Modeling a Target-selection Motion by Leveraging an Optimal Feedback Control Mechanism

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Outline

- Introduction
- Method
- Experiment
- Result and Analysis

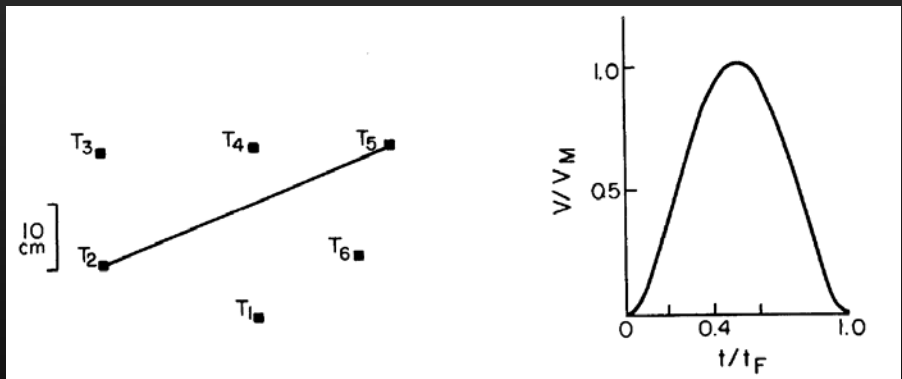
Introduction

Introduction

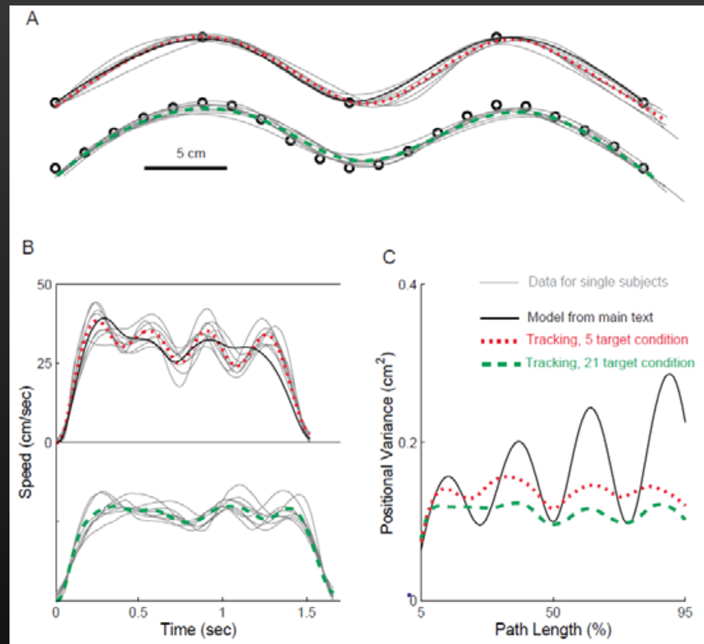
- Target Selection
  - one of the most fundamental interaction tasks in HCI
- Existing Models
  - predict duration and endpoint distribution
- User's continuous motion during the interaction process remains unclear

Introduction

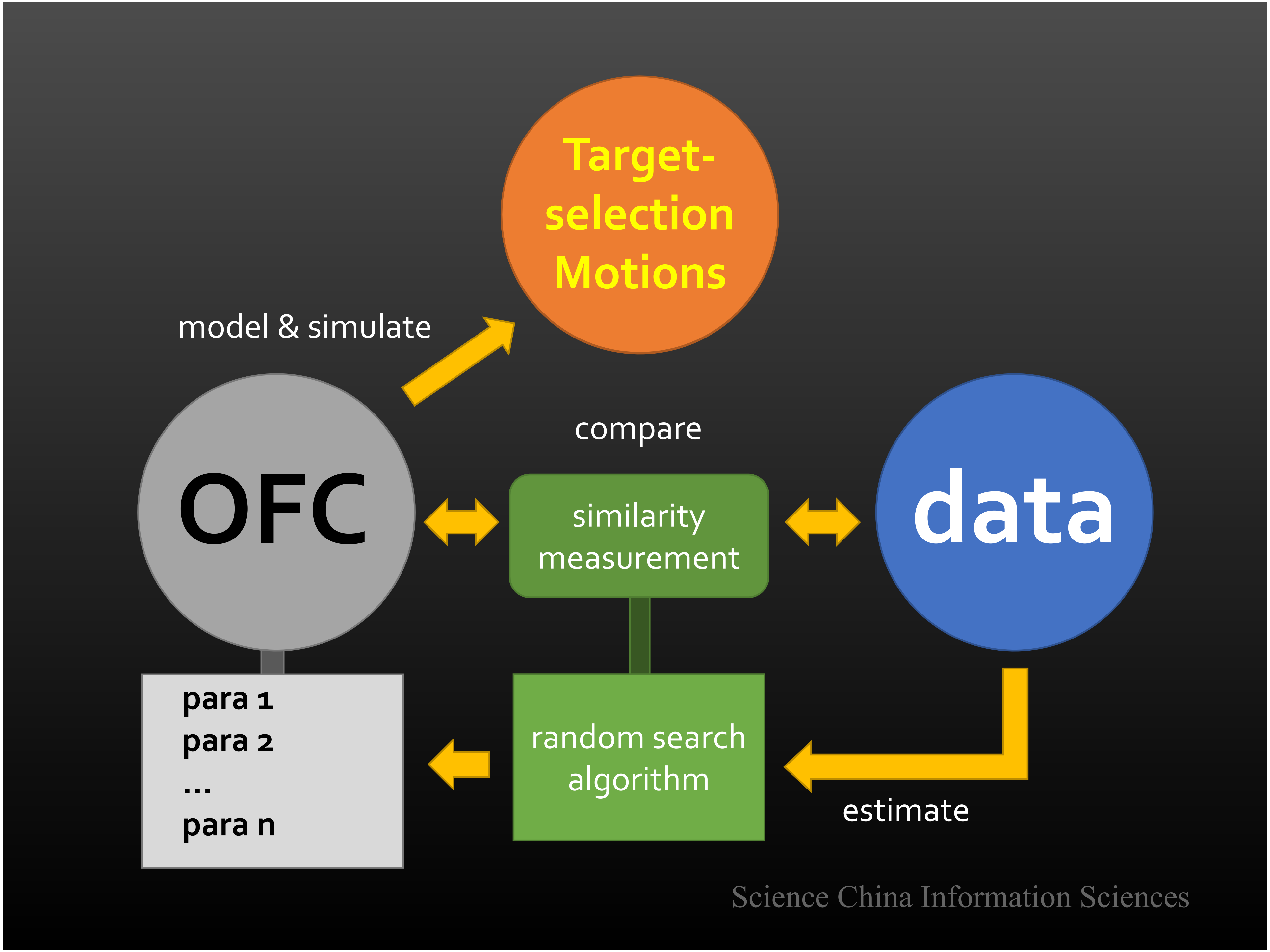
- Optimal Control Theory
  - Open-loop, Close-loop



Flash T and Hogan N (1985)



Todorov E and Jordan M I (2002)


Target-selection Motions
model & simulate
compare
OFC
similarity measurement
data
para 1
para 2
...
para n
random search algorithm
estimate
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Methods

OFC

key parameters

$$\begin{aligned} Dynamics \quad & \boldsymbol{x}_{t+1} = A\boldsymbol{x}_t + B\boldsymbol{u}_t + \boldsymbol{\xi}_t + \varepsilon_t C\boldsymbol{u}_t \\ Feedback \quad & \boldsymbol{y}_{t+1} = H\boldsymbol{x}_t + \boldsymbol{\omega}_t \\ Cost \quad & \sum_{t=1}^{n} \boldsymbol{x}_t^T Q_t \boldsymbol{x}_t + \boldsymbol{u}_t^T R \boldsymbol{u}_t \end{aligned}$$

$$\theta = [\underbrace{\sigma_c}_{\text{control dependent noise}}, \underbrace{\sigma_p, \sigma_v, \sigma_f}_{\text{position velocity \& force noise}}, \underbrace{r, w_v, w_f}_{\text{penalty of efforts, velocity \& force}}]$$

$$\boldsymbol{x}_t = [p_x(t); p_y(t); \dot{p}_x(t); \dot{p}_y(t);$$ position and velocity

$$f_x(t); f_y(t); g_x(t); g_y(t);$$ force related terms

$$p_x^*(t); p_y^*(t)]$$ target position

affects

$$\boldsymbol{u}_t = [u_x(t); u_y(t)]$$

observes

$$\boldsymbol{y}_t = [p_x(t); p_y(t); \dot{p}_x(t); \dot{p}_y(t); f_x(t); f_y(t)]$$

The Newtonian mechanics system with the following discrete-time form to simulate the cursor's movement (d = x/y):

$$
\begin{aligned}
p_d(t+1) &= p_d(t) + \dot{p}_d(t)\Delta,\\
\dot{p}_d(t+1) &= \dot{p}_d(t) + f_d(t)\Delta/m,\\
f_d(t+1) &= f_d(t)(1-\Delta/\tau_2) + g_d(t)\Delta/\tau_2,\\
g_d(t+1) &= g_d(t)(1-\Delta/\tau_1) + u_d(t)(1+\sigma_c\varepsilon_t)\Delta/\tau_1.
\end{aligned}
$$

which can be transformed into the overall OFC formulation using the following matrixes:

$$
A = \begin{bmatrix}
1 & . & \Delta & . & . & . & . & . & 0_{8\times 2}\\
. & 1 & . & \Delta & . & . & . & . & \\
. & . & 1 & . & \Delta/m & . & . & . & \\
. & . & . & 1 & . & \Delta/m & . & . & \\
. & . & . & . & 1-\Delta/\tau_2 & . & \Delta/\tau_2 & . & \\
. & . & . & . & . & 1-\Delta/\tau_2 & . & \Delta/\tau_2 & \\
. & . & . & . & . & . & 1-\Delta/\tau_1 & . & \\
. & . & . & . & . & . & . & 1-\Delta/\tau_1 & \\
0_{2\times 8} & & & & & & & & I_{2\times 2}
\end{bmatrix},
\quad
B = \begin{bmatrix} 0_{6\times 2}\\ \begin{bmatrix} \Delta/\tau_1 & . \\ . & \Delta/\tau_1 \end{bmatrix}\\ 0_{2\times 2}\end{bmatrix},
$$

$$
\boxed{C = B\sigma_c,}
$$

$$
H = \begin{bmatrix} I_{6\times 6} & 0_{6\times 4}\end{bmatrix}.
$$

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We set the covariances of white noise ξ_t and ω_t in the OFC system as follows:

$$\Omega^{\xi} = 0;\ \Omega^{\omega} = (diag[\boxed{\sigma_p; \sigma_p; \sigma_v; \sigma_v; \sigma_f; \sigma_f}])^2$$

which assumes that the white noise in the control process is ignorable and that in the observation process has the standard deviations σ_p, σ_v and σ_f with respect to position, velocity, and force.

The state cost:

$$(p_x(t) - p_x^*(t))^2 + (p_y(t) - p_y^*(t))^2 +$$

$$(\dot{p}_x(t)w_v)^2 + (\dot{p}_y(t)w_v)^2 +$$

$$(f_x(t)w_f)^2 + (f_y(t)w_f)^2$$

The control cost:

$$r({u_x}^2 + {u_y}^2)$$

The total cost can be transformed into the OFC formulation by the matrixes $\boxed{R = r}$ and Q_t, as follows:

$$Q_t = \frac{1}{N}\boldsymbol{q}^T\boldsymbol{q};\ \ \boldsymbol{q} = \begin{bmatrix} -1 & . & . & . & . & . & . & . & 1 & . \\ . & -1 & . & . & . & . & . & . & . & 1 \\ . & . & w_v & . & . & . & . & . & . & . \\ . & . & . & w_v & . & . & . & . & . & . \\ . & . & . & . & w_f & . & . & . & . & . \\ . & . & . & . & . & w_f & . & . & . & . \end{bmatrix}$$

similarity measurement

To find the θ*, we defined a measurement for similarity.

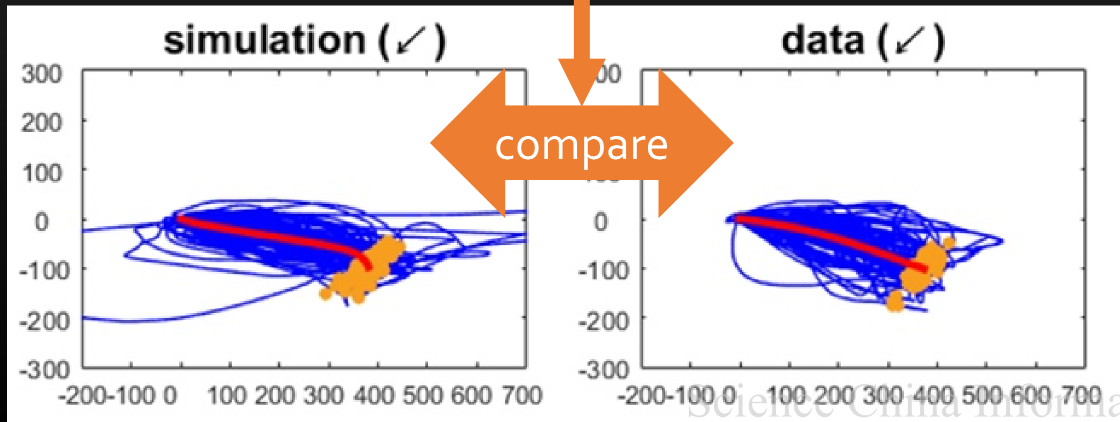
$$J(\theta) = trE \times vaE,$$

$$trE = \frac{1}{n}\sum_{i=1}^{n} ||\overline{p_i^s} - \overline{p_i^e}||,$$

$$vaE = \frac{1}{n}\sum_{i=1}^{n} ||var_i^s - var_i^e||$$

$$\overline{p_i} = (\overline{x_i}, \overline{y_i}) = (\frac{1}{M}\sum_{j=1}^{M} x_{ij}, \frac{1}{M}\sum_{j=1}^{M} y_{ij})$$

$$var_i = (\sqrt{\frac{1}{M}\sum_{j=1}^{M}(x_{ij} - \overline{x_i})^2}, \sqrt{\frac{1}{M}\sum_{j=1}^{M}(y_{ij} - \overline{y_i})^2})$$



random search algorithm

We developed a random search algorithm to obtain θ* by the following major steps:

(a) Given a searching region μi– Δi< θi< μi+ Δi(i = 1,2,…n),
(b) (b) find T θ in this region that satisfies J(θ(1)) > J(θ(2)) > …J(θ(T)).
(c) Determine the new μi and Δi as follows:

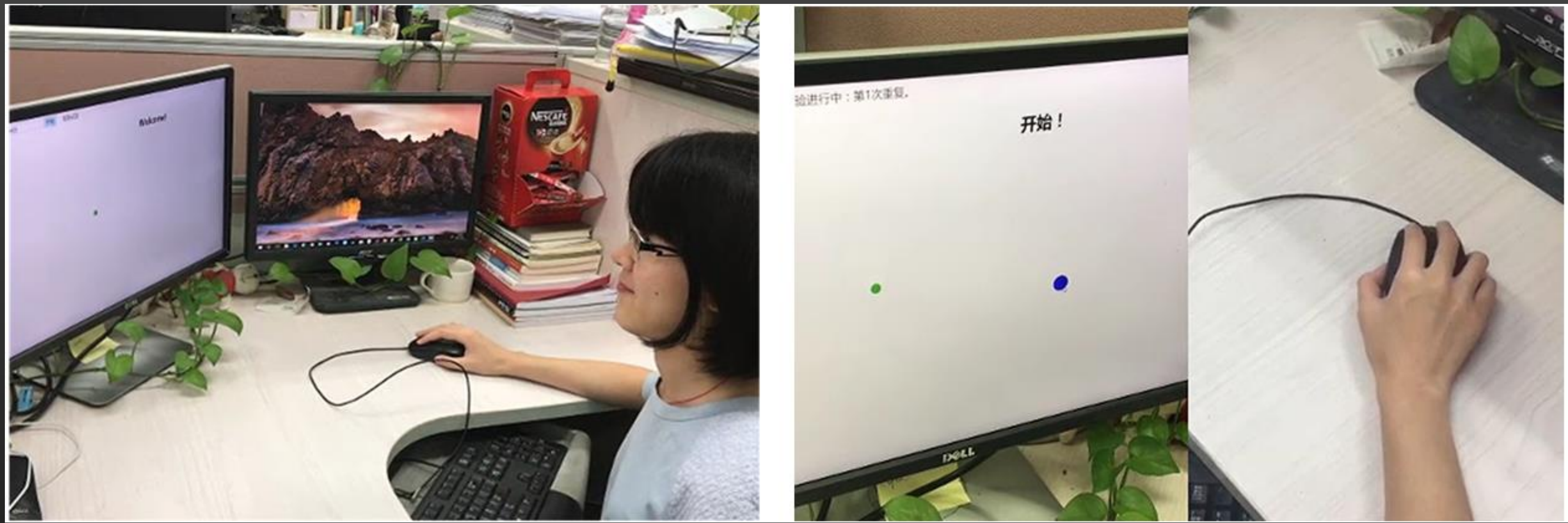
$$\mu_i = \frac{\sum_{t=1}^{T} W^{(t)} \theta_i^{(t)}}{\sum_{t=1}^{T} W^{(t)}}, \quad \Delta_i = C\sqrt{\frac{\sum_{t=1}^{T} W^{(t)} (\theta_i^{(t)} - \mu_i)^2}{\sum_{t=1}^{T} W^{(t)}}}, \quad W^{(t)} = \frac{J(\theta^{(T)})}{J(\theta^{(t)})}$$

(d) search in the new region until satisfying the following convergence condition or reaching the maximum number of iterations.

$$\left| \frac{J(\theta^{(1)}) - J(\theta^{(T)})}{J(\theta^{(T)})} \right| < \varphi$$

Experiment

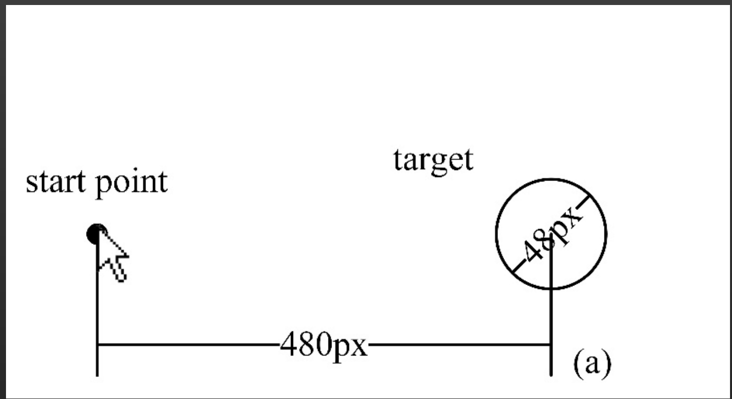
Experimental Process:

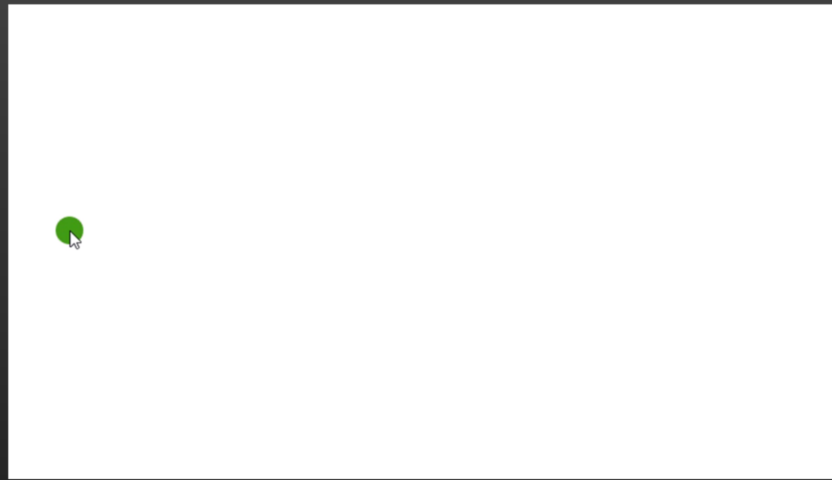


Twenty subjects (6 females and 14 males, with an average age of 26.3 years).
All were right-handed and had more than 2 years of experience using a computer and mouse.
One static target-selection task, and 8 moving target-selection tasks corresponding to 8 moving directions.
Six selections per subject per task, that is 1080 trials in total.

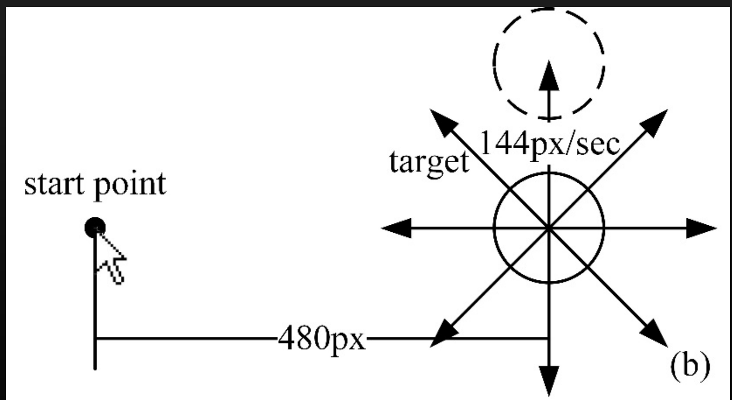
Experimental Process:

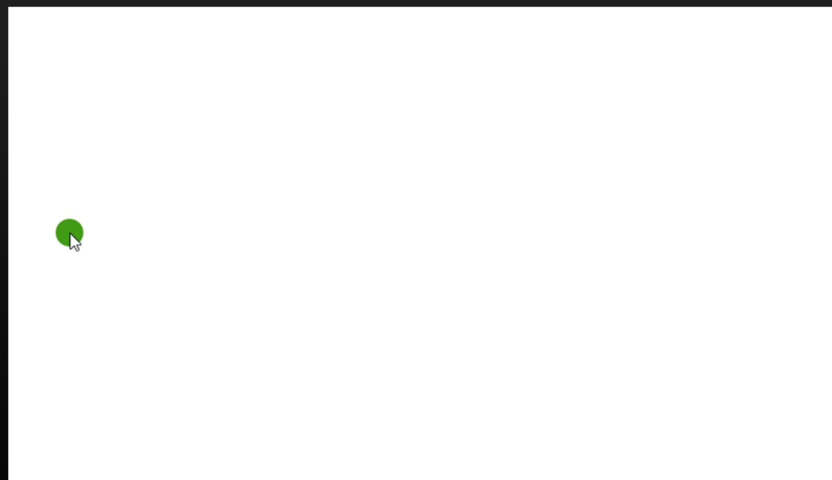
Static target:



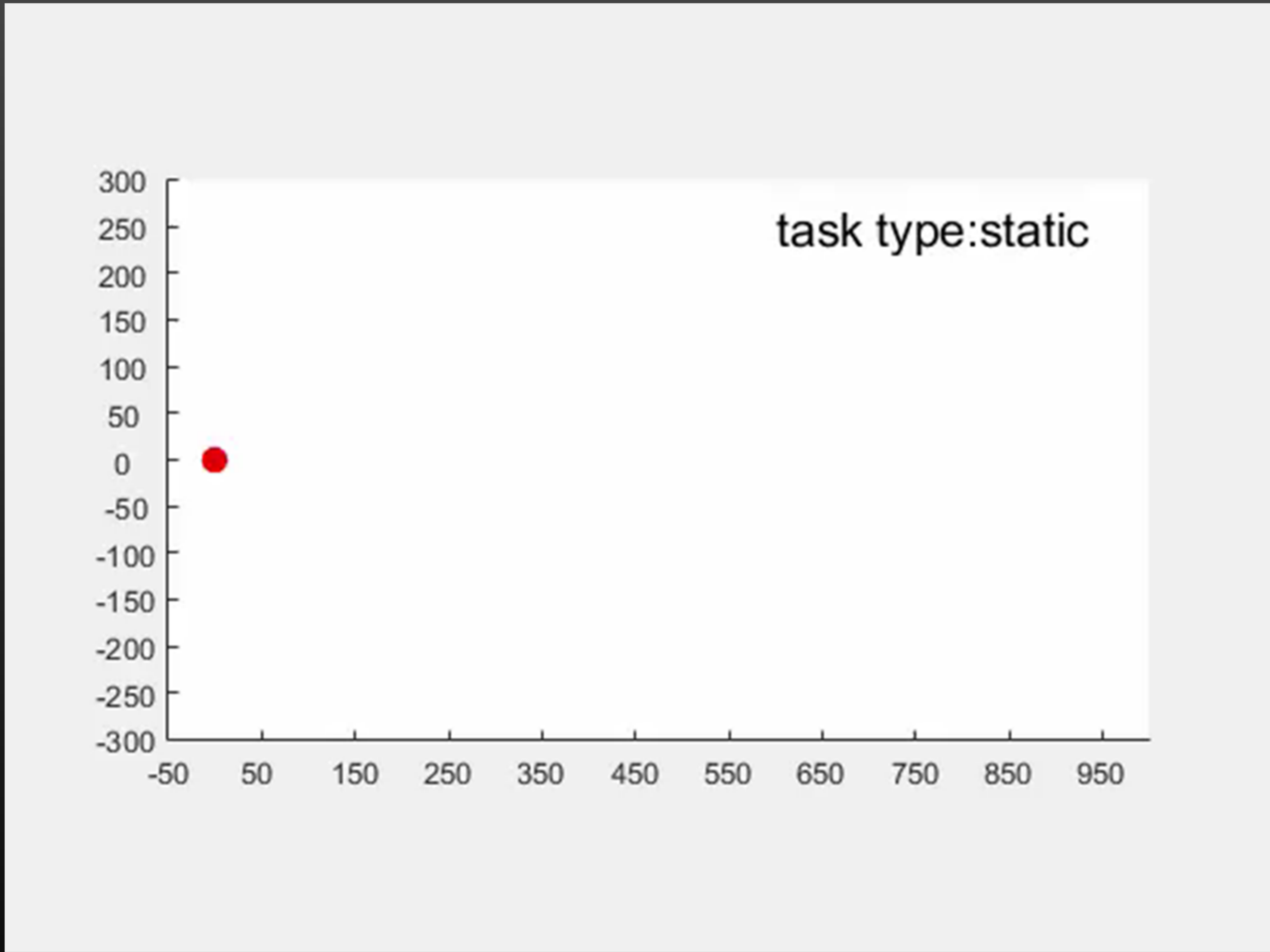


Moving target:





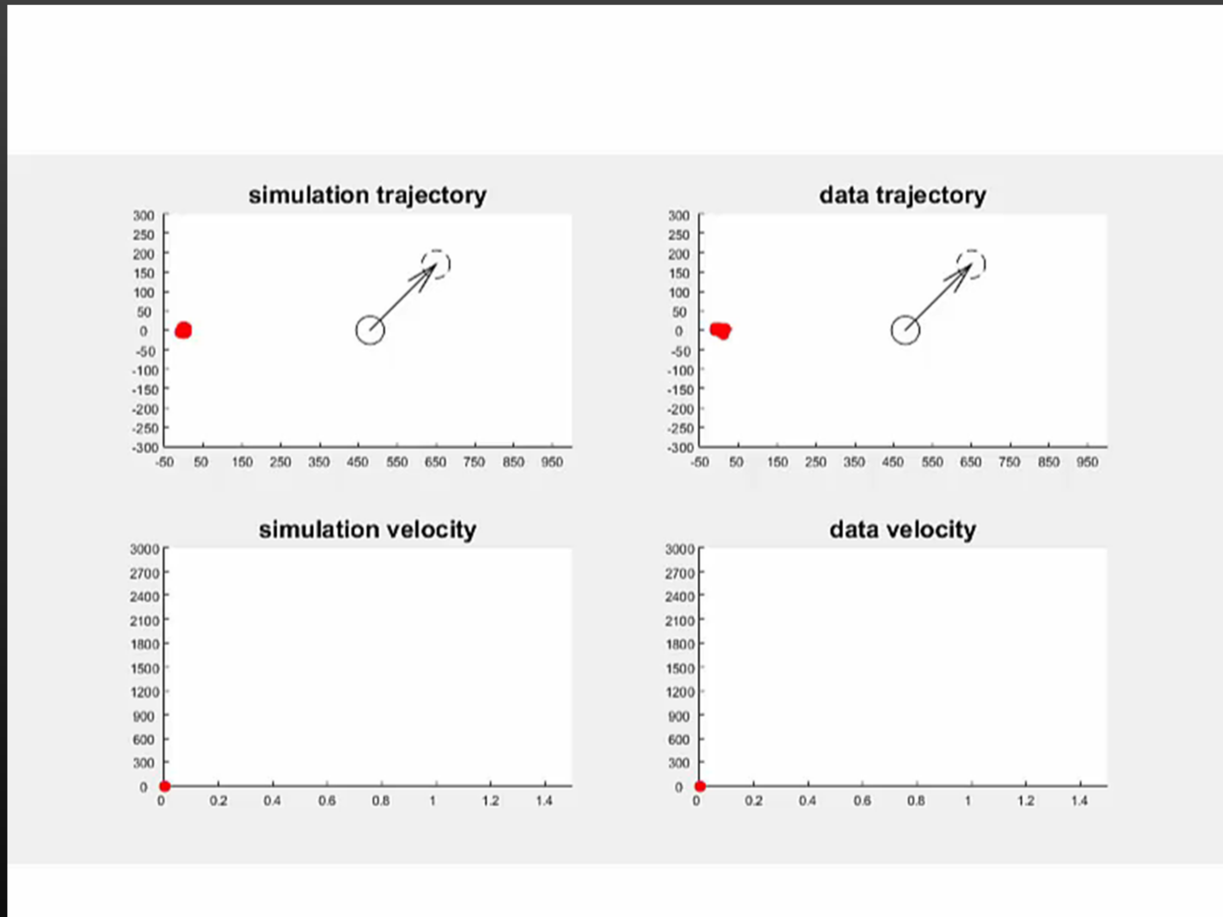
The collected data



data from static and eight moving selection tasks were plot in the same time

The random search algorithm is estimating key parameters of OFC system:

the iteration process of a moving target task (↗)



the algorithm iteratively find a parameter set that make the simulation has highest similarity to the data

Result and Analysis

The estimated parameters sets

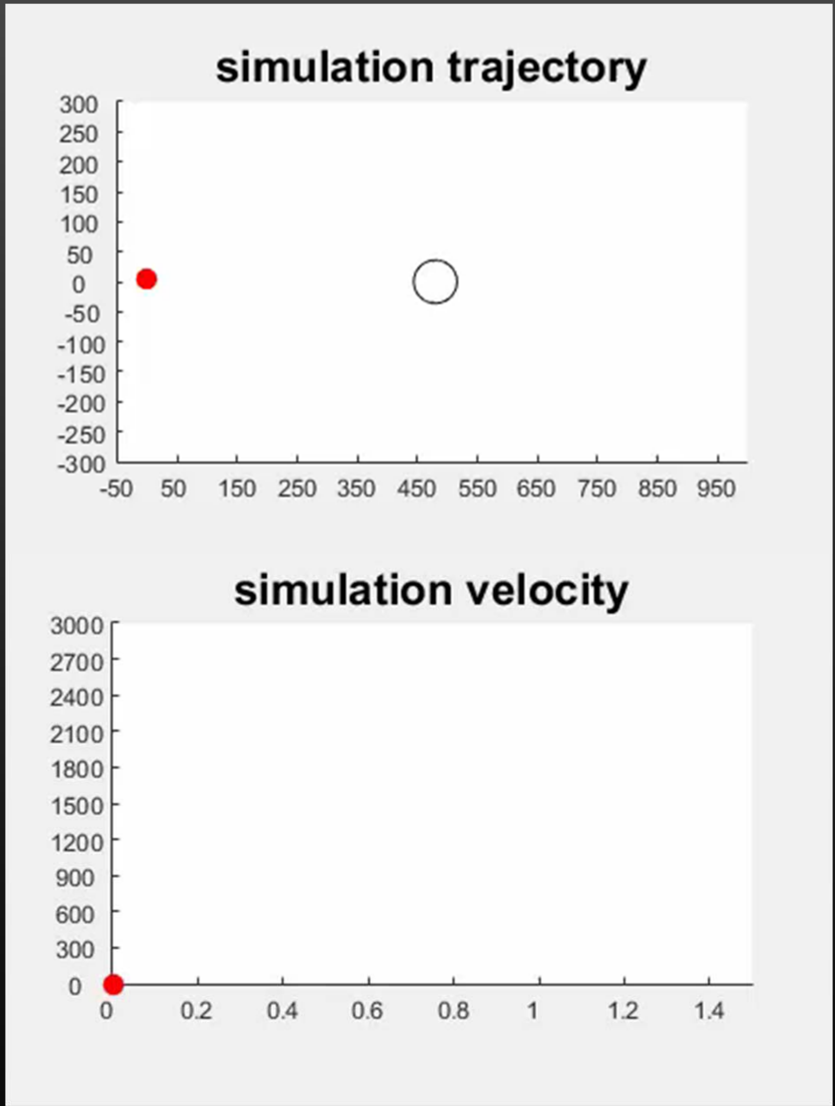
The final estimated parameters for static target selection and moving target selection are shown.

Static target selection: θ_s

Moving target selection: θ_m

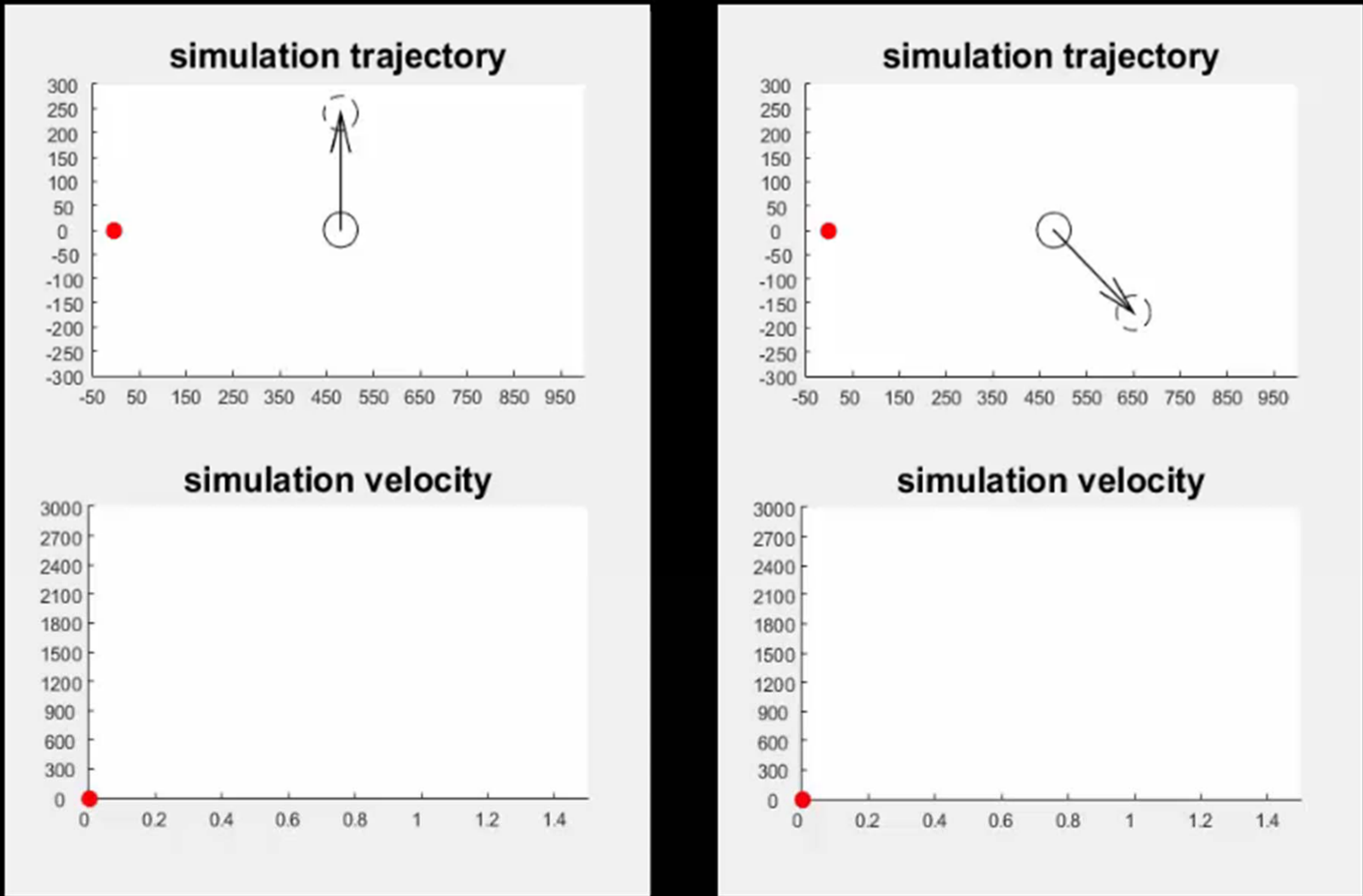
parameters	σ_c	σ_p	σ_v	σ_f	r	w_v	w_f
θ_s	5.4132	68.05	474.68	381.10	690.90	820.35	592.20
θ_m	10.3310	147.29	241.49	147.82	1149.71	646.96	176.89

For moving target selection, there are 8 sets of parameters obtained from 8 directions, we use the mean values of them as the final estimated parameters.

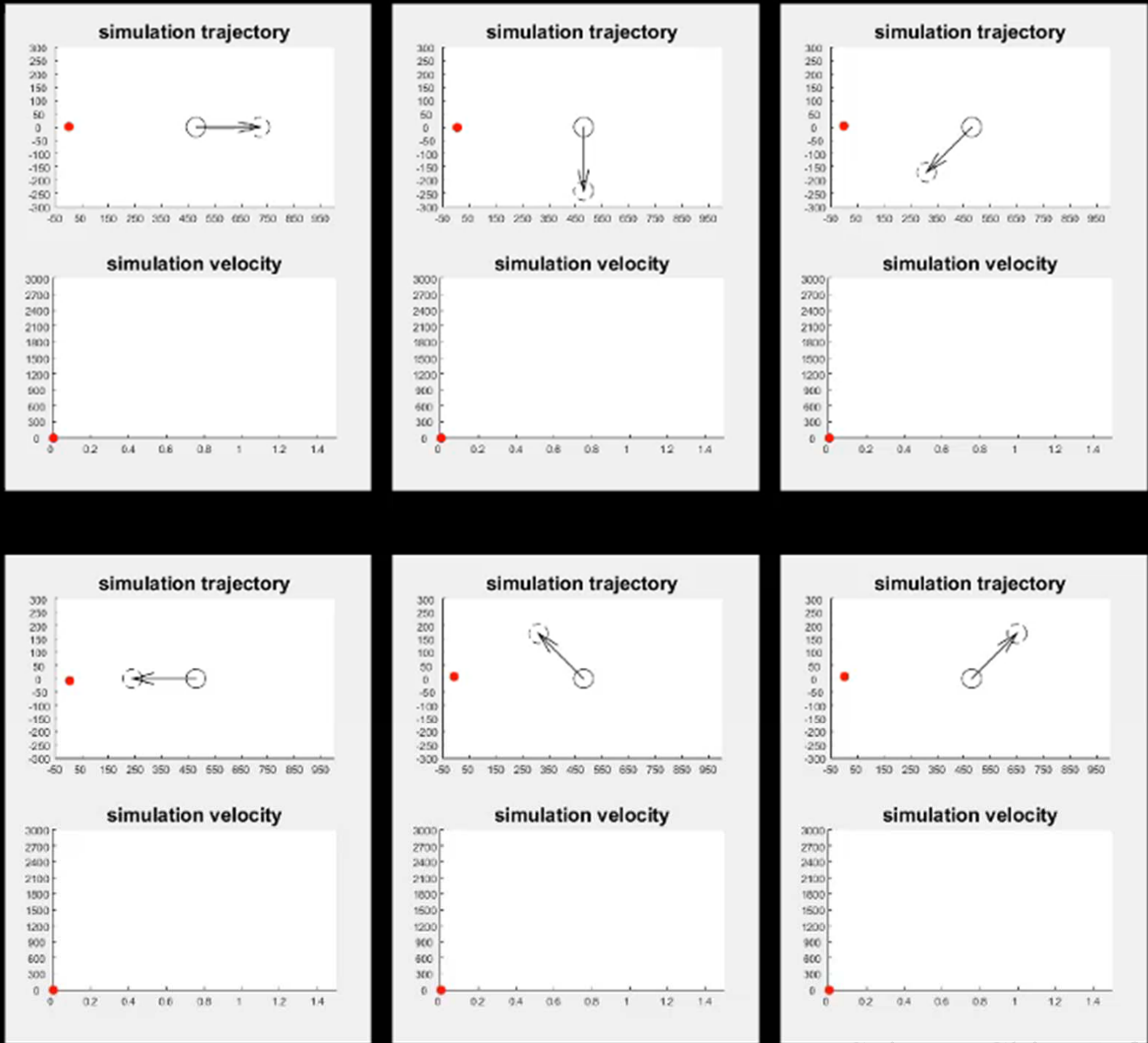


simulation of static target task

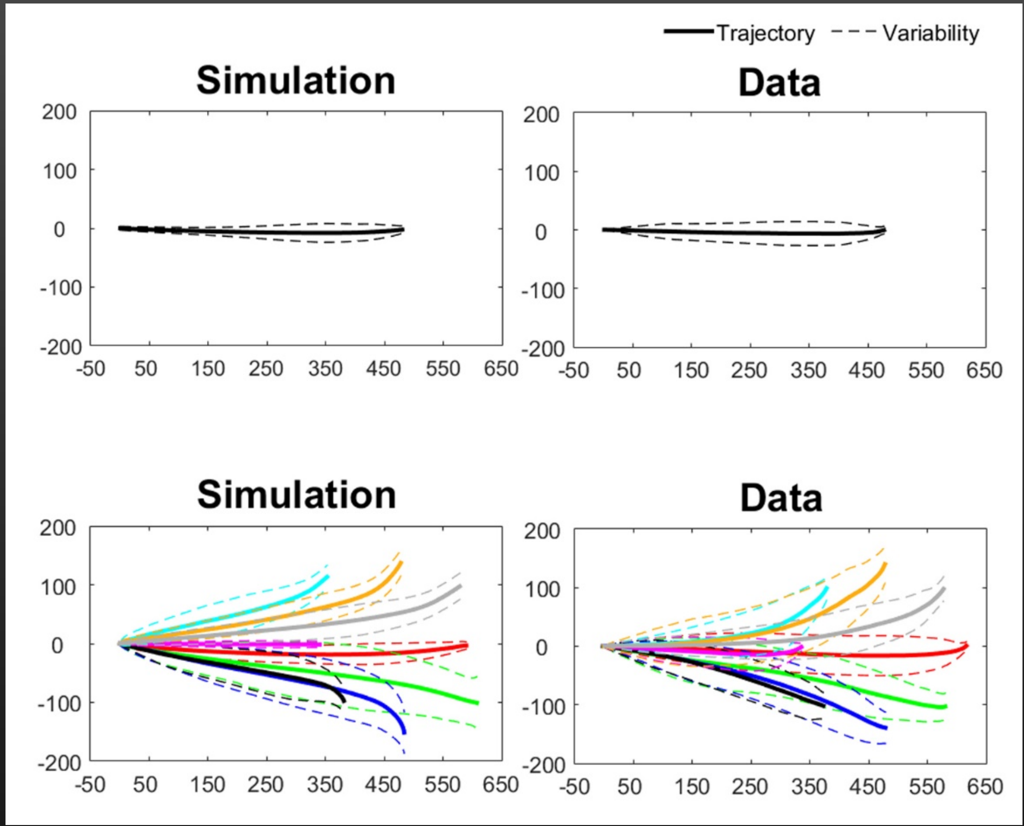
Using the above two parameter sets, we simulated 120 trajectories for each of the 9 target selection tasks.
Left is simulation of the one static target task.



simulation of 2 moving target tasks



simulation of other 6 moving target tasks

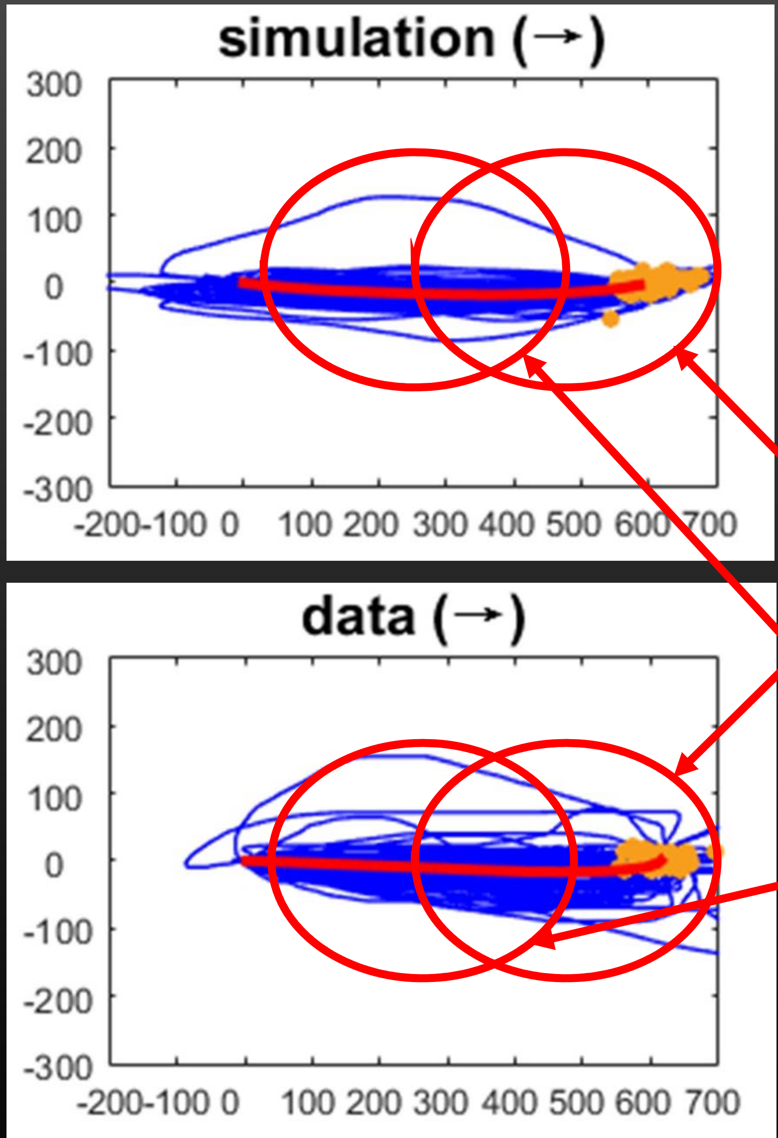


This figure shows the mean trajectory and trajectory variability of simulation and data of the 9 tasks, respectively.

As shown, our system closely simulated the mean trajectories and trajectory variabilities of all the experimental conditions.

measures	static	moving (→)	moving (↘)	moving (↓)	moving (↙)	moving (←)	moving (↖)	moving (↑)	moving (↗)
trE	9.86	30.51	45.02	52.07	22.14	23.13	37.04	38.86	44.93
vaE	17.89	46.02	40.32	30.98	27.03	27.14	28.80	49.75	29.82

The quantitative similarity between our simulation and the data.

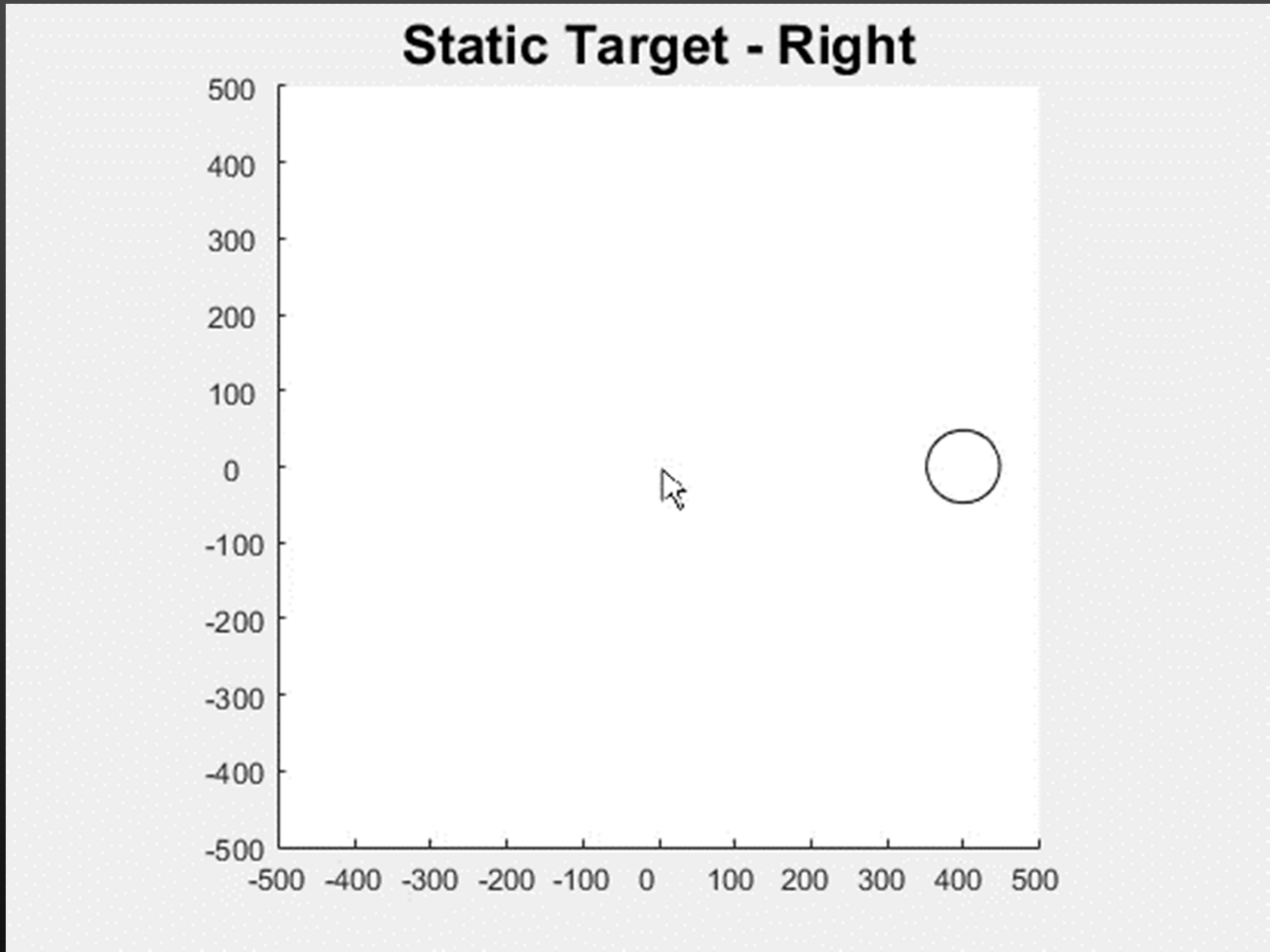


The system simulated important characteristics of selection motions such as:

speed-accuracy tradeoff

and

feedback-decision uncertainty



With this approach, we can further simulate various other selection motions by simply changing the target's parameters

Thanks