

A stochastic logical model-based approximate solution for energy management problem of HEVs

Jiangyan ZHANG¹ & Yuhu WU^{2*}

¹College of Mechanical and Electronic Engineering, Dalian Minzu University, Dalian 116600, China;

²School of Control Science and Engineering, Dalian University of Technology, Dalian 116024, China

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The development of control algorithms to improve the energy efficiency of hybrid electric vehicles (HEVs) has attracted extensive attentions due to the advancement of the powertrain on fuel economy and emission reduction. The objective of the energy management control for HEVs is to distribute the driver demand power between the engine and the motor to maximize the efficiency. Deterministic dynamic programming (DP) has been applied to solve the problem, but it just provides a theoretical optimal solution which cannot guarantee the practical requirements. To deal with the involved time-varying characteristics, significant efforts have been paid to develop model predictive control (MPC)-based online algorithms [1, 2]. However, the nonlinearity in the system causes strong limits to present a general framework to get an exact optimal solution. Moreover, the computational burden is a considerable issue to apply online optimization solutions. The vehicle system explicitly involves the driver behavior which generates stochastic influence to the energy efficiency performance. Recently, extended predictive-based algorithms that integrate the theoretical tools of stochastic system control are investigated [3–5].

In this article, a case study for optimal control of hybrid electrical vehicle (HEV) is discussed based on the theoretical framework of stochastic logical dynamical system. With the mathematical tool of semi-tensor product denoted by \ltimes and the Markov

chain model, the contributions on synthesis of logical dynamical system show that simple optimization can be achieved in the logical domain [6, 7]. This work introduces a logical stochastic control algorithm that generates an approximate optimal solution for the HEV control problem. A simulation result is presented to indicate the potential of applying the logical model-based control design strategy to the HEV.

Problem formulation. Consider a parallel HEV with a five-speed manual transmission system. In the energy management problem of HEV, the main system dynamics is the battery state of charge (SoC) which can be modeled as

$$\text{SoC} = \frac{-U_{oc} + \sqrt{U_{oc}^2 - 4R_b\eta_m T_m \omega_m}}{2Q_{bmax}R_b}, \quad (1)$$

where U_{oc} , R_b , Q_{bmax} denote the open-circuit voltage, internal resistance and maximum charge capacity of the battery, respectively, and T_m , ω_m and η_m denote the torque, speed and efficiency of the motor, respectively. Using a simple logic for gear shifting decision, the control variable can be chosen as the engine torque, i.e., $u = T_e$. For the powertrain system, the stochastic disturbances include the driver demand driving torque T_d and the vehicle speed v . Denote the disturbance vector as $w = [T_d, v]^T$. The objective is to find a control policy $u(t)$, $t \in [0, T]$ to minimize a given cost function, where T denotes the driving duration. This

* Corresponding author (email: wuyuhu@dlut.edu.cn)

work focuses on minimizing the equivalent energy consumption which is defined by

$$J = \int_0^T \Gamma_E Q_{\text{lhv}} \dot{m}_f(x, u, w) + \dot{m}_e(x, u, w) dt, \quad (2)$$

where \dot{m}_f and \dot{m}_e denote the fuel flow rate and the change rate of the battery internal energy, respectively, Γ_E denotes the factor for converging the fuel consumption into equivalent energy and Q_{lhv} denotes the fuel lower heating value. Meanwhile, the minimization of cost function (2) subjects to the constraints of the system dynamics (1) which is simply expressed as $\dot{x} = f(x, u, w)$ ($x = \text{SoC}$), and the physical limits of the state x and the control variable u . Furthermore, the probability distributions $P(T_d)$ and $P(v)$ of the stochastic variables are estimated according to a set of real driving cycles, respectively.

To develop the logical-based control algorithm, the optimal control problem is discretized as

$$\begin{aligned} \min_u J = & \quad E \\ & \quad w_k \quad \left\{ \sum_{k=0}^{N-1} g(x_k, u_k) \right\} \\ & \quad k = 0, \dots, N-1 \\ \text{s.t.} \quad & \begin{cases} x_{k+1} = \bar{f}(x_k, u_k, w_k), \\ T_d \in \Omega(P(T_d)), \\ v \in \Omega(P(v)), \\ x \in [x_{\min}, x_{\max}], \\ u \in [u_{\min}, u_{\max}], \end{cases} \end{aligned} \quad (3)$$

with $g(x, u) = \Gamma_E Q_{\text{lhv}} \Delta m_f + \Delta m_e$, where \bar{f} denotes the forward difference with respect to f . Note that it is difficult to exactly solve the above stochastic optimal control problem. The logical system-based design method proposed in [7] is investigated to get an approximate solution.

Choose proper quantitative factors σ_x ($\sigma_x n_x = x_{\max} - x_{\min}$) and σ_u ($\sigma_u n_u = u_{\max} - u_{\min}$), respectively. The effective ranges of the state are qualified to finite intervals $\mathcal{S}^i = [x^{i-1}, x^i]$ ($i = 1, \dots, n_x$), and obtain finite control variable u^j ($j = 1, \dots, n_u$). δ_x^i and δ_u^j represent the i -th and j -th columns of the identify matrixes I_{n_x} and I_{n_u} , respectively, as the corresponding logical variables. In this case, $\delta_x^i \sim x \in [x^{i-1}, x^i]$ and $\delta_u^j \sim u^j$. Moreover, the cost function is approximately redefined with the following relation:

$$\bar{g}(\delta_x^i, \delta_u^j) \sim g\left(\frac{x^{i-1} + x^i}{2}, \frac{u^{j-1} + u^j}{2}\right). \quad (4)$$

Denote $\Delta_x = \{\delta_x^1, \dots, \delta_x^{n_x}\}$ and $\Delta_u = \{\delta_u^1, \dots, \delta_u^{n_u}\}$ as the logical spaces of the state and control variable, respectively. The cost function in the logical domain can be represented as

$$\tilde{g}(\delta_x, \delta_u) = \delta_x^T G \delta_u, \quad \forall \delta_x \in \Delta_x, \delta_u \in \Delta_u \quad (5)$$

with $G = [G_{i,j}]_{n_x \times n_u} = [\bar{g}(\delta_x^i, \delta_u^j)]_{n_x \times n_u}$.

Furthermore, the following Markov chain model is proposed:

$$P_{i,j}^\gamma = \{x_{k+1} = \delta_x^j \mid x_k = \delta_x^i, u_k = \delta_u^\gamma\}, \quad (6)$$

which is calculated with

$$\begin{aligned} P_{i,j}^\gamma = & \sum_{\beta=1}^{n_v} \sum_{\alpha=1}^{n_T} \frac{\mathcal{M}(f^{-1}(\mathcal{S}^i, T_e^\gamma, T_d^\alpha, v^\beta) \cap \mathcal{S}^j)}{\mathcal{M}(\mathcal{S}^i)} \\ & P(T_d^\alpha) \cdot P(v^\beta), \end{aligned} \quad (7)$$

where \mathcal{M} denotes the Lebesgue measure on real number field \mathbb{R} , and n_v and n_T denote the sample numbers of T_d and v , respectively. According to (7), for a δ_u^j , the following transition probability matrix can be obtained:

$$P_{\delta_u^j} = \begin{pmatrix} p_{1,1}^j & \cdots & p_{1,n_x}^j \\ \vdots & \vdots & \vdots \\ p_{n_x,1}^j & \cdots & p_{n_x,n_x}^j \end{pmatrix}, \quad j = 1, \dots, n_u. \quad (8)$$

Then, the problem (3) is reformulated in the logical domain. Find a control policy such that

$$\min_{\delta_u} J = \quad E \\ \quad w_k \quad \left\{ \sum_{k=0}^{N-1} \tilde{g}(\delta_{x,k}, \delta_{u,k}) \right\}, \quad (9)$$

which subjects to the Markov model (6).

Optimization algorithm. Define a matrix $\mathbb{P} = [(P_{\delta_u^1}^T)^T, (P_{\delta_u^2}^T)^T, \dots, (P_{\delta_u^{n_u}}^T)^T]^T$ where

$$P_{\delta_u^k} = \begin{pmatrix} p_{1,1}^k & \cdots & p_{1,n_x}^k \\ \vdots & \vdots & \vdots \\ p_{n_x,1}^k & \cdots & p_{n_x,n_x}^k \end{pmatrix}, \quad k = 1, \dots, n_u.$$

The proposed control problem (9) can be solved according to Algorithm 1.

Note that in the proposed control scheme, more intervals to discretize the continuous variables can increase the control precision, while cause heavy computational burden. Hence, tread-off should be involved based on the validation testings.

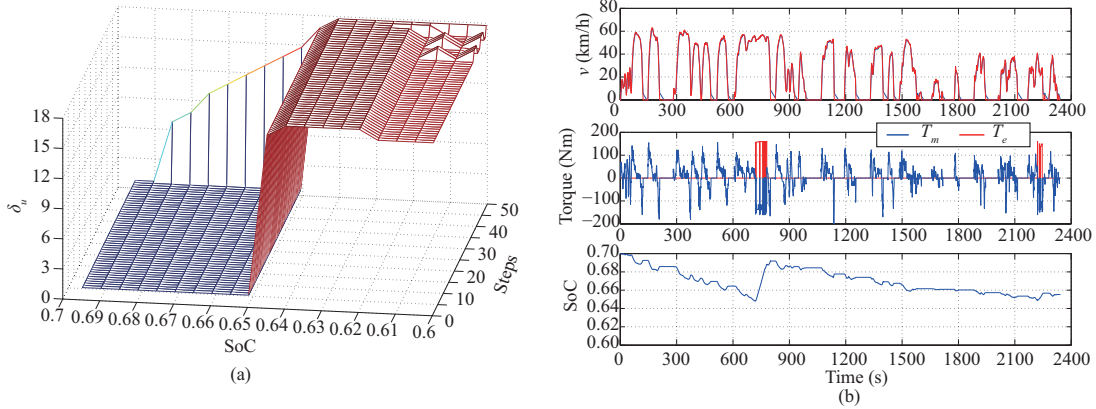


Figure 1 (Color online) A validation result. (a) Control law; (b) instantaneous vehicle speed, torque commands and SoC.

Algorithm 1

S1: Set $k = 0$, $\mathcal{K} = [0 \cdots 0]_{n_x}^T$;
S2: Find the feedback control law

$$\Phi_{N-k+1}^* = L_{n_u} [q_1, \dots, q_{n_x}],$$

which is the solution of the following cost function associated matrix:

$$TK = \begin{pmatrix} \min_{j=1, \dots, n_u} \{G_{1,j} + (\delta_x^1)^T \times (\delta_u^j)^T \mathbb{P} \mathcal{K}\} \\ \vdots \\ \min_{j=1, \dots, n_u} \{G_{n_x,j} + (\delta_x^{n_x})^T \times (\delta_u^j)^T \mathbb{P} \mathcal{K}\} \end{pmatrix},$$

i.e., for all $i = 1, \dots, n_x$, $q_i = \operatorname{argmin}_{j=1, \dots, n_u} \{G_{i,j} + (\delta_x^i)^T \times (\delta_u^j)^T \mathbb{P} \mathcal{K}\}$;

S3: Set $k = k + 1$, $\mathcal{K}_k^* = TK$, and $\mathcal{K} = TK$;

S4: If $k < N$, then go to **S2**; else, $J^* = \mathcal{K}_N^*$ and stop.

Simulation and verification. A group of driving cycles under city driving condition are analyzed to get the $P(T_d)$ and $P(v)$. According to (7), the matrix \mathbb{P} is calculated. The proposed control algorithm is applied to an HEV simulator composed by Matlab/Simulink. A PI controller is used to generate the driver demand torque. Furthermore, note that for the Algorithm 1, the equivalent factor Γ_E is a key turning parameter that effects the control performance. A bigger Γ_E means that the fuel consumption will be more than the electricity. In this case, the HEV runs in the charge-sustaining mode. Otherwise, it will be a plug-in like HEV that runs in a charge-depleting mode.

Set $n_x = 20$ with respect to the range that $T_e \in [0, 170]$ Nm. The effective state range is set as $x \in [0.6, 0.7]$, and $n_u = 18$. The control period is 60 s. By conducting series tests, it is found that the selected quantitative factors guarantee acceptable control performance and cost few computing time. Figure 1 shows a result with $\Gamma_E = 0.27$. The control law given by Figure 1(a) shows when the SoC is higher than a value (0.65 in the testing case), the HEV runs as an electric vehicle, other-

wise, the engine provides assistant driving torque to recover the battery charge as can be observed from Figure 1(b). This indicates that the control law guarantees that the HEV can run in an almost charge-sustaining mode under the considered constraints. Furthermore, note that by applying the proposed control scheme, the HEV operating mode depends on the constraint conditions which affects the solution of the problem (3). Finally, the validation result demonstrates the performance of the control system, and shows that the logical-based design method is potentially applicable to deal with the considered control problem.

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