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Special Focus on Learning and Real-time Optimization of Automotive Powertrain Systems

Relevance between fractional-order hybrid model and unified equivalent circuit model of electric vehicle power battery

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Lithium-ion battery has gradually become the main power battery for electric vehicles. Battery models play an important role in battery design and safe operation. However, an accurate, intuitive, and efficient model is not available easily, because it is highly complex, nonlinear and uncertain. Battery models are divided into electrochemical model, thermal model, stochastic model, neural network model, and equivalent circuit model (ECiBaM) [1]. ECiBaMs use different physical components, such as voltage source, resister and capacitor to simulate the battery I-V characteristics. Due to the intuitive simulation and electrical design, ECiBaMs have been widely used, such as Thevenin, PNGV, and n-order RC. However, the traditional ECiBaMs did not take into account the battery nonlinear capacity. Thus, it is difficult to describe the battery run time accurately [2].

In fact, battery power is not available like water in a bucket. The available capacity is nonlinearly related to the discharge rate, mainly manifested as capacity effect and recovery effect [1, 3]. The former is that the greater the discharge rate, the smaller the available capacity, and the latter is that the battery available capacity will rise up when discharge is stopped. Kinetic battery model (KiBaM) was used to describe the battery nonlinear capacity performance. It is intuitionistic, and easy to understand [4]. In [5], a hybrid battery model was proposed, in which ECiBaM and KiBaM were both used for the I-V performance and nonlinear capacity. However, this model is limited to integer-order calculus. Due to the specific material and chemical properties, lithium-ion batteries show a typical fractional-order dynamic behavior [2, 6].

The commonly used definitions of fractional derivative include Grunwald-Letnikov definition, Riemann-Liouville definition and Caputo definition [7]. Caputo definition is

$$^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{1+\alpha-n}} \mathrm{d}\tau, \quad (1)$$

where $\Gamma()$ is Gamma function, and it is defined as

$$\Gamma(z) = \int_0^{\infty} t^{z-1} \mathrm{e}^{-t} \mathrm{d}t, \quad z \in C. \quad (2)$$

And it is usually used in the following transform:

$$L\left[\frac{t^{\alpha-1}}{\Gamma(\alpha)}\right] = \frac{1}{s^{\alpha}}. \quad (3)$$

The Laplace transform of Caputo definition is

$$L[^{C}D_t^{\alpha} f(t)] = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0). \quad (4)$$

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The proposed fractional-order hybrid model is shown in Figure 1(a). The right part (ECiBaM) has been studied in [8]. The left part (fractional-order KiBaM) is proposed, where y_1 is the available capacity that can be obtained directly at discharge, and its height is h_1; y_2 is the temporary capacity, and its height is h_2; k represents the flow from y_1 to y_2; and c represents the proportion of the battery capacity between the two wells.

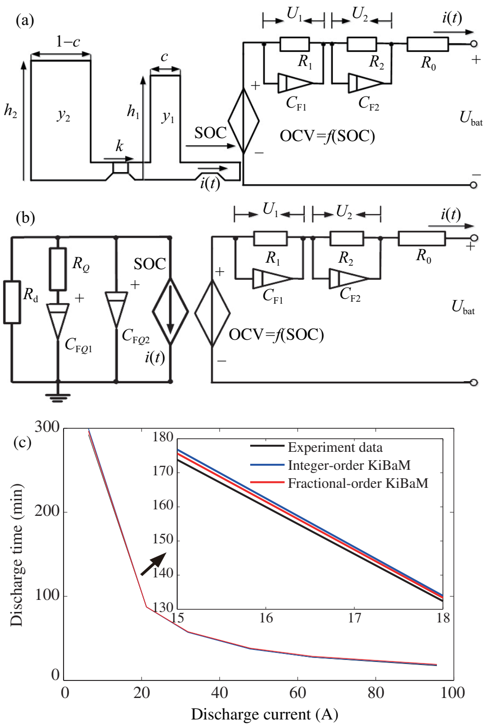

Figure 1 (Color online) (a) The proposed fractional-order hybrid model; (b) the proposed unified fractional-order ECiBaM; (c) comparison of the discharge time.

The unavailable capacity and the available capacity of battery can be expressed as

$$\begin{cases} C_{\text{total}}(t) = y_1(t) + y_2(t), \\ C_{\text{unava}}(t) = (1-c)\delta_h(t) \\ \qquad = (1-c)(h_2(t) - h_1(t)), \\ C_{\text{avail}}(t) = C_{\text{total}}(t) - C_{\text{unava}}(t). \end{cases} \tag{5}$$

The capacity nonlinear characteristics of battery should be a fractional differential,

$$\begin{cases} \dfrac{\mathrm{d}^\alpha y_1(t)}{\mathrm{d}t^\alpha} = -i(t) + k\left[\dfrac{y_2(t)}{1-c} - \dfrac{y_1(t)}{c}\right], \\ \dfrac{\mathrm{d}^\alpha y_2(t)}{\mathrm{d}t^\alpha} = -k\left[\dfrac{y_2(t)}{1-c} - \dfrac{y_1(t)}{c}\right], \end{cases} \tag{6}$$

where fractional order $0 < \alpha < 1$.

If the battery discharges in a constant current, the Laplace transform of (6) will be

$$\begin{cases} s^\alpha Y_1(s) = -\dfrac{I}{s} + k\left[\dfrac{Y_2(s)}{1-c} - \dfrac{Y_1(s)}{c}\right], \\ s^\alpha Y_2(s) = -k\left[\dfrac{Y_2(s)}{1-c} - \dfrac{Y_1(s)}{c}\right]. \end{cases} \tag{7}$$

Assuming that the initial conditions $t_0 = 0$ and SOC=1, the following will be obtained.

$$\begin{cases} Y_1(s) = \dfrac{cC}{s^\alpha} - \dfrac{cI}{s^{\alpha+1}} - \dfrac{(1-c)I}{s(s^\alpha + k')}, \\ Y_2(s) = \dfrac{(1-c)C}{s^\alpha} - \dfrac{(1-c)I}{s^{\alpha+1}} + \dfrac{(1-c)I}{s(s^\alpha + k')}, \end{cases} \tag{8}$$

where $k' = \frac{k}{c(1-c)}$.

The inverse Laplace transform of fractional calculus transfer function can be obtained by Mittag-Leffler function, a basic function in fractional calculus defined as

$$E_{\alpha,\beta}(z) = \sum_{j=0}^{\infty} \frac{z^j}{\Gamma(\alpha j + \beta)}, \quad \alpha > 0, \quad \beta > 0. \tag{9}$$

To facilitate the inverse Laplace transform, a new function is defined here.

$$\varepsilon(t, m, \alpha, \beta) = t^{\beta-1} E_{\alpha,\beta}(mt^\alpha). \tag{10}$$

And the Laplace transform of this function is

$$L[\varepsilon(t, \pm m, \alpha, \beta)] = \frac{s^{\alpha-\beta}}{s^\alpha \mp m}. \tag{11}$$

From the form of transfer function (8), it is clear that $\beta = \alpha + 1$ in the function (11), and the following inverse Laplace transform can be derived.

$$\begin{cases} y_1(t) = cC\dfrac{t^{\alpha-1}}{\Gamma(\alpha)} - cI\dfrac{t^\alpha}{\Gamma(\alpha+1)} \\ \qquad - (1-c)It^\alpha E_{\alpha,\alpha+1}(-k't^\alpha), \\ y_2(t) = (1-c)C\dfrac{t^{\alpha-1}}{\Gamma(\alpha)} - (1-c)I\dfrac{t^\alpha}{\Gamma(\alpha+1)} \\ \qquad + (1-c)It^\alpha E_{\alpha,\alpha+1}(-k't^\alpha). \end{cases} \tag{12}$$

Then we can obtain

$$\delta_h(t) = \frac{I}{c} t^\alpha E_{\alpha,\alpha+1}(-k't^\alpha). \tag{13}$$

Obviously, when y_1=0 (h_1=0), the battery is discharged completely. A battery module of 32.5 Ah was chosen for experimental verification. As shown in Figure 1(c), the discharge time of fractional KiBaM is more accurate when considering the unavailable capacity at different current.

An equivalent circuit was proposed to describe the nonlinear capacity in [9]. From the perspective of unified modeling, a unified fractional-order ECiBaM can be achieved rather than to patch different kinds of models together. As shown in

Figure 1(b), the proposed model is equivalent to the fractional-order hybrid model in a sense. The battery capacity is also divided into available capacity and unavailable capacity, where fractional-order capacitor C_{FQ1} stores a portion of available capacity, and C_{FQ2} stores the unavailable capacity and the other part of available capacity. The discharge current of C_{FQ2} depends on R_Q and the voltage difference between the two capacitors. R_{d} is the self-discharge resistor.

The relationship between voltage and current of capacitors in traditional model is

$$i(t)=\frac{\mathrm{d}Q(t)}{\mathrm{d}t}=C\frac{\mathrm{d}u(t)}{\mathrm{d}t}. \tag{14}$$

In fact, capacitors are fractional-order element (FOE). In 1889, Curie's empirical law stated that the current through a capacitor is

$$i(t)=\frac{U_0}{h_1 t^n},\quad 0<n<1,\quad t>0, \tag{15}$$

where U_0 is the voltage applied at $t=0$; h_1 is a constant and related to the capacitance and the type of dielectric; n is a constant and related to the losses of capacitor. The lower the losses, the closer n to be 1.0.

In 1994, a new capacitor model, actually a fractional-order model, was proposed in capacitor theory by Westerlund and Ekstam,

$$i(t)=C_{\mathrm{F}}\frac{\mathrm{d}^n u(t)}{\mathrm{d}t^n},\quad 0<n<1,\quad t>0, \tag{16}$$

where C_{F} is the capacitance; n is related to the losses of the capacitor.

In the zero initial conditions, the Laplace transform of the fractional-order capacitor is

$$\frac{U(s)}{I(s)}=\frac{1}{C_{\mathrm{F}}s^n}. \tag{17}$$

Ignoring self-discharge, the capacity relationship of the fractional-order ECiBaM in Figure 1(b) is

$$\begin{cases} C_{\mathrm{total}}(t)=Q_1(t)+Q_2(t),\\ C_{\mathrm{unava}}(t)=C_{\mathrm{FQ2}}\delta(t)\\ \qquad\qquad\;\, =C_{\mathrm{FQ2}}(U_2(t)-U_1(t)),\\ C_{\mathrm{avail}}(t)=C_{\mathrm{total}}(t)-C_{\mathrm{unava}}(t), \end{cases} \tag{18}$$

where Q_1 and Q_2 are the charge of capacitor C_{FQ1} and C_{FQ2}; U_1, U_2 and $\delta(t)$ are the terminal voltage and voltage difference of capacitors, respectively.

When capacitors are FOE, the battery capacity is expressed as

$$\begin{cases} \dfrac{\mathrm{d}^\alpha U_1(t)}{\mathrm{d}^\alpha t}=-\dfrac{i(t)}{C_{\mathrm{FQ1}}}+\dfrac{U_2(t)-U_1(t)}{R_Q C_{\mathrm{FQ1}}},\\ \dfrac{\mathrm{d}^\beta U_2(t)}{\mathrm{d}^\beta t}=-\dfrac{U_2(t)-U_1(t)}{R_Q C_{\mathrm{FQ2}}}. \end{cases} \tag{19}$$

If the initial conditions is the same as (6), the Laplace transform of (19) is

$$\begin{cases} s^\alpha U_1(s)=-\dfrac{I}{C_{\mathrm{FQ1}}s}+\dfrac{U_2(s)-U_1(s)}{R_Q C_{\mathrm{FQ1}}},\\ s^\beta U_2(s)=-\dfrac{U_2(s)-U_1(s)}{R_Q C_{\mathrm{FQ2}}}. \end{cases} \tag{20}$$

Although the parameters and the meanings of two models are different, the parameter identification is also different according to the experimental data. From the basic capacity relationships (5) and (18), (6) and (19), (7) and (20), they are equivalent to each other in a sense. k and R_Q reflect the battery nonlinear capacity effect; (y_1, y_2) and (Q_1, Q_2) reflect the two parts of battery capacity; $(1-c)\delta_h(t)$ and $C_{\mathrm{FQ2}}\delta(t)$ reflect the unavailable capacity of battery at discharge; α and (α,β) reflect the battery fractional characteristics. Thus, a unified fractional-order ECiBaM is built to describe the different characteristics of battery, including I-V characteristics, nonlinear capacity effect, and recovery effect. Moreover, it gains more degrees of freedom, greater flexibility, and more clear physical meanings.

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