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Guaranteed cost boundary control for cluster synchronization of complex spatio-temporal dynamical networks with community structure

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Abstract This paper discusses the problem for cluster synchronization control of a nonlinear complex spatio-temporal dynamical network (CSDN) with community structure. Initially, a collocated boundary controller with boundary measurement is studied to achieve the cluster synchronization of the CSDN. After that, a guaranteed cost boundary controller is further developed based on the obtained results. Furthermore, the suboptimal control design is addressed by minimizing an upper bound of the cost function. Finally, a numerical example is given to demonstrate the effectiveness of the proposed methods.

Keywords cluster synchronization, complex dynamical networks, boundary control, guaranteed control, LMIs

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1 Introduction

Complex dynamical networks (CDNs), including sensor networks [1], multi-agent systems [2] and neural networks [3, 4], have attracted a great deal of attention over the past few decades. Synchronization or consensus, one of the most important dynamical behaviors of CDNs, has been extensively applied to engineering fields, such as automatic control [5–7], secure communication [8] and brain cognition [9]. In the past few decades, many synchronization notions have been extensively presented, such as asymptotical synchronization [10], complete synchronization [11], projective synchronization [12], lag synchronization [13], phase synchronization [14], approximate synchronization [15], pinning synchronization [16], fixed-time synchronization [17], mixed synchronization [18], and cluster synchronization [19–22]. As a particular one, cluster synchronization in community networks demands that synchronization occurs in each group rather than among different groups. Currently, cluster synchronization has attracted increasing attention due to its applications in biological science and communication engineering [23]. In [24],

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cluster synchronization in community network was proposed and several sufficient conditions were presented. Cluster synchronization of coupled nonidentical dynamical systems was studied in [25]. In [26], using pinning control and adaptive coupling strength control, cluster synchronization in linear community networks was studied. Generally speaking, the study of cluster synchronization in community networks has a significant importance.

The spatial density of the species, a node of food webs, is spatio-temporal; diffusion effect of coupled chemical reactions is spatio-temporal; and infight hose-and-drogue aerial refueling is spatio-temporal [27]. There are also many other CDNs in nature and discipline fields with spatio-temporal characteristics [28, 29]. As a result, it is necessary to research synchronization of CDNs with spatio-temporal characteristics, named complex spatio-temporal dynamical networks (CSDNs).

Although the study of synchronization control of CSDNs is hard due to the infinite dimensional characteristics, it has attracted many researchers over the past few decades, e.g., scalar proportional control [28], matrix proportional control [29], P-sD control [30], impulsive control [31], intermittent control [32], adaptive control [33–36]. These controllers are state feedback, requiring sensors and actuators distributing all over the spatial domain. These characteristics make it difficult for these controllers to be applied in practice when states are unknown.

Luckily, boundary control can solve this problem. In the past few years, boundary control has been managed to be applied to synchronization of CSDNs. Two boundary controllers were studied for synchronization of coupled linear partial differential systems [37, 38]. The two used All spatial information, requiring sensors distributing all over the spatial domain. However, to authors' best knowledge, boundary control with boundary measurement has not been considered for cluster synchronization of CSDNs with community structure, only requiring few sensors and actuators locating at the spatial boundary positions. This is the first main motivation of this paper.

From another perspective, networks are expected to achieve not only synchronization, but also with an adequate level of synchronization performance. Guaranteed cost control can effectively deal with this problem [39–42] and many important relevant results over CDNs have been obtained [43, 44]. However, guaranteed cost control of CSDNs has not been addressed yet to authors' best knowledge. A guaranteed cost boundary control with boundary measurement for cluster synchronization of CSDNs is further studied, which is the second main motivation of this paper.

This paper studies cluster synchronization control of nonlinear CSDNs with community structure. In CSDNs, the local dynamics in each community are identical while those in different communities are nonidentical. Initially, a collocated boundary controller with boundary measurement is studied to ensure synchronization of CSDNs, requiring sensors and actuators simultaneously locating only at one end of one-dimensional spatial domain. Therefore, only several sensors and actuators are needed in the proposed controller. Sufficient conditions of the existence of the boundary controller are presented as a convex feasibility of linear matrix inequalities (LMIs). After that, a guaranteed cost collocated boundary controller with boundary measurement is investigated. Sufficient conditions of the guaranteed cost boundary controller are obtained as a convex feasibility of spatial algebraic linear matrix inequalities (SALMIs). In addition, a suboptimal controller is addressed to minimize an upper bound of the cost function. Finally, numerical simulation demonstrates the effectiveness of the proposed methods.

Notation. The following notations will be used in this paper. $\|\cdot\|$ denotes the standard Euclidean norm. $\otimes$ stands for the Kronecker product of matrices. Identity matrix of $n \times n$ dimension is denoted by I_n. Matrix $M > (<, \leqslant)\ 0$ means it is symmetric positive definite (negative definite, semi-negative definite, respectively). The superscript "T" stands for the transpose of a vector or a matrix. The symbol "$*$" stands for the ellipsis of transpose blocks in symmetric matrices.

2 Preliminaries and problem formulation

This paper considers a nonlinear CSDN in one spatial dimension with N nonidentical nodes and l communities, $2 \leqslant l \leqslant N$. The dynamics of the i-th ($i \in \mathcal{N} \triangleq \{1, 2, \ldots, N\}$) node are described by the coupled semi-linear PDE model as

$$\begin{cases} y_{i,t}(x,t) = \Theta_{\phi_i} y_{i,xx}(x,t) + A_{\phi_i} y_i(x,t) + f_{\phi_i}(y_i(x,t)) + c\sum\limits_{k=1}^{l}\sum\limits_{j\in V_k} g_{ij} y_j(x,t), \\ y_{i,x}(x,t)|_{x=0} = B_{\phi_i} u_i(t), \quad y_{i,x}(x,t)|_{x=L} = 0, \\ y_i(x,0) = y_{i,0}(x), \quad i \in \mathcal{N}, \end{cases} \tag{1}$$

where $y_i(x,t) \triangleq [y_{i1}(x,t), y_{i2}(x,t), \ldots, y_{in}(x,t)]^{\rm T} \in \mathbb{R}^n$ are the states. $x \in [0,L] \in \mathbb{R}$ and $t \in [0,\infty)$ are respective the spatial and time variables. The subscripts x and t respectively mean the partial derivatives with respect to x and t. $u_i(t) \in \mathbb{R}^m$ are the boundary control inputs. Dispersal matrices $\Theta_{\phi_i} \in \mathbb{R}^{n\times n}$ are assumed to be positive. $A_{\phi_i} \in \mathbb{R}^{n\times n}$, and $B_{\phi_i} \in \mathbb{R}^{n\times m}$ respectively represent the connection matrices and the control input matrices. $f_{\phi_i}(y_i(x,t))$ are time and spatial variable nonlinear perturbation. c is a known scalar coupling strength. $G \triangleq (g_{ij})_{N\times N}$ describes the diffusive topological structure of the CSDN, g_{ij} defined as $g_{ij} > 0 (i \neq j)$, $g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}$, $i \in \mathcal{N}$. $V_k (k = 1, 2, \ldots, l)$ are node sets of the k-th community. The function ϕ is defined as ϕ: $\{1, 2, \ldots, N\} \to \{1, 2, \ldots, l\}$, if node $i \in V_k$, then $\phi_i = k$. n-dimensional real vector functions.

It is assumed that there are differences among different communities, i.e., $\Theta_{\phi_i} \neq \Theta_{\phi_j}$, $A_{\phi_i} \neq A_{\phi_j}$, $B_{\phi_i} \neq B_{\phi_j}$, and $f_{\phi_i} \neq f_{\phi_j}$, if $\phi_i \neq \phi_j$. Set $s_{\phi_i}(x,t) \in \mathbb{R}^n$ be an isolated node in the ϕ_i-th community, i.e.,

$$\begin{cases} s_{\phi_i,t}(x,t) = \Theta_{\phi_i} s_{\phi_i,xx}(x,t) + A_{\phi_i} s_{\phi_i}(x,t) + f_{\phi_i}(s_{\phi_i}(x,t)), \\ s_{\phi_i,x}(x,t)|_{x=0} = s_{\phi_i,x}(x,t)|_{x=L} = 0, \\ s_{\phi_i}(x,0) = s_{\phi_i,0}(x), \quad i \in \mathcal{N}, \end{cases} \tag{2}$$

where $s_{\phi_i}(x,t)$ may be an equilibrium point, a periodic orbit, or a chaotic orbit.

This paper aims to study a boundary controller to achieve cluster synchronization of the CSDN (1) with the isolated node (2). Denote the synchronization errors $e_i(x,t) \triangleq y_i(x,t) - s_{\phi_i}(x,t), i \in \mathcal{N}$. The synchronization error system of the i-th node can be got from (1) and (2) as

$$\begin{cases} e_{i,t}(x,t) = \Theta_{\phi_i} e_{i,xx}(x,t) + A_{\phi_i} e_i(x,t) + F_{\phi_i}(e_i(x,t)) + c\sum\limits_{j=1}^{N} g_{ij} e_j(x,t), \\ e_{i,x}(x,t)|_{x=0} = B_{\phi_i} u_i(t), \quad e_{i,x}(x,t)|_{x=L} = 0, \\ e_i(x,0) = e_{i,0}(x), \quad i \in \mathcal{N}, \end{cases} \tag{3}$$

where $F_{\phi_i}(e_i(x,t)) \triangleq f_{\phi_i}(y_i(x,t)) - f_{\phi_i}(s_{\phi_i}(x,t))$ and $e_{i,0}(x) \triangleq y_{i,0}(x) - s_{\phi_i,0}(x)$.

The synchronization error system (3) can be rewritten in a compact way as

$$\begin{cases} e_t(x,t) = \Theta e_{xx}(x,t) + (A + c\bar{G}) e(x,t) + F(e(x,t)), \\ e_x(x,t)|_{x=0} = Bu(t), \quad e_x(x,t)|_{x=L} = 0, \\ e(x,0) = e_0(x), \end{cases} \tag{4}$$

where $e(x,t) \triangleq [e_1^{\rm T}(x,t), e_2^{\rm T}(x,t), \ldots, e_N^{\rm T}(x,t)]^{\rm T}$, $\Theta \triangleq {\rm diag}\{\Theta_{\phi_1}, \Theta_{\phi_2}, \ldots, \Theta_{\phi_N}\}$, $A \triangleq {\rm diag}\{A_{\phi_1}, A_{\phi_2}, \ldots, A_{\phi_N}\}$, $B \triangleq {\rm diag}\{B_{\phi_1}, B_{\phi_2}, \ldots, B_{\phi_N}\}$, $u(t) \triangleq [u_1^{\rm T}(t), u_2^{\rm T}(t), \ldots, u_N^{\rm T}(t)]^{\rm T}$, $\bar{G} \triangleq G \otimes I_n$, and $F(e(x,t)) \triangleq [F_{\phi_1}^{\rm T}(e_1(x,t)), F_{\phi_2}^{\rm T}(e_2(x,t)), \ldots, F_{\phi_N}^{\rm T}(e_N(x,t))]^{\rm T}$.

Some definition, lemmas and assumption are first given to be used in the subsequent sections.

Definition 1. The CSDN (1) is said to achieve cluster synchronization with the isolated node (2), if its solution satisfies $\lim_{t\to\infty} ||y_i(x,t) - s(x,t)||_2 \to 0,\ i \in \{1, 2, \ldots, N\}$ for any initial condition.

Lemma 1 (Wirtinger's inequality [45])**.** Given a square integrable vector function $z(x)$ with $z(0) = 0$ or $z(L) = 0$, for any symmetric matrix $S > 0$, the following inequality holds:

$$\int_0^L z^{\rm T}(s) S z(s) {\rm d}s \leqslant 4L^2\pi^{-2} \int_0^L ({\rm d}z(s)/{\rm d}s)^{\rm T} S ({\rm d}z(s)/{\rm d}s) {\rm d}s. \tag{5}$$

Lemma 2 ([46]). For two square integrable vector functions $a(x)$, $b(x)$, $x \in [0,L]$, the following inequality holds for any scalar $\alpha > 0$:

$$2\int_0^L a^{\mathrm{T}}(x)b(x)\mathrm{d}x \leqslant \alpha \int_0^L a^{\mathrm{T}}(x)a(x)\mathrm{d}x + \alpha^{-1}\int_0^L b^{\mathrm{T}}(x)b(x)\mathrm{d}x. \tag{6}$$

Assumption 1. Assume the nonlinear functions $f_{\phi_i}(\cdot)$ satisfy the Lipschitz condition, i.e., for any ξ, η, there exist scalars $\chi_i > 0$ satisfying

$$\|f_{\phi_i}(\xi) - f_{\phi_i}(\eta)\| \leqslant \chi_i \|\xi - \eta\|, \quad i \in \mathcal{N}.$$

Setting $\mathcal{A} \triangleq \Theta e_{xx}$, $\mathcal{A}$ is the infinitesimal generator of a C_0 semigroup. Using Theorem 1.7 of Chapter 6 in [47], the system (4) possesses a unique solution under Assumption 1. The Lipschitz condition of Assumption 1 has been widely used in many other literatures [48–50].

3 Boundary control with boundary measurement

In this section, the following boundary controller with boundary measurement is employed for cluster synchronization of the CSDN (1) as

$$u_i(t) = K_i e_i(0,t), \quad i \in \mathcal{N}, \tag{7}$$

where $K_i \in \mathbb{R}^{m\times n}$ are control gain parameters to be determined.

Remark 1. The controller (7) is referred to a collocated boundary one. The implementation of such a controller requires few sensors and actuators only at the boundary of spatial domain. Especially in practice, the merit makes it easy to be employed.

Substituting (7) into (3), the synchronization error system is obtained as

$$\begin{cases} e_{i,t}(x,t) = \Theta_{\phi_i} e_{i,xx}(x,t) + A_{\phi_i} e_i(x,t) + F_{\phi_i}(e_i(x,t)) + c\sum\limits_{j=1}^{N} g_{ij} e_j(x,t), \\ e_{i,x}(x,t)|_{x=0} = B_{\phi_i} K_i e_i(0,t), \quad e_{i,x}(x,t)|_{x=L} = 0, \\ e_i(x,0) = e_{i,0}(x). \end{cases} \tag{8}$$

Consider the Lyapunov functional candidate for the synchronization error system (8) as

$$V(t) = \sum_{i=1}^{N}\int_0^L e_i^{\mathrm{T}}(x,t)P_i e_i(x,t)\mathrm{d}x, \tag{9}$$

where $P_i \in \mathbb{R}^{n\times n}$ are positive symmetric matrices to be determined. The time derivative of $V(t)$ of (9) is

$$\begin{aligned} \dot{V}(t) &= 2\int_0^L \sum_{i=1}^{N} e_i^{\mathrm{T}}(x,t)P_i e_{i,t}(x,t)\mathrm{d}x \\ &= 2\int_0^L \sum_{i=1}^{N} e_i^{\mathrm{T}}(x,t)P_i\Theta_{\phi_i} e_{i,xx}(x,t)\mathrm{d}x + \int_0^L \sum_{i=1}^{N} e_i^{\mathrm{T}}(x,t)[P_i A_{\phi_i} + *]e_i(x,t)\mathrm{d}x \\ &\quad +2\int_0^L \sum_{i=1}^{N} e_i^{\mathrm{T}}(x,t)P_i F_{\phi_i}(e_i(x,t))\mathrm{d}x + 2c\int_0^L \sum_{i=1}^{N} e_i^{\mathrm{T}}(x,t)P_i \sum_{j=1}^{N} g_{ij}e_j(x,t)\mathrm{d}x. \end{aligned} \tag{10}$$

Making use of integrating by parts and considering the boundary condition of (8),

$$\begin{aligned} &2\int_0^L \sum_{i=1}^{N} e_i^{\mathrm{T}}(x,t)P_i\Theta_{\phi_i} e_{i,xx}(x,t)\mathrm{d}x \\ &= -2\sum_{i=1}^{N} e_i^{\mathrm{T}}(0,t)P_i\Theta_{\phi_i}B_{\phi_i}K_{\phi_i}e_i(0,t) - 2\int_0^L \sum_{i=1}^{N} e_{i,x}^{\mathrm{T}}(x,t)P_i\Theta_{\phi_i}e_{i,x}(x,t)\mathrm{d}x \\ &= -e^{\mathrm{T}}(0,t)[P\Theta BK + *]e(0,t) - \int_0^L e_x^{\mathrm{T}}(x,t)[P\Theta + *]e_x(x,t)\mathrm{d}x, \end{aligned} \tag{11}$$

where $P \triangleq \mathrm{diag}\{P_1, P_2, \ldots, P_N\}$, $K \triangleq \mathrm{diag}\{K_1, K_2, \ldots, K_N\}$, and $e(x,t)$, Θ, B are defined in (4).

Since $P_i > 0$ and $\Theta_{\phi_i} > 0$, $i \in \mathcal{N}$, then

$$[P\Theta + *] > 0. \tag{12}$$

Denote $\bar{e}(x,t) \triangleq e(x,t) - e(0,t)$, then $\bar{e}(0,t) = 0$. Using Lemma 1 and considering (12),

$$-\int_0^L e_x^{\mathrm{T}}(x,t)[P\Theta + *]e_x(x,t)\mathrm{d}x \leqslant -0.25L^{-2}\pi^2 \int_0^L \bar{e}^{\mathrm{T}}(x,t)[P\Theta + *]\bar{e}(x,t)\mathrm{d}x. \tag{13}$$

According to Lemma 2 and Assumption 1, for any positive scalar $\alpha_i \in \mathbb{R}$,

$$\begin{aligned}
&2\int_0^L \sum_{i=1}^N e_i^{\mathrm{T}}(x,t)P_iF_{\phi_i}(e_i(x,t))\mathrm{d}x \\
&\leqslant \alpha_i \int_0^L \sum_{i=1}^N e_i^{\mathrm{T}}(x,t)P_iP_ie_i(x,t)\mathrm{d}x + \alpha_i^{-1}\int_0^L \sum_{i=1}^N F_{\phi_i}^{\mathrm{T}}(e_i(x,t))F_{\phi_i}(e_i(x,t))\mathrm{d}x \\
&\leqslant \alpha_i \int_0^L \sum_{i=1}^N e_i^{\mathrm{T}}(x,t)P_iP_ie_i(x,t)\mathrm{d}x + \alpha_i^{-1}\chi_i^2\int_0^L \sum_{i=1}^N e_i^{\mathrm{T}}(x,t)e_i(x,t)\mathrm{d}x \\
&= \alpha \int_0^L e^{\mathrm{T}}(x,t)PPe(x,t)\mathrm{d}x + \alpha^{-1}\chi\int_0^L e^{\mathrm{T}}(x,t)e(x,t)\mathrm{d}x,
\end{aligned} \tag{14}$$

where $\alpha \triangleq \mathrm{diag}\{\alpha_1 \otimes I_n, \alpha_2 \otimes I_n, \ldots, \alpha_N \otimes I_n\}$, $\chi \triangleq \mathrm{diag}\{\chi_1^2 \otimes I_n, \chi_2^2 \otimes I_n, \ldots, \chi_N^2 \otimes I_n\}$.

Substituting (11), (13) and (14) into (10),

$$\begin{aligned}
\dot{V}(t) \leqslant\; & -e^{\mathrm{T}}(0,t)[P\Theta BK + *]e(0,t) - 0.25L^{-2}\pi^2\int_0^L \bar{e}^{\mathrm{T}}(x,t)[P\Theta + *]\bar{e}(x,t)\mathrm{d}x \\
& + \int_0^L e^{\mathrm{T}}(x,t)[PA + *]e(x,t)\mathrm{d}x + \alpha\int_0^L e^{\mathrm{T}}(x,t)PPe(x,t)\mathrm{d}x + \alpha^{-1}\chi\int_0^L e^{\mathrm{T}}(x,t)e(x,t)\mathrm{d}x \\
& + \int_0^L e^{\mathrm{T}}(x,t)[cP\bar{G} + *]e(x,t)\mathrm{d}x \\
=\; & \int_0^L \tilde{e}^{\mathrm{T}}(x,t)\bar{\Psi}\tilde{e}(x,t)\mathrm{d}x,
\end{aligned} \tag{15}$$

where $\tilde{e}(x,t) \triangleq [e^{\mathrm{T}}(0,t), e^{\mathrm{T}}(x,t)]^{\mathrm{T}}$, and

$$\bar{\Psi} \triangleq \begin{bmatrix} \bar{\Psi}_{11} & \bar{\Psi}_{12} \\ * & \bar{\Psi}_{22} \end{bmatrix}, \tag{16}$$

in which $\bar{\Psi}_{11} \triangleq [-L^{-1}P\Theta BK - 0.25L^{-2}\pi^2P\Theta + *]$, $\bar{\Psi}_{12} \triangleq 0.5L^{-2}\pi^2P\Theta$, and $\bar{\Psi}_{22} \triangleq [PA + cP\bar{G} - 0.25L^{-2}\pi^2P\Theta + *] + \alpha^{-1}\chi + \alpha PP$.

From the above analysis, the following result can be obtained.

Theorem 1. For the CSDN (1) under Assumption 1, using the collocated boundary controller (7), the CSDN (1) achieves cluster synchronization with the isolated node (2), if there exist scalars $\alpha_i > 0$, $n \times n$ matrices $Q_i > 0$ and $m \times n$ matrices Z_i, $i \in \mathcal{N}$, satisfying the following LMI:

$$\Psi \triangleq \begin{bmatrix} \Psi_{11} & \Psi_{12} & 0 \\ * & \Psi_{22} & Q \\ * & * & -\alpha\chi^{-1}I \end{bmatrix} < 0, \tag{17}$$

where

$$\begin{aligned}
&\Psi_{11} \triangleq [-L^{-1}\Theta BZ - 0.25L^{-2}\pi^2\Theta Q + *], \\
&\Psi_{12} \triangleq 0.5L^{-2}\pi^2\Theta Q, \\
&\Psi_{22} \triangleq [AQ + c\bar{G}Q - 0.25L^{-2}\pi^2\Theta Q + *] + \alpha I, \\
&Q \triangleq \mathrm{diag}\{Q_1, Q_2, \ldots, Q_N\}, \\
&Z \triangleq \mathrm{diag}\{Z_1, Z_2, \ldots, Z_N\}.
\end{aligned}$$

In this case, the gain matrices of the controller (7) could be given as

$$K_i = Z_i Q_i^{-1}, \quad i \in \mathcal{N}. \tag{18}$$

Proof. Assume there exist scalars $\alpha_i > 0$, $n \times n$ matrices $Q_i > 0$ and $m \times n$ matrices Z_i, $i \in \mathcal{N}$ satisfying LMI (17). Set

$$Q_i = P_i^{-1}, \quad Z_i = K_i Q_i. \tag{19}$$

Considering (19), pre- and post-multiply Ψ respectively by $\mathrm{diag}\{Q^{-1}, Q^{-1}, I\}$,

$$\begin{aligned} \hat{\Psi} &\triangleq \mathrm{diag}\{Q^{-1}, Q^{-1}, I\}\Psi\mathrm{diag}\{Q^{-1}, Q^{-1}, I\} \\ &= \begin{bmatrix} \bar{\Psi}_{11} & 0.5L^{-2}\pi^2 P\Theta & 0 \\ * & \hat{\Psi}_{22} & I \\ * & * & -\alpha\chi^{-1}I \end{bmatrix}, \end{aligned} \tag{20}$$

where $\bar{\Psi}_{11}$ is defined in (16) and $\hat{\Psi}_{22} \triangleq [PA + cP\bar{G} - 0.25L^{-2}\pi^2 P\Theta + *]$.

Since $\mathrm{diag}\{Q^{-1}, Q^{-1}, I\} > 0$, it follows from (17) and (20) that $\hat{\Psi} < 0$, by Schur complement, equivalent to

$$\bar{\Psi} < 0. \tag{21}$$

Substituting (21) into (15), $\dot{V}(t) \leqslant -\lambda||\tilde{e}(\cdot,t)||_2^2 \leqslant -\lambda||e(\cdot,t)||_2^2$, for all non-zero $e(\cdot,t)$, where λ is the minimal eigenvalue of $-\Psi$, which ensures the asymptotic stability of cluster synchronization error system (5), i.e., the CSDN (1) achieves cluster synchronization with the isolated node (2). Moreover, control gain parameters K_i can be obtained by (9). This completes the proof.

Remark 2. Notice that boundary control methods were studied in [51,52] for semi-linear PDE systems. Based on these boundary control methods, a boundary controller is studied for cluster synchronization of a semi-linear CSDN with N nonidentical nodes.

Remark 3. Compared with existing synchronization control methods of CSDNs [27,30,31], the major contribution of this paper lies in studying the boundary controller with boundary measurement, requiring few sensors and actuators locating only at the spatial boundary domain. Distributed controllers in references [27,30,31] were employed, where sensors and actuators distribute all over the spatial domain. When the states are unknown, this characteristic makes it hard for these controllers to be implemented in practice.

4 Guaranteed cost boundary control with boundary measurement

This section studies guaranteed cost boundary control with boundary measurement for cluster synchronization of CSDN (1). The quadratic cost function for the CSDN (1) is considered as

$$J = \int_0^\infty v(t)\mathrm{d}t, \tag{22}$$

where $v(t) = \sum_{i=1}^N v_i(t)$, $v_i(t) = \int_0^L e_i^{\mathrm{T}}(x,t)R_i e_i(x,t)\mathrm{d}x + u_i^{\mathrm{T}}(t)S_i u_i(t)$, and $R_i \in \mathbb{R}^{n\times n}$, $S_i \in \mathbb{R}^{m\times m}$ are known weighting matrices.

A boundary controller (7) is further studied to determine a cost bound $\bar{J} < +\infty$ as small as possible for cluster synchronization of the CSDN (1).

Combining (15) and (22),

$$\begin{aligned} \dot{V}(t) + v(t) &\leqslant \int_0^L \tilde{e}^{\mathrm{T}}(x,t)\bar{\Psi}\tilde{e}(x,t)\mathrm{d}x + \int_0^L e^{\mathrm{T}}(x,t)Re(x,t)\mathrm{d}x + e^{\mathrm{T}}(0,t)K^{\mathrm{T}}SKe(0,t) \\ &= \int_0^L \tilde{e}^{\mathrm{T}}(x,t)\bar{\Xi}\tilde{e}(x,t)\mathrm{d}x, \end{aligned} \tag{23}$$

where

$$\bar{\Xi} \triangleq \begin{bmatrix} \bar{\Xi}_{11} & 0.5L^{-2}\pi^2 P\Theta \\ * & \bar{\Xi}_{22} \end{bmatrix}, \tag{24}$$

in which $\bar{\Xi}_{11} \triangleq \Psi_{11} + L^{-1}K^{\mathrm{T}}SK$ and $\bar{\Xi}_{22} \triangleq [PA + cP\bar{G} - 0.25L^{-2}\pi^2 P\Theta + *] + \alpha^{-1}\chi I + \alpha PP + R$.

From the above analysis, the following result can be obtained.

Theorem 2. For the CSDN (1) under Assumption 1, given any $n \times n$ matrices $R_i > 0$ and $m \times m$ matrices $S_i > 0$, if there exist scalars $\alpha_i > 0$, $n \times n$ matrices $Q_i > 0$ and $m \times n$ matrices Z_i, $i \in \mathcal{N}$ satisfying the following LMI:

$$\Xi \triangleq \begin{bmatrix} \Psi_{11} & \Psi_{12} & 0 & 0 & Z^{\mathrm{T}} \\ * & \Psi_{22} & Q & Q & 0 \\ * & * & -\alpha\chi^{-1}I & 0 & 0 \\ * & * & * & \Xi_{44} & 0 \\ * & * & * & * & \Xi_{55} \end{bmatrix} < 0, \tag{25}$$

where $\Psi_{11}, \Psi_{12,}, \Psi_{22}$ are given in (17), $\Xi_{44} \triangleq [-0.5R^{-1} + *]$, $\Xi_{55} \triangleq [-0.5LS^{-1} + *]$, $R \triangleq \mathrm{diag}\{R_1, R_2, \ldots, R_N\}$, and $S \triangleq \mathrm{diag}\{S_1, S_2, \ldots, S_N\}$, then the controller (7) is guaranteed cost and satisfies

$$J < \left\langle e_0(x), Q^{-1}e_0(x)\right\rangle. \tag{26}$$

In this case, the gain parameters of the controller (7) could be given as

$$K_i = Z_i Q_i^{-1}, \quad i \in \mathcal{N}. \tag{27}$$

Proof. For any $n \times n$ matrices $R_i > 0$ and $m \times m$ matrices $S_i > 0$, assume there exist scalars $\alpha_i > 0$, $n \times n$ matrices $Q_i > 0$ and $m \times n$ matrices Z_i satisfying LMI (25). Set

$$Q_i = P_i^{-1}, \quad Z_i = K_i Q_i, \quad i \in \mathcal{N}. \tag{28}$$

Pre- and post-multiply Ξ respectively by $\mathrm{diag}\{Q^{-1}, Q^{-1}, I, I, I\}$,

$$\begin{aligned} \hat{\Xi} &\triangleq \mathrm{diag}\{Q^{-1}, Q^{-1}, I, I, I\}\Xi\mathrm{diag}\{Q^{-1}, Q^{-1}, I, I, I\} \\ &= \begin{bmatrix} \bar{\Psi}_{11} & \bar{\Psi}_{12} & 0 & 0 & K^{\mathrm{T}} \\ * & \bar{\Psi}_{22} & I & I & 0 \\ * & * & -\alpha\chi^{-1}I & 0 & 0 \\ * & * & * & \Xi_{44} & 0 \\ * & * & * & * & \Xi_{55} \end{bmatrix}. \end{aligned} \tag{29}$$

Since $\mathrm{diag}\{Q^{-1}, Q^{-1}, I, I, I\} > 0$, it follows from (25) and (29) that

$$\hat{\Xi} < 0. \tag{30}$$

Substituting $\bar{\Xi} < 0$ into (12), $\dot{V}(t) \leqslant -\lambda||\tilde{e}(\cdot,t)||_2^2 \leqslant -\lambda||e(\cdot,t)||_2^2$, for all non-zero $e(\cdot,t)$, where λ is the minimal eigenvalue of $-\Psi$, which ensures the asymptotic stability of cluster synchronization error system (5), i.e., the CSDN (1) achieves cluster synchronization with the isolated node (2) according to Definition 1. Using Schur complement, the inequality (30) is equivalent to $\bar{\Xi} < 0$, implying

$$\dot{V}(t) + v(t) < 0, \tag{31}$$

for all non-zero $\tilde{e}(\cdot,t)$. Rearrange and integrate of (31),

$$J = \int_0^{\infty} v(t)\mathrm{d}t < -\left. V(t)\right|_{t=0}^{t=\infty} < \bar{J}, \tag{32}$$

where $\bar{J} \triangleq V(0) = \left\langle e_0(x), Q^{-1}e_0(x)\right\rangle$. Moreover, Eq. (28) implies (27). The proof is complete.

Remark 4. Theorem 2 implies Theorem 1, since that $\Xi < 0$ implies $\Psi < 0$ by using matrix property.

Theorem 3. For the CSDN (1) under Assumption 1, if there exist a set of solutions $\gamma, Q_i, Z_i, i \in \mathcal{N}$ such that the following SALMI optimization problem:

$$\min_{\gamma, Q_i, Z_i, i \in \mathcal{N}} \gamma, \tag{33}$$

satisfying LMI (25) and SALMI

$$\begin{bmatrix} \gamma L^{-1} & e_0^{\mathrm{T}}(x) \\ * & Q \end{bmatrix} > 0, \tag{34}$$

where $Q > 0$ is given in (17) and $e_0(x)$ is given in (11), then the controller (7) with control gain parameters (27) is guaranteed cost in the sense of minimizing the upper bound of the cost function (22).

Proof. By Theorem 2, the boundary controller (7) with control gain parameters (25) via any feasible solutions $\gamma, Q_i, Z_i, i \in \mathcal{N}$ to LMI (25) is guaranteed cost for the CSDN (1) with the cost function (22). Using Schur complement, for each $x \in [0, L]$, Eq. (34) gives

$$e_0^{\mathrm{T}}(x) Q^{-1} e_0(x) < \gamma L^{-1}. \tag{35}$$

Integrating above inequality from 0 to L, yields $\bar{J} < \gamma$ and Eq. (22) is obtained. The proof is complete.

To solve SALMI (34), the space interval $[0, L]$ is discretized into space instances $\{x_k, k \in \mathcal{S} \triangleq \{1, 2, \dots, S\}, x_0 = 0, x_0 = L\}$ of the same distance, where

$$x_k - x_{k-1} = L/S \triangleq \varepsilon, \quad k \in \mathcal{S}. \tag{36}$$

With (36), Theorem 3 can be formulated as the approximate LMI optimization problem

$$\min_{\gamma, Q_i, Z_i, i \in \mathcal{N}} \gamma, \tag{37}$$

subject to LMI (25), $Q > 0$ and LMIs

$$\begin{bmatrix} \gamma L^{-1} & e_0^{\mathrm{T}}(x_k) \\ * & Q \end{bmatrix} > 0, \quad k \in S, \tag{38}$$

which can be solved by LMI optimization techniques.

Remark 5. There are the similar results in Theorems 1–3 for other boundary conditions, such as $y_{i,x}(x,t)|_{x=0} = 0$ and $y_{i,x}(x,t)|_{x=L} = B_{\phi_i} u_i(t)$, $y_{i,x}(x,t)|_{x=0} = B_{\phi_i} u_i(t)$ and $y_i(L,t) = 0$, or $y_i(0,t) = 0$ and $y_{i,x}(x,t)|_{x=L} = B_{\phi_i} u_i(t)$, $i \in \mathcal{N}$.

Remark 6. Refs. [39–42] considered guaranteed cost controllers for ability of several systems, which is different from this paper considering synchronization of CSDNs.

Remark 7. There are many important results on guaranteed cost control of CDNs [43, 44], which are different from this paper studying a boundary controller designed at the boundary positions of CSDNs.

5 Numerical example

In this section, a numerical example is given to demonstrate the effectiveness of the proposed methods. Consider a nonlinear CSDN (1) composed of 6 nodes with the first community $V_1 = \{1, 2, 3\}$, the second

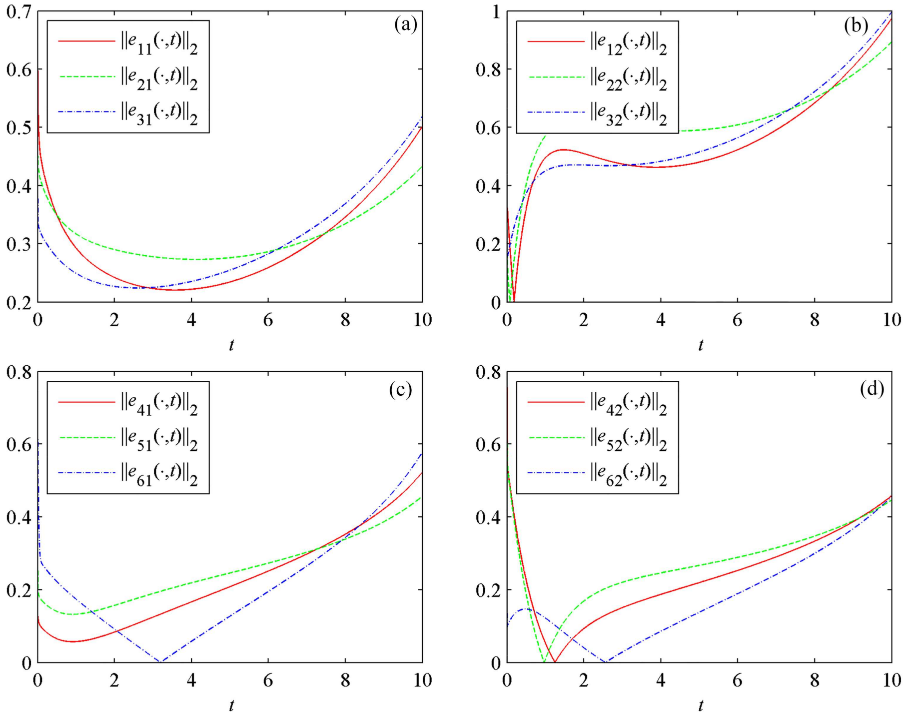

Figure 1 (Color online) The open-loop trajectories of synchronization errors $\|e_i(\cdot,t)\|$, $i \in \{1,2,\ldots,6\}$. (a) $\|e_{i,1}(\cdot,t)\|, i \in V_1$; (b) $\|e_{i,2}(\cdot,t)\|, i \in V_1$; (c) $\|e_{i,1}(\cdot,t)\|, i \in V_2$; (d) $\|e_{i,2}(\cdot,t)\|, i \in V_2$.

community $V_2 = \{4,5,6\}$ and the following coefficients,

$$
\begin{aligned}
&y_i(x,t) = [y_{i1}(x,t), y_{i2}(x,t)]^{\mathrm{T}}, \quad B_1 = B_2 = [1,0]^{\mathrm{T}},\\
&f_1(y_i(x,t)) = f_2(y_i(x,t)) = [-\sin(y_{i1}(x,t)), 0]^{\mathrm{T}},\\
&\Theta_1 = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}, \quad \Theta_2 = \begin{bmatrix} 2 & 0 \\ 0 & 7 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.6 & 0.2 \\ 3.6 & -1.6 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0.3 \\ 0.9 & -1 \end{bmatrix}, \quad c = 0.3, \quad L = 1,\\
&G = \begin{bmatrix} -1.1099 & 0.0738 & 0.3480 & 0.1971 & 0.1609 & 0.3301 \\ 0.1083 & -0.7412 & 0.4071 & 0.0334 & 0.0205 & 0.1719 \\ 0.0464 & 0.5404 & -1.4337 & 0.5080 & 0.2374 & 0.1015 \\ 0.2583 & 0.2497 & 0.4373 & -1.7602 & 0.2439 & 0.5711 \\ 0.5472 & 0.5708 & 0.2076 & 0.1741 & -2.0318 & 0.5320 \\ 0.1260 & 0.0785 & 0.3123 & 0.5433 & 0.2415 & -1.3016 \end{bmatrix}.
\end{aligned} \tag{39}
$$

The initial conditions take

$$
\begin{aligned}
y_{1,0}(x) &= [0.5 + 0.3\cos(\pi x), 0.2\cos(\pi x + \pi/4)]^{\mathrm{T}},\\
y_{2,0}(x) &= [0.4 + 0.1\sin(2\pi x + \pi/6), 0.1 + 0.2\cos(\pi x)]^{\mathrm{T}},\\
y_{3,0}(x) &= [0.3 - 0.2\cos(3\pi x), 0.5\sin(\pi x + \pi/12)]^{\mathrm{T}},\\
y_{4,0}(x) &= [0.1 + 0.3\cos(\pi x), -0.3 + 0.2\cos(5\pi x)]^{\mathrm{T}},\\
y_{5,0}(x) &= [0.2 + 0.3\sin(2\pi x + \pi/6), -0.3 + 0.5\cos(\pi x)]^{\mathrm{T}},\\
y_{6,0}(x) &= [-0.2 + 0.6\cos(2\pi x + \pi/3), 0.1 - 0.2\sin(3\pi x)]^{\mathrm{T}},\\
s_0(x) &= [0.2\cos(\pi x), 0.3\sin(\pi x)]^{\mathrm{T}}, \quad x \in [0,1].
\end{aligned} \tag{40}
$$

Figure 1 illustrates open-loop trajectories of synchronization errors $\|e_i(\cdot,t)\|$ of the CSDN (1), $i \in \{1,2,\ldots,6\}$, clearly showing that the CSDN (1) is not synchronized with the isolated node (2).

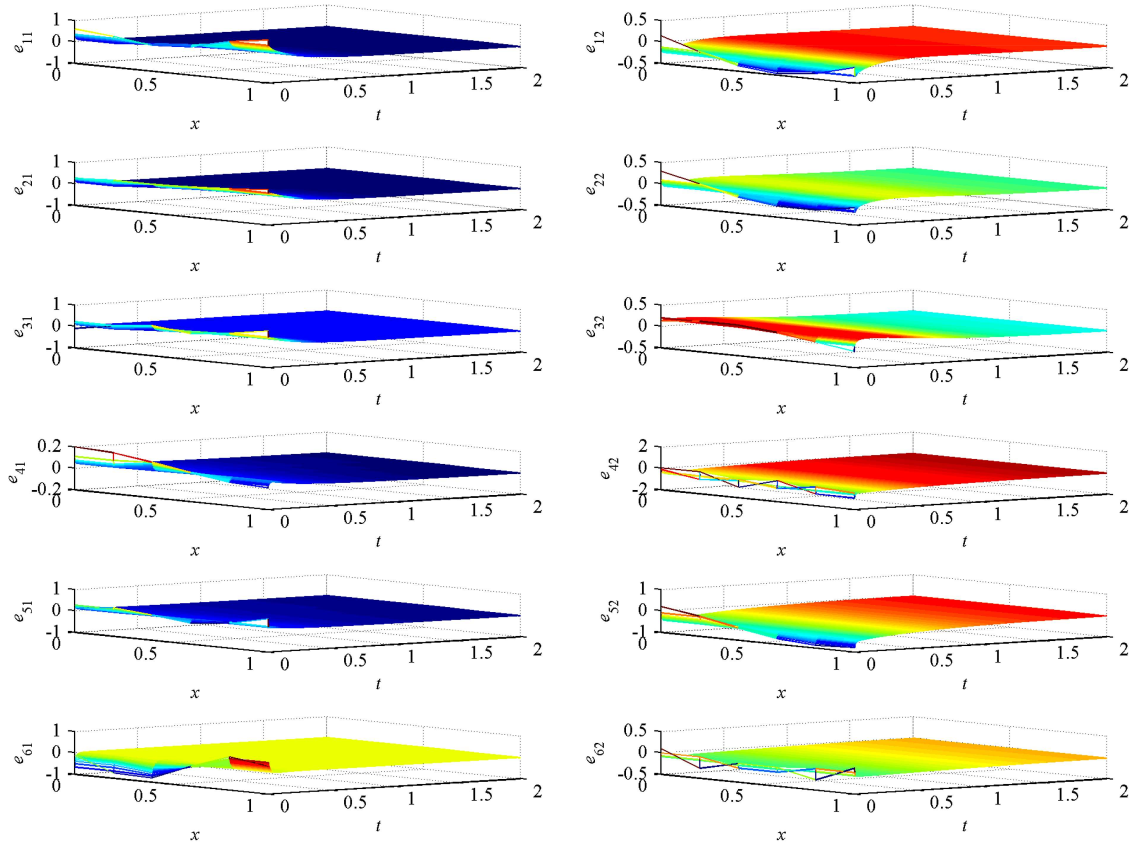

Figure 2 (Color online) The closed-loop profiles of synchronization errors $e_i(X,t), i \in \{1,2,\ldots,6\}$.

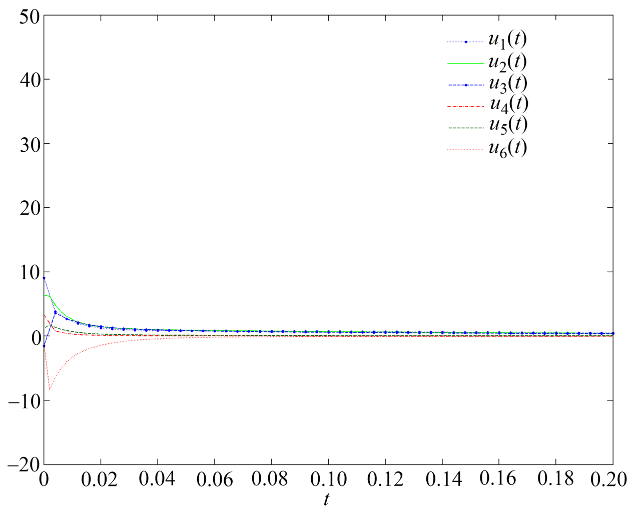

Figure 3 (Color online) The trajectories of control inputs $u_i(t)$, $i \in \{1,2,\ldots,6\}$.

It is not difficult to verify that $f_{\phi_i}(y_i(x,t))$ satisfies the Lipschitz condition with $\chi_i = 1, i \in \{1,2,\ldots,6\}$. Set $R_i = 0.2I_2$, $S_i = 0.2$. Divide the spatial domain $[0,1]$ into space instances $\{x_k | k \in \{0,1,2,\ldots,5\}, x_0 = 0, x_5 = 1\}$ of the same distance with $\varepsilon = 0.2$. With the controller (7), employing the mincx solver to solve the optimization of (37) such that LMIs (25) and (38) in Theorem 3, the control gain parameters K_i, $i \in \{1,2,\ldots,6\}$ are derived from (27) as $K_1 = [14.7512, 1.7759]$, $K_2 = [22.3141, 2.6487]$, $K_3 = [18.2428, 2.2607]$, $K_4 = [17.4976, 1.4607]$, $K_5 = [6.8930, 0.8276]$, $K_6 = [12.4123, 1.8343]$.

Applying the controller (7) with the these control gain parameters, Figure 2 shows the closed-loop profiles of synchronization errors $e_i(X,t)$, $i \in \{1, 2, \ldots, 6\}$. The numerical result shows that the proposed control law (7) can guide the CSDN (1) to achieve cluster synchronization with the isolated node (2). Moreover, Figure 3 shows the corresponding trajectories of control inputs $u_i(t)$, $i \in \{1, 2, \ldots, 6\}$.

6 Conclusion

In this paper, a boundary controller with boundary measurement was studied for cluster synchronization of a nonlinear CSDN with community structure. Only at the spatial boundary positions does the boundary controller require few sensors and actuators. Taking use of Lyapunov's direct method and Wirtinger's inequality, a sufficient condition was obtained in terms of LMIs to guarantee the existence of the boundary controller. The control gain parameters were obtained via feasible solutions to the given LMIs. After that, a guaranteed cost boundary controller and then a suboptimal one were developed, and two sufficient conditions were obtained in terms of SALMIs to guarantee the existence of the controllers. Finally, simulation results verified the effectiveness of theoretical results.

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Conflict of interest The authors declare that they have no conflict of interest.

References

1 Garone E, Gasparri A, Lamonaca F. Clock synchronization for wireless sensor network with communication delay. Automatica, 2015, 59: 60–72

2 Li H Q, Liao X F, Huang T W, et al. Event-triggering sampling based leader-following consensus in second-order multi-agent systems. IEEE Trans Autom Control, 2015, 60: 1998–2003

3 Cao J D, Rakkiyappan R, Maheswari K, et al. Exponential H_∞ filtering analysis for discrete-time switched neural networks with random delays using sojourn probabilities. Sci China Technol Sci, 2016, 59: 387–402

4 Huang T W, Li C D, Duan S K, et al. Robust exponential stability of uncertain delayed neural networks with stochastic perturbation and impulse effects. IEEE Trans Neural Netw Learn Syst, 2012, 23: 866–875

5 Li H Q, Huang C C, Chen G, et al. Distributed consensus optimization in multiagent networks with time-varying directed topologies and quantized communication. IEEE Trans Cybern, 2017, 47: 2044–2057

6 Li H Q, Chen G, Huang T W, et al. Event-triggered distributed average consensus over directed digital networks with limited communication bandwidth. IEEE Trans Cybern, 2016, 46: 3098–3110

7 Li H Q, Chen G, Huang T W, et al. High-performance consensus control in networked systems with limited bandwidth communication and time-varying directed topologies. IEEE Trans Neural Netw Learn Syst, 2017, 28: 1043–1054

8 Wu X J, Zhu C J, Kan H B. An improved secure communication scheme based passive synchronization of hyperchaotic complex nonlinear system. Appl Math Comput, 2015, 252: 201–214

9 Fan D G, Wang Q Y. Synchronization and bursting transition of the coupled hindmarsh-rose systems with asymmetrical time-delays. Sci China Technol Sci, 2017, 60: 1019–1031

10 Li X D, Rakkiyappan R, Sakthivel N. Non-fragile synchronization control for markovian jumping complex dynamical networks with probabilistic time-varying coupling delays. Asian J Control, 2015, 17: 1678–1695

11 Yao C G, Zhao Q, Yu J. Complete synchronization induced by disorder in coupled chaotic lattices. Phys Lett A, 2013, 377: 370–377

12 Mahmoud G, Mahmoud E. Complex modified projective synchronization of two chaotic complex nonlinear systems. Nonlinear Dyn, 2013, 73: 2231–2240

13 Rajagopal K, Vaidyanathan S. Adaptive lag synchronization of a modified rucklidge chaotic system with unknown parameters and its labview implementation. Sensor Transducers, 2016, 200: 37–44

14 Ferrari F A, Viana R L, Lopes S R, et al. Phase synchronization of coupled bursting neurons and the generalized kuramoto model. Neural Netw, 2015, 66: 107–118

15 Li T, Rao B P. Criteria of kalman's type to the approximate controllability and the approximate synchronization for a coupled system of wave equations with dirichlet boundary controls. SIAM J Control Opt, 2016, 54: 49–72
16 Yu W W, Chen G R, Lü J H. On pinning synchronization of complex dynamical networks. Automatica, 2009, 45: 429–435
17 Cao J D, Li R X. Fixed-time synchronization of delayed memristor-based recurrent neural networks. Sci China Inf Sci, 2017, 60: 032201
18 Nagornov R, Osipov G, Komarov M, et al. Mixed-mode synchronization between two inhibitory neurons with post-inhibitory rebound. Commun Nonlinear Sci Numer Simul, 2016, 36: 175–191
19 Sorrentino F, Pecora L M, Hagerstrom A M, et al. Complete characterization of the stability of cluster synchronization in complex dynamical networks. Sci Adv, 2016, 2: e1501737
20 Wang Y L, Cao J D. Cluster synchronization in nonlinearly coupled delayed networks of non-identical dynamic systems. Nonlinear Anal Real World Appl, 2013, 14: 842–851
21 Kang Y, Qin J H, Ma Q C, et al. Cluster synchronization for interacting clusters of nonidentical nodes via intermittent pinning control. IEEE Trans Neural Netw Learn Syst, 2017. doi: 10.1109/TNNLS.2017.2669078
22 Cao J D, Li L L. Cluster synchronization in an array of hybrid coupled neural networks with delay. Neural Netw, 2009, 22: 335–342
23 Kaneko K. Relevance of dynamic clustering to biological networks. Phys D Nonlinear Phenom, 1994, 75: 55–73
24 Wang K H, Fu X C, Li K Z. Cluster synchronization in community networks with nonidentical nodes. Chaos Interdiscipl J Nonlinear Sci, 2009, 19: 023106
25 Lu W L, Liu B, Chen T P. Cluster synchronization in networks of coupled nonidentical dynamical systems. Chaos Interdiscipl J Nonlinear Sci, 2010, 20: 013120
26 Wu Z Y, Fu X C. Cluster mixed synchronization via pinning control and adaptive coupling strength in community networks with nonidentical nodes. Commun Nonlinear Sci Numer Simul, 2012, 17: 1628–1636
27 Wang J W, Wu H N, Li H X. Guaranteed cost distributed fuzzy observer-based control for a class of nonlinear spatially distributed processes. AIChE J, 2013, 59: 2366–2378
28 Sheng L, Yang H Z, Lou X Y. Adaptive exponential synchronization of delayed neural networks with reaction-diffusion terms. Chaos Soliton Fract, 2009, 40: 930–939
29 Yu F, Jiang H J. Global exponential synchronization of fuzzy cellular neural networks with delays and reaction–diffusion terms. Neurocomputing, 2011, 74: 509–515
30 Yang C D, Qiu J L, He H B. Exponential synchronization for a class of complex spatio-temporal networks with space-varying coefficients. Neurocomputing, 2015, 151: 40–47
31 Hu C, Jiang H J, Teng Z D. Impulsive control and synchronization for delayed neural networks with reaction-diffusion terms. IEEE Trans Neural Netw, 2010, 21: 67–81
32 Hu C, Yu J, Jiang H J, et al. Exponential synchronization for reaction-diffusion networks with mixed delays in terms of p-norm via intermittent driving. Neural Netw, 2012, 31: 1–11
33 Gan Q T. Adaptive synchronization of stochastic neural networks with mixed time delays and reaction-diffusion terms. Nonlinear Dyn, 2012, 69: 2207–2219
34 Wang J L, Wu H N. Synchronization and adaptive control of an array of linearly coupled reaction-diffusion neural networks with hybrid coupling. IEEE Trans Cyber, 2014, 44: 1350–1361
35 Wang J L, Wu H N, Guo L. Novel adaptive strategies for synchronization of linearly coupled neural networks with reaction-diffusion terms. IEEE Trans Neural Netw Learn Syst, 2014, 25: 429–440
36 Shafi Y, Arcak M. An adaptive algorithm for synchronization in diffusively-coupled systems. In: Proceedings of American Control Conference, Portland, 2014. 2220–2225
37 Wu K N, Tian T, Wang L M. Synchronization for a class of coupled linear partial differential systems via boundary control. J Franklin Inst, 2016, 353: 4062–4073
38 Wu K N, Tian T, Wang L M, et al. Asymptotical synchronization for a class of coupled time-delay partial differential systems via boundary control. Neurocomputing, 2016, 197: 113–118
39 Wu Z G, Dong S L, Shi P, et al. Fuzzy-model-based nonfragile guaranteed cost control of nonlinear markov jump systems. IEEE Trans Syst Man Cybern Syst, 2017, 47: 2388–2397
40 Lu R Q, Cheng H L, Bai J J. Fuzzy-model-based quantized guaranteed cost control of nonlinear networked systems. IEEE Trans Fuzzy Syst, 2015, 23: 567–575
41 Wang J W, Li H X, Wu H N. Fuzzy guaranteed cost sampled-data control of nonlinear systems coupled with a scalar reaction-diffusion process. Fuzzy Set Syst, 2016, 302: 121–142
42 Wang Z P, Wu H N. Finite dimensional guaranteed cost sampled-data fuzzy control for a class of nonlinear distributed parameter systems. Inf Sci, 2016, 327: 21–39
43 Lee T H, Ji D H, Park J H, et al. Decentralized guaranteed cost dynamic control for synchronization of a complex dynamical network with randomly switching topology. Appl Math Comput, 2012, 219: 996–1010
44 Feng J W, Yang P, Zhao Y. Cluster synchronization for nonlinearly time-varying delayed coupling complex networks

with stochastic perturbation via periodically intermittent pinning control. Appl Math Comput, 2016, 291: 52–68

45 Seuret A, Gouaisbaut F. Wirtinger-based integral inequality: application to time-delay systems. Automatica, 2013, 49: 2860–2866

46 Yang C D, Zhang A C, Chen X, et al. Stability and stabilization of a delayed PIDE system via SPID control. Neural Comput Appl, 2016. doi: 10.1007/s00521-016-2297-5

47 Pazy A. Semigroups of Linear Operators and Applications to Partial Differential Equations. Berlin: Springer, 1983

48 Zhou J, Lu J N, Lü J H. Adaptive synchronization of an uncertain complex dynamical network. IEEE Trans Autom Control, 2006, 51: 652–656

49 Chen Z, Fu X L, Zhao D H. Anti-periodic mild attractor of delayed hopfield neural networks systems with reaction-diffusion terms. Neurocomputing, 2013, 99: 372–380

50 Lu J Q, Ho D W, Cao J D, et al. Single impulsive controller for globally exponential synchronization of dynamical networks. Nonlinear Anal Real World Appl, 2013, 14: 581–593

51 Wu H N, Wang J W, Li H X. Fuzzy boundary control design for a class of nonlinear parabolic distributed parameter systems. IEEE Trans Fuzzy Syst, 2014, 22: 642–652

52 Wang J W, Wu H N, Sun C Y. Boundary controller design and well-posedness analysis of semi-linear parabolic PDE systems. In: Proceedings of 2014 American Control Conference, Beijing, 2014. 3369–3374