+t+1) = 1$, we have

$$\mathrm{E}[P(s+j)] = \sum_{i=0}^{j-1} (\alpha^i - \alpha^{i+1}) H_i + \alpha^j H_j, \tag{14}$$

where $H_i = \sum_{l=0}^{i-1} (A^l) \Sigma_w (A^l)'$ and $j = 1, 2, \ldots, t$.

Proof. Note that $\gamma(k) = 0$ or 1, $\theta(\gamma(k)) = 0$ or 1 are both dual variables. According to the duality theory, we can transform from (5) to the dual problem statement. The consequence of dormancy action $\gamma(k) = 0$ is then

$$\theta(\gamma(k)) = \begin{cases} 1, & \text{with probability } 1-\alpha, \\ 0, & \text{with probability } \alpha. \end{cases} \tag{15}$$

If $\gamma(k) = 1$, $\theta(\gamma(k)) = 1$. Then the continuity is similar to [42].

From Lemma 1, for a given consecutive dormancy action schedule time interval $[s+1, s+t]$, $\mathrm{E}[P(s+j)]$, $j = 1, 2, \ldots, t$ does not depend on s but only depends on the time difference. Thus we denote $\Gamma_\alpha(t)$ by the sum of expected error covariance with any t times consecutive dormancy action in the given dormancy action period $[s+1, s+t]$. In the same manner, any communication schedule with consecutive dormancy sequence $t_1, t_2, \ldots, t_s$ in a given period will contribute to the same sum of expected error covariance. Then we denote $\Gamma_\alpha(t_1 \oplus t_2 \oplus \cdots \oplus t_s)$ by the sum of expected error covariance with consecutive dormancy action sequence $t_1, t_2, \ldots, t_s$ in the given communication period $[1, T]$.

$$\Gamma_\alpha(t) = \sum_{j=1}^{t} \mathrm{E}[P(s+j)] = \sum_{j=1}^{t} \left[\sum_{i=0}^{j-1} (\alpha^i - \alpha^{i+1}) H_i + \alpha^j H_j \right]. \tag{16}$$

A communication action schedule with given communication number $T - n$ (dormancy number n) can be divided into the following consecutive dormancy sequences, $k_1, k_2, \ldots, k_s$ i.e.,

$$(1, \ldots, 1, \underbrace{0, \ldots, 0}_{k_1 \text{ times}}, 1, \ldots, 1, \underbrace{0, \ldots, 0}_{k_2 \text{ times}}, 1, \ldots, 1, \ldots, \underbrace{0, \ldots, 0}_{k_s \text{ times}}, 1, \ldots, 1),$$

where $\sum_{i=1}^{s} k_i = n$. The dormancy action schedule is also denoted as $\gamma^{(k_1 \oplus k_2 \oplus \cdots \oplus k_s)}$. Note that each neighboring sequence of 0s denoting dormancy is divided by at least one communication action 1. Thus, we have

$$J_\alpha^{(k_1 \oplus k_2 \oplus \cdots \oplus k_s)}(\gamma) = \frac{1}{T} \sum_{k=1}^{T} \mathrm{E}[P(\gamma(k))] = \frac{1}{T} \sum_{i=1}^{s} \sum_{n_i=1}^{k_i} \left[\sum_{m=0}^{j_i - 1} (\alpha^m - \alpha^{m+1}) H_m + \alpha^{j_i} H_{j_i} \right]. \tag{17}$$

Remark 2. The dormancy action schedule and the corresponding cost are independent of the dormancy action start time. That is to say, any permutation of k_i $i=1,2,\ldots,s$ will generate the same estimation error when the horizon $[1,T]$ is pre-given.

Lemma 2 ([43])**.** The following statements are true.

(1) $J_\alpha^{(n_1)} \leqslant J_\alpha^{(n_2)}$ where $n_1 \leqslant n_2$;

(2) $J_\alpha^{(n_1 \oplus n_2)} \leqslant J_\alpha^{(n)}$ where $n = n_1 + n_2$;

(3) $J_\alpha^{(n_1 \oplus n_2 \oplus \cdots \oplus n_s)} \leqslant J_\alpha^{(n)}$ where $n = n_1 + n_2 + \cdots + n_s$;

(4) $J_\alpha^{(m_1 \oplus m_2)} \leqslant J_\alpha^{(n_1 \oplus n_2)}$ where $m_1 + m_2 = n_1 + n_2$ and $\max\{m_1, m_2, n_1, n_2\}$ is n_1 or n_2.

Remark 3. From the statements above, we know that the fewer dormancy times gathered together, the smaller the corresponding system cost becomes. Moreover, scattered dormancy action is much better than consecutive dormancy action in the sense of average expected estimation error covariance from the viewpoint of the communication generator.

3.2 Optimal communication scheduling design

From Lemmas 1 and 2 and from the different perspectives adopted to understand scheduling, the scattered communication action schedule is better than the grouped schedule, which will lead to a better estimation.

Definition 2 (Completely scattered)**.** Denote a communication action as 1 and denote a dormancy action as 0. Then, there are $T-n$ communication actions (1) and n dormancy actions (0). Thus every two dormancy actions are separated by one or more communication actions, if and only if the actions satisfy the relation $T-n+1 \geqslant n$, which leads to $n \leqslant \frac{T+1}{2}$.

Theorem 1. If $n \leqslant \frac{T+1}{2}$, the optimal communication schedule γ^* for Problem 1 is any n times scattered communication action, referred to as completely scattered

$$(1,\ldots,1),0,(1,\ldots,1),0,\ldots(1,\ldots,1)\ldots,0,(1,\ldots,1).$$

The corresponding cost function is given by

$$J_\alpha(\gamma)_{\min} = \frac{1}{T}\sum_{i=1}^{s}\sum_{n_i=1}^{k_i}[\alpha\Sigma_w] = \frac{n\alpha\Sigma_w}{T}. \tag{18}$$

Proof. See Appendix A for the proof.

Theorem 2. If $n > \frac{T+1}{2}$, the optimal communication schedule γ^* for Problem 1 is

$$\underbrace{\underbrace{(0,\ldots,0)}_{\text{Qt times}},1,\underbrace{(0,\ldots,0)}_{\text{Qt times}},1,\ldots}_{(T-n+1)-\text{Rm times}}\underbrace{\underbrace{(0,\ldots,0)}_{\text{Qt}+1\text{ times}}\ldots,1,\underbrace{(0,\ldots,0)}_{\text{Qt}+1\text{ times}}}_{\text{Rm times}},$$

where we take the quotient of $\frac{n}{T-n+1}$ as Qt, where $\text{Qt} = \llcorner \frac{n}{T-n+1} \lrcorner$, $\text{Rm} = n-(T-n+1)\text{Qt}$.

The corresponding cost function is given by

$$\begin{aligned} J_\alpha(\gamma)_{\min} = {} & \frac{1}{T}\text{Rm}\left[\sum_{i=1}^{\text{Qt}-1}(\text{Qt}-i)(\alpha^i-\alpha^{i-1})H_i + \sum_{i=1}^{\text{Qt}}(\text{Qt}+1-i)\alpha^i H_i\right] \\ & + \frac{1}{T}[(T-n+1)-\text{Rm}] \\ & \times \left[\sum_{i=1}^{\text{Qt}}(\text{Qt}+1-i)(\alpha^i-\alpha^{i-1})H_i + \sum_{i=1}^{\text{Qt}+1}(\text{Qt}+2-i)\alpha^i H_i\right]. \end{aligned} \tag{19}$$

Proof. See Appendix B for the proof.

Remark 4. Note that $t_1, t_2, \ldots, t_s$ satisfy commutative law so the optimal communication schedule may not be unique but the corresponding cost function is unique for both conditions of the two theorems above. From Theorems 1 and 2, one can see that scattered communication action taking place as uniformly as possible leads to the minimal cost, in other words, the schedule that reduces the number of dormancy actions gathered together leads to the minimal cost. And in this case, it will therefore also produce the optimized estimation result.

4 Distributed filter design for networked sensor system

In this section, we first propose a distributed filter design under the optimal communication protocol for networked sensor systems, and then analyze its stability under the optimal centralized cyclic finite-time communication protocol scheme.

4.1 Distributed filter design under optimal communication protocol

In this subsection, based on the optimized state estimation of each sensor node, we present a method to design distributed filters over networked sensor systems to achieve the overall performance optimized with an energy constraint, so that the filtering state is unbiased and the filtering error covariance is minimized conditionally.

Proposition 1. Distributed filtering for a networked system involving multiple sensors is equivalent to the filtering process over the communication channel between the target system and each sensor node in the networked sensor system.

Proof. The structure of each sensor filtering over the communication channel between the target system and one sensor node [44] is

$$\hat{x}(k+1) = A(k)\hat{x}(k) + K(k)\beta(k)[y(k) - C(k)\hat{x}(k)], \tag{20}$$

where $K(k)$ is the filtering gain and $\beta(k)$ denotes the on or off state of the communication channel.

The structure of distributed filtering for a networked system involving multiple sensors is

$$\hat{x}_i(k+1) = A(k)\hat{x}_i(k) + \sum_{j\in\mathcal{N}_i} H_{ij}(k)a_{ij}(k)[y_j(k) - m_j(k)C_j(k)\hat{x}_j(k)], \tag{21}$$

where $\hat{x}_i(k+1)$ is the state estimate from each node and $H_{ij}(k)$ denotes the multiple sensor filtering gain which corresponds to $K(k)$. The on or off states of distributed filtering communication channels for the networked sensor system are all included in $a_{ij}(k)$ which corresponds to $\beta(k)$. Moreover, the two structures (20) and (21) above can be combined into a unified structure as (21) when there is only one sensor in the networked system with the subscript $i = 1$ omitted. The proposition is thus complete.

The specific distributed filter design explanations are proposed as follows. First, it can be verified that the unbiased condition is true for $k = 0$ according to $\hat{x}_i(0) = \mathrm{E}\{x(0)\}$ for all $i \in \mathcal{V}$. Letting $\tilde{x}_i(k) = x_i(k) - \hat{x}_i(k)$, we have the following system that reflects the filtering error dynamics [26]:

$$\begin{aligned}\tilde{x}_i(k+1) =& A(k)\tilde{x}_i(k) - \sum_{j\in\mathcal{N}_i} H_{ij}(k)d_{ij}\gamma(k)m_j(k)C_j(k)\tilde{x}_j(k) - \sum_{j\in\mathcal{N}_i} H_{ij}(k)d_{ij}\gamma(k)v_j(k)\\ &- \sum_{j\in\mathcal{N}_i} H_{ij}(k)d_{ij}\gamma(k)[\lambda_j(k) - m_j(k)]C_j(k)x(k) + w(k),\end{aligned} \tag{22}$$

for $i = 1, 2, \ldots, n$. Then for any $i \in \mathcal{V}$, it follows from the above that

$$\begin{aligned}\mathrm{E}\{\tilde{x}_i(k+1)\} =& A(k)\mathrm{E}\{\tilde{x}_i(k)\} - \sum_{j\in\mathcal{N}_i} H_{ij}(k)d_{ij}\gamma(k)m_j(k)C_j(k)\mathrm{E}\{\tilde{x}_j(k)\}\\ &- \sum_{j\in\mathcal{N}_i} H_{ij}(k)d_{ij}\gamma(k)\mathrm{E}\{v_j(k)\}\\ &- \sum_{j\in\mathcal{N}_i} H_{ij}(k)d_{ij}\gamma(k)[\mathrm{E}\{\lambda_j(k)\} - m_j(k)]C_j(k)\mathrm{E}\{x(k)\} + \mathrm{E}\{w(k)\}.\end{aligned} \tag{23}$$

Then considering the fact that $w(k)$, $\tilde{x}_i(k)$, and $v_i(k)$ are all zero-mean, and $\mathrm{E}\{\lambda_j(k)\} = m_j(k)$, it can be proven that $\mathrm{E}\{\tilde{x}_i(k+1)\} = 0$ for any $i \in \mathcal{V}$. At this point, the filter is unbiased.

In simple form, we denote

$$\begin{aligned}
&\tilde{x}(k) := \mathrm{vec}_n^{\mathrm{T}}\{\tilde{x}_i^{\mathrm{T}}(k)\}, && \bar{x}(k) := \mathrm{vec}_n^{\mathrm{T}}\{\bar{x}^{\mathrm{T}}(k)\},\\
&\bar{M}(k) := \mathrm{diag}_n\{m_i(k)I\}, && H(k) := [H_{ij}(k)]_{n\times n},\\
&\bar{C}(k) := \mathrm{diag}_n\{C_i(k)\}, && \bar{A}(k) := \mathrm{diag}_n\{A(k)\},\\
&\bar{w}(k) := \mathrm{vec}_n^{\mathrm{T}}\{w^{\mathrm{T}}(k)\}, && \bar{v}(k) := \mathrm{vec}_n^{\mathrm{T}}\{v_i^{\mathrm{T}}(k)\},\\
&\hat{A}(k) := \mathrm{diag}_n\{\tilde{A}(k)\}, && T_i(k) := \mathrm{diag}\{d_{i1}\gamma(k)I, \ldots, d_{in}\gamma(k)I\},\\
&\bar{\Lambda}(k) := \mathrm{diag}_n\{\lambda_i(k)I\}, && E_i := \mathrm{diag}\{\underbrace{0,\ldots,0}_{i-1}, I, \underbrace{0,\ldots,0}_{i-1}\}.
\end{aligned}$$

Then the respective presentations above can be rewritten as the following augmented form:

$$\begin{aligned}
\tilde{x}(k+1) = {}& \left[\bar{A}(k) - \sum_{i=1}^{n} E_i H(k) T_i(k) \bar{M}(k) \bar{C}(k)\right] \tilde{x}(k)\\
& - \sum_{i=1}^{n} E_i H(k) T_i(k) \left[\bar{\Lambda}(k) - \bar{M}(k)\right] \bar{C}(k) \bar{x}(k) - \sum_{i=1}^{n} E_i H(k) T_i(k) \bar{v}(k) + \bar{w}(k).
\end{aligned} \tag{24}$$

Define the optimization object

$$P(k) = \mathrm{E}\{\tilde{x}(k)\tilde{x}^{\mathrm{T}}(k)\}. \tag{25}$$

This is the objective function to be optimized. Here, we plan to design unbiased filters which are guaranteed by the previous derivation, so that the filtering estimation error covariance $P(k+1)$ is conditionally minimized with the constrained communication energy at each time k.

There are some denotations available for convenience. We define

$$\begin{aligned}
&\Omega(0) := \mathrm{E}\{x(0)x^{\mathrm{T}}(0)\}, \quad \Omega(k) := \mathrm{E}\{x(k)x^{\mathrm{T}}(k)\},\\
&U(k) := \mathrm{E}\left\{[\bar{\Lambda}(k) - \bar{M}(k)]\bar{C}(k)\bar{x}(k)\bar{x}_T(k)\bar{C}^{\mathrm{T}}(k)[\bar{\Lambda}(k) - \bar{M}(k)]^{\mathrm{T}}\right\} = \mathrm{diag}_n\left\{l_i(k)C_i(k)\Omega(k)C_i^{\mathrm{T}}(k)\right\},\\
&W(k) := \mathrm{E}\{\bar{w}(k)\bar{w}^{\mathrm{T}}(k)\}, \quad V(k) := \mathrm{E}\{\bar{v}(k)\bar{v}^{\mathrm{T}}(k)\},\\
&Y(k) := \bar{M}(k)\bar{C}(k)P(k)\bar{C}^{\mathrm{T}}(k)\bar{M}^{\mathrm{T}}(k) + V(k) + U(k), \quad Z(k) := \bar{M}(k)\bar{C}(k)P(k)\bar{A}^{\mathrm{T}}(k),\\
&\bar{\Omega}(k) := \mathrm{E}\{\bar{x}(k)\bar{x}^{\mathrm{T}}(k)\}, \quad \mathcal{H}(k) := \bar{Z}^{\mathrm{T}}(k)Y^{-1}(k).
\end{aligned}$$

Theorem 3. If the networked sensor system cannot communicate completely, an conditionally optimized solution for the parameters of the filter is given by

$$H_{ij}(k) = \begin{cases} \mathcal{H}_{ij}(k) d_{ij}^{-1} \gamma^{-1}(k), & \text{if } d_{ij} \neq 0 \text{ and } \gamma(k) \neq 0,\\ 0, & \text{otherwise,} \end{cases} \tag{26}$$

and $P(k)$ obeys the following equation:

$$\begin{aligned}
P(k+1) = {}& \left[\sum_{i=1}^{n} E_i \mathcal{H}(k) T_i(k) - Z^{\mathrm{T}}(k) Y^{-1}(k)\right] Y(k) \left[\sum_{i=1}^{n} E_i \mathcal{H}(k) T_i(k) - Z^{\mathrm{T}}(k) Y^{-1}(k)\right]^{\mathrm{T}}\\
& - Z^{\mathrm{T}}(k) Y^{-1}(k) Z(k) + \bar{A}(k) P(k) \bar{A}^{\mathrm{T}}(k) + W(k).
\end{aligned} \tag{27}$$

Proof. See Appendix C.

Remark 5. The distributed filtering parameters $H_{ij}(k)$ are calculated to minimize the overall filtering error covariance at each time step, under the condition that the communication energy is limited. It should be noted that some statistical characteristics of stochastic sensor gain degradations, process noise, and measurement noises are required in advance with the current changeable physical network topology, the proposed optimal communication schedule, and the state transition and measurement matrices, to determine the filtering gain matrix $H_{ij}(k)$. It is worth noting that the changeable topology is obtained by the information triggering technique among the nodes when new nodes are deployed or disappear from the networked system due to failure. Thus, the up-to-date values of $P(k)$ and $H_{ij}(k)$ at each sensor node need not obtain the global measurements from each sensor.

4.2 Stability analysis under optimal communication protocol

Stability is the most important requirement in distributed filter design for a networked sensor system with an energy constraint [45]. Because the information cannot be shared to the extent possible in an ideal system, the stability of the system is therefore of greater practical interest.

Theorem 4. The networked sensor system with an energy constraint under the optimal centralized cyclic finite-time communication protocol scheme is stable in the sense that the covariance of the system state is bounded when $n \neq 0$.

Proof. In the sense of bounded covariance [46], we can see that

$$\mathrm{Var}(x(k)) = \mathrm{E}[P(k)]. \tag{28}$$

Because $n \neq 0$ occurs in a certain period T, we can see that

$$\mathrm{Var}(x(k)) \leqslant \max_{p} \left\{ \sum_{i=0}^{p-1} (\alpha^i - \alpha^{i+1}) H_i + \alpha^p H_p \right\} = C^*, \tag{29}$$

where p is the maximum value of consecutive dormancy actions. For a finite value of p, the covariance of the state is bounded. If transmission times are $n = 0$ per period, the value of p tends to be infinite. Thus the covariance is unbounded. The proof is complete.

Remark 6. Theorem 4 shows that under the proposed communication protocol scheme, the networked sensor system with an energy constraint remains stable even if $n \neq 0$ which means that there is still communication in the finite horizon. However, if $n = 0$, the communication generator remains idle, and in this case, the generator cannot regulate the state and this leads to system instability.

5 An illustrative example

Consider the following discrete time-varying networked sensor system first introduced by [40]. The target system is described with the parameters below:

$$A(k) = \begin{bmatrix} 0.1315 + 0.0054\sin(k) & 0.0537 \\ 0.0201 & -0.1007 \end{bmatrix}.$$

Assume that $w(k)$ is a zero-mean Gaussian white noise and its covariance is $0.1I$. The initial value of the state $x(0)$ distributes over $[-0.1, 0]$ uniformly, and therefore $\mathrm{E}\{x(0)\} = [-0.05, 0.05]^{\mathrm{T}}$.

The networked sensor system containing 4 sensors is modeled and described with a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{D})$ with the set of nodes $\mathcal{V} = \{1, 2, 3, 4\}$, the set of edges denoting communication channels $\mathcal{E} = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,3), (4,1), (4,4)\}$, where we use $a_{ij} = 1$ to simultaneously represent the physical communication channel and the communication action generated.

The sensor nodes are described with the quantized parameters below:

$$C_1(k) = [0.82, 0.62], \quad C_2(k) = [0.75, 0.80],$$
$$C_3(k) = [0.74, 0.75], \quad C_4(k) = [0.75, 0.70].$$

The measurement noises $v_i(k)$ are uncorrelated Gaussian sequences and their covariances are $0.05I$. $\lambda_i(k)$ distributed uniformly over $[0.45 + 0.1i, 0.95 + 0.1i]$, for $i = 1, \ldots, 4$, respectively.

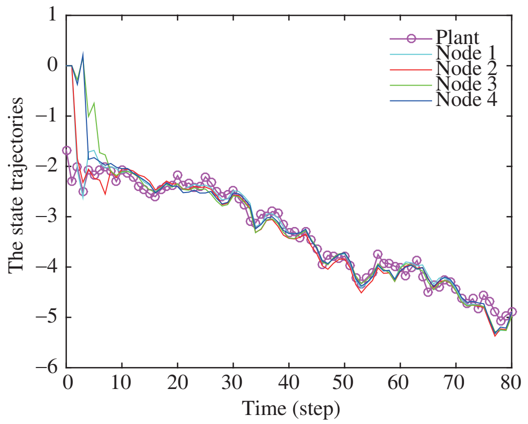

Figure 2 (Color online) The state trajectories of the plant and the sensor nodes under the optimal communication schedule γ^*.

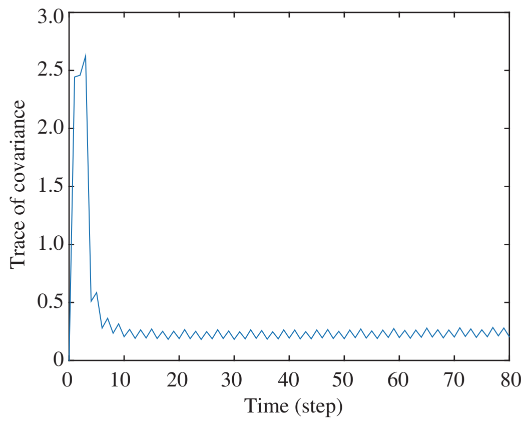

Figure 3 (Color online) The trace of estimation error covariance for the state under the optimal communication schedule γ^*.

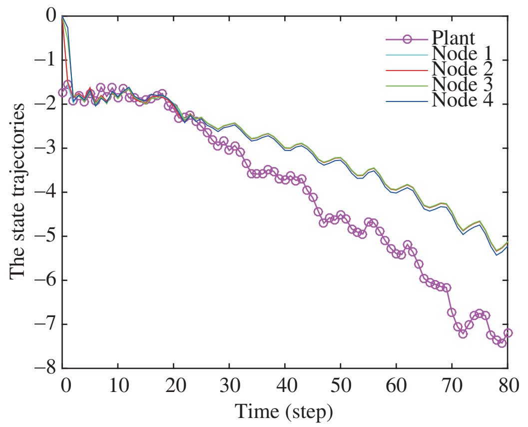

Figure 4 (Color online) The state trajectories of the plant and the sensor nodes under an extra communication schedule γ^{-*}.

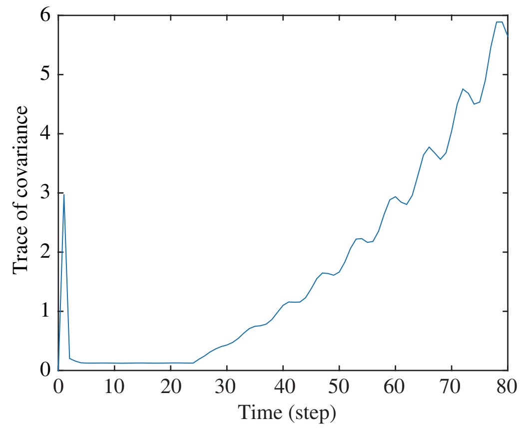

Figure 5 (Color online) The trace of estimation error covariance for the state under an extra communication schedule γ^{-*}.

Consider a network whose communication rate is 50%, $T = 50$ and $n = 25$. According to the theorems of optimal communication protocol design, we already know that the optimal schedule is communication at intervals, which is called schedule γ^* and can be represented as

$$\gamma^* = (1, 0, 1, 0, 1, 0, \ldots, 1, 0, 1, 0, 1, 0).$$

For comparison, we choose an extra communication schedule γ^{-*} randomly as

$$\gamma^{-*} = (1, 1, 1, 1, 1, 1, \ldots, 0, 0, 0, 0, 0, 0).$$

Monte-Carlo simulations have already been performed in the example that follows. The trace of the estimation error covariance is applied for evaluation of the filtering performance. Figure 2 shows the state trajectories of the plant and the sensor nodes under the optimal communication schedule γ^*, while Figure 3 shows the trace of estimation error covariance for the state under the optimal communication schedule γ^*.

As a comparison, Figure 4 shows the state trajectories of the plant and the sensor nodes under an extra communication schedule γ^{-*}, while Figure 5 shows the trace of estimation error covariance for the state under an extra communication schedule γ^{-*}.

To evaluate the effect of different communication protocols on the filtering performance, we introduce a common physical quantity state estimate relative error (RE) to quantify the relative state estimation

Table 1 State estimate relative error (RE)

Schedule	Node 1 (%)	Node 2 (%)	Node 3 (%)	Node 4 (%)	Average (%)
γ^*	4.59	4.78	4.21	4.35	4.49
γ^{-*}	45.8	46.2	44.6	42.7	44.8

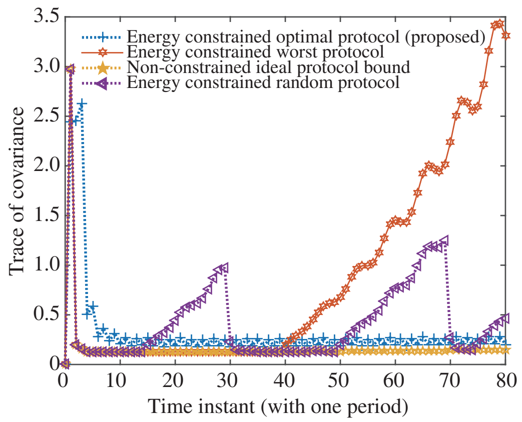

Figure 6 (Color online) Comparison of proposed energy constrained optimal protocol, energy constrained worst protocol, non-constrained ideal protocol, and energy constrained random protocol schemes in one period of finite time.

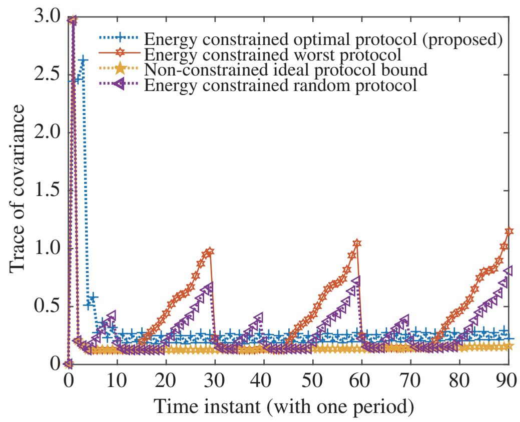

Figure 7 (Color online) Comparison of proposed energy constrained optimal protocol, energy constrained worst protocol, non-constrained ideal protocol, and energy constrained random protocol schemes in three periods of time.

error to evaluate the state estimate. Here, RE is equal to the ratio between the state estimate error and the actual state (see Table 1).

Next, we perform comparison simulations of the trace of filtering covariance in different schemes, which are the proposed energy constrained optimal protocol, energy constrained worst protocol, non-constrained ideal protocol, and energy constrained random protocol. In one period of finite time, Figure 6 verifies that our proposed algorithm has a smaller overall trace of filtering covariance than the others and is close to the bound of the ideal trace of filtering covariance. To evaluate results over several periods, we take three periods as an example and Figure 7 demonstrates the same results which are consistent with both our analysis and verification. Moreover, the results indicate that our proposed algorithm and strategy are suitable for the finite-time and cyclic communication protocol scheme.

From the numerical simulation of the example, we can find the optimal centralized cyclic finite-time communication protocol and know that under the optimal communication schedule, the designed distributed filter has better performance of state estimation and better error covariance estimation than any other schedules for a networked sensor system with energy constraints, and at the same time, we make it clear that the system is stable under such schedule.

6 Conclusion

In this paper, considering the optimal communication protocol problem under the centralized cyclic finite-time communication scheme, we have developed a distributed filter design with sensor gain degradation. We proved that scattered communication actions minimize the expected average error covariance conditionally with constrained energy. The optimized distributed filters have been designed under the optimal communication schedule for networked sensor systems. The stability condition of the system has also been demonstrated under this communication protocol. Finally, numerical examples have been provided, to demonstrate and clarify what we have proposed. In the future, we will consider sensors with limited computation capacity and the decentralized finite-time communication protocol excepting the parallelization and synchronization assumption, and will verify the effectiveness and feasibility of such a protocol in the future work.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61490701, 61210012, 61290324, 61473164), and Research Fund for the Taishan Scholar Project of Shandong Province of China (Grant No. LZB2015-162).

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Appendix A Proof of Theorem 1

Owing to limited space, the proof line is given below. We consider the permutation and combination theory and use the interpolation method. From Lemmas 1 and 2 in the main paragraph, we need to separate every 0 between two 1s in any configuration available. Because it satisfies the completely scattered condition, we are able to separate every 0 between two 1s. Intuitively, there are several communication schedules as long as two or more 0s are not adjacent.

Appendix B Proof of Theorem 2

Owing to limited space, the proof line is given below and the process is similar with the proof of Theorem 1. We consider the permutation and combination theory and apply the interpolation method. From Lemmas 1 and 2 in the main paragraph, we want to separate 0s between two 1s on average. Here because it does not satisfy the completely scattered condition, there must be at least two adjacent 0s. We can obtain $(n+1-\mathrm{Rm})$ segments where there are Qt 0s in each while we can obtain Rm segments where there are $(\mathrm{Qt}+1)$ 0s in each. Thus there are $\binom{n+1}{\mathrm{Rm}}$ combinations, which is the number of optimal communication schedules. And the corresponding cost function is available when substituted into the formula.

Appendix C Proof of Theorem 3

We have the form

$$\tilde{x}(k+1) = \left[\bar{A}(k) - \sum_{i=1}^{n} E_i H(k) T_i(k) \bar{M}(k) \bar{C}(k)\right] \tilde{x}(k)$$
$$- \sum_{i=1}^{n} E_i H(k) T_i(k) \left[\bar{\Lambda}(k) - \bar{M}(k)\right] \bar{C}(k) \bar{x}(k) - \sum_{i=1}^{n} E_i H(k) T_i(k) \bar{v}(k) + \bar{w}(k). \tag{C1}$$

We also have the definition

$$P(k+1) = \mathrm{E}\left\{\tilde{x}(k+1)\tilde{x}^{\mathrm{T}}(k+1)\right\}. \tag{C2}$$

Then it follows that

$$P(k+1) = \left[\bar{A}(k) - \sum_{i+1}^{n} E_i H(k) T_i(k) \bar{M}(k) \bar{C}(k)\right] P(k) \left[\bar{A}(k) - \sum_{i+1}^{n} E_i H(k) T_i(k) \bar{M}(k) \bar{C}(k)\right]^{\mathrm{T}}$$
$$+ \left[\sum_{i=1}^{n} E_i H(k) T_i(k)\right] V(k) \left[\sum_{i=1}^{n} E_i H(k) T_i(k)\right]^{\mathrm{T}}$$
$$+ \left[\sum_{i=1}^{n} E_i H(k) T_i(k)\right] U(k) \left[\sum_{i=1}^{n} E_i H(k) T_i(k)\right]^{\mathrm{T}} + W(k). \tag{C3}$$

Additionally, $\Omega(k)$ can be calculated as follows:

$$\begin{aligned}\Omega(k+1) &= \mathrm{E}\{x(k+1)x^{\mathrm{T}}(k+1)\} \\ &= \mathrm{E}\left\{\{A(k)x(k) + w(k)\}\{A(k)x(k) + w(k)\}^{\mathrm{T}}\right\} \\ &= A(k)\Omega(k)A^{\mathrm{T}}(k) + S(k). \end{aligned} \tag{C4}$$

According to the notation, it follows that

$$\begin{aligned}P(k+1) &= \left[\sum_{i=1}^{n} E_i H(k) T_i(k)\right] Y(k) \left[\sum_{i=1}^{n} E_i H(k) T_i(k)\right]^{\mathrm{T}} \\ &- Z^{\mathrm{T}}(k) \left[\sum_{i=1}^{n} E_i H(k) T_i(k)\right]^{\mathrm{T}} \\ &- \left[\sum_{i=1}^{n} E_i H(k) T_i(k)\right] Z(k) + \bar{A}(k) P(k) \bar{A}^{\mathrm{T}}(k) + W(k). \end{aligned} \tag{C5}$$

Because $Y(k) = Y^{\mathrm{T}}(k) > 0$, the previous equation can be written as

$$\begin{aligned}P(k+1) &= \left[\sum_{i=1}^{n} E_i H(k) T_i(k) - Z^{\mathrm{T}}(k) Y^{-1}(k)\right] Y(k) \left[\sum_{i=1}^{n} E_i H(k) T_i(k) - Z^{\mathrm{T}}(k) Y^{-1}(k)\right]^{\mathrm{T}} \\ &- Z^{\mathrm{T}}(k) Y^{-1}(k) Z(k) + \bar{A}(k) P(k) \bar{A}^{\mathrm{T}}(k) + W(k). \end{aligned} \tag{C6}$$

In the case when the networked sensor systems can not communicate completely, we determine that $P(k+1)$ is minimized when

$$\sum_{i=1}^{n} E_i H(k) T_i(k) = \mathcal{H}(k). \tag{C7}$$

It is known that $T_i(k)$ is not invertible for $i \in \mathcal{V}$ for the reason of sparse topology and matrix irreversibility. Under this circumstance, a means to determine the filter gains is to calculate $H(k)$ as above. We denote $T_i^{\dagger}(k)$ as the Moore-Penrose pseudo inverse of $T_i(k)$. Thus, it can be assured that

$$[H_{i1}(k), \ldots, H_{in}(k)] = [\mathcal{H}_{i1}(k), \ldots, \mathcal{H}_{in}(k)] T_i^{\dagger}(k). \tag{C8}$$

Moreover, it can be easily proven that $P(k)$ can be calculated recursively. The proof is thus complete.