

# Probabilistic-constrained robust secure transmission for energy harvesting over MISO channels

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**Abstract** In this paper, we consider a system supporting simultaneous wireless information and power transfer (SWIPT), where the transmitter delivers private message to a destination receiver (DR) and powers to multiple energy receivers (ERs) with multiple single-antenna external eavesdroppers (Eves). We study secure robust beamformer and power splitting (PS) design under imperfect channel state information (CSI). The artificial noise (AN) scheme is further utilized at the transmitter to provide strong wireless security. We aim at maximizing the energy harvested by ERs subject to the transmission power constraint, a range of outage constraints concerning the signal-to-interference-plus-noise ratio (SINR) recorded at the DR and the Eves, as well as concerning the energy harvested at the DR. The energy harvesting maximization (EHM) problem is challenging to directly solve, we resort to Bernstein-type inequality restriction technique to reformulate the original problem as a tractable approximated version. Numerical results show that our robust beamforming scheme outperforms the beamforming scheme relying on the worst-case design philosophy.

**Keywords** secure communication, wireless power transfer, power splitting, Bernstein-type inequality, robust beamforming

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## 1 Introduction

In order to support green energy applications in emerging 5G wireless communications (such as heterogeneous cellular networks and massive MIMO), simultaneous wireless information and power transfer (SWIPT) is envisioned as a popular solution to power the wireless devices [1, 2], in which mobile terminals can harvest energy from surrounding electromagnetic radiation [3, 4]. In particular, there are two strategies are widely prevailing for SWIPT [5]. For power splitting (PS) strategy, the receiver splits the received energy into two streams for energy harvesting (EH) and information decoding (ID) separately. For time switching (TS) strategy, the receiver divides the time between EH and ID. It has been reported in [6] that PS strategy in general can obtain better transmission compared with the TS strategy. In this paper, we focus on the PS strategy.

Since the energy receiver (ER) may have better fading channel than the destination receiver (DR) in SWIPT systems, which gives rise to a higher probability to overhear the messages intended for DR. Therefore, secure wireless transmissions are identified as a critical challenge facing SWIPT systems. Compared to conventional methods, physical-layer security (PLS) was considered as a promising approach to achieve secure transmission [7, 8]. PLS based SWIPT systems operating on multiple antennas setups

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[9, 10], relay networks [11, 12], cognitive radio networks [13], OFDMA [14], have been studied, where the transmitter can perfectly obtain the channel state information (CSI).

Practical implementation of PLS critically relies on the CSI associated with DR and eavesdropper (Eve). However, the CSI is very hard to be perfectly known at base station due to quantization errors, channel estimation errors and non-cooperation with hostile Eves [15, 16]. Hence, robust secure transmission is a critical issue for SWIPT systems with channel uncertainties. Generally, there are two uncertainty models for characterizing imperfect CSI, namely deterministic uncertainty and stochastic uncertainty. Robust secure transmission for SWIPT using the deterministic uncertainty model has been studied in [17–19], where the CSI quantization errors were modeled in the bounded ellipsoidal form. However, since the deterministic uncertainty model characterizes the worst-case scenario, which may rarely happen, the performance achieved using the deterministic uncertainty model can be too conservative. By contrast, the stochastic uncertainty model is a better choice for handling the estimation errors that have certain statistical properties. For example, an outage-based beamformer for SWIPT over MISO channel was designed in [20]. However, authors did not consider secure communications.

As far as secure SWIPT systems are concerned, most existing contributions primarily focus on secrecy rate maximization or power minimization problems, and only limited studies [21–23] have been considered to maximize the harvested energy such that achieving a trade-off between energy and security, which is of paramount importance as well. Responding to this, robust secure beamformer design over MISO channel was presented in [21, 22], in which the worst-case deterministic channel uncertainty model was considered. However, the stochastic channel uncertainty model for pursuing EHM problem in secure SWIPT systems has not been explored, which motivates our paper.

In this paper, we address a more general scenario where one transmitter sends confidential information to one DR and powers to multiple ERs in the presence of multiple external Eves, where the exact CSI of all wireless channels is not known at the transmitter. Unlike [22, 24], the PS scheme is adopted at the DR in our design such that the received energy is split into two different power levels, which is used by ID and wireless EH separately. We aim at maximizing the harvested energy at ERs under the max-min fairness criterion [25], subject to a range of probabilistic outage constraints concerning the communication security that is characterized by the SINRs recorded at the IR and Eves, the probabilistic outage constraints concerning the energy harvested at the DR and ERs, and the transmission power constraint. For the sake of clarify, the main contributions of this paper are summarized as follows.

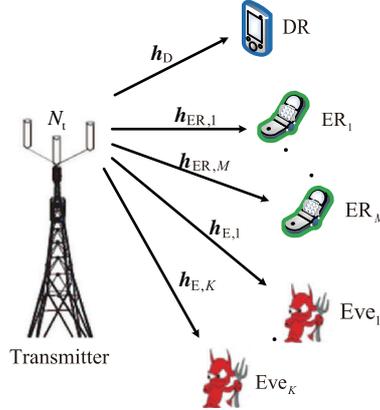
- We investigate a more general scenario for an SWIPT system. Different from previous works, this paper aims at maximizing the energy harvested to ERs, and the system model also treats ERs as being legitimate so that secrecy leakage is not prevented from these users. Furthermore, the artificial noise (AN) is injected at the transmitter to destroy the reception of Eves for supporting secure transmission.
- For imperfect CSI, the robust EHM problem is non-convex and challenging to solve. To address it, we invoke the Bernstein-type inequality restriction<sup>1)</sup> techniques to formulate many efficiently tractable linear matrix inequality (LMI) constraints. Furthermore, a rank-1 solution is reconstructed by employing the quadratic equality constraint, and the computational complexity of the proposed scheme is analyzed.

The remaining of this article is organized as follows. The system model supporting SWIPT is introduced in Section 2. In Section 3, we design secure robust beamformer under probabilistic CSI-error model. In Section 4, simulation results are provided to reveal the validity of the proposed robust secure scheme. Finally, the conclusion is remarked in Section 5.

Notations. Let  $\mathbf{A}^T$ ,  $\mathbf{A}^H$ ,  $\text{rank}(\mathbf{A})$  and  $\text{tr}(\mathbf{A})$  respectively denote the transpose, conjugate transpose, rank and trace of the matrix  $\mathbf{A}$ . Positive semidefinite of matrix  $\mathbf{A}$  is represented by  $\mathbf{A} \succeq \mathbf{0}$ . The Euclidean norm of a scalar is indicated by  $\|\cdot\|$ .  $E[\cdot]$  stands for the expectation operation,  $\text{vec}(\cdot)$  means a vectorization by stacking the columns of a matrix.  $\Pr\{\cdot\}$  represents the probability of an event.  $\mathbb{H}_n^+$  denotes the set of  $n \times n$  Hermitian matrices.

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1) The Bernstein-type inequality restriction is a tractable approximation method converting probabilistic constraints into deterministic ones.


**Figure 1** (Color online) System model.

## 2 System model

We consider a downlink MISO SWIPT system, which consists of one transmitter, one DR,  $M > 1$  ERs and  $K > 1$  external Eves, as shown in Figure 1. It is presumed that the transmitter has  $N_t > 1$  antennas, and each node except the transmitter has single receiving antenna. Suppose that Eves overhear the signal transmitted from transmitter without any interference, at the same time the ERs only harvest energy and do not overhear the message intended for the DR. For ease of description, we denote the set of Eves as  $\mathcal{K} \triangleq \{1, \dots, K\}$  and the set of ERs as  $\mathcal{M} \triangleq \{1, \dots, M\}$ , respectively.

The linear beamforming is performed at the transmitter to deliver the information-bearing signal  $\mathbf{x}$  to the DR while keeping it secret from Eves. Hence, the received signals at the DR and Eve $_k$  can be written as

$$y_D = \mathbf{h}_D^H \mathbf{x} + n_D, \quad (1)$$

$$y_{E,k} = \mathbf{h}_{E,k}^H \mathbf{x} + n_{E,k}, \quad (2)$$

respectively, where  $\mathbf{h}_D \in \mathbb{C}^{N_t \times 1}$  and  $\mathbf{h}_{E,k} \in \mathbb{C}^{N_t \times 1}$  represent the channel vectors between transmitter and DR, and Eve $_k$ , respectively. We assume that all channel coefficients undergo Rayleigh flat-fading, while  $n_D$  and  $n_{E,k}$  denote additive white Gaussian noises (AWGNs) at DR and Eve $_k$  with  $n_D \sim \mathcal{CN}(0, \sigma_D^2)$  and  $n_{E,k} \sim \mathcal{CN}(0, \sigma_{E,k}^2)$ , respectively.

To further support secure communication and to facilitate EH at the DR, an AN-aided beamforming scheme is adopted at the transmitter. Hence, the transmitted signal vector is denoted by

$$\mathbf{x} = \mathbf{w}s + \mathbf{v}, \quad (3)$$

where  $s$  and  $\mathbf{w} \in \mathbb{C}^{N_t \times 1}$  denote the data symbol and beamforming vector, respectively. Hence,  $\mathbf{w}s$  carries the confidential information intended for the DR. We assume  $\mathbb{E}[ss^T] = 1$  for simplicity.  $\mathbf{v}$  is the Gaussian AN vector invoked by the transmitter following  $\sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ . Since ERs harvest energy from the transmitter, the AN also constitutes a source of the energy to be harvested.

In our design the PS strategy of [6, 18] is exploited at the DR to process the EH and ID. To be specific, the received signals at the DR are divided into two power portions, one for ID and the other for EH. Let  $0 < \rho \leq 1$  denote the PS factor of the DR, then the receiving signal for ID at the DR is expressed as

$$y_D^{\text{ID}} = \sqrt{\rho}(\mathbf{h}_D^H \mathbf{x} + n_D) + n_{D,\text{SP}}, \quad (4)$$

where  $n_{D,\text{SP}} \sim \mathcal{CN}(0, \sigma_{D,\text{SP}}^2)$  is AWGN introduced by information processing at the DR. We first define a new matrix  $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ , which satisfies  $\mathbf{W} \succeq \mathbf{0}$  and  $\text{rank}(\mathbf{W}) = 1$ . Then, the instantaneous SINRs at both the DR and Eve $_k$  in terms of  $\mathbf{W}$  are respectively given by

$$\text{SINR}_D = \frac{\rho \mathbf{h}_D^H \mathbf{W} \mathbf{h}_D}{\rho(\mathbf{h}_D^H \mathbf{\Sigma} \mathbf{h}_D + \sigma_D^2) + \sigma_{D,\text{SP}}^2}, \quad (5)$$

$$\text{SINR}_{E,k} = \frac{\mathbf{h}_{E,k}^H \mathbf{W} \mathbf{h}_{E,k}}{\mathbf{h}_{E,k}^H \boldsymbol{\Sigma} \mathbf{h}_{E,k} + \sigma_{E,k}^2}. \quad (6)$$

The energy harvested at the DR and ER<sub>m</sub> are expressed as

$$E_D = (1 - \rho)\eta (\mathbf{h}_D^H (\mathbf{W} + \boldsymbol{\Sigma}) \mathbf{h}_D + \sigma_D^2), \quad (7)$$

$$E_{ER_m} = \eta \mathbf{h}_{ER,m}^H (\mathbf{W} + \boldsymbol{\Sigma}) \mathbf{h}_{ER,m}, \quad (8)$$

respectively, where  $\mathbf{h}_{ER,m}$  indicates the channel vector between the transmitter and ER<sub>m</sub>.  $\eta \in (0, 1]$  indicates the energy conversion efficiency standing for the loss from RF energy to electrical energy. In this case, the harvested energy is used for charging the batteries of DR/ERs to extend the operating time<sup>2)</sup>. Throughout this paper, we assume  $\eta = 1$ .

### 3 Robust design for secure SWIPT

#### 3.1 Optimization problem formulation

In practical secure communication systems threatened by Eves, perfect CSI is very hard to extract at the transmitter due to non-cooperation with the Eves, as well as to the channel estimation and feedback errors. In a probabilistic CSI-error model, the CSI error is stochastic and follows a certain distribution. We assume that the actual channels  $\mathbf{h}_D, \mathbf{h}_{E,k}, \mathbf{h}_{ER,m}$  can be modeled by

$$\mathbf{h}_D = \hat{\mathbf{h}}_D + \Delta \mathbf{h}_D, \quad (9)$$

$$\mathbf{h}_{E,k} = \hat{\mathbf{h}}_{E,k} + \Delta \mathbf{h}_{E,k}, \quad (10)$$

$$\mathbf{h}_{ER,m} = \hat{\mathbf{h}}_{ER,m} + \Delta \mathbf{h}_{ER,m}, \quad (11)$$

respectively, where  $\hat{\mathbf{h}}_D, \hat{\mathbf{h}}_{E,k}, \hat{\mathbf{h}}_{ER,m}$  are the estimated CSI, which are available to the transmitter. Assume that each CSI-error vector obeys a complex-valued Gaussian distribution, i.e.,  $\Delta \mathbf{h}_D \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Omega}_D)$ ,  $\Delta \mathbf{h}_{E,k} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Omega}_{E,k})$ ,  $\Delta \mathbf{h}_{ER,m} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Omega}_{ER,m})$ , where  $\boldsymbol{\Omega}_D \succeq \mathbf{0}$ ,  $\boldsymbol{\Omega}_{E,k} \succeq \mathbf{0}$  and  $\boldsymbol{\Omega}_{ER,m} \succeq \mathbf{0}$  are the covariance matrices associated with  $\Delta \mathbf{h}_D$ ,  $\Delta \mathbf{h}_{E,k}$  and  $\Delta \mathbf{h}_{ER,m}$ .

In order to maximize the energy harvested at ERs, we aim at seeking the optimal transmit matrix  $\mathbf{W}$ , AN covariance matrix  $\boldsymbol{\Sigma}$  and PS ratio  $\rho$ , while guaranteeing secure requirement between the transmitter and the DR. The constraints considered include the transmit power constraint and a range of outage constraints, namely the minimum probability for the SINR recorded at the DR to be no lower than a given threshold, the minimum probability for the SINR recorded at the Eves to be no higher than a given threshold, and the minimum probability for the amount of the harvested energy at the DR to be no less than a given threshold. Hence, the optimization problem is formulated as

$$\max_{\mathbf{W}, \boldsymbol{\Sigma}, \rho} \min_m \mathbf{h}_{ER,m}^H (\mathbf{W} + \boldsymbol{\Sigma}) \mathbf{h}_{ER,m} \quad (12a)$$

$$\text{s.t. } \Pr_{\Delta \mathbf{h}_D} \{\text{SINR}_D \geq \gamma_D\} \geq 1 - p_D, \quad (12b)$$

$$\Pr_{\Delta \mathbf{h}_{E,k}} \{\text{SINR}_{E,k} \leq \gamma_{E,k}\} \geq 1 - p_{E,k}, \forall k \in \mathcal{K}, \quad (12c)$$

$$\Pr_{\Delta \mathbf{h}_D} \{E_D \geq \Gamma_D\} \geq 1 - q_D, \quad (12d)$$

$$\text{tr}(\mathbf{W} + \boldsymbol{\Sigma}) \leq P_{\text{th}}, \quad (12e)$$

$$\mathbf{W} \succeq \mathbf{0}, \boldsymbol{\Sigma} \succeq \mathbf{0}, 0 < \rho < 1, \quad (12f)$$

$$\text{rank}(\mathbf{W}) = 1, \quad (12g)$$

where  $\gamma_D$  and  $\Gamma_D$  represent the minimum SINR and EH requirements at the DR, respectively;  $\gamma_{E,k}$  denotes the maximum tolerable SINR recorded at the Eves to eavesdrop the information between transmitter and

<sup>2)</sup> Note that we mainly focus on the EHM problem, how to use the harvested power is beyond this paper.

DR, and  $P_{\text{th}}$  denotes the maximum transmit power;  $p_D \in (0, 1]$ ,  $p_{E,k} \in (0, 1]$ ,  $q_D \in (0, 1]$  and  $q_{ER,m} \in (0, 1]$  stand for the prescribed maximal outage probability of the SINR recorded at the DR, of the SINR recorded at Eve $_k$ , as well as of the amount of the energy harvested at the DR and ER $_m$ , respectively. It is in sense that the PS ratio  $\rho$  should guarantee  $\text{SINR}_D > 0$  and  $E_D > 0$  as given in constraints (12b) and (12d), thus  $0 < \rho < 1$  is met. Since the secrecy rate is increasing with the SINR recorded at the DR but decreasing with that recorded at the Eves monotonically [22, 26], we consider the minimum guaranteed SINR target at the DR and maximum tolerable SINR targets at the Eves when formulating Problem (12). The constraints (12b) and (12c) together ensure that the lower bound of secrecy rate by  $R_{\text{sec}} = [\log_2(1 + \text{SINR}_D) - \max_{k \in \mathcal{K}} \log_2(1 + \text{SINR}_{E,k})] \geq [\log_2(1 + \gamma_D) - \max_{k \in \mathcal{K}} \log_2(1 + \gamma_{E,k})]$ . By adjusting the values of  $\gamma_D$  and  $\gamma_{E,k}$ , the required lower bound on secrecy rate can be guaranteed. Throughout the rest of this paper, we assume that the above outage-constrained optimization Problem (12) is feasible.

It is readily checked that Problem (12) is nonconvex, since the outage constraints in (12b)–(12d) do not carry out simple closed-form expressions and the rank-1 constraint (12g) is nonconvex. Therefore, it is particularly challenging to directly deal with (12). To begin with, we introduce a real-valued variable  $t$ , Problem (12) is reformulated as

$$\max_{\mathbf{W}, \boldsymbol{\Sigma}, \rho, t} t \quad (13a)$$

$$\text{s.t. } \Pr_{\Delta \mathbf{h}_D} \{ \text{SINR}_D \geq \gamma_D \} \geq 1 - p_D, \quad (13b)$$

$$\Pr_{\Delta \mathbf{h}_{E,k}} \{ \text{SINR}_{E,k} \leq \gamma_{E,k} \} \geq 1 - p_{E,k}, \forall k \in \mathcal{K}, \quad (13c)$$

$$\Pr_{\Delta \mathbf{h}_D} \{ E_D \geq \Gamma_D \} \geq 1 - q_D, \quad (13d)$$

$$\Pr_{\Delta \mathbf{h}_{ER,m}} \{ \mathbf{h}_{ER,m}^H (\mathbf{W} + \boldsymbol{\Sigma}) \mathbf{h}_{ER,m} \geq t \} \geq 1 - q_{ER,m}, \forall m \in \mathcal{M}, \quad (13e)$$

$$\text{tr}(\mathbf{W} + \boldsymbol{\Sigma}) \leq P_{\text{th}}, \quad (13f)$$

$$\mathbf{W} \succeq \mathbf{0}, \boldsymbol{\Sigma} \succeq \mathbf{0}, 0 < \rho < 1, \quad (13g)$$

$$\text{rank}(\mathbf{W}) = 1, \quad (13h)$$

where  $t \geq 0$  is the minimum harvested energy amongst the ERs obeying max-min fairness criterion [22, 23]. The constraint (13e) represents that the probability of the energy harvested at ER $_m$  being larger than  $t$  must be no less than  $1 - q_{ER,m}$ , which can be easily followed by the fact

$$\begin{aligned} \Pr_{\Delta \mathbf{h}_{ER,m}} \left\{ \min_{m \in \mathcal{M}} \mathbf{h}_{ER,m}^H (\mathbf{W} + \boldsymbol{\Sigma}) \mathbf{h}_{ER,m} \geq t \right\} &= \prod_{m=1}^M \Pr_{\Delta \mathbf{h}_{ER,m}} \{ \mathbf{h}_{ER,m}^H (\mathbf{W} + \boldsymbol{\Sigma}) \mathbf{h}_{ER,m} \geq t \} \\ &\geq \prod_{m=1}^M (1 - q_{ER,m}). \end{aligned} \quad (14)$$

To circumvent this predicament of Problem (13), we have to transform the outage constraints (13b)–(13e) into tractable forms. In what follows we will develop a conservative approximation approach to suboptimally solve (13) with manageable computational complexity.

### 3.2 Robust beamforming design

The key to solve probabilistic-constraint based optimization problems is constituted by an appropriate restriction method. Following the idea in [20], we invoke the Bernstein-type inequality restriction technique to formulate an approximated version of (12).

Since  $\Delta \mathbf{h}_D \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Omega}_D)$ ,  $\Delta \mathbf{h}_{E,k} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Omega}_{E,k})$  and  $\Delta \mathbf{h}_{ER,m} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Omega}_{ER,m})$ , the CSI error vectors are rewritten as

$$\Delta \mathbf{h}_D = \boldsymbol{\Omega}_D^{1/2} \mathbf{r}_D, \quad (15)$$

$$\Delta \mathbf{h}_{E,k} = \boldsymbol{\Omega}_{E,k}^{1/2} \mathbf{r}_{E,k}, \quad (16)$$

$$\Delta \mathbf{h}_{ER,m} = \boldsymbol{\Omega}_{ER,m}^{1/2} \mathbf{r}_{ER,m}, \quad (17)$$

where we have  $\mathbf{r}_D \sim \mathcal{CN}(0, \mathbf{I}_{N_t})$ ,  $\mathbf{r}_{E,k} \sim \mathcal{CN}(0, \mathbf{I}_{N_t})$  and  $\mathbf{r}_{ER,m} \sim \mathcal{CN}(0, \mathbf{I}_{N_t})$ .  $\Omega_D = \Omega_D^{1/2} \Omega_D^{1/2}$ ,  $\Omega_{E,k} = \Omega_{E,k}^{1/2} \Omega_{E,k}^{1/2}$  and  $\Omega_{ER,m} = \Omega_{ER,m}^{1/2} \Omega_{ER,m}^{1/2}$ .

Then, by substituting  $\text{SINR}_D$ ,  $\text{SINR}_{E,k}$ ,  $E_D$  into (12), Problem (12) is recast as

$$\max_{\mathbf{W}, \boldsymbol{\Sigma}, \rho, t} \quad (18a)$$

$$\text{s.t. } \Pr \{ \mathbf{r}_D^H \mathbf{A}_D \mathbf{r}_D + 2\text{Re}(\mathbf{r}_D^H \mathbf{u}_D) + c_D \geq 0 \} \geq 1 - p_D, \quad (18b)$$

$$\Pr \{ \mathbf{r}_{E,k}^H \mathbf{A}_{E,k} \mathbf{r}_{E,k} + 2\text{Re}(\mathbf{r}_{E,k}^H \mathbf{u}_{E,k}) + c_{E,k} \leq 0 \} \geq 1 - p_{E,k}, \quad (18c)$$

$$\Pr \{ \mathbf{r}_D^H \mathbf{B}_D \mathbf{r}_D + 2\text{Re}(\mathbf{r}_D^H \mathbf{v}_D) + s_D \geq 0 \} \geq 1 - q_D, \quad (18d)$$

$$\Pr \{ \mathbf{r}_{ER,m}^H \mathbf{B}_{ER,m} \mathbf{r}_{ER,m} + 2\text{Re}(\mathbf{r}_{ER,m}^H \mathbf{v}_{ER,m}) + s_{ER,m} \geq 0 \} \geq 1 - q_{ER,m}, \quad (18e)$$

$$\text{tr}(\mathbf{W} + \boldsymbol{\Sigma}) \leq P_{\text{th}}, \quad (18f)$$

$$\mathbf{W} \succeq \mathbf{0}, \boldsymbol{\Sigma} \succeq \mathbf{0}, 0 < \rho < 1, \quad (18g)$$

$$\text{rank}(\mathbf{W}) = 1. \quad (18h)$$

For ease of exposition, let  $\mathbf{F}_D = \frac{1}{\gamma_D} \mathbf{W} - \boldsymbol{\Sigma}$ ,  $\mathbf{F}_{E,k} = \frac{1}{\gamma_{E,k}} \mathbf{W} - \boldsymbol{\Sigma}$  and  $\mathbf{T}_D = \mathbf{W} + \boldsymbol{\Sigma}$ . Hence, the relevant parameters in Problem (18) are formulated as

$$\begin{cases} \mathbf{A}_D = \Omega_D^{1/2} \mathbf{F}_D \Omega_D^{1/2}, \mathbf{u}_D = \Omega_D^{1/2} \mathbf{F}_D \hat{\mathbf{h}}_D, \\ \mathbf{A}_{E,k} = \Omega_{E,k}^{1/2} \mathbf{F}_{E,k} \Omega_{E,k}^{1/2}, \mathbf{u}_{E,k} = \Omega_{E,k}^{1/2} \mathbf{F}_{E,k} \hat{\mathbf{h}}_{E,k}, \\ \mathbf{B}_D = \Omega_D^{1/2} \mathbf{T}_D \Omega_D^{1/2}, \mathbf{v}_D = \Omega_D^{1/2} \mathbf{T}_D \hat{\mathbf{h}}_D, \\ \mathbf{B}_{ER,m} = \Omega_{ER,m}^{1/2} \mathbf{T}_D \Omega_{ER,m}^{1/2}, \mathbf{v}_{ER,m} = \Omega_{ER,m}^{1/2} \mathbf{T}_D \hat{\mathbf{h}}_{ER,m}, \\ c_D = \hat{\mathbf{h}}_D^H \mathbf{F}_D \hat{\mathbf{h}}_D + \sigma_D^2 - \frac{\sigma_{D,SP}^2}{\rho}, c_{E,k} = \hat{\mathbf{h}}_{E,k}^H \mathbf{F}_{E,k} \hat{\mathbf{h}}_{E,k} - \sigma_{E,k}^2, \\ s_D = \hat{\mathbf{h}}_D^H \mathbf{T}_D \hat{\mathbf{h}}_D + \sigma_D^2 - \frac{I_D}{1-\rho}, s_{ER,m} = \hat{\mathbf{h}}_{ER,m}^H \mathbf{T}_D \hat{\mathbf{h}}_{ER,m} - t, \end{cases}$$

where  $\frac{1}{\rho}$  and  $\frac{1}{1-\rho}$  are convex functions over  $0 < \rho < 1$  [6]. To proceed, let us transform the constraints (18b)–(18e) into convex forms using the following lemmas [27].

**Lemma 1** (Bernstein-type inequality restriction). For an arbitrary 3-tuple of (deterministic) variables  $(\mathbf{Q}, \mathbf{r}, c) \in \mathbb{H}^n \times \mathbb{C}^n \times \mathbb{R}$ ,  $\mathbf{a} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_n)$  and  $\eta \in (0, 1]$ , the sufficient condition for

$$\Pr \{ \mathbf{a}^H \mathbf{Q} \mathbf{a} + 2\text{Re}(\mathbf{a}^H \mathbf{r}) + c \geq 0 \} \geq 1 - \eta \quad (19)$$

can be equivalently reformulated as LMI and second-order cone (SOC) constraints:

$$\begin{cases} \text{tr}(\mathbf{Q}) - \sqrt{-2 \ln \eta} t_1 + t_2 \ln \eta + c \geq 0, \\ \left\| \begin{bmatrix} \text{vec}(\mathbf{Q}) \\ \sqrt{2} \mathbf{r} \end{bmatrix} \right\| \leq t_1, \\ t_2 \mathbf{I}_n + \mathbf{Q} \succeq \mathbf{0}, t_2 \geq 0, \end{cases}$$

where  $t_1$  and  $t_2$  are the introduced slack variables.

*Proof.* Please refer to [27].

**Lemma 2.** For an arbitrary 3-tuple of (deterministic) variables  $(\mathbf{Q}, \mathbf{r}, c) \in \mathbb{H}^n \times \mathbb{C}^n \times \mathbb{R}$ ,  $\mathbf{a} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_n)$  and  $\eta \in (0, 1]$ , the sufficient condition for

$$\Pr \{ \mathbf{a}^H \mathbf{Q} \mathbf{a} + 2\text{Re}(\mathbf{a}^H \mathbf{r}) + c \leq 0 \} \geq 1 - \eta \quad (20)$$

can be equivalently reformulated as LMI and second-order cone (SOC) constraints:

$$\begin{cases} \text{tr}(\mathbf{Q}) + \sqrt{-2 \ln \eta} t_1 - t_2 \ln \eta + c \leq 0, \\ \left\| \begin{bmatrix} \text{vec}(\mathbf{Q}) \\ \sqrt{2} \mathbf{r} \end{bmatrix} \right\| \leq t_1, \\ t_2 \mathbf{I}_n - \mathbf{Q} \succeq \mathbf{0}, t_2 \geq 0. \end{cases}$$

By applying Lemma 1 and introducing the slack variables  $\lambda_D, \mu_D$ , an approximation to the outage constraint of (18b) is given by

$$\begin{cases} \text{tr}(\mathbf{A}_D) - \sqrt{-2 \ln p_D} \lambda_D + \mu_D \ln p_D + c_D \geq 0, \\ \left\| \begin{bmatrix} \text{vec}(\mathbf{A}_D) \\ \sqrt{2} \mathbf{u}_D \end{bmatrix} \right\| \leq \lambda_D, \\ \mu_D \mathbf{I}_{N_t} + \mathbf{A}_D \succeq \mathbf{0}, \mu_D \geq 0. \end{cases} \quad (21)$$

By applying Lemma 2 and introducing the slack variables  $\{\lambda_{E,k}\}_{k=1}^K, \{\mu_{E,k}\}_{k=1}^K$ , the constraint of (18c) is expressed as

$$\begin{cases} \text{tr}(\mathbf{A}_{E,k}) + \sqrt{-2 \ln p_{E,k}} \lambda_{E,k} - \mu_{E,k} \ln p_{E,k} + c_{E,k} \leq 0, \\ \left\| \begin{bmatrix} \text{vec}(\mathbf{A}_{E,k}) \\ \sqrt{2} \mathbf{u}_{E,k} \end{bmatrix} \right\| \leq \lambda_{E,k}, \\ \mu_{E,k} \mathbf{I}_{N_t} - \mathbf{A}_{E,k} \succeq \mathbf{0}, \mu_{E,k} \geq 0, \forall k \in \mathcal{K}. \end{cases} \quad (22)$$

Similarly, using Lemma 1, the constraints of (18d) and (18e) are also respectively expressed as

$$\begin{cases} \text{tr}(\mathbf{B}_D) - \sqrt{-2 \log q_D} \alpha_D + \beta_D \ln q_D + s_D \geq 0, \\ \left\| \begin{bmatrix} \text{vec}(\mathbf{B}_D) \\ \sqrt{2} \mathbf{v}_D \end{bmatrix} \right\| \leq \alpha_D, \\ \beta_D \mathbf{I}_{N_t} + \mathbf{B}_D \succeq \mathbf{0}, \beta_D \geq 0; \end{cases} \quad (23)$$

$$\begin{cases} \text{tr}(\mathbf{B}_{ER,m}) - \sqrt{-2 \ln q_{ER,m}} \alpha_{ER,m} + \beta_{ER,m} \ln q_{ER,m} + s_{ER,m} \geq 0, \\ \left\| \begin{bmatrix} \text{vec}(\mathbf{B}_{ER,m}) \\ \sqrt{2} \mathbf{v}_{ER,m} \end{bmatrix} \right\| \leq \alpha_{ER,m}, \\ \beta_{ER,m} \mathbf{I}_{N_t} + \mathbf{B}_{ER,m} \succeq \mathbf{0}, \beta_{ER,m} \geq 0, \forall m \in \mathcal{M}, \end{cases} \quad (24)$$

where  $\alpha_D, \beta_D, \{\alpha_{ER,m}\}_{m=1}^M$  and  $\{\beta_{ER,m}\}_{m=1}^M$  are the introduced slack variables. Replacing outage constraints (18b)–(18e) with (21)–(24), Problem (18) is reformulated as

$$\max_{\mathbf{W}, \boldsymbol{\Sigma}, \rho, t, \lambda_D, \mu_D, \alpha_D, \beta_D, \{\lambda_{E,k}, \mu_{E,k}\}_{k=1}^K, \{\alpha_{ER,m}, \beta_{ER,m}\}_{m=1}^M} t \quad (25a)$$

$$\text{s.t. Eqs. (21) } \sim \text{(24),} \quad (25b)$$

$$\text{tr}(\mathbf{W} + \boldsymbol{\Sigma}) \leq P_{\text{th}}, \quad (25c)$$

$$\mathbf{W} \succeq \mathbf{0}, \boldsymbol{\Sigma} \succeq \mathbf{0}, 0 < \rho < 1, \quad (25d)$$

$$\text{rank}(\mathbf{W}) = 1. \quad (25e)$$

Notice that Problem (25) is still nonconvex involving the rank-1 constraint (25e). By ignoring constraint (25e) via the SDR technique of [26, 28], Problem (25) constitutes a convex semidefinite program (SDP) with LMI and SOC constraints, thus it is solved efficiently by existing interior point method [29]. A key

issue getting the relaxation of Problem (25) is that the rank of  $\mathbf{W}$  is one. However, the optimal solution for solving Problem (25) may be high-dimension. To obtain the rank-1 solution of  $\mathbf{W}$ , we rewrite Problem (25) as

$$\max_{\mathbf{W}, \boldsymbol{\Sigma}, \rho, t, \lambda_D, \mu_D, \alpha_D, \beta_D, \{\lambda_{E,k}, \mu_{E,k}\}_{k=1}^K, \{\alpha_{ER,m}, \beta_{ER,m}\}_{m=1}^M} t \quad (26a)$$

$$\text{s.t. Eqs. (21) } \sim \text{(24), (25c) } \sim \text{(25e),} \quad (26b)$$

$$\mathbf{W} = \mathbf{w}\mathbf{w}^H. \quad (26c)$$

It can be observed that Problem (26) involves a quadratic equality constraint (26c). In this regard, we have the following lemma.

**Lemma 3.** The equality constraint (i.e.,  $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ ) is equivalent to the following constraints:

$$\begin{cases} \left\| \begin{bmatrix} \mathbf{A}_1 & \mathbf{W} & \mathbf{w} \\ \mathbf{W}^H & \mathbf{A}_2 & \mathbf{w} \\ \mathbf{w}^H & \mathbf{w}^H & 1 \end{bmatrix} \right\| \succeq \mathbf{0}; \\ \text{tr}(\mathbf{A}_1 - \mathbf{w}\mathbf{w}^H) \leq 0, \end{cases} \quad (27)$$

where  $\mathbf{A}_1 \in \mathbb{C}^{N_t \times N_t}$  and  $\mathbf{A}_2 \in \mathbb{C}^{N_t \times N_t}$  are the newly introduced slack matrices.

*Proof.* Please refer to [30].

Applying Lemma 3, Problem (26) can be approximated as

$$\max_{\mathbf{W}, \boldsymbol{\Sigma}, \mathbf{w}, \mathbf{A}_1, \mathbf{A}_2, \rho, t, \lambda_D, \mu_D, \alpha_D, \beta_D, \{\lambda_{E,k}, \mu_{E,k}\}_{k=1}^K, \{\alpha_{ER,m}, \beta_{ER,m}\}_{m=1}^M} t \quad (28a)$$

$$\text{s.t. Eqs. (21) } \sim \text{(24), (25c), (25d), and (25e),} \quad (28b)$$

$$\begin{cases} \left\| \begin{bmatrix} \mathbf{A}_1 & \mathbf{W} & \mathbf{w} \\ \mathbf{W}^H & \mathbf{A}_2 & \mathbf{w} \\ \mathbf{w}^H & \mathbf{w}^H & 1 \end{bmatrix} \right\| \succeq \mathbf{0}, \\ \text{tr}(\tilde{\mathbf{w}}(n)\tilde{\mathbf{w}}(n)^H) + 2\text{Re}\{\text{tr}(\mathbf{w} - \tilde{\mathbf{w}}(n))\tilde{\mathbf{w}}(n)^H\} \geq \text{tr}(\mathbf{A}_1), \end{cases} \quad (28c)$$

where  $\tilde{\mathbf{w}}(n)$  is the variable at the  $n$ -th iteration by using first-order Taylor series approximation. An initial value  $\tilde{\mathbf{w}}(0)$  can be randomly generated until satisfying the feasible conditions for Problem (28) [31, 32].

### 3.3 Complexity analysis

In this subsection, we evaluate the complexity of the proposed robust secure scheme. According to [27], the computational complexity per iteration mainly stems from the number of optimization variables, the number of SDP linear inequalities and SDP size. For stochastic channel uncertainty, Problem (25) has  $2N_t^2$  design variables and  $(2K + 2M + 6)$  slack variables,  $(K + M + 4)$  SDP linear inequalities of size  $N_t$ , and  $(2K + 2M + 6)$  scalar linear inequalities.

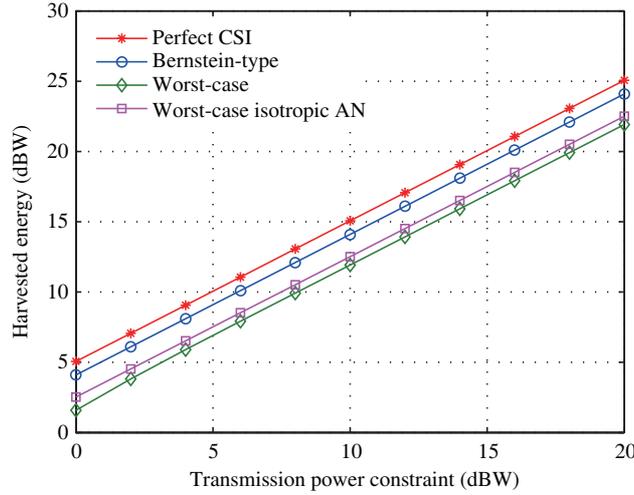
**Remark 1.** From Table 1 we can see that the computational complexity of our probabilistic-constraint based Bernstein-type method is slightly higher than that of the deterministic-constraint based worst-case method [22], because the probabilistic constraints are more complex. However, the proposed Bernstein-type beamforming scheme exhibits better performance than the worst-case scheme, as demonstrated by our simulation results provided in Section 4.

## 4 Numerical results

In this section, we examine the performance of our proposed probabilistic-constrained robust design for a secure SWIPT system. All channels are supposed to be Rayleigh flat-fading obeying  $\mathcal{CN}(0, 1)$ . For

**Table 1** Computational complexity analysis

Methods	Computation complexity order (ignoring $\ln(1/\epsilon)$ in $\mathcal{O}(\cdot)\ln(1/\epsilon)$ , where $\epsilon$ denotes the accuracy requirement); $n = \mathcal{O}(KMN_t^2)$ .
Bernstein-type	$\mathcal{O}(\sqrt{(K+M+4)N_t+2M+2K+6}[\{(K+M+2)N_t^3+2\}+n\{(K+M+2)N_t^2+2\}+(K+M+2)(N_t^2+N_t+1)^2+n^2])$
Worst-case	$\mathcal{O}(\sqrt{(K+M+4)N_t+2M+2K+4}[\{(K+M+2)(N_t+1)^3+2N_t^3+2\}+n\{(K+M+2)(N_t+1)^2+2N_t^2+2\}+n^2])$

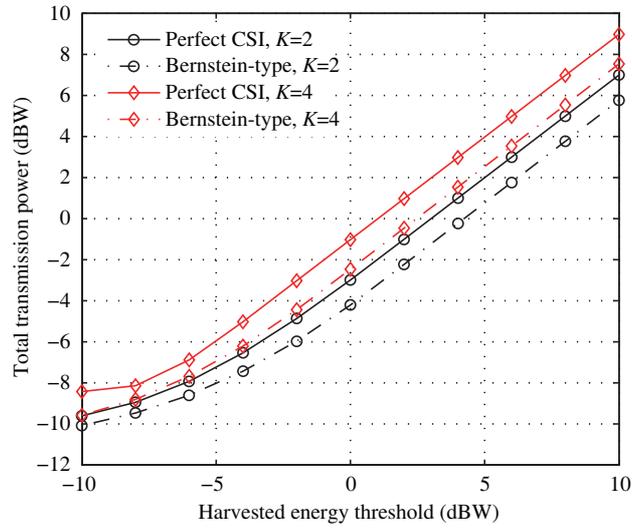

**Figure 2** (Color online) The harvested energy versus the transmission power constraint  $P_{th}$  under  $N_t = 4$  and  $K = 2$ .

simplicity, the channel-error vectors are recognized as the same covariances of  $\mathbf{\Omega}_D = \mathbf{\Omega}_{E,k} = \mathbf{\Omega}_{ER,m} = 0.2\mathbf{I}_{N_t}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}$ . The number of ERs is  $M = 2$ ,  $\gamma_D = 0$  dB,  $\gamma_{E,k} = -5$  dB. The outage probabilities are set to be  $p_D = p_{E,k} = q_D = q_{ER,m} = q$ . In our simulations, all results were averaged over 1000 times.

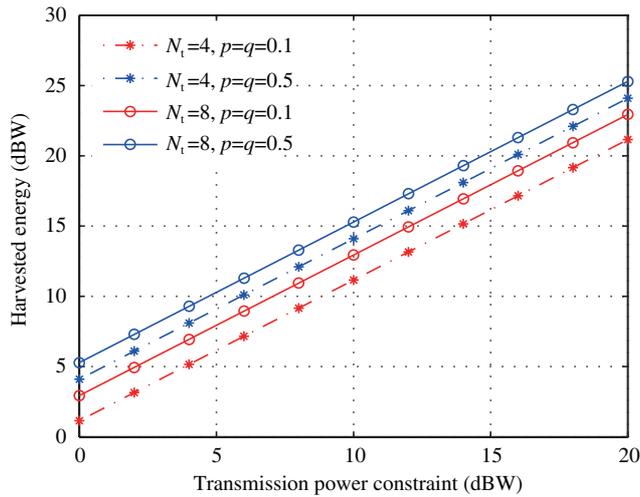
First, we compare the performance of the robust design scheme proposed in this paper (labeled as “Bernstein-type”), the deterministic-constraint based worst-case scheme [22] (labeled as “Worst-case”), the deterministic-constraint based worst-case isotropic scheme (labeled as “Worst-case isotropic AN”), and the perfect CSI scheme under  $N_t = 4$  and  $K = 2$ . The perfect CSI scheme shows the upper bound of performance, which is obtained by  $\Delta\mathbf{h}_D = \mathbf{0}$ ,  $\Delta\mathbf{h}_{E,k} = \mathbf{0}$  and  $\Delta\mathbf{h}_{ER,m} = \mathbf{0}$ . The worst-case isotropic AN scheme represents the AN is in the null space of  $\mathbf{h}_D$ . As is shown in Figure 2, our proposed robust scheme always outperforms the worst-case schemes. This is because the  $S$ -procedure-based scheme is more conservative than the Bernstein-type inequality-based scheme [22, 26]. Furthermore, the proposed robust design scheme is closer to the perfect scheme. In addition, the worst-case isotropic AN scheme harvests more energy than the worst-case scheme, which can be explained by benefiting from no generated AN to the DR.

Then, we exhibit the transmit power  $\text{tr}(\mathbf{W} + \mathbf{\Sigma})$  as a function of the harvested energy threshold  $\Gamma_D$  prescribed for each ER. It is observed from Figure 3 that, with the increasing of  $\Gamma_D$  as well as  $K$ , the required total transmit power increases. This is due to more information-bearing power is required for restraining the maximal outage probability of the SINR recorded at the DR.

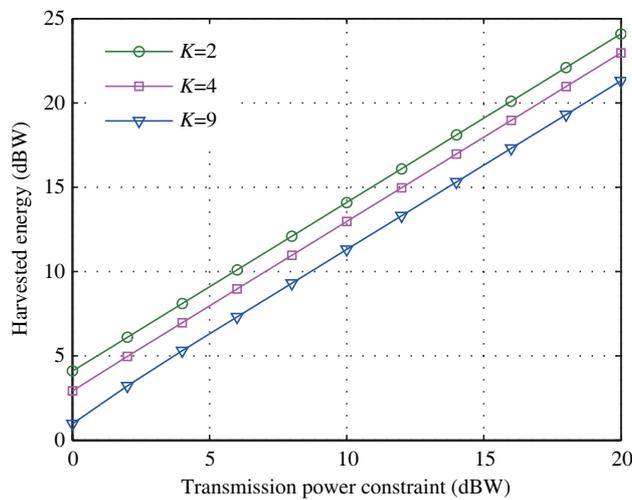
To further observe the behaviors of the robust scheme proposed in this paper, in next simulations we study the impacts of transmit antenna  $N_t$  and outage probability on the harvested energy. From Figure 4, we can see that increasing the number of  $N_t$  and  $p = q$ , the harvested energy tends to increase. Figure 5 evaluates the effect of the number of Eves  $K$  on the proposed robust design scheme under  $N_t = 4$ . It can be found that the harvested energy is very sensitive to the number of Eves  $K$ . When  $K$  increases, the performance loss becomes large, which confirms the importance of secure communication.



**Figure 3** (Color online) The total transmission power  $\text{tr}(\mathbf{W} + \mathbf{\Sigma})$  versus the harvested energy threshold  $\Gamma_D$  prescribed for each ER under  $N_t = 4$  and  $K = \{2, 4\}$ .



**Figure 4** (Color online) The harvested energy versus the transmission power constraint  $P_{th}$  for different  $N_t$  and  $p = q$ .



**Figure 5** (Color online) The harvested energy versus the transmission power constraint  $P_{th}$  for different  $K$ .

## 5 Conclusion

A probabilistically robust secure beamformer for a SWIPT network was designed where imperfect CSI was available to the transmitter. Both the AN and PS schemes were adopted in our design. Our objective was to maximize the harvested energy at ERs subject to a range of outage constraints. The formulated EHM problem was nonconvex, which was converted into an SDP with LMI and SOC constraints by employing the Bernstein-type inequality restriction technique. Furthermore, a rank-1 solution was reconstructed. Numerical results have showed that the proposed robust beamforming scheme has better performance than the worst-case beamforming scheme.

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**Conflict of interest** The authors declare that they have no conflict of interest.

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