

Appendix A: The Proof of Theorem 2

Proof. Using (5), it can be easily shown that

$$\begin{aligned} P_{\text{out}}^C &= 1 - \Pr(\gamma_{S,C} \geq t_C, \gamma_{D,C} \geq t_C, \gamma_{C,C} \geq t_C) \\ &= 1 - I. \end{aligned} \quad (\text{A-1})$$

Defining $Z = |g_3|^2$, from (6), (7), (8) and (A-1), one obtains

$$\begin{cases} (a - b\tau_C)Y \leq b\tau_C X - \frac{1}{\gamma_X} \\ (a - b\tau_C)X \leq b\tau_C Y - \frac{1}{\gamma_Y} \\ \gamma Z (b\tau_C - a)(X + Y) \geq 1 \end{cases}. \quad (\text{A-2})$$

If $\tau_C \leq a/b$, $\gamma_{C,C} \geq t_C$ will not hold, and therefore, $P_{\text{out}}^C = 1$. Otherwise, I can be obtained as follows

$$\begin{aligned} I &= \Pr(\gamma_{S,C} \geq t_C, \gamma_{D,C} \geq t_C, \gamma_{C,C} \geq t_C) \\ &= \underbrace{\int_0^{X_0} \int_{gx + \frac{1}{hx}}^{\infty} \int_{Z_0}^{\infty} f_X(x) f_Y(y) f_Z(z) dz dy dx}_{I_1} \\ &\quad + \underbrace{\int_0^{X_0} \int_{gy + \frac{1}{hy}}^{\infty} \int_{Z_0}^{\infty} f_Y(y) f_X(x) f_Z(z) dz dx dy}_{I_2} \\ &\quad + \underbrace{\int_{X_0}^{\infty} \int_{X_0}^{\infty} \int_{Z_0}^{\infty} f_X(x) f_Y(y) f_Z(z) dz dy dx}_{I_3}, \end{aligned} \quad (\text{A-3})$$

where $g = \frac{b\tau_C}{a - b\tau_C}$, $h = \gamma(b\tau_C - a)$, $Z_0 = 1/[h(x + y)]$. Moreover, integral I_1

can be evaluated as

$$\begin{aligned}
I_1 &= \sum_{k=0}^{m_3-1} \frac{\omega_3^k}{h^k k!} \\
&\times \int_0^{X_0} f_X(x) \int_{gx+\frac{1}{hx}}^{\infty} \exp\left(-\frac{\omega_3}{h(x+y)}\right) \frac{1}{(x+y)^k} f_X(x) f_Y(y) dy dx \\
&= \frac{\omega_2^{m_2}}{\Gamma(m_2)} \sum_{k=0}^{m_3-1} \frac{\omega_3^k}{h^k k!} \sum_{p=0}^{m_2-1} (-1)^{m_2-1-p} \binom{m_2-1}{p} \\
&\times \int_0^{X_0} x^{m_2-1-p} \exp(\omega_2 x) f_X(x) \int_{x+gx+\frac{1}{hx}}^{\infty} v^{p-k} \exp\left(-\frac{\omega_3}{hv} - \omega_2 v\right) dv \\
&\approx \frac{\omega_2^{m_2}}{\Gamma(m_2)} \sum_{k=0}^{m_3-1} \frac{\omega_3^k}{h^k k!} \sum_{p=0}^{m_2-1} (-1)^{m_2-1-p} \binom{m_2-1}{p} \\
&\times \int_0^{X_0} x^{m_2-1-p} \exp(\omega_2 x) f_X(x) \int_{x+gx+\frac{1}{hx}}^{\infty} v^{p-k} \exp(-\omega_2 v) dv \\
&= \frac{\omega_1^{m_1}}{\Gamma(m_1)} \frac{\omega_2^{m_2}}{\Gamma(m_2)} \sum_{k=0}^{m_3-1} \sum_{p=0}^{m_2-1} \frac{\omega_3^k}{h^k k!} (-1)^{m_2-1-p} \omega_2^{-(p-k+1)} \binom{m_2-1}{p} \\
&\times \int_0^{X_0} x^{m_1+m_2-2-p} e^{(-\omega_1 x + \omega_2 x)} \Gamma\left(p-k+1, \omega_2 \left(x + gx + \frac{1}{hx}\right)\right) dx.
\end{aligned} \tag{A-4}$$

Similarly, I_2 and I_3 can also be deduced, yielding (12), which concludes the proof. \square

References

- [1] Gradshteyn I S, Ryzhik I M. Table of Integrals, Series, and Products. 7th ed. Academic Press, 2007