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Fractional full duplex cellular network: a stochastic geometry approach

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Abstract In-band full-duplex (FD) communication has been considered as a promising technology to enhance the spectral efficiency (SE) for the next generation of wireless system. However, the severe interference in FD cellular network may largely degrade the system performance, especially for cell edge users. In this paper, multi-cell cellular network consisting of BSs with FD capability and legacy half duplex (HD) users is studied. In order to relieve the received interference and enhance the system SE, a new resource allocation strategy named fractional FD (FFD) is proposed and its main idea and procedure are described as follows. Firstly, the frequency and time resources are partitioned into FD resource blocks (RBs) and HD RBs based on the network topology. Then all the users are classified into two groups named cell center users (CCUs) and cell edge users (CEUs) based on their channel conditions. At last, in each cell, each FD RB is allocated to a pair of CCUs with one in uplink and the other in downlink transmission direction, while only one CEU in the uplink (or downlink) transmission direction is scheduled over each HD RB. Tractable results of both the coverage probabilities and the ergodic rates of FFD, FD and HD systems are derived using stochastic geometry method. Numerical results show that, FFD can significantly improve the coverage probability of all users especially for CEUs compared with FD cellular system, and higher system SE is obtained compared with FD and HD cellular network. With proper design of the classification criterion and under the simulation settings of this paper, the SE of FFD system outperforms FD and HD system by 1.25 and 1.38 times, respectively.

Keywords full duplex, half duplex, stochastic geometry, radio resources allocation, small cell

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1 Introduction

The ever-increasing demand for mobile data traffic with limited frequency spectrum resources has pushed researchers to find various technologies to enhance the spectrum efficiency (SE), among which in-band full duplex (FD) communication is one of the hot topics [1, 2]. Traditional wireless transceivers operate either in time division duplex (TDD) or frequency division duplex (FDD) mode, which is also called half duplex (HD) mode, so as to protect the receiver from being overwhelmed by the self-interference (SI) generated by itself transmitter. Recent advances in interference cancelation technologies and hardware developments [3–5] have made radios possible to work in FD mode, where the radios can transmit and receive signal over the same frequency simultaneously offering the potential to double the SE. Up to 110 dB SI attenuation has been achieved in [3].

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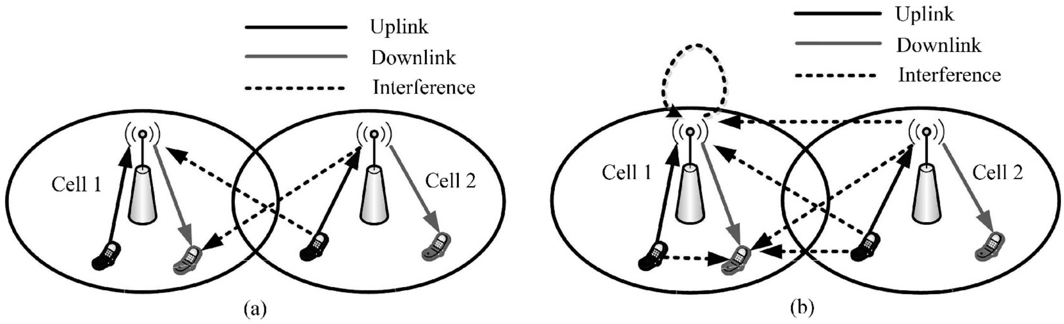

Figure 1 An exemplary interference scenario of HD/FD cellular network. (a) Synchronous TDD cellular network; (b) FD cellular network.

There are various applications of FD communication, including cognitive radio system [6], relay network [7, 8], bi-directional communication system [9], device to device communication system [10] and cellular network [11–19]. Among them, FD cellular network draws more and more attention, especially in ultra-dense network (UDN) [20] where the transmission power is low and the coverage range is small.

The researches of FD cellular system include single cell system [11–14] and multi-cell network [15–19]. In single cell system, besides SI, the inter-user interference [11, 12] and the precoder design in multi-user multiple input multiple output (MU-MIMO) system [13, 14] are both comprehensively investigated.

In multi-cell FD cellular system, more interference is involved. Taking a two cells interference scenario as an example, as illustrated in Figure 1(a) and (b), the dotted arrows show the interference experienced by the user and BS in Cell 1. As shown in Figure 1(a), in HD cellular network which is specified as synchronous TDD system in this paper, the downlink user only gets interference from the transmission of neighboring BSs. Similarly, in the uplink, the BS is only interfered by the transmission of uplink users in neighboring cells. However, in FD cellular system illustrated in Figure 1(b), besides the described interference in HD cellular system, downlink user suffers additional interference generated from the uplink transmission of intra-cell and inter-cell users, and FD BS suffers from SI and inter-BS interference. On the basis of above discussion, it can be observed that the interference in FD cellular system is much more severe and harmful.

Therefore, proper interference management methods are needed to improve the system performance.

Authors in [19] applied Coordinated Multi-Point technology in FD network to deal with the severe interference where perfect BS to BS interference cancellation is assumed. However, this assumption implies the need of sufficient capability for interconnection between BSs which may be far from reality when deployment cost is considered in many scenarios. In [17], a mixed full/half duplex cellular network, which consists of FD and HD cells, has been proposed so as to maintain the interference generated from FD cells within a moderate level. However, all the users in the same cell are served with the same duplex mode resources, which may result in system SE degradation and serious interference for cell edge users nearby FD cells. Authors in [18] proposed an "α duplex network", where α ($1 \geqslant \alpha \geqslant 0$) proportion of users are served with FD radio resources and the rest users are served with HD radio resources in the same cell, however the radio resources are randomly assigned to the users without taking the performance of individual users into consideration, which implies that a cell edge user may be served with FD radio resources resulting in severe interference.

In this paper, we concentrate on a multi-cell network consisting of BSs with FD capability and legacy HD users. All the available radio resources are partitioned into orthogonal resource blocks (RBs). Each RB can be selected to operate either in FD mode or in HD mode. For briefness, we call the RBs operating in FD (HD) mode as FD (HD) RBs.

Intuitively, the cell center users who experience higher signal power and lower interference, are more suitable to be served in FD RBs, while the cell edge users who suffer severe interference and experience lower signal to interference plus noise ratio (SINR) may be better to be served in HD RBs.

Inspired by this idea, we propose a new resource allocation strategy named fractional FD (FFD).

Different from the existing researches, each BS assigns different duplex mode radio resources to different users according to the channel condition of individual users. The main idea of the proposed FFD strategy is described as follows. Firstly all the RBs are divided into FD RBs and HD RBs according to certain metric. Then users are classified into two groups named cell center users (CCUs) whose channel condition is better and cell edge users (CEUs) whose channel condition is poorer. Finally FD RBs are allocated to CCUs while the HD RBs are assigned to CEUs. Possible criteria for partitioning RBs and classifying CCUs and CEUs are also proposed in this paper.

The benefits of our proposed FFD strategy are as follows. For CCUs both higher sum throughput and higher quality communication can be achieved. For CEUs, the severe interference can be well relieved compared with that if CEUs are served with FD RBs.

The contributions of this paper are two folds. Firstly, a new resource allocation strategy FFD is proposed. To our best knowledge, there is no existing work focusing on partitioning different duplex mode RBs to users based on the channel condition of individual users. Possible process and metrics for partitioning RBs and classifying users are also proposed. Secondly, analytical evaluation of FFD cellular system is conducted using stochastic geometry method. Tractable expressions of both coverage probabilities and ergodic rates are derived along with the network topology.

The rest of the paper is organized as follows. In Section 2, we introduce the system model and the proposed FFD method including the process and criteria. In Section 3, the laplace transforms of the system interference are derived for performance evaluation using stochastic geometry. The coverage probabilities and ergodic rates are derived for different duplex schemes in Section 4. Numeral results and analysis are presented in Section 5. Section 6 is the conclusion of the paper.

2 System model and proposed FFD scheme

2.1 Network assumptions

In this paper, we focus on a single tier multi-cell network that consists of BSs with FD capability and HD users. Over a FD RB two HD users with one in the uplink and one in downlink transmission, can be simultaneously served , while over a HD RB, only an uplink user or a downlink user is scheduled. In order to prevent the received signal in HD RBs from being interfered by the transmission in FD RBs due to the non-linearity of transceiver, we assume that HD and FD RBs are allocated in different time slots.

The proposed FFD scheme which will be described in the next subsection, is suitable for any BS deployment and any user distribution. In order to provide tractable performance evaluation of the proposed FFD scheme, stochastic geometry method is applied [21] in this paper, where the location of BSs is assumed to follow homogeneous spatial Poisson point process (PPP) $\Phi_{\rm s}$ with density $\lambda_{\rm s}$, and users are also assumed to be randomly located in the network according to PPP $\Phi_{\rm u}$ with density $\lambda_{\rm u}$. $\lambda_{\rm u}$ is assumed to be sufficient large so as to guarantee that BSs can randomly select users to serve. We also assume that each BS randomly selects one active user scheduled over a HD RB or one pair users scheduled over a FD RB in each cell. Due to the limited space, user scheduling is not taken into consideration for the user pairs working over the same FD RBs. Additional gain from user scheduling will be discussed in our future work.

Users are assumed to communicate with their closest BSs, which follows the association principle of maximum average received power. Then, the probability density function (PDF) of the user at distance R from its serving BS can be derived as [21]

$$f_R(R) = 2\pi\lambda_{\rm s} R{\rm e}^{-\lambda_{\rm s}\pi R^2}. \tag{1}$$

2.2 Proposed fractional full duplex scheme

In the following, the procedure of FFD is firstly described to outline the basic rules for FFD, then a possible detail design of the radio resource partition method and user classification metric is given.

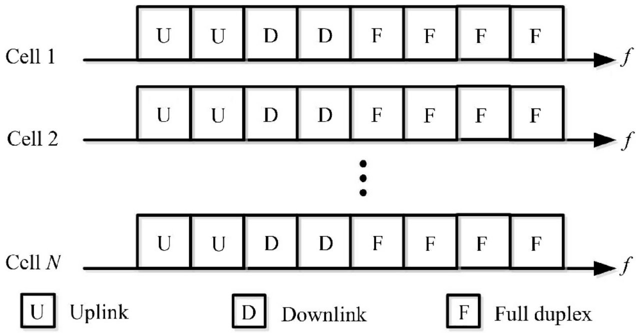

Figure 2 Resource blocks allocation.

2.2.1 *Procedure of FFD*

(1) **Radio resource partition.** All the RBs are divided into three groups $\psi_{\rm f}$, $\psi_{\rm u}$, $\psi_{\rm d}$ according to certain classification metric, where the BS operates in FD mode, HD mode uplink transmission and HD mode downlink transmission, respectively. In this paper, synchronized RB partition among all cells is assumed. As shown in Figure 2, once an RB is decided to operate in FD or HD mode, then all the BSs operate in the same mode over this RB[1)].

(2) **User classification.** Users are classified into CCUs and CEUs in the uplink and downlink transmission respectively based on the channel condition of individual users.

(3) **RBs assignment.** In each cell, for each RB belonging to $\psi_{\rm f}$, we select randomly (or in other scheduling scheme manner) two CCUs to serve with one working in the uplink and the other in downlink transmission. One uplink (downlink) CEU is scheduled randomly (or in other scheduling scheme manner) over each of the RB belongs to $\psi_{\rm u}$ ($\psi_{\rm d}$).

2.2.2 *The detail design of FFD*

The design of FFD can be various based on different system requirements. The adopted classification metrics and detail design in this paper are detailed as following.

We denote $\beta_{\rm c}$ and $\beta_{\rm e}$ as the partitioned proportions of FD RBs and HD RBs, respectively, where $\beta_{\rm c}+\beta_{\rm e}=1$. When the traffic for uplink and downlink is symmetric, the proportions of the RBs partitioned to group $\psi_{\rm f}$, $\psi_{\rm u}$, $\psi_{\rm d}$ are $\beta_{\rm c}$, $\beta_{\rm e}/2$ and $\beta_{\rm e}/2$, respectively. In this paper, the metric for determining $\beta_{\rm c}$ and $\beta_{\rm e}$ is given as

$$\beta_{\rm c}=\frac{{\rm E}\left(N_{\rm dc}\right)}{{\rm E}\left(N_{\rm d}\right)},\quad \beta_{\rm e}=\frac{{\rm E}\left(N_{\rm de}\right)}{{\rm E}\left(N_{\rm d}\right)},\tag{2}$$

where ${\rm E}(\cdot)$ represents the operation of expectation. $N_{\rm d}$ is the total number of the downlink users in a cell who are randomly selected by the BS to serve. $N_{\rm dc}$ and $N_{\rm de}$ are the numbers of selected downlink CCUs and CEUs in a cell, respectively. Obviously, $N_{\rm d}=N_{\rm dc}+N_{\rm de}$.

The classification of CCUs and CEUs is based on the following criteria. Downlink user i is classified as a downlink CCU if $\tau_{{\rm d}i}\geqslant\gamma_{\rm d}$, otherwise user i is treated as a downlink CEU, where $\gamma_{\rm d}$ is a predefined threshold and $\tau_{{\rm d}i}$ is the instant temporary SINR[2)] for downlink user i which is defined as

$$\tau_{{\rm d}i}=\frac{s_{{\rm d}i}}{I_{{\rm d}i}+\sigma^2},\tag{3}$$

where $s_{{\rm d}i}$ is the desired received signal power. $I_{{\rm d}i}$ denotes the received instant interference that generated from all the cells when the RB used by user i is assumed to be a FD RB in all cells and is assigned to a pair of randomly selected users in each cell.

1) Although a fully dynamic RB assignment for each cell may bring some flexibility, the interference management may be more complexity, which is out of scope of this paper. In addition, universal frequency reuse across all the cells is applied.

2) Note that both $\tau_{{\rm d}i}$ and $I_{{\rm d}i}$ are intermediate variables for classifying CCUs and CEUs which are different from the SINR and interference I_m defined in (5) when the decision of resource allocation is made.

In the same way, the uplink users are also divided into CCUs and CEUs based on the classification threshold $\gamma_{\rm u}$, which is carefully designed so as to make the following proportions satisfy:

$$\beta_{\rm c} = \frac{{\rm E}\left(N_{\rm uc}\right)}{{\rm E}\left(N_{\rm u}\right)}, \quad \beta_{\rm e} = \frac{{\rm E}\left(N_{\rm ue}\right)}{{\rm E}\left(N_{\rm u}\right)}, \tag{4}$$

where $N_{\rm u}$ is the total number of the selected uplink users in a cell. $N_{\rm uc}$ and $N_{\rm ue}$ are the numbers of selected uplink CCUs and CEUs in a cell, respectively.

2.3 Channel model and SINR characterization

Without loss of generality, in the performance evaluation of FFD we focus on a receiver node located at the origin in cell o and the received SINR can be given in unified form:

$$\text{SINR}_m = \frac{P_m G_m}{\sigma^2 + I_m + \delta I_{\rm SI}}, \tag{5}$$

where $m \in \{{\rm fu, fd, hu, hd}\}$ represents the type of transmission. ${\rm fu, fd, hu, hd}$ denote FD uplink, FD downlink, HD uplink and HD downlink transmission, respectively. P_m and G_m are the transmission power of the transmitter and the channel gain of the desired signal in a given RB, respectively. Power control is not taken into consideration in this paper and will be discussed in our future work.

In this paper, block fading channel model is adopted for all the channels. Then the channel gain G_m can be expressed as $G_m = h_m k_m {R_m}^{-\alpha_{\rm bu}}$, where h_m and $k_m R_m^{-\alpha_{\rm bu}}$ represent small scale fading and large scale fading, respectively. R is the propagation distance between the focused user and its serving BS. k_m denotes the part of path loss model that is invariant with the propagation distance. $\alpha_{\rm bu}$ is the path loss exponent between the user and BS. The small scale fading h_m is assumed to be rayleigh fading, i.e, $h_m \sim \exp(1)$.

$I_{\rm SI}$ represents the residual SI that only exist in FD uplink transmission in the given FD RB. Therefore, δ equals 1 only when FD uplink transmission is regarded, otherwise, $\delta = 0$. In this paper, the residual SI $I_{\rm SI}$ after interference cancellation is assumed to be a Gaussian random variable with zero mean and variance proportional to transmission power of the FD BS $P_{\rm fd}$ as [13] does. Then $I_{\rm SI}$ can be written as

$$I_{\rm SI} \sim \mathcal{N}(0, {\sigma_{\rm I}}^2), \quad {\sigma_{\rm I}}^2 = P_{\rm fd}|\varepsilon|^2, \tag{6}$$

where $0 \leqslant |\varepsilon|^2 \leqslant 1$ refers to the SI cancellation capability. $|\varepsilon|^2 = 1$ implies nothing is done to SI and $|\varepsilon|^2 = 0$ indicates that perfect SI cancelation is assumed.

I_m denotes the sum of received interference in the given RB except SI and can be expressed as

$$I_{\rm fd} = \sum_{z \in \Phi_{\rm fd} \backslash b_o} P_{\rm fd} G_{z u_o, {\rm bu}} + \sum_{x \in \Phi_{\rm fu}} P_{\rm fu} G_{x u_o, {\rm uu}}, \qquad I_{\rm fu} = \sum_{z \in \Phi_{\rm fd} \backslash b_o} P_{\rm fd} G_{z b_o, {\rm bb}} + \sum_{x \in \Phi_{\rm fu} \backslash u_o} P_{\rm fu} G_{x b_o, {\rm bu}}, \tag{7}$$

$$I_{\rm hd} = \sum_{z \in \Phi_{\rm hd} \backslash b_o} P_{\rm hd} G_{z u_o, {\rm bu}}, \qquad I_{\rm hu} = \sum_{x \in \Phi_{\rm hu} \backslash u_o} P_{\rm hu} G_{x b_o, {\rm bu}}, \tag{8}$$

where Φ_m denotes the interfering node set working in mode m. b_o and u_o are the BS and user located in cell o, respectively. $\Phi_m \backslash b_o$ ($\Phi_m \backslash u_o$) denotes that b_o (u_o) is excluded from Φ_m. z and x denote interfering BSs and users working either over FD or HD RBs. $G_{*,{\rm bu}} = h_* k_* {D_*}^{-\alpha_{\rm bu}}$, $G_{*,{\rm uu}} = h_* k_* {D_*}^{-\alpha_{\rm uu}}$ and $G_{*,{\rm bb}} = h_* k_* {D_*}^{-\alpha_{\rm bb}}$ denote the interfering channel gain between BS and user, two users and two BSs, respectively. h_*, D_* and k_* represent the small scale fading, interfering distance and large scale fading attenuation constant, respectively. $\alpha_{\rm uu}$, $\alpha_{\rm bb}$, $\alpha_{\rm bu}$ denote the path loss exponents of the channels between two users, two BSs, BS and user, respectively.

The user u_o located at cell o is also included in the interfering node sets in FD downlink transmission shown in the second item on the right side of $I_{\rm fd}$ defined in (7). The main reason is that, in FD cells, two HD users are simultaneously served over one FD RB. Therefore, the downlink user gets interference from the uplink users in all the cells including its own cell.

2.4 Interference cancellation technology

Over FD RBs, there are two kinds of severe interference: the inter-BS interference and the inter-user interference in the same cell. Because neighboring nodes can be arbitrarily close to each other when the nodes are distributed based on PPP model. Therefore, the performance gain of FD communication can be largely degraded. In this occasion, it is better but not necessary to take some interference cancellation strategies into account.

In the analysis of this paper, the influence of applying successive interference cancellation (SIC) [22] on the inter-BS and inter-user interference is analysed. As an usual assumption [23], the interference can be assumed to be perfectly cancelled using SIC if the interference signal is strong enough compared with the desired signal. In order to make the results tractable, we assume that when SIC is applied, the interference is completely removed if the interfering distance is shorter than ηR_m, where η $(1 \geqslant \eta \geqslant 0)$ is the SIC capacity and R_m denotes the transmission distance of the desired signal. While nothing is done to the interference with distance larger than ηR_m. Obviously, $\eta = 0$ means that no interference cancellation strategy is applied.

3 Laplace transform of the interference

Stochastic geometry method is adopted to evaluate the system performance and both the coverage probability and ergodic rate are derived. The coverage probability and ergodic rate can be expressed as the function of laplace transforms (LT) of the received interference I_m $(m \in \{\text{fu}, \text{fd}, \text{hu}, \text{hd}\})$. Hence, in order to obtain the coverage probability and ergodic rate, the LT of I_m is firstly derived.

As shown in (7) and (8), the uplink and downlink interference forms with the same duplex mode are almost the same except for some parameter settings. To save the space of the paper, the LT for uplink and downlink transmission in unified form is presented here.

3.1 Laplace transform of interference over HD RB

We denote the interference suffered over HD RBs as $I_{\text{h}q}$, where $q \in \{u, d\}$ denotes the transmission direction. u and d represent uplink and downlink transmission, respectively. Then based on the interference expression in (8), we can get the LT of the interference in HD RBs in the following lemma.

Lemma 1. When the location of HD BSs follows the spatial PPP distribution with density λ_{s}, the LT of interference $I_{\text{h}q}$ can be derived as

$$L_{I_{\text{h}q}}(s) = \text{E}_{I_{\text{h}q}}(\text{e}^{-sI_{\text{h}q}}) = \exp\left\{\pi\lambda_{\text{s}}R^2 - \pi\lambda_{\text{s}}(s')^{\frac{2}{\alpha_{\text{bu}}}}\omega\left(\alpha_{\text{bu}}, R'\right)\right\}, \tag{9}$$

where $s' = sP_{\text{h}q}k_{\text{h}q}$ and $R' = s'R^{-\text{bu}}$. $\omega(\cdot,\cdot)$ is defined as

$$\omega\left(\alpha_{\text{bu}}, R'\right) = \left\{\frac{2}{\alpha_{\text{bu}}}\left\{J\left(\frac{2}{\alpha_{\text{bu}}}+1, 1, -\frac{2}{\alpha_{\text{bu}}}, R'\right) - \Gamma\left(-\frac{2}{\alpha_{\text{bu}}}\right)\Gamma\left(\frac{2}{\alpha_{\text{bu}}}+1\right)\right\}\right\}, \tag{10}$$

where $J(\cdot,\cdot,\cdot,\cdot)$ has already been given by [17] and is defined as

$$J(\mu,\beta,\nu,\gamma) = \int_0^{\infty} x^{\mu-1}\text{e}^{-\beta x}\Gamma(\nu,\gamma x)\text{d}x = \frac{\gamma^{\nu}\Gamma(\mu+\nu)}{\mu(\gamma+\beta)^{\mu+\nu}}{}_2F_1\left(1, \mu+\nu; \mu+1; \frac{\beta}{\gamma+\beta}\right) \tag{11}$$

with constrains $\text{Re}(\gamma+\beta) > 0, \text{Re}(\mu) > 0, \text{Re}(\mu+\nu) > 0$. ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$, $\Gamma(a) = \int_0^{\infty} t^{a-1}\text{e}^{-t}\text{d}t$ and $\Gamma(x,a) = \int_x^{\infty} t^{a-1}\text{e}^{-t}\text{d}t$ denote the hypergeometric function, standard gamma function and incomplete gamma function, respectively.

Proof. Please see Appendix A.

3.2 Laplace transform of interference over FD RBs

The LT of interference in FD RBs is defined as

$$L_{I_{\mathrm{f}q}}(s) = \mathbb{E}_{I_{\mathrm{f}q}}(\mathrm{e}^{-sI_{\mathrm{f}q}}). \tag{12}$$

As shown in (7), the interference generates from both the BSs set Φ_{fd} and uplink users set Φ_{fu}. According to the user association principle, the users are located in the voronoi cells formed by the FD BSs. Therefore, Φ_{fd} and Φ_{fu} are correlated with each other. However, the relationship of them is still unknown and makes the results untractable. Fortunately, as shown in [24], the correlation is weak and has little impact on the final results. Hence, in this paper, we also assume these two Poisson processes are independent from each other.

Lemma 2. If the locations of BSs and users are distributed based on independent spatial PPP with density λ_{s}, the LT of interference $I_{\mathrm{f}q}$ over FD RBs can be derived as

$$L_{I_{\mathrm{f}q}}(s) = \exp\left\{\pi\lambda_{\mathrm{s}}{R_{\mathrm{d}}}^2 - \pi\lambda_{\mathrm{s}}(s_{\mathrm{d}})^{\frac{2}{\alpha_{\mathrm{d}}}}\omega\left(\alpha_{\mathrm{d}}, s_{\mathrm{d}}R_{\mathrm{d}}\right)\right\}\exp\left\{\pi\lambda_{\mathrm{s}}{R_{\mathrm{u}}}^2 - \pi\lambda_{\mathrm{s}}(s_{\mathrm{u}})^{\frac{2}{\alpha_{\mathrm{u}}}}\omega\left(\alpha_{\mathrm{u}}, s_{\mathrm{u}}R_{\mathrm{u}}\right)\right\}, \tag{13}$$

where $s_{\mathrm{d}} = sP_{\mathrm{fd}}k_{z,o}$ and $s_{\mathrm{u}} = sP_{\mathrm{fu}}k_{z,o}$. $\omega(\cdot,\cdot)$ and $J(\cdot,\cdot,\cdot,\cdot)$ are defined as (10) and (11), respectively. In addition, the other parameters are defined as follows:

- when $q = d$, $\alpha_{\mathrm{d}} = \alpha_{\mathrm{bu}}$, $\alpha_{\mathrm{u}} = \alpha_{\mathrm{uu}}$, $R_{\mathrm{d}} = R$ and $R_{\mathrm{u}} = \eta_{\mathrm{uu}}R$;
- when $q = u$, $\alpha_{\mathrm{d}} = \alpha_{\mathrm{bb}}$, $\alpha_{\mathrm{u}} = \alpha_{\mathrm{bu}}$, $R_{\mathrm{d}} = \eta_{\mathrm{bb}}R$ and $R_{\mathrm{u}} = R$,

where η_{uu} and η_{bb} denote the SIC capability of inter-user and inter-BS interference, respectively.

Proof. Please see Appendix B.

3.3 Joint Laplace transform

The joint LT of downlink users is discussed here. Due to the RBs allocation mechanism described in Subsection 2.2, the users are assumed to work in FD RBs at first and regrouped based on their temporary SINR. The interfering BSs remain the same after the downlink users are recognized as CEUs. We use I_{d} and $\hat{I}_{\mathrm{d}}$ to denote the interference suffered by a downlink user before and after it is classified as CEUs. Therefore, the interference I_{d} and $\hat{I}_{\mathrm{d}}$ is correlated with each other. The joint LT of downlink users is defined as

$$\mathcal{L}_{\hat{I}_{\mathrm{d}},I_{\mathrm{d}}}(s_{\mathrm{h}}, s_{\mathrm{f}}) = \mathrm{E}_{\hat{I}_{\mathrm{d}},I_{\mathrm{d}}}\left(\mathrm{e}^{-s_{\mathrm{h}}\hat{I}_{\mathrm{d}} - s_{\mathrm{f}}I_{\mathrm{d}}}\right). \tag{14}$$

Lemma 3. When a downlink user is classified as CEU, the joint LT of interference $L_{\hat{I}_{\mathrm{fd}},I_{\mathrm{fd}}}(s_{\mathrm{h}}, s_{\mathrm{f}})$ before and after it is allocated HD RB is given by

$$\begin{cases} (1)\text{ when } s_{\mathrm{fd}} \neq s_{\mathrm{hd}}, \\ \exp\left\{\pi\lambda_{\mathrm{s}}(1+\eta_{\mathrm{uu}})R^2 + \dfrac{2\pi\lambda_{\mathrm{s}}}{\alpha_{\mathrm{bu}}(s_{\mathrm{fd}} - s_{\mathrm{hd}})}\left\{\Gamma\left(-\dfrac{2}{\alpha_{\mathrm{bu}}}\right)\Gamma\left(\dfrac{2}{\alpha_{\mathrm{bu}}}+1\right)\left({s_{\mathrm{fd}}}^{\frac{2}{\alpha_{\mathrm{bu}}}+1} - {s_{\mathrm{hd}}}^{\frac{2}{\alpha_{\mathrm{bu}}}+1}\right)\right.\right. \\ \quad \left. - J\left(\dfrac{2}{\alpha_{\mathrm{bu}}}+1, \dfrac{1}{s_{\mathrm{fd}}}, -\dfrac{2}{\alpha_{\mathrm{bu}}}, R^{-\alpha_{\mathrm{bu}}}\right) + J\left(\dfrac{2}{\alpha_{\mathrm{bu}}}+1, \dfrac{1}{s_{\mathrm{hd}}}, -\dfrac{2}{\alpha_{\mathrm{bu}}}, R^{-\alpha_{\mathrm{bu}}}\right)\right\} \\ \quad \left. -\pi\lambda_{\mathrm{s}}(s_{\mathrm{fu}})^{\frac{2}{\alpha_{\mathrm{uu}}}}\omega\left(\alpha_{\mathrm{uu}}, R_{\mathrm{ju}}\right)\right\}; \\ (2)\text{ when } s_{\mathrm{fd}} = s_{\mathrm{hd}} = s, \\ \exp\left\{\pi\lambda_{\mathrm{s}}(1+\eta_{\mathrm{uu}})R^2 - 2\pi\lambda_{\mathrm{s}}\left\{-\dfrac{1}{\alpha_{\mathrm{bu}}}\Gamma\left(-\dfrac{2}{\alpha_{\mathrm{bu}}}\right)s^{\frac{2}{\alpha_{\mathrm{bu}}}}\Gamma\left(\dfrac{2}{\alpha_{\mathrm{bu}}}+2\right)\right.\right. \\ \quad \left.\left. + \dfrac{1}{\alpha_{\mathrm{bu}}}s^2 J\left(\dfrac{2}{\alpha_{\mathrm{bu}}}+2, \dfrac{1}{s}, -\dfrac{2}{\alpha_{\mathrm{bu}}}, R^{-\alpha_{\mathrm{bu}}}\right) + \dfrac{1}{2}(s_{\mathrm{fu}})^{\frac{2}{\alpha_{\mathrm{uu}}}}\omega\left(\alpha_{\mathrm{uu}}, R_{\mathrm{ju}}\right)\right\}\right\}, \end{cases} \tag{15}$$

where $\eta_{\rm uu}$ denotes the SIC capability of inter-user interference. $\omega(\cdot,\cdot)$ and $J(\cdot,\cdot,\cdot,\cdot)$ are defined as (10) and (11), respectively. The other parameters are defined as follows:

$$s_{\rm fd}=s_{\rm f}P_{\rm fd}k_{\rm fd},\quad s_{\rm hd}=s_{\rm h}P_{\rm hd}k_{\rm hd},\quad s_{\rm fu}=s_{\rm f}P_{\rm fu}k_{\rm fu},\quad R_{\rm ju}=s_{\rm fu}\eta_{\rm uu}R.$$

Proof. Please refer to Appendix C.

4 Coverage probability and ergodic rate

Coverage probability is the probability that the received SINR of a receiver is larger than the target SINR T and it is defined as

$$F_{\theta,q}(T)=\mathbb{P}(\text{SINR}>T), \tag{16}$$

where $\theta\in\{\text{CEU},\text{CCU},\text{FD},\text{HD}\}$ and q denote the type of users and transmission direction, respectively.

In the following subsections, the coverage probabilities of FFD, FD and HD systems are all derived. Since we treat CCUs and CEUs differently in FFD strategy, the coverage probabilities of CEUs and CCUs in FFD system are also derived in Subsections 4.1 and 4.2 respectively. The coverage probability of a random user (it may be a CEU or a CCU) in FD (HD) system, which is called the coverage probability of FD (HD) system in this paper, is derived in Subsection 4.3.

4.1 Coverage probability of CEUs in FFD cellular system

In this section, the coverage probability of CEUs in FFD cellular system is derived. Based on the classification rule, all the users are assumed to be served in FD RBs and calculate the received temporary SINR. The users, whose temporary SINR is lower than the predefined SINR threshold γ_q, are regarded as CEUs and HD RBs are assigned to them.

We should note that due to the different interference scenarios between uplink and downlink transmission, different proportions of CCUs can be obtained base on the uplink and downlink categorization criterions. However, CCUs are scheduled in FD RBs and the proportions of CCUs in uplink and downlink transmission direction should be equal. In this paper, $\gamma_{\rm u}$ will be carefully designed according to $\gamma_{\rm d}$ in order to make the proportions of CCUs in the uplink and downlink transmission to be the same.

For simplification, we also derive the CEU coverage probability for both uplink and downlink transmission in unified form. Based on the assumptions above, coverage probability of CEUs can be given as Theorem 1.

Theorem 1. The coverage probability defined in (16) of a typical CEU employing FFD scheme can be derived as

$$F_{\text{CEU},q}(T)=\frac{\int_{R=0}^{\infty}\left(\mathrm{e}^{-s_{\rm h}\sigma^2}\left(L_{I_{{\rm h}q}}(s_{\rm h})-\mathrm{e}^{-s_{\rm f}\sigma'^2}L_U(s_{\rm h},s_{\rm f})\right)\right)f_R(R)\,\mathrm{d}R}{\int_{R=0}^{\infty}\left(1-\mathrm{e}^{-s_{\rm f}\sigma'^2}L_{I_{{\rm f}q}}(s_{\rm f})\right)f_R(R)\,\mathrm{d}R}, \tag{17}$$

where T represents the target SINR. $q\in\{\rm u,d\}$ represents uplink or downlink transmission direction. $f_R(R)$ is defined as (1). For saving the space, we define $\sigma'^2=\left(\sigma^2+\delta I_{\rm SI}\right)$, $s_{\rm h}=\frac{TR^{\alpha_{\rm bu}}}{P_{{\rm h}q}k_{{\rm h}q}}$ and $s_{\rm f}=\frac{\gamma_q R^{\alpha_{\rm bu}}}{P_{{\rm f}q}k_{{\rm f}q}}$. γ_q represents the threshold SINR for classifying CCUs or CEUs. L_{I_m} is the LT of interference I_m ($m\in\{{\rm h}q,{\rm f}q\}$) and they are defined in (9) and (13). L_U which is defined in (18), denotes the joint LT of interference $I_{{\rm f}q}$ and $I_{{\rm h}q}$:

$$L_U=\begin{cases}L_{\rm hu}(s_{\rm h})\,L_{\rm fu}(s_{\rm f}), & \text{if } q=\rm u,\\ L_{\hat{I}_{\rm fd},I_{\rm fd}}(s_{\rm h},s_{\rm f}), & \text{if } q=\rm d,\end{cases} \tag{18}$$

where $L_{\rm hu}(\cdot)$, $L_{\rm fu}(\cdot)$ and $L_{\hat{I}_{\rm fd},I_{\rm fd}}(\cdot,\cdot)$ are defined in (9), (13) and (15), respectively.

Proof. Please see Appendix D.

4.2 Coverage probability of CCUs in FFD cellular system

In this section, the coverage probability of CCUs in FFD cellular system is derived. Based on the definition, the temporary SINR of CCUs is larger than the predefined threshold γ_q. For saving the space, we also give the coverage probability for both uplink and downlink transmission in unified form.

Theorem 2. The coverage probability defined in (16) of a typical CCU adopting FFD scheme can be derived as

$$F_{\mathrm{CCU},q}(T) = \frac{\int_{R=0}^{\infty} \left(\exp\left(-s_{\mathrm{c}}\sigma'^2\right) L_{I_{\mathrm{f}q}}\left(s_{\mathrm{c}}\right)\right) f_R\left(R\right) \mathrm{d}R}{\int_{R=0}^{\infty} \left(\exp\left(\frac{-\gamma_q R^{\alpha_{\mathrm{bu}}} \sigma'^2}{P_{\mathrm{f}q} k_{\mathrm{f}q}}\right) L_{I_{\mathrm{f}q}}\left(\frac{\gamma_q R^{\alpha_{\mathrm{bu}}}}{P_{\mathrm{f}q} k_{\mathrm{f}q}}\right)\right) f_R\left(R\right) \mathrm{d}R}, \tag{19}$$

where γ_q and T represent the threshold SINR and the target SINR, respectively. $q \in \{u, d\}$ represents uplink or downlink transmission direction. Due to the limited space, we define $s_{\mathrm{c}} = \frac{R^{\alpha_{\mathrm{bu}}} \max(T,\gamma_q)}{P_{\mathrm{f}q} k_{\mathrm{f}q}}$ and $\sigma'^2 = \left(\sigma^2 + \delta I_{\mathrm{SI}}\right)$. L_{I_m} is the LT of interference I_m ($m \in \{\mathrm{h}q, \mathrm{f}q\}$) and they are defined in (9) and (13), respectively.

Proof. The proof is similar with the proof of Theorem 1 and we omit it here.

4.3 Coverage probability for FD and HD cellular system

For comparison, we also obtain the coverage probability for FD and HD cellular systems in this section. The interference suffered in FD and HD cellular systems can be shown in (7) and (8) and we can get their coverage probabilities as the following theorem.

Theorem 3. The coverage probabilities defined in (16) of an user served in FD and HD cellular networks can be derived as (20) and (21), respectively:

$$F_{\mathrm{FD},q}\left(T\right) = \mathbb{P}\left(\mathrm{SINR}_{\mathrm{FD},q} > T\right) = \int_{R=0}^{\infty} \left(\mathrm{e}^{-s_{\mathrm{f}}\left(\sigma^2 + \delta I_{\mathrm{SI}}\right)} L_{I_{\mathrm{f}q}}\left(s_{\mathrm{f}}\right)\right) f_R\left(R\right) \mathrm{d}R, \tag{20}$$

$$F_{\mathrm{HD},q}\left(T\right) = \mathbb{P}\left(\mathrm{SINR}_{\mathrm{HD},q} > T\right) = \int_{R=0}^{\infty} \left(\mathrm{e}^{-s_{\mathrm{h}}\left(\sigma^2\right)} L_{I_{\mathrm{h}q}}\left(s_{\mathrm{h}}\right)\right) f_R\left(R\right) \mathrm{d}R, \tag{21}$$

where $s_{\mathrm{f}} = \frac{\gamma_q R^{\alpha_{\mathrm{bu}}}}{P_{\mathrm{f}q} k_{\mathrm{f}q}}$ and $s_{\mathrm{h}} = \frac{T R^{\alpha_{\mathrm{bu}}}}{P_{\mathrm{h}q} k_{\mathrm{h}q}}$. T is the target SINR. $F(T)$ is the coverage probability which is defined in (16). $q \in \{\mathrm{u}, \mathrm{d}\}$ represents uplink or downlink transmission direction. $L_{I_{\mathrm{f}q}}$ is the LT of interference $I_{\mathrm{f}q}$ defined in (13). $L_{I_{\mathrm{h}q}}$ is the LT of interference $I_{\mathrm{h}q}$ defined in (9).

Proof. The proof is similar with the proof of Theorem 1 and we omit it here.

4.4 Ergodic rate

Spectrum efficiency of the network is a key performance indicator in order to support high-capacity communication. Based on the expectation properties and the coverage probability we can obtain, the average ergodic rate as [21]

$$\mathcal{R}_{\theta,\mathrm{II}} = \mathrm{E}\left(\ln(1 + \mathrm{SINR}_{\theta,q})\right) = \int_{t>0} F_{\theta,q}\left(\mathrm{e}^t - 1\right) \mathrm{d}t, \tag{22}$$

where $\theta \in \{\mathrm{CEU}, \mathrm{CCU}, \mathrm{FD}, \mathrm{HD}\}$ and q denote the type of users and transmission direction, respectively. Then, according to (22) and the coverage probability $F_{\theta,q}(T)$ we have already solved in Theorems 1–3, we can get the average user rates of the cellular systems utilizing different duplex schemes.

5 Numerical evaluation

In this section, numerical evaluations are conducted to evaluate the system performance. Firstly, we confirm the validity of analytical results by simulations. Then, the coverage probability and SE of the system applying HD, FD, and the proposed FFD scheme are compared under different system parameters.

In this section, unless otherwise specified, parameters used in the simulations are presented in Table 1, where $\eta = \eta_{\mathrm{uu}} = \eta_{\mathrm{bb}} = 0$ implies that no interference cancellation method is applied.

Table 1 Simulation parameters

RB band width	Number of RB	Thermal noise	P_b	$P_{\rm u}$	η	$\lambda_{\rm s}$	$\lambda_{\rm u}$	$\gamma_{\rm d}$	$\lvert\varepsilon\rvert^2$	Path loss [25]
1 MHz	128	-174 dBm·Hz^{-1}	0.1 W	0.1 W	0	10^{-3} m^{-2}	1 m^{-2}	0 dB	-110 dB	$140.7 + 36.7\lg R$ (R in km)

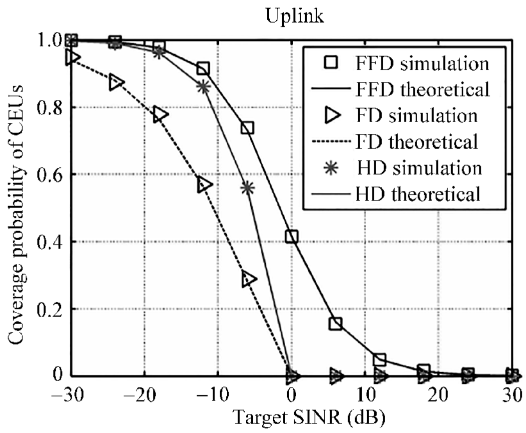

Figure 3 Coverage probability for CEUs.

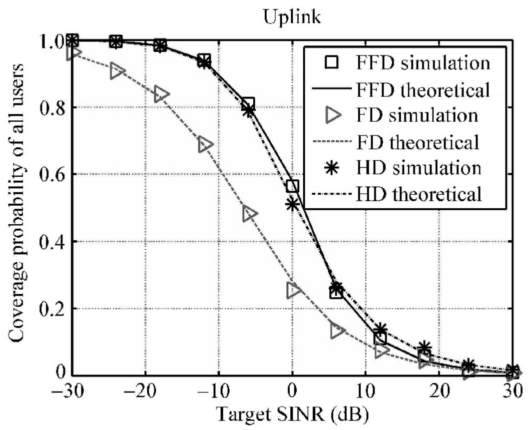

Figure 4 Coverage probability for all users.

5.1 Coverage probability

5.1.1 *Coverage probability for CEUs*

Firstly, we compare the coverage probabilities of CEUs employing HD, FD and FFD modes. Without loss of generality, only the uplink performance is shown and similar results can be obtained in downlink transmission. As illustrated in Figure 3, the lines and the marks are analytical and simulation results, respectively. It can be observe that the simulation results match the analytical results perfectly, which verifies the validity of the analytical results obtained in Section 4. We can also observe that FFD can effectively improve the coverage probabilities of the CEUs. For instance, the SINR of all the CEUs is below 0 dB in FD and HD cellular system, while after applying FFD scheme, the SINR of more than 40% CEUs is higher than 0 dB. The performance gain mainly comes from the fact that when using FFD scheme CEUs are assigned with new HD RBs, then the interference suffered by the CEUs can be largely relieved which results in higher SINR.

5.1.2 *Coverage probability for all users*

Next, as illustrated in Figure 4, we compare the coverage probabilities of all users. Similar results can be obtained in downlink transmission, so only the results in uplink are shown due to the limited space. We can observe that, the coverage probability of FD cellular network are very poor and large proportion of users in FD cellular system may not be able to be served properly. The reason is that serious inter-BS and inter-user interference largely degrades the system performance in FD cellular network. As shown in Figure 4, the coverage probability of FFD scheme is almost the same as HD mode system and much better than FD cellular system. Because the CEUs who suffer severe interference in FD RBs are reallocated HD RBs and the severe interference is greatly reduced. But at higher target SINR, the coverage probability is a little poorer than HD system, which result from the fact that CCUs are served in FD RBs in FFD scheme and more interference is received.

5.2 Spectrum efficiency

SE is a key performance criterion and the system SE performance employing different duplex schemes is evaluated here.

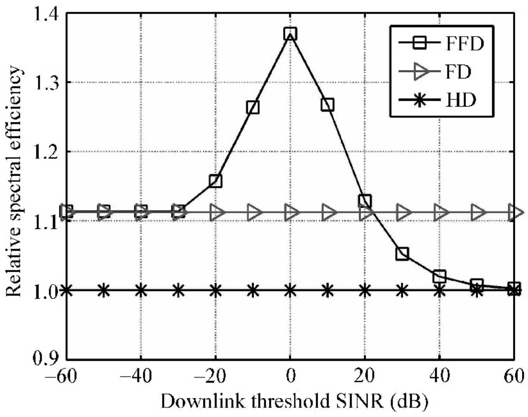

Figure 5 SE vs. threshold SINR.

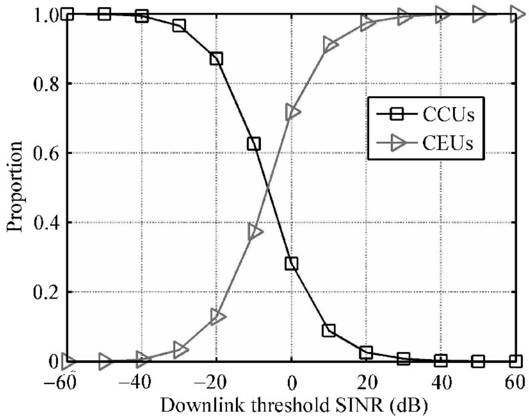

Figure 6 The proportions of CCUs and CEUs.

5.2.1 *Influence of threshold SINR*

As to SE, we first evaluate how the system SE behaves along with the threshold SINR $\gamma_{\rm d}$, which is used to partition the RBs and classify CCUs and CEUs. As illustrated in Figure 5, the SE is normalized based on the SE of HD cellular network. We can observe that although SI can be almost omitted compared with the other interference, the sum SE gain in FD system over HD system is very limited, which mainly results from the severe inter-BS and inter-user interference. When applying our proposed FFD scheme, the SE is significantly improved. For instance, when SINR threshold equals to 0 dB, the SE of FFD system is 1.38 and 1.25 times higher than HD and FD system, respectively.

In addition, SINR threshold is another very important design parameter for FFD. If SINR threshold is sufficiently small, all the users will be classified as CCUs. While, when SINR threshold is large enough, all the users will be sorted as CEUs. This phenomenon is also confirmed by the simulations shown in Figures 5 and 6. When SINR threshold is smaller than -55 dB, almost all the users are regarded as CCUs and the SE of FFD is equal to FD cellular system. On the contrary, when SINR threshold is larger than 55 dB, all the users are treated as CEUs and the SE of FFD is the same as HD system. Therefore, FD and HD cellular systems are two special cases of FFD system. We should point out that Figure 6 also shows the proportions of RBs allocated to CCUs and CEUs.

5.2.2 *Influence of BS density*

Then we analyse how the SE changes along with the BS density. With the cellular network becoming denser, on one hand, the transmission distance gets shorter which results in higher received power, on the other hand, the interference also becomes stronger. Therefore, as shown in Figure 7, at first, the SE grows fast with enhanced BS density. When the BS density is large enough, the SE will become constant afterwards at the settings of this paper. The main reason is that when the cell radius further becomes smaller, the increase of signal strength is counter-balanced by the stronger interference. Hence, UDN is one of key technologies to enhance the SE. We can observe that when BS density is larger than 10^{-6} m^{-2}, SE of FD system is better than HD system, which indicates that dense cellular network is more suitable to deploy FD BSs. While when the BS is sparse, HD mode is more favorable. Moreover, the SE of FFD system outperforms HD and FD system regardless how BS density changes. The SE gain of FFD system over HD and FD system is more attractive when network is denser. Therefore, FFD scheme is an effective method to enhance the SE no matter the BS density is and can achieve higher SE when using in future UDN.

5.2.3 *Influence of SI*

In the following, the influence of SI is analysed. As illustrated in Figure 8, SI is a key influencer of the SE of both FD and FFD system. Because SI can largely degrade the FD system performance, then serving

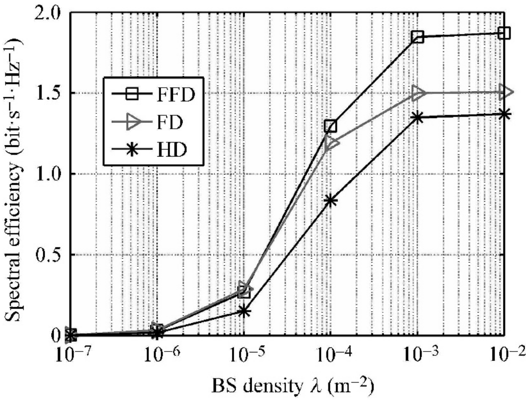

Figure 7 SE vs. BS density.

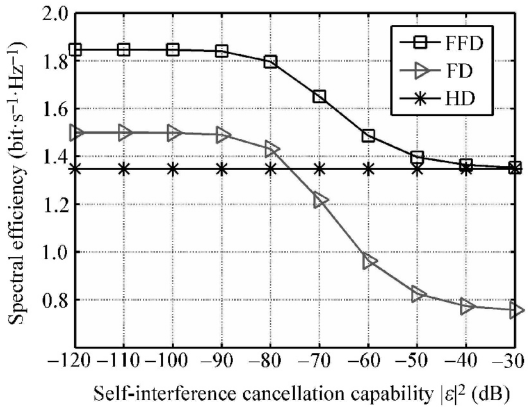

Figure 8 SE vs. self-interference cancelation capability.

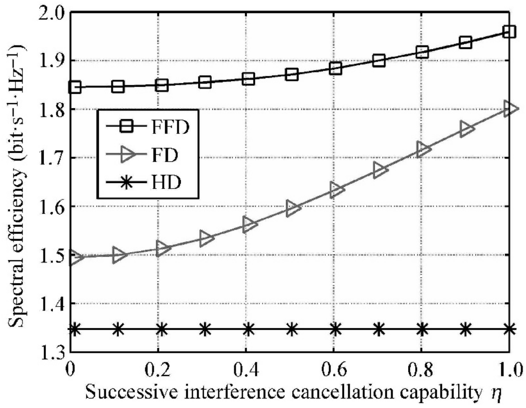

Figure 9 SE vs. successive interference cancellation capability.

some CCUs in FD RBs is even worse than arranging them to be served in HD mode RBs. Therefore, when SI is very strong the SE is the same as HD cellular system, which indicates that all the users are served in HD RBs. In addition, in order to achieve the same SE, the SI attenuation needed for FFD is far smaller than FD system. Under the assumptions of this paper, in order to outperform HD system regarding SE, the minimum SI attenuation for FD system is about −77 dB, while it is about −38 dB for FFD system. Therefore, SI cancelation capability is not very strict for FFD and the SI cancellation algorithm can be easier and simple to be realized.

5.2.4 *Influence of SIC capability*

At last, the SE is evaluated under different SIC capability. Here we assume $\eta_{\rm uu} = \eta_{\rm bb} = \eta$. Just as discussed in Subsection 2.4, we assume the inter-user and inter-BS interference within ηR_m can be completely removed and nothing is done to the interference with distance larger than ηR_m, where R_m is the transmission distance between the focused user and its serving BS. As shown in Figure 9, the SE of FD system is effectively improved with increasing η, which means inter-user and inter-BS interference are crucial part of the overall system interference. Therefore, these two kinds of interference must be well handled in the future FD cellular network. We can also observe that FFD also benefits from the effective SIC scheme and always outperforms other duplex schemes, but it is not sensitive to the SIC capability parameter.

6 Conclusion

In this paper, a new resources allocation method called FFD is proposed. Feasible metrics for partitioning RBs and classifying users are also proposed. Analytical results are derived for both the coverage probabilities and ergodic rates using stochastic geometry method. In the simulations, we compare the system performance under different system parameters. The coverage probability of FFD system is almost the same as HD system and much better than FD system, while its SE can be much higher than both FD and HD system. Under the simulation settings of this paper and proper design of the criterion for classifying CCUs and CEUs, the SE of FFD system is 1.38 and 1.25 times of HD and FD system respectively. Numerical results also show that inter-user and inter-BS interference have a great influence on FD system performance, but are not sensitive for FFD cellular system.

Through numerical results, it can be observed that the criterion for classifying users is a key design parameter to obtain higher SE. In this paper, we mainly focus on the design of FFD scheme and analysing the benefit that FFD scheme can provide when proper criterion is given. Therefore, one of the future research topics is to find the optimal criterion for classifying users. Besides, lower bound of FFD performance is analyzed in this paper with full power transmission at each node and it can be foreseen that proper power control will definitely improve the performance more and provide more space for future system optimization. In addition, RB partition across all the cells is assumed to be synchronous in this paper and more flexible RBs partition scheme will be another interesting topic of the future work.

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Conflict of interest The authors declare that they have no conflict of interest.

References

1 Sabharwal A, Schniter P, Guo D, et al. In-band full-duplex wireless: challenges and opportunities. IEEE J Sel Area Commun, 2014, 32: 1637–1652
2 Kim D, Lee H, Hong D. A survey of in-band full-duplex transmission: from the perspective of phy and mac layers. IEEE Commun Surv Tut, 2015, 17: 2017–2046
3 Bharadia D, McMilin E, Katti S. Full duplex radios. SIGCOMM Comput Commun Rev, 2013, 43: 375–386
4 Duarte M, Dick C, Sabharwal A. Experiment-driven characterization of full-duplex wireless systems. IEEE Trans Wirel Commun, 2012, 11: 4296–4307
5 Goyal S, Liu P, Panwar S, et al. Full duplex cellular systems: will doubling interference prevent doubling capacity? IEEE Commun Mag, 2015, 53: 121–127
6 Liao Y, Song L, Han Z, et al. Full duplex cognitive radio: a new design paradigm for enhancing spectrum usage. IEEE Commun Mag, 2015, 53: 138–145
7 Liu G, Yu F, Ji H, et al. In-band full-duplex relaying: a survey, research issues and challenges. IEEE Commun Surv Tut, 2015, 17: 500–524
8 Riihonen T, Werner S, Wichman R. Hybrid full-duplex/half-duplex relaying with transmit power adaptation. IEEE Trans Wirel Commun, 2011, 10: 3074–3085
9 Ahmed E, Eltawil A, Sabharwal A. Rate gain region and design tradeoffs for full-duplex wireless communications. IEEE Trans Wirel Commun, 2013, 12: 3556–3565
10 Chai X, Liu T, Xing C, et al. Throughput improvement in cellular networks via full-duplex based device-to-device communications. IEEE Access, 2015, 4: 7645–7657
11 Shao S, Liu D, Deng K, et al. Analysis of carrier utilization in full-duplex cellular networks by dividing the co-channel interference region. IEEE Commun Lett, 2014, 18: 1043–1046
12 Bai J, Sabharwal A. Distributed full-duplex via wireless side-channels: bounds and protocols. IEEE Trans Wirel Commun, 2013, 12: 4162–4173
13 Nguyen D, Tran L, Pirinen P, et al. Precoding for full duplex multiuser mimo systems: spectral and energy efficiency maximization. IEEE Trans Signal Process, 2013, 61: 4038–4050
14 Nguyen D, Tran L, Pirinen P, et al. On the spectral efficiency of full-duplex small cell wireless systems. IEEE Trans Wirel Commun, 2014, 13: 4896–4910
15 Boudreau G, Panicker J, Guo N, et al. Interference coordination and cancellation for 4G networks. IEEE Commun Mag, 2009, 47: 74–81

16 Shen Z, Khoryaev A, Eriksson E, et al. Dynamic uplink-downlink configuration and interference management in TD-LTE. IEEE Commun Mag, 2012, 50: 51–59
17 Lee J, Quek T. Hybrid full-/half-duplex system analysis in heterogeneous wireless networks. IEEE Trans Wirel Commun, 2015, 14: 2883–2895
18 AlAmmouri A, ElSawy H, Amin O, et al. In-band α-duplex scheme for cellular networks: a stochastic geometry approach. IEEE Trans Wirel Commun, 2016, 15: 6797–6812
19 Thomsen H, Popovski P, Carvalho E, et al. Compflex: comp for in-band wireless full duplex. IEEE Wirel Commun Lett, 2016, 5: 144–147
20 Li Y, Fan P Z, Leukhin A, et al. On the spectral and energy efficiency of full-duplex small cell wireless systems with massive MIMO. IEEE Trans Veh Tech, 2017, 66: 2339–2353
21 Andrews J, Baccelli F, Ganti R. A tractable approach to coverage and rate in cellular networks. IEEE Trans Commun, 2011, 59: 3122–3134
22 Simeone O, Erkip E, Shamai S. Full-duplex cloud radio access networks: an information-theoretic viewpoint. IEEE Wirel Commun Lett, 2014, 3: 413–416
23 Sato H. The capacity of the gaussian interference channel under strong interference. IEEE Trans Inf Theory, 1982, 27: 786–788
24 Novlan T, Dhillon H, Andrews J. Analytical modeling of uplink cellular networks. IEEE Trans Wirel Commun, 2013, 12: 2669–2679
25 Access E U T R. Further advancements for E-UTRA physical layer aspects. 3GPP Technical Specification TR, 2010, 36: V2

Appendix A Proof of Lemma 1

Starting with the definition of laplace transform, we can get

$$L_{I_{\mathrm{h}q}}(s) \overset{\text{(a)}}{=} \mathrm{E}_{\Phi_{\mathrm{h}q}}\left\{\prod_{z\in\Phi_{\mathrm{h}q}\backslash c_o} \mathrm{E}_{h_{z,d_o}} \exp\left(-s' h_{z,d_o} D_{z,d_o}^{-\alpha_{\mathrm{bu}}}\right)\right\} \overset{\text{(b)}}{=} \exp\left\{-2\pi\lambda_{\mathrm{s}}\int_R^\infty \left\{1-\mathrm{E}_{\mathrm{h}}\left[\mathrm{e}^{\left(-s' h v^{-\alpha_{\mathrm{bu}}}\right)}\right]\right\} v\mathrm{d}v\right\}, \quad \text{(A1)}$$

where c_o and d_o are b_o (u_o) and u_o (b_o) if $q = d(u)$, respectively. (a) follows the fact that $\Phi_{\mathrm{h}q}$ and h_{z,d_o} are independent from each other. Therefore, the expectation order can be exchanged. (b) follows from the probability generating function of PPP. Carrying on the proof in (A1), the followings can be obtained

$$\begin{aligned}&\exp\left\{-2\pi\lambda_{\mathrm{s}}\int_R^\infty \left\{1-\mathrm{E}_{\mathrm{h}}\left[\exp\left(-s' h v^{-\alpha_{\mathrm{bu}}}\right)\right]\right\} v\mathrm{d}v\right\}\\ &\overset{\text{(c)}}{=} \exp\left\{-\frac{2}{\alpha_{\mathrm{bu}}}\pi\lambda_{\mathrm{s}}\mathrm{E}_{\mathrm{h}}\left\{-(s'h)^{\frac{2}{\alpha_{\mathrm{bu}}}}\Gamma\left(-\frac{2}{\alpha_{\mathrm{bu}}}\right)-\frac{\alpha_{\mathrm{bu}}}{2}R^2+(hs')^{\frac{2}{\alpha_{\mathrm{bu}}}}\Gamma\left(-\frac{2}{\alpha_{\mathrm{bu}}}, hs'R^{-\alpha_{\mathrm{bu}}}\right)\right\}\right\}, \quad \text{(A2)}\end{aligned}$$

where (c) is proofed in [17]. Based on the Rayleigh fading assumption, $h \sim \exp(1)$, we can obtain the expectations as

$$\mathrm{E}_{\mathrm{h}}\left\{h^{\frac{2}{\alpha_{\mathrm{bu}}}}\right\} = \int_0^\infty x^{\frac{2}{\alpha_{\mathrm{bu}}}}\mathrm{e}^{-x}\mathrm{d}x \overset{\text{(d)}}{=} \Gamma\left(\frac{2}{\alpha_{\mathrm{bu}}}+1\right), \quad \text{(A3)}$$

$$\mathrm{E}_{\mathrm{h}}\left\{h^{\frac{2}{\alpha_{\mathrm{bu}}}}\Gamma\left(-\frac{2}{\alpha_{\mathrm{bu}}}, hs'R^{-\alpha_{\mathrm{bu}}}\right)\right\} = \int_0^\infty x^{\frac{2}{\alpha_{\mathrm{bu}}}}\Gamma\left(-\frac{2}{\alpha_{\mathrm{bu}}}, xs'R^{-\alpha_{\mathrm{bu}}}\right)\mathrm{e}^{-x}\mathrm{d}x \overset{\text{(e)}}{=} J\left(\frac{2}{\alpha_{\mathrm{bu}}}+1, 1, -\frac{2}{\alpha_{\mathrm{bu}}}, s'R^{-\alpha_{\mathrm{bu}}}\right), \text{(A4)}$$

where $J(\mu,\beta,\nu,\gamma)$ is defined in (11) and proofed in [17]. (d) and (e) can be obtained by Eqs. (3.478) and (3.381)[3)], respectively. Substitute (A3) and (A4) into (A2) and we can get the result of Lemma 1.

Appendix B Proof of Lemma 2

Starting with the definition of laplace transform, we can get

$$\begin{aligned}L_{I_{\mathrm{f}q}}(s) &\overset{\text{(f)}}{=} \mathrm{E}_{D,h,\Phi_{\mathrm{fd}}}\left\{\prod_{z\in\Phi_{\mathrm{fd}}\backslash b_o}\exp\left(-sP_{\mathrm{fd}}h_{z,d_o}k_{z,d_o}D_{z,d_o}^{-\alpha_{\mathrm{d}}}\right)\right\}\mathrm{E}_{D,h,\Phi_{\mathrm{fu}}}\left\{\prod_{x\in\Phi_{\mathrm{fu}}\backslash c_o}\exp\left(-sP_{\mathrm{fu}}h_{x,d_o}k_{x,d_o}D_{x,d_o}^{-\alpha_{\mathrm{u}}}\right)\right\}\\ &\overset{\text{(g)}}{=} \exp\left\{-2\pi\lambda_{\mathrm{s}}\mathrm{E}_{\mathrm{h}}\int_{R_{\mathrm{d}}}^\infty\left\{1-\left[\exp\left(-s_{\mathrm{d}}hv^{-\alpha_{\mathrm{d}}}\right)\right]\right\}v\mathrm{d}v\right\}\exp\left\{-2\pi\lambda_{\mathrm{s}}\mathrm{E}_{\mathrm{h}}\int_{R_{\mathrm{u}}}^\infty\left\{1-\left[\exp\left(-s_{\mathrm{u}}hv^{-\alpha_{\mathrm{u}}}\right)\right]\right\}v\mathrm{d}v\right\}, \quad \text{(B1)}\end{aligned}$$

where c_o and d_o are $\emptyset$ (u_o) and u_o (b_o) if $q = d(\mathrm{u})$, respectively. (f) follows the fact that Φ_{fu} and Φ_{fd} are assumed to be independent. In Step (g), when we focus on the downlink, the inter-user interference distance can be arbitrarily close, therefore the integration lower bound R_{u} is $\eta_{\mathrm{uu}}R$ when taking the interference cancelation strategy into consideration. Based on the association principle, the interfering distance between focused downlink user and neighbor BSs is at least R, so $R_{\mathrm{d}} = R$. While when considering uplink transmission, the inter-BS interference distance between focused BS and the neighbor BSs can also be arbitrarily close, as a consequence R_{d} is $\eta_{\mathrm{bb}}R$ when SIC is applied. In the same way, the distance from the neighbor uplink users to the focused BS is at least R, which gives $R_{\mathrm{u}} = R$. Then following the proof in Appendix A, we can get the result in (13).

3) Gradshteyn I, Ryzhik I. Table of Integrals, Series, and Products. Manhattan: Academic Press, 2014. 346–370.

Appendix C Proof of Lemma 3

Before solving joint LT of $I_{\rm d}$ and $\hat{I}_{\rm d}$, some properties of the sum of two exponential distribution random variables are analysed.

We define G as the sum of two exponential distribution random variables as follows:

$$G = s_2 h_1 + s_2 h_2, \tag{C1}$$

where both h_1 and h_2 follow exponential distribution, i.e, h_1 and $h_2 \sim \exp(1)$. Then, when $s_1 = s_2 = s$, G is the sum of two exponential random variables with same rate. As presented in [17], G follows Erlang distribution. While, if $s_1 \neq s_2$, G follows the hypo-exponential distribution.

Based on the PDF of G, we can have

$$\mathrm{E}_G\left(G^{\delta}\right) = \begin{cases} s^{\delta}\Gamma\left(\delta+2\right), & \text{if } s_1 = s_2 = s, \\ \dfrac{1}{s_2 - s_1}\Gamma\left(\delta+1\right)\left({s_2}^{\delta+1} - {s_1}^{\delta+1}\right), & \text{otherwise}, \end{cases} \tag{C2}$$

where (C2) can be obtained by Eq. (3.478)[3]. Furthermore, according to Eq. (3.381)[3], we can also obtain

$$\mathrm{E}_G\left(G^{\mu-1}\Gamma\left(\nu,\gamma G\right)\right) = \begin{cases} s^2 J\left(\mu{+}1, s^{-1}, \nu, \gamma\right), & \text{if } \mathrm{s_1{=}s_2{=}\,s}, \\ \frac{1}{s_2 - s_1}\left(J\left(\mu, \frac{1}{s_2}, \nu, \gamma\right) - J\left(\mu, \frac{1}{s_1}, \nu, \gamma\right)\right), & \text{otherwise}, \end{cases} \tag{C3}$$

where $J(\cdot,\cdot,\cdot,\cdot)$ is defined in Eq. (11). We also start with the definition of joint LT.

$$\begin{aligned}
&\mathcal{L}_{\hat{I}_{\rm d}, I_{\rm d}}\left(s_{\rm h}, s_{\rm f}\right) = \mathrm{E}_{\hat{I}_{\rm d}, I_{\rm d}}\left(\mathrm{e}^{-s_{\rm h}\hat{I}_{\rm d} - s_{\rm f} I_{\rm d}}\right) \\
&\overset{(k)}{=} \mathrm{E}\left(\exp\left(-s_{\rm hd}\left(\sum_{z\in\Phi_{\rm hd}\backslash b_o} h_{z,u_o} {D_{z,u_o}}^{-\alpha_{\rm bu}}\right) - \left(\sum_{z\in\Phi_{\rm fd}\backslash b_o} s_{\rm fd} h_{z,u_o} {D_{z,u_o}}^{-\alpha_{\rm bu}} + \sum_{x\in\Phi_{\rm fu}} s_{\rm fu} h_{x,u_o} {D_{x,u_o}}^{-\alpha_{\rm uu}}\right)\right)\right) \\
&\overset{(l)}{=} \mathrm{E}\left(\exp\left(-\left(\sum_{z\in\Phi_{\rm fd}\backslash b_o}\left(s_{\rm hd} h_{z,u_o} + s_{\rm fd} h_{z,u_o}\right) {D_{z,u_o}}^{-\alpha_{\rm bu}}\right) - s_{\rm fu}\left(\sum_{x\in\Phi_{\rm fu}} h_{x,u_o} (D_{x,u_o})^{-\alpha_{\rm uu}}\right)\right)\right) \\
&\overset{(m)}{=} \exp\left\{-2\pi\lambda_{\rm s}\mathrm{E}\int_R^{\infty} 1 - \mathrm{e}^{\left(-\left(s_{\rm fd}h_{\rm fd} + s_{\rm hd}h_{\rm hd}\right)v^{-\alpha_{\rm bu}}\right)} v\mathrm{d}v\right\} \exp\left\{-2\pi\lambda_{\rm s}\mathrm{E}\int_{\eta_{\rm uu}R}^{\infty} 1 - \mathrm{e}^{\left(-s_{\rm fu}h_{\rm fu}v^{-\alpha_{\rm uu}}\right)} v\mathrm{d}v\right\} \\
&\overset{(n)}{=} \exp\left\{-2\pi\lambda_{\rm s}\mathrm{E}_{G_{\rm u}}\left\{-\frac{1}{\alpha_{\rm bu}} {G_{\rm u}}^{\frac{2}{\alpha_{\rm bu}}}\Gamma\left(-\frac{2}{\alpha_{\rm bu}}\right) - \frac{1}{2}R^2 + \frac{1}{\alpha_{\rm bu}} {G_{\rm u}}^{\frac{2}{\alpha_{\rm bu}}}\Gamma\left(-\frac{2}{\alpha_{\rm bu}}, \frac{G_{\rm u}}{R^{\alpha}}\right)\right\}\right\}\omega\left(\alpha_{\rm uu}, s_{\rm fu}\eta_{\rm uu}R\right),
\end{aligned} \tag{C4}$$

where $s_{\rm fd} = s_{\rm f}P_{\rm fd}k_{\rm fd}$, $s_{\rm hd} = s_{\rm h}P_{\rm hd}k_{\rm hd}$ and$s_{\rm fu} = s_{\rm f}P_{\rm fu}k_{\rm fu}$ in Step (k). In Step (l), the interference BS sets in HD RBs and FD RBs are the same, therefore, the interfering distances between the focused downlink user and the BSs after classifying the user as CEU is the same as the distances when the focused user is regarded as cell CCU. Hence, we can write them together as the first sum item of Step (l). $G_{\rm u} = s_{\rm fd}h_{\rm fd} + s_{\rm hd}h_{\rm hd}$ which is the same as (C1) and its properties have already been given by (C2) and (C3). Then the conclusion of Lemma 3 is obtained.

Appendix D Proof of Theorem 1

The proof begins with the definition of the CEU and coverage probability, then we can obtain

$$\begin{aligned}
&F_{{\rm edge},q}\left(T\right) = \mathbb{P}\left(\mathrm{SINR}' > T | \mathrm{SINR} < \gamma_q\right) \\
&\overset{\rm (h)}{=} \frac{\mathrm{E}\left(\mathbb{P}\left(\mathrm{SINR}' > T, \mathrm{SINR} < \gamma_q | R, I_{{\rm f}q}, I_{{\rm h}q}\right)\right)}{\mathrm{E}\left(\mathbb{P}\left(\mathrm{SINR} < \gamma_q | R, I_{{\rm f}q}\right)\right)} \overset{(I)}{=} \frac{\mathrm{E}\left(\mathrm{e}^{\left(-s_{\rm h}\left(\sigma^2 + \hat{I}_{\rm e}\right)\right)}\left(1 - \mathrm{e}^{\left(-s_{\rm f}\left(\sigma^2 + I_{\rm e} + \delta I_{\rm SI}\right)\right)}\right)\right)}{\mathrm{E}\left(\left(1 - \mathrm{e}^{\left(-s_{\rm f}\left(\sigma^2 + I_{\rm e} + \delta I_{\rm SI}\right)\right)}\right)\right)},
\end{aligned} \tag{D1}$$

where SINR and $I_{\rm e}$ denote the received SINR and interference when the user is regarded as CCU. While SINR$'$ and $\hat{I}_{\rm e}$ represent the received SINR and interference after the user is classified as CEU. SINR is defined as (5). Step (h) follows Bayes' theorem. Step (I) is obtained based on two facts. one is the independency of the small scale fading over FD and HD RBs, the other fact is that the small scale fading is assumed to be Rayleigh, i.e, $h \sim \exp(1)$, so $\mathbb{P}(h < t) = 1 - \exp(-t)$. Then carry on the proof in Step (I) and we have

$$\begin{aligned}
&F_{{\rm edge},q}\left(T\right) = \mathbb{P}\left(\mathrm{SINR}' > T | \mathrm{SINR} < \gamma_q\right) = \frac{\int_{R=0}^{\infty}\mathrm{E}\left(\mathrm{e}^{\left(-s_{\rm h}\sigma^2\right)}\left(\mathrm{e}^{-s_{\rm h}\hat{I}_{\rm e}} - \mathrm{e}^{\left(-s_{\rm f}\sigma'^2 + s_{\rm h}\hat{I}_{\rm e} + s_{\rm f}I_{\rm e}\right)}\right)\right) f_R\left(R\right)\mathrm{d}R}{\int_{R=0}^{\infty}\mathrm{E}\left(1 - \mathrm{e}^{\left(-s_{\rm f}\left(\sigma'^2 + I_{\rm e}\right)\right)}\right) f_R\left(R\right)\mathrm{d}r} \\
&\overset{\rm (j)}{=} \frac{\int_{R=0}^{\infty}\left(\mathrm{e}^{-s_{\rm h}\sigma^2}\left(L_{I_{{\rm h}q}}\left(s_{\rm h}\right) - \mathrm{e}^{-s_{\rm f}\sigma'^2} L_U\left(s_{\rm h}, s_{\rm f}\right)\right)\right) f_R\left(R\right)\mathrm{d}R}{\int_{R=0}^{\infty}\left(1 - \mathrm{e}^{-s_{\rm f}\sigma'^2} L_{I_{{\rm f}q}}\left(s_{\rm f}\right)\right) f_R\left(R\right)\mathrm{d}R},
\end{aligned} \tag{D2}$$

where $f_R(R)$ is defined in (1). We can obtain Step (j) according to the definition of LT. L_{I_m} is the LT of interference I_m ($m \in \{{\rm h}q, {\rm f}q\}$) and they are defined in (9) and (13). The joint LT L_U is different in the downlink and uplink transmission. In downlink transmission, the interfering BSs are the same before and after allocating HD RBs to the CEUs, which results in $\hat{I}_{\rm e}$ and $I_{\rm e}$ are correlated with each other. As discussed in Lemma 3, L_U is defined in 18. While in the uplink, the uplink interfering user sets have already changed when the serving RBs of CEU switch from FD RBs to HD RBs. Therefore, in uplink L_U can be written as $L_{\rm hu}\left(s_{\rm h}\right)L_{\rm fu}\left(s_{\rm f}\right)$. Then the conclusion of Theorem 1 is obtained.