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Economic power dispatch in smart grids: a framework for distributed optimization and consensus dynamics

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Abstract By using the distributed consensus theory in multi-agent systems, the strategy of economic power dispatch is studied in a smart grid, where many generation units work cooperatively to achieve an optimal solution in a local area. The relationship between the distributed optimization solution and consensus in multi-agent systems is first revealed in this paper, which can serve as a general framework for future studies of this topic. First, without the constraints of capacity limitations, it is found that the total cost for all the generators in a smart grid can achieve the minimal value if the consensus can be reached for the incremental cost of all the generation units and the balance between the supply and demand of powers is kept. Then, by designing a distributed consensus control protocol in multi-agent systems with appropriate initial conditions, incremental cost consensus can be realized and the balance for the powers can also be satisfied. Furthermore, the difficult problem for distributed optimization of the total cost function with bounded capacity limitations is also discussed. A reformulated barrier function is proposed to simplify the analysis, under which the total cost can reach the minimal value if consensus can be achieved for the modified incremental cost with some appropriate initial values. Thus, the distributed optimization problems for the cost function of all generation units with and without bounded capacity limitations can both be solved by using the idea of consensus in multi-agent systems, whose theoretical analysis is still lacking nowadays. Finally, some simulation examples are given to verify the effectiveness of the results in this paper.

Keywords power dispatch strategy, consensus, incremental cost, capacity limitations, distributed protocol, multi-agent systems

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1 Introduction

Economic power dispatch in smart grids has been widely discussed recently in the literature, which is actually one of the important problems in smart grids. The objective is to find a optimal solution for the total generation cost or operation [1]. Recently, many existing optimization methods have been investigated,

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for example, the genetic algorithm was proposed in [2]. By utilizing evolutionary programming and sequential quadratic programming, a hybrid method was constructed in [3]. Some algorithms were designed to solve the economic power dispatch problem by using particle swarm optimization approach [4, 5].

However, in the conventional optimization algorithms for economic power dispatch, there are still some disadvantages. First, in all these algorithms [2–5], a control center is required, which can access information data of all the generation units. Obviously this cannot work effectively for the generation units with a very large-scale size. Actually, in the real world, there are many generation units in a smart grid and it is quite impossible to process all the data in such a center [6]. Second, the information data exchange may not be accurately measured due to the external disturbances, failure of devices, unreliability of information channel, and time delay. Then, it is needed to design a robust algorithm to overcome these shortcomings for solving this optimization problem. Third, a smart grid can be regarded as a complex network, where the nodes represent the generation units and the edges describe the information exchange between the nodes [7]. From the network point of view, how the network structure affects the optimization problem of economic power dispatch is still unclear.

Thus, distributed consensus strategy is a good way to solve the economic power dispatch optimization problem by only utilizing local information based on the network structure. Recently, distributed cooperative control of multi-agent systems has been widely studied [8–10], where multiple agents work cooperatively with local information and have the ability of self-organization. In particular, each agent can only communicate with its own local neighbors and a collective behavior can still be reached even under these local interactions. Actually, consensus is one of the most basic collective behaviors in multi-agent systems [9, 11–18].

Very recently, the distributed optimization problem has been widely investigated based on the discrete-time and continuous-time algorithms [19–21]. Then, some results have been proposed for applying consensus in multi-agent systems into distributed power dispatch in a smart grid [22–24]. Although these studies are very interesting and provide some good ideas, there are still several shortcomings. First, the designed protocol is not fully distributed where the leader can access all the information of the whole network [22–24]. Second, it is unknown whether the consensus state is the optimal solution for the economic dispatch problem. Third, some studies for distributed economic power dispatch in a smart grid with capacity limitations were given [25–27]. However, the rigorous analysis for the convergence of the algorithm was not discussed, which lacks theoretical analysis and this is actually the most important issue. Without addressing this issue, it would be unconvincing to apply such algorithms. In addition, it was found that complete consensus state cannot be achieved from the simulations in [25, 26]. Furthermore, in [28], an external penalty based distributed consensus approach was introduced to solve the economic dispatch problem, where a distributed allocation algorithm was employed to handle generator initialization. All incremental costs of generations can achieve consensus. However, during the dynamical evolution, some generations run beyond their maximal capacity due to external penalty.

The main contribution of this paper is four folds. First, it is theoretically proved that if the consensus state for the incremental cost or its modified version in a smart grid is achieved, the total cost function will reach its minimal value, which solves the fundamental problem for the relationship between the consensus and optimal value. Second, two fully distributed protocols are designed to reach consensus of the incremental cost for all the generation units with and without capacity limitations. It is first theoretically analyzed that complete consensus for the incremental cost can be reached, which solves the optimization problem for the total cost and contributes to the theoretical gap for the previous research work. Third, the nonlinear coupled protocol for power dispatch with capacity limitations is proposed, which is very difficult to be analyzed. Fourth, the most important contribution is that a fundamental framework for studying distributed economic power dispatch by using consensus is established.

In this paper, the objective is to derive the rigorous analysis of two distributed consensus strategies for economic power dispatch without and with capacity limitations. It is found that the minimal value of the total cost for generators can be obtained if the incremental cost consensus for all generation units can be reached and the balance between supply and demand of powers is kept. By using theories of multi-agent systems, some distributed consensus protocols are designed for incremental cost with appropriate initial conditions.

2 Preliminaries

Some preliminaries results about algebraic graph theory and distributed protocols of consensus in multi-agent systems are introduced in this section.

2.1 Algebraic graph theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, G)$ be a weighted undirected network of order N, with the set of vertices $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, the set of undirected edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $G = (G_{ij})_{N\times N}$. An undirected edge $\mathcal{E}_{ij}$ in the network $\mathcal{G}$ is denoted by the unordered pair of vertices (v_i, v_j), which means that vertices v_i and v_j can communicate with each other. In view of the definition of adjacency matrices for weighted graphs [29], $G_{ij} = G_{ij} > 0$ if and only if there is an undirected edge (v_i, v_j) in $\mathcal{G}$. The degree of vertex v_i is defined by $k_i = \sum_{j=1,j\neq i}^N G_{ij}$, which is the total weights between vertex v_i and all the other vertices. In this paper, only positively weighted undirected networks are considered, which indicates that communication between two nodes are mutual. As usual, we assume there is no self-loop in $\mathcal{G}$.

An undirected path between vertices v_i and v_j is a sequence of edges $(v_i, v_{i_1}), (v_{i_1}, v_{i_2}), \ldots, (v_{i_l}, v_j)$ in the undirected network with distinct vertices v_{i_m}, $m = 1, 2, \ldots, l$ [30, 31]. An undirected graph $\mathcal{G}$ is connected if between any pair of distinct vertices v_i and v_j in $\mathcal{G}$, there exists an undirected path between v_i and v_j, $i, j = 1, 2, \ldots, N$. For simplicity, assume that the network is connected throughout the paper.

The Laplacian matrix L is defined by

$$L_{ij} = -G_{ij}, \quad i \neq j; \quad L_{ii} = -\sum_{j=1,j\neq i}^N L_{ij}, \tag{1}$$

which ensures the diffusion that $\sum_{j=1}^N L_{ij} = 0$.

The following well-known results are introduced here [17, 29]:

(1) In an undirected network $\mathcal{G}$, the Lapacian matrix L has a simple eigenvalue 0 and all the other eigenvalues are positive if and only if the directed network is connected.

(2) The second smallest eigenvalue $\lambda_2(L)$ of the Laplacian matrix L in an undirected network $\mathcal{G}$ satisfies

$$\lambda_2(L) = \min_{x^{\mathrm{T}}\mathbf{1}_N=0,x\neq 0} \frac{x^{\mathrm{T}}Lx}{x^{\mathrm{T}}x}. \tag{2}$$

(3) For any $\eta = (\eta_1, \ldots, \eta_N)^{\mathrm{T}} \in \mathbb{R}^N$, $\eta^{\mathrm{T}}L\eta = \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N G_{ij}(\eta_i - \eta_j)^2$.

(4) If the Laplacian matrix L is irreducible, then the general algebraic connectivity $a(L) > 0$, where

$$a(L) = \min_{x^{\mathrm{T}}\xi=0,x\neq 0} \frac{x^{\mathrm{T}}\widehat{L}x}{x^{\mathrm{T}}\Xi x}, \tag{3}$$

with $\widehat{L} = \frac{\Xi L + L^{\mathrm{T}}\Xi}{2}$, $\Xi = \mathrm{diag}(\xi_1, \ldots, \xi_N)$, $\xi = (\xi_1, \xi_2, \ldots, \xi_N) > 0$, $\xi^{\mathrm{T}}L = 0$, and $\sum_{i=1}^N \xi_i = 1$.

2.2 Consensus in multi-agent systems

The distributed consensus protocol in a multi-agent system is described as follows [9, 13, 16]:

$$\dot{x}_i(t) = u_i(t), \quad u_i(t) = \sum_{j=1,j\neq i}^N G_{ij}(x_j(t) - x_i(t)) = -\sum_{j=1}^N L_{ij}x_j(t), \quad i = 1, 2, \ldots, N, \tag{4}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector of the ith vertex which can represent any physical quantity including voltage, output power, or incremental cost, $u_i(t)$ is the control input, and $G = (G_{ij})_{N\times N}$ is the coupling configuration matrix representing the topological structure of the

the network. Note that consensus protocol in (4) is distributed where each agent can only use the local information of its neighboring agents.

Now, the concept of consensus in multi-agent systems is given.

Consensus in multi-agent system (4) is said to be achieved if for any initial conditions,

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j = 1, 2, \ldots, N.$$

A classical study for consensus in multi-agent system (4) is proposed in [9, 13, 16]:

Consensus in multi-agent system (4) can be achieved if the undirected network is connected, and $\lim_{t\to\infty} \|x_i(t) - x^*(t)\| = 0,\ \forall i,\ j = 1, 2, \ldots, N$, where $x^*(t) = \frac{1}{N}\sum_{k=1}^{N} x_k(t)$ is the average state center of all the agents.

3 Distributed economic power dispatch strategy in a smart grid without capacity limitations

By using the idea of consensus in multi-agent systems, a fully distributed economic power dispatch strategy is proposed in this section to achieve the minimal value of the total cost of all the generation units in a smart grid without capacity limitations.

3.1 Problem formulation

First, we show that global optimal solution of the total cost function can be obtained when the incremental consensus can be reached. Then, a fully distributed consensus protocol is designed to solve the economic power dispatch problem in a smart grid, which is different from the conventional centralized algorithms [2, 4, 5].

Assume the generation unit i is in the form of a quadratic cost function [23, 24]:

$$C_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2, \quad i = 1, 2, \ldots, N, \tag{5}$$

where P_{Gi} denotes the output power of generator i, and the parameters $\alpha_i, \beta_i, \gamma_i > 0$. Then, the total cost C of the whole generator systems is

$$C = \sum_{i=1}^{N} C_i(P_{Gi}). \tag{6}$$

Economic power dispatch problem aims to minimize the total cost of N generators under the balance condition between supply and demand of powers,

$$\min_{P_{Gi}} \sum_{i=1}^{N} C_i(P_{Gi}) = \min_{P_{Gi}} \sum_{i=1}^{N} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) \quad \text{s.t.} \quad \sum_{i=1}^{N} P_{Gi} = P_D, \tag{7}$$

where P_D denotes the total power demand. Let $P_G = (P_{G1}, \ldots, P_{GN})^{\mathrm{T}}$ and the constraint can be rewritten as $P_G^{\mathrm{T}} 1_N = P_D$ where 1_N is the N-dimensional vector with all elements being 1.

3.2 Optimization by consensus

In this subsection, the main results will be given to solve optimization problem (7). First, the Incremental Cost (IC) of generator i is defined by $\mathrm{IC}_i = \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi}$, $i = 1, 2, \ldots, N$. Next, we will derive the result as follows.

The optimization problem (7) can have its optimal solution under the following two conditions:

$$\mathrm{IC}_i = \mathrm{IC}_j, \quad \forall i, j = 1, \ldots, N, \tag{8}$$

and

$$\sum_{i=1}^{N} P_{Gi} = P_D, \tag{9}$$

which indicates that consensus for the incremental cost of all generators can be reached and the balance between the supply and demand powers can be satisfied.

In order to solve the optimization problem (7), the conventional Lagrange multiplier method is utilized. Consider the following Lagrange function:

$$\begin{aligned} f(P_G, \lambda) &= \sum_{i=1}^{N} C_i(P_{Gi}) + \lambda \left(P_D - \sum_{i=1}^{N} P_{Gi} \right) \\ &= \sum_{i=1}^{N} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \lambda \left(P_D - \sum_{i=1}^{N} P_{Gi} \right), \end{aligned} \tag{10}$$

where λ is the Lagrange multiplier. Then, the optimal solution of (7) can be obtained if

$$\frac{\partial f}{\partial P_{Gi}} = \frac{\partial C_i}{\partial P_{Gi}} - \lambda = \beta_i + 2\gamma_i P_{Gi} - \lambda = 0, \tag{11}$$

and

$$\frac{\partial f}{\partial \lambda} = P_D - \sum_{i=1}^{N} P_{Gi} = 0. \tag{12}$$

Thus, at the optimal value, one has $\frac{\partial C_i}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi} = \lambda$ for all $i = 1, \ldots, N$ and $\sum_{i=1}^{N} P_{Gi} = P_D$.

From the above discussion, one can see that the total cost of all the generation units in a smart grid can obtain its optimal value if the incremental cost consensus (8) and balance (9) conditions are satisfied. The condition (9) is easy to understand, which requires the balance between supply and demand powers. As for condition (8), one has that the smaller γ_i, the more effectiveness of the generator unit i, which indicates that the generator unit i can produce more output powers under the same cost.

In the existing literature, many centralized economic power dispatch algorithms have been proposed to solve problem (7). However, in a large-scale smart grid, it is quite impossible to deal with all the units' information data in a processing center. Then, a distributed strategy using only local information could be very desirable and robust. In particular, distributed consensus algorithms in multi-agent systems can be utilized. In multi-agent systems, each agent can only communicate with its neighbors locally for the incremental cost state. Then, it can processor its local information to reach consensus for the incremental cost of all generators.

3.3 Distributed consensus protocol design for power dispatch without capacity limitations

From the above subsection, the framework of incremental cost consensus is the same as the previous consensus studies literature [11, 13, 15], where the incremental cost is regarded as the system states. However, the big difference is the global constraint (9). This actually makes the problem totally difficult. In [22–24], though the consensus protocol in multi-agent systems was utilized, there is a leader which should measure the information of output power of the whole network. Actually, this is a centralized protocol which utilizes the global information.

As usual, the network is assumed to be connected. That is, each generation unit can communicate with its neighbors, for example, the network in Figure 1, where the nodes with big circles are local leaders processing the information of local neighboring agents.

Denote the incremental cost of generator i by $y_i(t) = \mathrm{IC}_i = \beta_i + 2\gamma_i P_{Gi}(t)$, $i = 1, 2, \ldots, N$. In order to design a fully distributed strategy for economic power dispatch in a smart grid under a global constraint

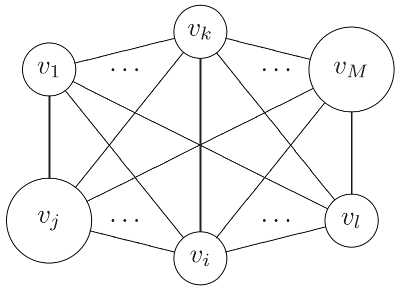

Figure 1 Illustration of networks with pinning control.

condition (9), a new distributed consensus algorithm is designed as follows:

$$\begin{aligned}\dot{y}_i(t) &= 2\gamma_i \sum_{j=1,j\neq i}^{N} G_{ij}(y_j(t)-y_i(t)) \\ &= -2c\gamma_i \sum_{j=1}^{N} L_{ij}y_j(t), \quad i=1,2,\ldots,N \end{aligned} \tag{13}$$

with initial conditions $y_i(0)=\beta_i+2\gamma_i P_{Gi}(0)$, $i=1,2,\ldots,N$, satisfying

$$\sum_{i=1}^{N} P_{Gi}(0)=P_D, \tag{14}$$

and $c>0$ being a constant representing the design gain of the convergence.

The condition (14) can be easily satisfied. Assume that there exist l hub generators $i_1,\ldots,i_l$ which are local leaders for their area regions. They are responsible for the regulation of their area and the demand of output power is first delivered into these leaders. Then, at each local area, the demand of output power is known and we can easily set

$$\sum_{j=1}^{l} P_{Gi_j}(0)=P_D, \quad i=1,2,\ldots,N. \tag{15}$$

Then, the optimization problem (7) can be solved if the distributed protocol is designed in (13) with initial conditions in (14). To give the detailed proof, the analysis can be performed by the following two steps.

Step 1. Balance condition between supply and demand of powers.

Let $\xi_k=\frac{1}{2\gamma_k}/\gamma$, $k=1,2,\ldots,N$, and $y^*(t)=\sum_{k=1}^{N}\xi_k y_k(t)=\frac{1}{\gamma}\sum_{k=1}^{N}(\beta_k/(2\gamma_k)+P_{Gk}(t))$ be the weighted average center of the incremental cost of all generators, where $\gamma=\sum_{k=1}^{N}\frac{1}{2\gamma_k}$. Then, due to the property that L is symmetric and zero row-sum, one has

$$\dot{y}^*(t)=-\frac{c}{\gamma}\sum_{i=1}^{N}\sum_{j=1}^{N}L_{ij}y_j(t)=0=\frac{1}{\gamma}\sum_{k=1}^{N}\dot{P}_{Gk}(t), \tag{16}$$

which indicates that

$$\sum_{k=1}^{N}P_{Gk}(t)=\sum_{k=1}^{N}P_{Gk}(0)=P_D. \tag{17}$$

From the above equality, one sees that the balance condition between the supply and demand of powers in (9) is satisfied, since the total output power of all generation units is a time-invariant constant.

Step 2. Consensus for incremental cost of generators.

Let $e_i = y_i - y^*$ be the error states between the ith generator and the weighted centroid of all the agents and $e = (e_1, \ldots, e_N)^{\rm T}$. It is easy to check that $\sum_{k=1}^N \xi_k = 1$ and $\sum_{k=1}^N \xi_k e_k = 0$. Then, one obtains the error system

$$\begin{aligned}\dot{e}_i(t) &= 2c\gamma_i \sum_{j=1, j\neq i}^N G_{ij}(e_j(t) - e_i(t)) \\ &= -2c\gamma_i \sum_{j=1}^N L_{ij} e_j(t), \quad i = 1, 2, \ldots, N.\end{aligned} \tag{18}$$

Consider the Lyapunov function candidate

$$V(t) = \sum_{i=1}^N e_i^{\rm T}(t) e_i(t)/(2\gamma_i). \tag{19}$$

From the results in Subsection 2.1, the derivative of $V(t)$ along the trajectories of (19) gives

$$\begin{aligned}\dot{V} &= 2\sum_{i=1}^N e_i^{\rm T}(t)\dot{e}_i(t)/(2\gamma_i) = -2c\sum_{i=1}^N\sum_{j=1}^N e_i^{\rm T}(t) L_{ij} e_j(t) \\ &= -2ce^{\rm T}(t)Le(t) = -c\sum_{i=1}^N\sum_{j=1}^N G_{ij}(e_i - e_j)^2,\end{aligned} \tag{20}$$

where $e(t) = (e_1^{\rm T}(t), e_2^{\rm T}(t), \ldots, e_N^{\rm T}(t))^{\rm T}$. By applying LaSalle invariance principle [32], consensus can be achieved for the incremental cost of all the generation units. From the results in Subsection 2.2, one can also derive the convergence rate of consensus.

A fully distributed power dispatch strategy is designed based on consensus in multi-agent systems, which is different from the centralized algorithms in the existing literature. In order to keep the balance between supply and demand powers, some initial conditions in (15) should be satisfied. Otherwise, the incremental cost consensus keeping the balance of powers cannot be reached. However, the designed protocol is distributed since one/several nodes should know the partial/total demand powers only at the initial states. Then, through distributed control, the optimal solution for the economic power dispatch in smart grids can be solved.

3.4 Convergence analysis

In the distributed algorithm for economic power dispatch strategy, the key performance index of convergence is very critical, which reflects the time for reaching consensus. From the above discussion, one has

$$\dot{V} = -2ce^{\rm T}(t)Le(t) \leqslant -ca(\Xi^{-1}L)e^{\rm T}(t)\Xi e(t) \leqslant -ca(\Xi^{-1}L)V. \tag{21}$$

Then, we can see that $e(t)$ exponentially converges to the consensus value with the convergence rate $-ca(\Xi^{-1}L)$, that is, $e_i(t) \sim \sqrt{\gamma_i} e^{-ca(\Xi^{-1}L)t}$, where $\Xi = {\rm diag}(\frac{1}{2\gamma_1}, \ldots, \frac{1}{2\gamma_N})$. Actually, this is still a theoretical value, and the states can converge to the consensus state very quickly.

4 Distributed power dispatch strategy in a smart grid with capacity limitations

In this section, a fully economic distributed power dispatch algorithm is designed to minimize the cost function of the generators in a smart grid with capacity limitations by applying consensus in multi-agent systems.

4.1 Problem formulation

Consider the same cost function as in (6) and optimization problem as in (7) with capacity limitations

$$\min_{P_{Gi}} \sum_{i=1}^{N} C_i(P_{Gi}) = \min_{P_{Gi}} \sum_{i=1}^{N} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2)$$
$$\text{s.t.} \sum_{i=1}^{N} P_{Gi} = P_D, \quad P_i^m \leqslant P_{Gi} \leqslant P_i^M, \tag{22}$$

where each power generation P_{Gi} is constrained by capacity limitation, that is, $P_i^m \leqslant P_{Gi} \leqslant P_i^M$, $i = 1, 2, \ldots, N$.

In order to use the above analysis for consensus strategy in a smart grid without capacity limitation, a new reformulated function called barrier function C_i^θ with a sufficiently small value θ is proposed, which can transform the original optimization problem into a new one without constraint. For example, θ logarithmic barrier function

$$\begin{aligned} C_i^\theta(P_{Gi}) &= C_i(P_{Gi}) - \theta \left(\ln(P_{Gi} - P_i^m) + \ln(P_i^M - P_{Gi})\right) \\ &= \sum_{i=1}^{N} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) - \theta \left(\ln(P_{Gi} - P_i^m) + \ln(P_i^M - P_{Gi})\right), \end{aligned} \tag{23}$$

which is a convex function and is valid if and only if the box capacity limitations in (22) is satisfied, that is, $P_i^m \leqslant P_{Gi} \leqslant P_i^M$ in its domain. Then, the original optimization problem in (22) is equivalent to the following new problem:

$$\min_{P_{Gi}} \textstyle\sum_{i=1}^{N} C_i^\theta(P_{Gi}) \quad \text{s.t.} \quad \sum_{i=1}^{N} P_{Gi} = P_D, \tag{24}$$

which is an optimization problem without capacity limitations. Thus, the analysis in the above section can be utilized.

4.2 Optimization by consensus

For a sufficiently small value θ, a modified incremental cost is defined, which is useful for the following analysis. The modified incremental cost of generator i is defined by

$$\mathrm{IC}_i^\theta = \frac{\partial C_i^\theta(P_{Gi})}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi} - \frac{\theta}{P_{Gi} - P_i^m} + \frac{\theta}{P_i^M - P_{Gi}}, \quad i = 1, 2, \ldots, N. \tag{25}$$

Next, by the defined modified incremental cost in (25), the result for solving the optimization problem (23) will be discussed later. Similarly, we will show that if the modified incremental cost of all generators can reach consensus, the optimization problem (24) can simultaneously obtain its global optimal solution.

The optimization problem (24) can obtain its optimal solution if the following two conditions are satisfied:

$$IC_i^\theta = IC_j^\theta, \quad \forall i, j = 1, \ldots, N, \tag{26}$$

and

$$\sum_{i=1}^{N} P_{Gi} = P_D, \tag{27}$$

which indicates that the modified incremental cost of all generators reach consensus.

Similarly, the Lagrange multiplier method is used with the following Lagrange function:

$$f^\theta(P_G, \lambda) = \sum_{i=1}^{N} C_i^\theta(P_{Gi}) + \lambda \left(P_D - \sum_{i=1}^{N} P_{Gi}\right)$$

$$= \sum_{i=1}^{N} \left(\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2\right) - \theta \left(\ln(P_{Gi} - P_i^m) + \ln(P_i^M - P_{Gi})\right)$$
$$+\lambda \left(P_D - \sum_{i=1}^{N} P_{Gi}\right), \tag{28}$$

where λ is the Lagrange multiplier. Then, it is well known that the minimum value of (28) can be reached if

$$\frac{\partial f}{\partial P_{Gi}} = \frac{\partial C_i^\theta}{\partial P_{Gi}} - \lambda = \mathrm{IC}_i^\theta - \lambda = 0, \tag{29}$$

and

$$\frac{\partial f}{\partial \lambda} = P_D - \sum_{i=1}^{N} P_{Gi} = 0.$$

Thus, at the optimal value, one has $\frac{\partial C_i^\theta}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi} - \frac{\theta}{P_{Gi}-P_i^m} + \frac{\theta}{P_i^M - P_{Gi}} = \lambda$ for all $i = 1, \ldots, N$ and $\sum_{i=1}^N P_{Gi} = P_D$.

Then, one similarly knows that the total cost of generators in a smart grid can reach its minimal value if the output power dispatch satisfies the conditions for the modified incremental cost consensus (26) and balance between supply and demand powers (27). Let the modified incremental cost $z_i(t) = \mathrm{IC}_i^\theta = \beta_i + 2\gamma_i P_{Gi} - \frac{\theta}{P_{Gi}-P_i^m} + \frac{\theta}{P_i^M - P_{Gi}}$, $i = 1, 2, \ldots, N$ and the consensus state be z^*. The following two cases can occur at the consensus state.

(1) If P_{Gi} does not sufficiently reach the bound P_i^m or P_i^M, then $z_i(t) \approx \beta_i + 2\gamma_i P_{Gi} \to z^*$ since θ is a sufficiently small value. This is consistent with power dispatch without capacity limitations as the powers do not reach the bounds.

(2) If P_{Gi} sufficiently reach the bound P_i^m or P_i^M, then the modified incremental cost can reach consensus as $z_i(t) = z^*$, where $P_{Gi} \to P_i^m$ or $P_{Gi} \to P_i^M$.

The above discussion is quite common in the optimization. The optimal value is achieved when the variables reach the consensus states in the box bounds or just arrive at the bounds. From above discussion, one can see that a smaller θ can induce a more accurate incremental cost consensus. However, this may cause a lower convergence rate.

4.3 Distributed consensus protocol design for power dispatch with capacity limitations

In order to design the distributed consensus protocol, the modified incremental cost of generator i for $z_i(t) = \mathrm{IC}_i^\theta = \frac{\partial C_i^\theta(P_{Gi})}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi} - \frac{\theta}{P_{Gi}-P_i^m} + \frac{\theta}{P_i^M - P_{Gi}}$ can be considered as a state variable, $i = 1, 2, \ldots, N$. Then, a distributed protocol for power dispatch in a smart grid with capacity limitations is designed as follows:

$$\dot{P}_{Gi}(t) = c \sum_{j=1, j\neq i}^{N} G_{ij}(z_j(t) - z_i(t))$$
$$= -c \sum_{j=1}^{N} L_{ij} z_j(t), \quad i = 1, 2, \ldots, N, \tag{30}$$

with initial conditions $P_{Gi}(0)$ satisfying

$$\sum_{i=1}^{N} P_{Gi}(0) = P_D, \quad i = 1, 2, \ldots, N, \tag{31}$$

and $c > 0$ being a constant affecting the convergence rate of the system.

Note that the designed distributed consensus protocol for economic power dispatch without capacity limitations (13) is a special case of (30) with capacity limitations, where the incremental cost $y_i(t)$ in (13) is replaced by the modified one $z_i(t)$ in (30). Next, a theorem is provided to see whether the distributed consensus protocol (30) can work.

Then, under the designed distributed protocol (30) with initial conditions in (31), the optimization problem (24) can be solved.

In order to show that the optimization problem (7) can be solved, one only needs to check if the conditions (8) and (9) are satisfied. Similarly, this analysis can be complected by the following two steps for the consensus condition of the modified incremental cost (26) and the balance condition (27).

Step 1. Balance between supply and demand of powers.

Since L is symmetric and zero row-sum, from (30), one has

$$\sum_{i=1}^{N} \dot{P}_{Gi}(t) = -c\sum_{i=1}^{N}\sum_{j=1}^{N} L_{ij} z_j(t) = 0, \tag{32}$$

which indicates that

$$\sum_{k=1}^{N} P_{Gk}(t) = \sum_{k=1}^{N} P_{Gk}(0) = P_D. \tag{33}$$

Thus, balance between supply and demand of powers is kept if the initial condition (31) is satisfied.

Step 2. Consensus for incremental cost of generators.

Since $z_i(t) = \beta_i + 2\gamma_i P_{Gi} - \frac{\theta}{P_{Gi}-P_i^m} + \frac{\theta}{P_i^M-P_{Gi}}$, system (30) is nonlinear-coupled in terms of variable P_{Gi} or z_i. In order to solve the consensus problem for the modified incremental cost, it is quite difficult to apply the results without capacity limitations directly. First, the relationship between P_{Gi} or z_i will be analyzed. By simple calculation, one has

$$\frac{\partial z_i}{\partial P_{Gi}} = 2\gamma_i + \frac{\theta}{(P_{Gi}-P_i^m)^2} + \frac{\theta}{(P_i^M-P_{Gi})^2} > 0, \tag{34}$$

which indicates that $z_i(t)$ is monotonically increasing for $P_{Gi}(t)$ on the interval (P_i^m, P_i^M).

Let $z_{\max} = \max_{1\leqslant i\leqslant N}\{z_1, z_2, \ldots, z_N\}$ and $z_{\min} = \min_{1\leqslant i\leqslant N}\{z_1, z_2, \ldots, z_N\}$ be the largest and smallest incremental cost of the units. Consider the distance function

$$\bar{V}(t) = z_{\max}(t) - z_{\min}(t). \tag{35}$$

Next, one aims to prove that $\bar{V}(t)$ tends to zero and thus consensus for the modified incremental cost can be reached.

Let $k = \arg\max_{1\leqslant i\leqslant N}\{z_1, z_2, \ldots, z_N\}$. Then, $z_k(t)$ is the maximum value at time t, that is, $z_k(t) \geqslant z_i(t)$ for all $i = 1, 2, \ldots, N$. From protocol (30), it is easy to see that $\dot{P}_{Gk}(t) \leqslant 0$, where the equality is satisfied if and only if consensus for the modified incremental cost $z_i(t)$ can be reached. Then, power of generator k P_{Gk} is decreasing, which induces the decrease of $z_k(t)$ at time t. Thus, the maximum value $z_k(t)$ decreases at any time t unless the consensus can be reached. Similarly, the minimum value also increases at any time except the consensus state. Then, one knows that $\bar{V}(t)$ in (35) is lower bounded and strictly monotonically decreasing at no consensus state. Therefore, $\bar{V}(t)$ converges to zero, where the modified incremental cost can reach consensus, that is, $z_i(t) - z_j(t)$ tends to zero for all $i, j = 1, 2, \ldots, N$.

Thus, a fully distributed power dispatch strategy is designed for a smart grid with capacity limitations based on consensus in multi-agent systems where consensus for the modified incremental cost of all the generators is reached and the information for balance between supply and demand powers is also kept. Note that this is quite different from the studies for power dispatch in a smart grid without capacity limitations. Even though the consensus state for the incremental cost can still be reached, the powers at some generators may achieve at the bounds. In addition, the nonlinear-coupled protocol makes analysis very difficult.

Table 1 Parameters of three-unit system

Unit	α_i	β_i	γ_i	$P_{Gi}(0)$
1	561	7.92	0.001562	300
2	310	7.85	0.00194	250
3	78	7.8	0.00482	100

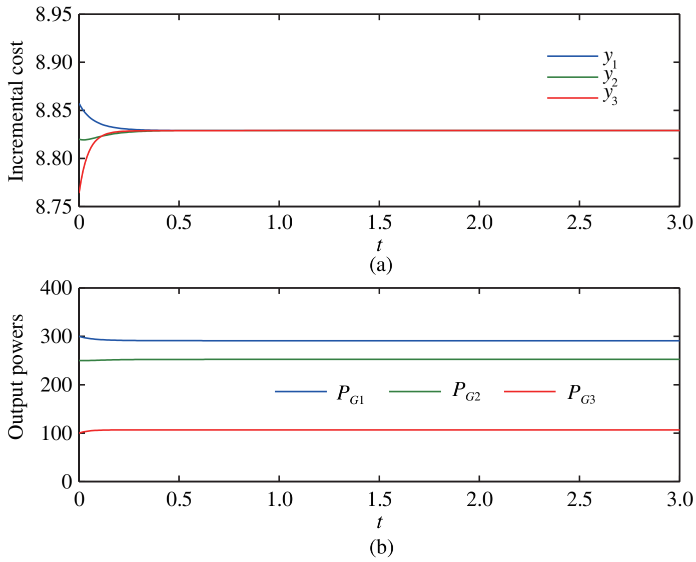

Figure 2 (Color online) (a) The incremental cost of all three generation units as well as (b) their corresponding output powers in Table 1.

5 Simulation examples

In this section, some simulation examples are given to show the effectiveness of the proposed distributed economic power dispatch algorithms.

5.1 Three-unit system with step input load

Case I. Three-unit system. In this subsection, the case in [23] with the same parameters is studied where there are three generation units, and the network structure is fully connected. The parameters and initial conditions are illustrated in Table 1.

By solving the optimal problem for the total cost, one can obtain the optimal solution for the incremental cost $\lambda = 8.8289$ \$/MWh when the total output power demand $P_D = 650$ as given in Table 1. Then, the corresponding output powers for these three generators are 290.9565 MWh, 252.3062 MWh, and 106.7373 MWh.

A fully distributed power dispatch strategy (13) is designed here. The incremental cost of all three generation units as well as their corresponding output powers are shown in Figure 2. Note that this is consistent with the exact optimal solution. In order to reach fast convergence, the control gain $c = 1000$ is chosen. From Figure 2, one has that consensus can be achieved at the optimal solution state $\lambda = 8.8289$ \$/MWh.

Case II. Three-unit system with step input load. Assume that the power demand has a change and increases from 650 MWh to 850 MWh at 1 s. Given that the power demand is 850 MWh, the optimal incremental cost is $\lambda = 9.1224$ \$/MWh. By simple calculation, the corresponding output powers of three generators are 384.8875 MWh, 327.9352 MWh, and 137.1772 MWh. Then, applying the distributed power dispatch protocol, one can see that consensus can be achieved even if there is a sudden change or jump for the total demand. The incremental cost of these three generation units as well as their corresponding output powers are illustrated in Figure 3.

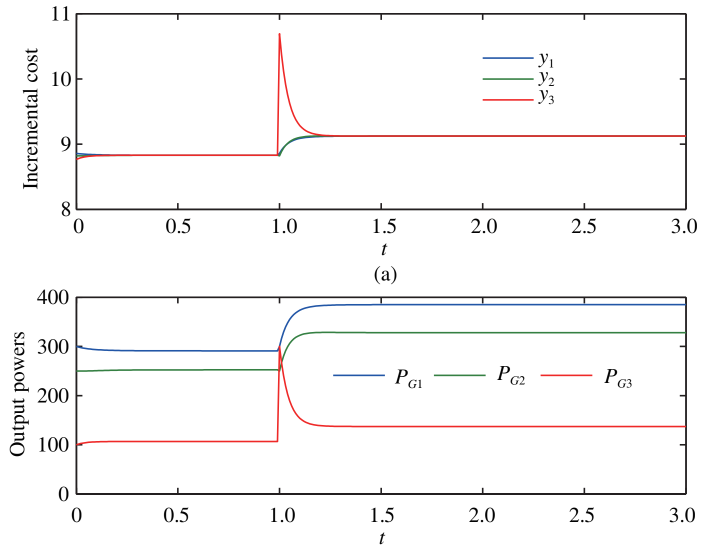

Figure 3 (Color online) (a) The incremental cost of all three generation units as well as (b) their corresponding output powers at power demands 650 MWh and 850 MWh.

Table 2 Parameters of 100-unit system in a scale-free network

Unit	α_i	β_i	γ_i	$P_{Gi}(0)$
i	[100, 500]	[7.5, 8]	[0.001, 0.004]	[175, 225]

5.2 100-unit system with scale-free complex network structure

Consider a well-known complex network with scale-free topology and there are 100 nodes describing all the generation units [33]. At each time step, a new node in the scale-free network is added into 5 existing nodes based on the preferential attachment of the degrees.

The total demand for the powers is 20 MWh for all these 100 generation units. In particular, the parameters of the cost function for all the generators are randomly selected from $\alpha_i \in [100, 500]$, $\beta_i \in [7.5, 8]$, and $\gamma_i \in [0.001, 0.004]$ as shown in Table 2. For simplicity, choose $P_{Gi}(0) \in [175, 225]$ satisfying the initial condition (14). Then, the optimal incremental cost is $\lambda = 99.7200$ \$/MWh if the total output power demand is P_D=20 MWh. The distributed power dispatch strategy can be designed accordingly. The incremental cost of all generation units as well as the output powers in a scale-free network are illustrated in Figure 4, which verify the effectiveness of the results in this paper.

5.3 Three-unit system with capacity limitations

Consider the same parameters as in case I with capacity limitations. Here, the capacity limitations for generation units are listed in Table 3 with initial powers. Note that the optimal values for all units in case I is 290.9565, 252.3062, and 106.7373 MWh, which actually are outside the capacity limitations for generator three. By designing protocol (30), the optimization problem for economic power dispatch can be solved where $\theta = 0.05$ is chosen.

From the analysis, one obtains the optimal modified incremental cost $\lambda = 8.7531$ \$/MWh when the total output power demand $P_D = 650$ as in Table 3. In addition, the corresponding output powers for these three generation units are 267.1343, 232.7648, and 150.1010 MWh. The states for the modified incremental cost of all three generation units and their corresponding output powers are shown in Figure 5, where the generation power for unit 3 reaches the lower bound.

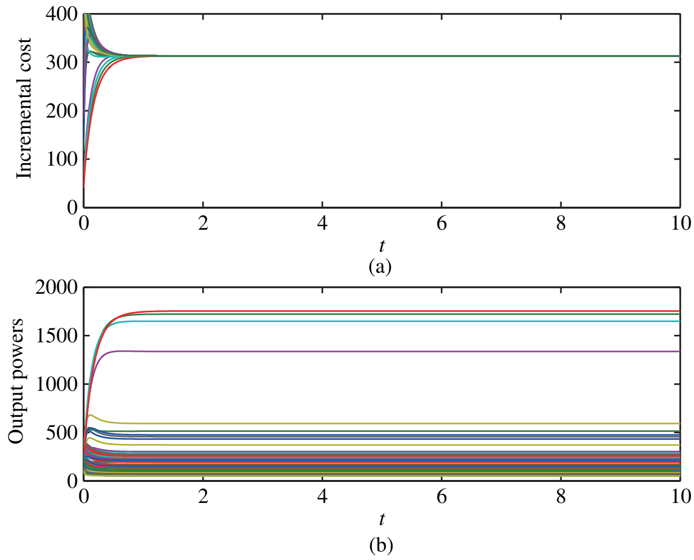

Figure 4 (Color online) (a) The incremental cost of all generation units as well as (b) their corresponding output powers in a scale-free network structure.

Table 3 Parameters of three-unit system

Unit	P_i^m	P_i^M	$P_{Gi}(0)$
1	250	300	260
2	200	300	220
3	150	200	170

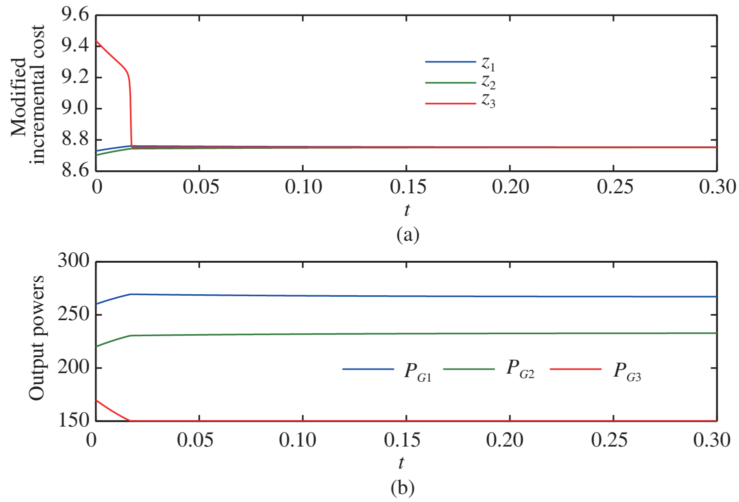

Figure 5 (Color online) (a) The modified incremental cost of all three generation units with capacity limitations as well as (b) their corresponding output powers in Table 3.

5.4 IEEE 30-bus

Optimal economic dispatch of IEEE 30-bus test case will be illustrated by our algorithm. The parameters of cost function are given in Table 4 and choose $\theta = 0.005$. Assume that the demand $P_D = 295.36$ and $P_0 = \{133.36, 70, 40, 20, 15, 17\}$. Given the IEEE 30-bus test diagram (Figure 6) with a communication topology in Figure 7, the optimal solution is obtained at 808.5405 and the optimal output powers are 193.0615, 48.5339, 19.5968, 12.0459, 10.0892, 12.0326.

The modified incremental cost is shown in Figure 8, where consensus is achieved. It is easy to see that the output powers of generators 11 and 13 only reach their minimal capacity at the optimal solution for

Table 4 Parameters of generations in IEEE 30-bus

Unit (Generator No.)	α_i	β_i	γ_i	$P_{\min}$	$P_{\max}$
1 (1)	0	2.00	0.00375	50	200
2 (2)	0	1.75	0.01750	20	80
3 (5)	0	1.00	0.06250	15	50
4 (8)	0	3.25	0.00834	10	35
5 (11)	0	3.00	0.02500	10	30
6 (13)	0	3.00	0.02500	12	40

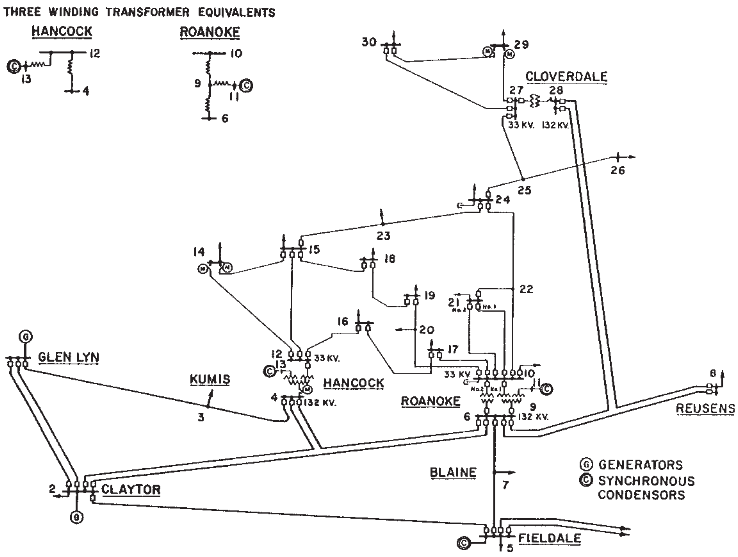

Figure 6 The diagram of IEEE 30-bus.

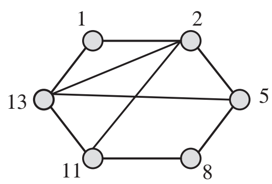

Figure 7 The communication topology of IEEE 30-bus.

the total cost function.

6 Conclusion

By using the distributed consensus theory in multi-agent systems, the strategy of economic power dispatch in a smart grid with and without capacity limitations has been discussed in this paper, where many generation units working cooperatively to achieve an optimal solution in a local area. It has been shown that the total cost for all the generators in a smart grid can reach the minimal value if the incremental cost for all the generation units can achieve consensus and the balance between the supply and demand of powers is kept. Then, two distributed consensus protocols for the generation powers in multi-agent systems have been designed with appropriate initial conditions. Thus, by using distributed consensus in multi-agent systems, the economic power dispatch optimization problem for the total cost function of all generation units in a smart grid with and without capacity limitations has been be carefully solved, which contributes the theoretical framework between distributed optimization and cooperative consensus control in multi-agent systems.

Compared with the conventional centralized optimization results for economic power dispatch strategies

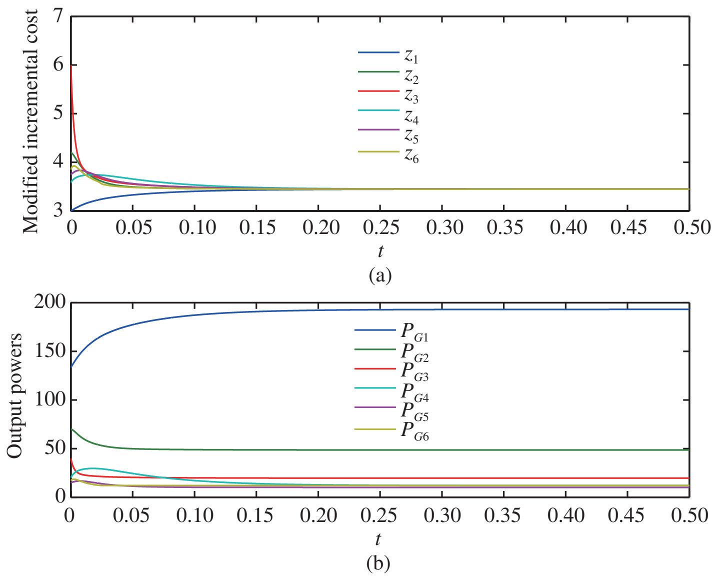

Figure 8 (Color online) (a) The states for the modified incremental cost of all generation units with capacity limitations and (b) their corresponding output powers in Table 4 for IEEE 30-bus test system.

in the existing literature, the distributed protocol designed here is very meaningful. Some realistic models on distributed economic power dispatch strategies in smart grids, for example, thermal constraints, network constrains, ramp up/down rates, power flows of network topology, reservers, and power losses, will be our future work.

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