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Resource allocation in Multiple-Relay Systems Exploiting Opportunistic Energy Harvesting

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Appendix A The optimal solution of the transmit power

If $\rho_l^*(\boldsymbol{\gamma}) \neq 0$ or $\hat{\rho}_l^*(\boldsymbol{\gamma}) \neq 0$, by letting $\frac{\partial \tilde{\mathcal{L}}(\boldsymbol{\gamma})}{\partial \tilde{P}_{sl}(\boldsymbol{\gamma})} = 0$ and $\frac{\partial \tilde{\mathcal{L}}(\boldsymbol{\gamma})}{\partial \tilde{P}_{rl}(\boldsymbol{\gamma})} = 0$, we have

$$P_{s0}^*(\boldsymbol{\gamma}) = \left\lceil \frac{1}{(\theta - \sum_{l=1}^{L} \eta \zeta_l \gamma_{sl}) \ln 2} - \frac{N_0}{\gamma_{s0}} \right\rceil, \tag{A1}$$

$$\frac{\nu_l \gamma_{s0}}{N_0 + \gamma_{s0} P_{sl}^*(\boldsymbol{\gamma})} + \frac{\mu_l \gamma_{sl}}{N_0 + \gamma_{sl} P_{sl}^*(\boldsymbol{\gamma})} = (\theta - \sum_{l'=1, l' \neq l}^{L} \eta \zeta_{l'} \gamma_{sl'}) \ln 2, \tag{A2}$$

$$P_{rl}^*(\boldsymbol{\gamma}) = \left\lceil \frac{\nu_l}{\zeta_l \ln 2} - \frac{N_0}{\gamma_{rl}} \right\rceil \tag{A3}$$

where $\lceil x \rceil = \max(x, 0)$. If $\rho_l^*(\boldsymbol{\gamma}) = 0$, $P_{sl}^*(\boldsymbol{\gamma}) = 0$ and if $\hat{\rho}_l^*(\boldsymbol{\gamma}) = 0$, $P_{rl}^*(\boldsymbol{\gamma}) = 0$. It can be proved that the equation (A2) can not have two positive roots. $P_{sl}^*(\boldsymbol{\gamma})$ is the positive root of (A2) when the equation has one negative and one positive root.

Appendix B Proof of Lemma 1

The partial derivatives $\frac{\partial \tilde{\mathcal{L}}(\boldsymbol{\gamma})}{\partial \rho_l(\boldsymbol{\gamma})}$ and $\frac{\partial \tilde{\mathcal{L}}(\boldsymbol{\gamma})}{\hat{\rho}_l(\boldsymbol{\gamma})}$ are given by

$$\frac{\partial \tilde{\mathcal{L}}(\boldsymbol{\gamma})}{\partial \rho_0(\boldsymbol{\gamma})} = \log\left(1 + \frac{\gamma_{s0} P_{s0}}{N_0}\right) - \frac{\gamma_{s0} P_{s0}}{(N_0 + \gamma_{s0} P_{s0}) \ln 2} - \kappa_{\boldsymbol{\gamma}}, \tag{B1}$$

$$\begin{aligned} \frac{\partial \tilde{\mathcal{L}}(\boldsymbol{\gamma})}{\partial \rho_l(\boldsymbol{\gamma})} &= \nu_l \left(\log(1 + \frac{\gamma_{s0} P_{sl}}{N_0}) - \frac{\gamma_{s0} P_{sl}}{(N_0 + \gamma_{s0} P_{sl}) \ln 2}\right) \\ &\quad + \mu_l \left(\log(1 + \frac{\gamma_{sl} P_{sl}}{N_0}) - \frac{\gamma_{sl} P_{sl}}{(N_0 + \gamma_{sl} P_{sl}) \ln 2}\right) - \kappa_{\boldsymbol{\gamma}}, \end{aligned} \tag{B2}$$

$$\frac{\partial \tilde{\mathcal{L}}(\boldsymbol{\gamma})}{\partial \hat{\rho}_l(\boldsymbol{\gamma})} = \nu_l \left(\log(1 + \frac{\gamma_{rl} P_{rl}}{N_0}) - \frac{\gamma_{rl} P_{rl}}{(N_0 + \gamma_{rl} P_{rl}) \ln 2}\right) - \kappa_{\boldsymbol{\gamma}}. \tag{B3}$$

By substituting (A1)-(A3) into (B1)-(B3) respectively, we can see that $\Pr(\frac{\partial \tilde{\mathcal{L}}(\boldsymbol{\gamma})}{\partial \rho_l(\boldsymbol{\gamma})} = 0) = 0$ and $\Pr(\frac{\partial \tilde{\mathcal{L}}(\boldsymbol{\gamma})}{\partial \hat{\rho}_l(\boldsymbol{\gamma})} = 0) = 0$. Therefore, for given $\boldsymbol{\gamma}$, $\tilde{\mathcal{L}}(\boldsymbol{\gamma})$ is an monotonic function of $\rho_l(\boldsymbol{\gamma}), \hat{\rho}_l(\boldsymbol{\gamma})$, and the optimal values of $\rho_l(\boldsymbol{\gamma}), \hat{\rho}_l(\boldsymbol{\gamma})$ must be achieved at the end point. Considering the constraint in (1), among $\rho_0^*(\boldsymbol{\gamma})$ and $\{\rho_l^*(\boldsymbol{\gamma}), \hat{\rho}_l^*(\boldsymbol{\gamma})\}_{l=1}^{L}$, only one variable is equal to 1 and other variables are equal to zero. The time-sharing factor with maximum partial derivative will equal 1, since we can make only one partial derivative greater than zero by choosing a suitable $\kappa_{\boldsymbol{\gamma}}$.

Appendix C Sub-gradients

Based on the optimal solutions in (7), (6) is solved with the sub-gradient method. According to analysis in the letter, dual variables are dual feasible when $\{\mu_l + \nu_l \geqslant 1\}_{l=1}^{L}$. Therefore, the Lagrange multipliers update equations are

$$\mu_l(n+1) = [\mu_l(n) + \frac{1}{n+m} \mathbb{E}_{\boldsymbol{\gamma}}\{r_{sl}(\boldsymbol{\gamma})\}]^+, \tag{C1}$$

$$\nu_l(n+1) = [\nu_l(n) + \frac{1}{n+m} \mathbb{E}_{\boldsymbol{\gamma}}\{r_{dl}(\boldsymbol{\gamma}) + r_{rl}(\boldsymbol{\gamma})\}]^+, \tag{C2}$$

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$$\zeta_l(n+1) = \lceil \zeta_l(n) + \frac{1}{n+m}(e_l - \mathbb{E}_{\boldsymbol{\gamma}}\{\tilde{P}_{rl}(\boldsymbol{\gamma})\})\rceil, \tag{C3}$$

$$\theta(n+1) = \lceil \theta(n) + \frac{1}{n+m}(P_a - \sum_{l=0}^{L} \mathbb{E}_{\boldsymbol{\gamma}}\{\tilde{P}_{sl}(\boldsymbol{\gamma})\})\rceil \tag{C4}$$

where $m > 0$, $[]^+$ denotes project on the closed convex set $\{u_l + v_l \geqslant 1\}_{l=1}^{L}$. Utilizing the gradient project method in [1], $\mu_l(n+1), \nu_l(n+1)$ can be determined by solving the following L subproblems in parallel

$$\begin{aligned} \min \quad & [\mu_l(n+1) - \mu_l(n) - \frac{1}{n+m}\mathbb{E}_{\boldsymbol{\gamma}}\{r_{sl}(\boldsymbol{\gamma})\}]^2 \\ & + [\nu_l(n+1) - \nu_l(n) - \frac{1}{n+m}\mathbb{E}_{\boldsymbol{\gamma}}\{r_{dl}(\boldsymbol{\gamma}) + r_{rl}(\boldsymbol{\gamma})\}]^2, \\ s.t. \quad & \mu_l(n+1) + \nu_l(n+1) \geqslant 1, \{\mu_l(n+1), \nu_l(n+1)\} \geqslant 0 \end{aligned} \tag{C5}$$

The optimal solution of (C5) is $\mu_l(n+1) = \lceil \frac{\xi_l}{2} + \mu_l(n) + \frac{1}{n+m} g_{\mu_l}(n)\rceil^+, \nu_l(n+1) = \lceil \frac{\xi_l}{2} + \nu_l(n) + \frac{1}{n+m} g_{\nu_l}(n)\rceil^+$ where $\xi_l \geqslant 0$ is the Lagrange multiplier. If the constraint cannot be satisfied when $\xi_l = 0$, the optimal ξ_l is determined by solving $\mu_l(n+1) + \nu_l(n+1) = 1$.

References

1 D. P. Bertsekas, "Nonlinear programming," Athena Scientific, 2009.