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Optimal control data scheduling with limited controller-plant communication

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Abstract This paper considers optimal control data scheduling for finite-horizon linear quadratic regulation (LQR) control of scalar systems with limited controller-plant communication. Both the single-system and multiple-system scenarios are studied. For the first scenario, we derive the necessary and sufficient condition for a comparison function to be positive. Using this condition, the optimality of an explicit schedule is extended from unstable systems in the existing work to general systems. For the second scenario, we are able to construct explicit optimal scheduling policies for three particular classes of problems. Numerical examples are provided to illustrate the proposed results.

Keywords control data scheduling, LQR control, optimal schedule, limited transmission energy, multiple systems

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1 Introduction

The advent of digital technologies has brought about a tight combination among communication, computational agents and physical processes. This new type of systems referred to as 'cyber-physical systems' (CPSs) [1] occur in various areas as energy, health care, military and transportation. However, the constraints in CPSs, such as limited sensor/controller energy, limited communication bandwidth and limited computational resource, restrict the system performance, which brings on several challenges in system analysis and design [2–5].

Recently, data scheduling problems for control and estimation of systems in CPSs have received considerable attention [6–8]. Lots of results focused on the stability of systems. Sinopoli et al. [9] indicated that there exists a critical value of the packet arrival rate for state estimation, below which the expected estimation error will become unbound. You and Xie [10] investigated the minimum data rate for mean square stabilizability of linear systems over a packet-dropping network. Wen and Guo [11] considered scheduling and controller co-design for simultaneously stabilizing a collection of plants in a CPS framework. To stabilize a time delay system under the communication bandwidth constraint, a combined dynamic-static scheduling method was proposed in [12]. Guo et al. [13] studied the system stability and controller design for linear systems where the selection of the sensors and actuators was driven by random events. In addition to the issues of system stability, another important problem is to find an optimal schedule to

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minimize a given performance index and the motivation arises from resource limitations caused by CPSs. This paper targets at finding the optimal control data schedule for limited controller-plant communication, which belongs to the latter. Optimal control/sensor data scheduling generally is NP-hard due to the huge solution space and the nonlinearity of the problem. Many existing studies focus on convex relaxation techniques but usually generate suboptimal schedules [14, 15]. However, we may obtain an optimal solution for some specific problems and some efforts have been attempted.

Imer and Başar [16] studied the optimal linear quadratic Gaussian control for scalar systems with limited controls where the controller only can act N times units within a time-horizon $M > N$. The authors showed that the optimal policy is a threshold type on the optimal state estimate generated by Kalman filter. Further, Bommannavar and Başar [17] considered a similar problem where the transmission channel was lossy. They showed that the optimal policy is of the threshold type again. Lincoln and Bernhardsson [18] considered an optimal LQR problem of switching systems. They proposed a branch-and-bound algorithm to efficiently prune the search tree and proved the optimality of a found candidate switch sequence. Savage and La Scala [19] considered an optimal measurement scheduling problem for a class of unstable scalar systems with a terminal estimation cost. Within a time-horizon T, they showed that a simple index policy is optimal under the constraint that only $d < T$ measurements can be taken. Moreover, Yang and Shi [20] proved that the index policy still is optimal for more general unstable scalar systems. The sensor scheduling problems with sensor energy constraint over a packet-dropping network were considered in [21, 22], where an optimal periodic and an optimal dynamic schedule were developed respectively to minimize the average estimation error covariance.

Optimal scheduling problems for multiple systems was also considered in [19, 23–26]. Savage and La Scala [19] showed that a simple index policy can be optimal to schedule multiple particular unstable scalar systems for certain problems with a terminal estimation cost. Howard et al. [23] considered a multiple-target tracking problem and the objective function is the sum of the track error covariances of each target generated by the Kalman filter. They presented that the optimal policy is greedy for scheduling multiple identical particular scalar systems. La Scala and Moran [24] further considered more general scalar systems. It is shown that the greedy policy still holds under certain conditions, with a constant gain filter instead of the Kalman filter. Cabrera [25] studied the estimation performance of the greedy policy for scheduling identical scalar systems. Shi and Zhang [26] considered scheduling two different high-order systems and proposed an explicit optimal periodic sensor schedule to minimize the average estimation error covariance.

In this paper, we consider optimal control data scheduling for finite-horizon LQR control of scalar systems with limited controller-plant communication. The closely related work to this paper is by Shi et al. [27], where the authors considered the finite-horizon LQR control for a class of scalar systems with limited controller-plant communication. Compared with [27], we have the following major differences.

(1) For the single-system scenario, we aim find an optimal control data schedule among general scalar systems, whereas Ref. [27] only analyzed a class of scalar systems.

(2) The optimal control data scheduling for multiple systems that is not considered by [27] is studied in this paper, which is more complex than that for the single-system scenario.

The main contributions are summarized as follows.

(1) For the single-system scenario, we extend the previous work Shi et al. did in [27] where an explicit optimal control data schedule was provided for unstable scalar systems. First, we derive the necessary and sufficient condition for a comparison function to be positive. Based on the condition, the optimality of the schedule presented by [27] is proved for both stable and unstable scalar systems.

(2) We consider the optimal control data scheduling problems involving multiple systems and present explicit optimal scheduling policies for several classes of problems.

The remainder of this paper is organized as follows. Section 2 formulates the mathematical framework of the problems considered in this paper. The problem of optimal control data scheduling for single system is studied in Section 3. Several results are presented for multiple systems in Section 4. Simulation examples are provided in Section 5. Some concluding remarks are provided in the end.

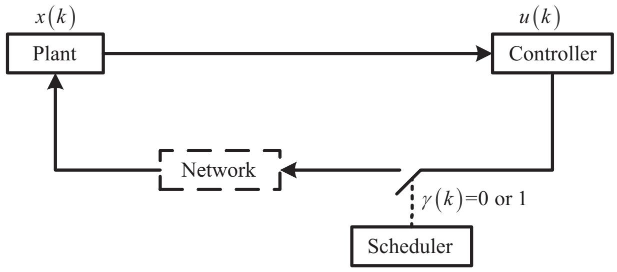

Figure 1 Control data scheduling diagram.

Notations. The positive integer k is the time index. $\mathbb{R}^+$ denotes the set of non-negative real numbers. $\mathbb{R}^{++}$ is set of positive real numbers. Let $m, n \in \mathbb{R}^{++}$; $\mathbb{R}^{m\times n}$ denotes the set of m by n real-valued matrices. For brevity, denote $\mathbb{R}^m := \mathbb{R}^{m\times 1}$. For $X, Y \in \mathbb{R}^{n\times n}$, $X > (\geqslant) Y$ means $X - Y$ is positive definite (semidefinite). For functions f, f_1, f_2 with the appropriate domains, $f_1 f_2(z) := f_1(f_2(z))$ and $f^t(z) := f(f^{t-1}(z))$ with $f^0(z) := z$.

2 Problem setup

2.1 System dynamics

Consider the following scalar discrete linear time-invariant system [27] (Figure 1):

$$x(k+1) = ax(k) + \gamma(k)bu(k), \quad k = 0, 1, \ldots, T-1, \tag{1}$$

where $x(k)$ is the system state, $u(k)$ is the control input, $a, b \neq 0$ are constant values, and $\gamma(k) \in \{0, 1\}$ is the decision variable whether the controller sends $u(k)$ to the plant. Here we omit the actuator node, which is generally co-located with the plant. In addition, the system (1) means that "zero control" is applied by the actuator for those times during which the control $u(k)$ is not sent. Assume the initial condition $x(0)$ is known to the controller. The cost function associated with system (1) is

$$\sum_{k=0}^{T-1} (qx^2(k) + \gamma(k)ru^2(k)) + q_T x^2(T), \tag{2}$$

where $q \in \mathbb{R}^{++}$ is the state weight, $r \in \mathbb{R}^{++}$ is the control weight and $q_T \in \mathbb{R}^+$ is the weight reflecting the penalty imposed on the terminal state.

For convenience, denote a sequence of binary-valued variables from time k to $T-1$ as

$$\Gamma(k, T-1) := [\gamma(k), \gamma(k+1), \ldots, \gamma(T-1)]. \tag{3}$$

Then for a given sequence $\Gamma(0, T-1)$, the optimal linear control law that minimizes the cost function can be solved by doing backward recursion of the cost function. Similar to the Proposition 3.1 from [27], we make use of the following proposition. The proof is omitted since it is simply based on dynamic programming.

Proposition 1. For a given sequence $\Gamma(0, T-1)$, the optimal control law is

$$u(k) = K_{\Gamma(k,T-1)} x(k), \tag{4}$$

where

$$K_{\Gamma(k,T-1)} = -\frac{abp(k+1)}{b^2 p(k+1) + r}. \tag{5}$$

If $\gamma(k) = 1$, $p(k)$ is computed recursively by

$$p(k) = a^2 p(k+1) - \frac{a^2 b^2 p^2(k+1)}{b^2 p(k+1) + r} + q, \tag{6}$$

and if $\gamma(k) = 0$,

$$p(k) = a^2 p(k+1) + q. \tag{7}$$

The above recursion starts from $p(T) = q_T$. Furthermore, the resulting minimal cost function is given by

$$J := p(0)x^2(0). \tag{8}$$

For simplicity, define $c := b^2/r$. Then Eq. (6) can be written as

$$p(k) = \frac{a^2 p(k+1)}{1 + cp(k+1)} + q. \tag{9}$$

The dynamical equations for updating $p(k)$ can be written as two maps of $\mathbb{R}^+ \to \mathbb{R}^+$, i.e.,

$$h(z) := a^2 z + q, \tag{10}$$

$$g(z) := \frac{a^2 z}{1 + cz} + q. \tag{11}$$

If $|a| < 1$, i.e., a is stable, the recursion $p(k+1) = h(p(k))$ converges to $\bar{p}$ for arbitrary non-negative initial condition, and $\bar{p}$ satisfies

$$\bar{p} = a^2 \bar{p} + q. \tag{12}$$

Hence, $\bar{p} = q/1 - a^2$.

2.2 Control data scheduling problems

Considering limited communication resource, such as the limited transmission energy at controller, the controller at most can communicate with the plant for $d(d \ll T)$ times, i.e.,

$$\sum_{k=0}^{T-1} \gamma(k) \leqslant d. \tag{13}$$

By Proposition 1, the best cost function can be obtained when the transmission time instants are given. The problems considered in this paper are finding the optimal schedule for both the single-system and multiple-system scenarios respectively in a control performance index context.

2.2.1 *Single-system scenario*

We wish to find the optimal schedule that satisfies (13) to minimize J defined by (8). Since $g(z) < h(z)$ and the fact that both h and g are increasing functions, systems being controlled is always beneficial for decreasing the cost. Therefore the optimal schedule remains the same if Eq. (13) is changed to

$$\sum_{k=0}^{T-1} \gamma(k) = d. \tag{14}$$

A schedule θ within a time-horizon T can be denoted as a set of transmission times: $\{k_1, k_2, \ldots, k_d\}$ with $0 \leqslant k_1 < \cdots < k_d \leqslant T - 1$. Clearly, $\gamma_\theta(k_i) = 1, i = 1, 2, \ldots, d$. Denote Θ as the set of all possible schedules, then we consider the following optimization problem.

Problem 1.

$$\begin{aligned}
\min_{\theta\in\Theta}\ & J_\theta = p(0)x^2(0) \\
\text{s.t.}\ & p(T) = q_T, \\
& p(k) = \begin{cases} g(p(k+1)), & \text{if } \gamma(k) = 1, \\ h(p(k+1)), & \text{if } \gamma(k) = 0. \end{cases} \\
& \sum_{k=0}^{T-1} \gamma_\theta(k) = d.
\end{aligned}$$

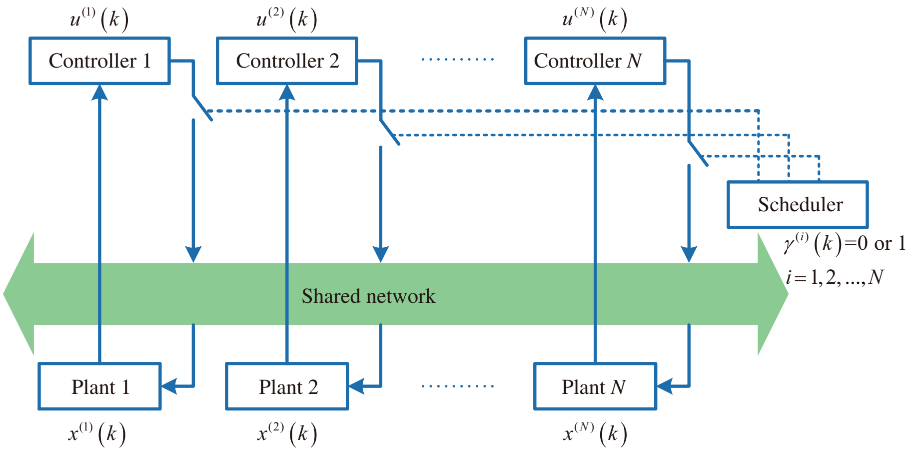

Figure 2 (Color online) Control data scheduling diagram with multiple systems.

2.2.2 *Multiple-system scenario*

In this case, besides the controller energy constraints, there also exists limited communication bandwidth, i.e., bandwidth is shared by many systems accessing the network at the same time and only a subset of controllers may communicate with the corresponding plants at each time (Figure 2). Generally, the control data are sent to the plant using the quadrature amplitude modulation (QAM) where the data are quantized into K bits. Thus the number of systems that can connect to the network simultaneously can be determined by the capacity of communication bandwidth. The dynamics of each system is given by

$$x^{(i)}(k+1) = a^{(i)}x^{(i)}(k) + \gamma^{(i)}(k)b^{(i)}u^{(i)}(k), \quad i = 1, 2, \ldots, N, \tag{15}$$

where $x^{(i)}(k)$, $u^{(i)}(k)$ and $\gamma^{(i)}(k) \in \{0, 1\}$ are the state, the control input and the decision variable of system i respectively. We denote the weights of each system as a tuple $\chi^{(i)} = \{q^{(i)}, q_T^{(i)}, r^{(i)}\}$. To represent the overall uncertainty of the systems, we may use the following performance index

$$V := \sum_{i=1}^{N} J^{(i)} \tag{16}$$

as the total cost function, where

$$\begin{aligned} J^{(i)} &= \min_{u(k)} \sum_{k=0}^{T-1} \left(q^{(i)}(x^{(i)}(k))^2 + \gamma^{(i)}(k)r^{(i)}(u^{(i)}(k))^2 \right) + q_T^{(i)}(x^{(i)}(T))^2 \\ &= p^{(i)}(0)(x^{(i)}(0))^2, \end{aligned} \tag{17}$$

according to Proposition 1, representing the minimal cost function of system $i \in \{1, 2, \ldots, N\}$ under given transmission time instants.

A schedule $\theta \in \Theta$ specifies the value of $\gamma^{(i)}(k)$ for $i \in \{1, 2, \ldots, N\}$ and $k \in \{0, 1, \ldots, T-1\}$. Then the problem of optimal control data scheduling involving multiple systems can be formulated as follows.

Problem 2.

$$\begin{aligned} &\min_{\theta \in \Theta} \ V_\theta = \sum_{i=1}^{N} J_\theta^{(i)} \\ &\text{s.t.} \ \sum_{i=1}^{N} \gamma^{(i)}(k) \leqslant B, \quad k = 0, 1, \ldots, T-1, \\ &\quad\ \ \sum_{k=0}^{T-1} \gamma^{(i)}(k) \leqslant d^{(i)}, \quad i =, 1, 2, \ldots, N, \end{aligned}$$

where $B > 0$ is the communication bandwidth constraint and $d^{(i)}$ is the limited communication times of each controller i.

To the best of our knowledge, it is difficult to obtain an explicit optimal schedule for this general problem. We will consider several concrete problems in Section 4.

3 Optimal control data scheduling for single system

Here we consider the single-system scenario. In [27] the authors showed that an optimal schedule to Problem 1 for unstable scalar systems is controlling the system at the first d time instants. However, they did not analyse whether this schedule is still optimal or is optimal under some conditions for stable scalar systems. In this section, we solve this problem.

From Proposition 1, we notice that $p(k)$ can be computed by multiple compositions of two maps h and g starting from $p(T)$. First, by (10) and (11), we have

$$hg(z) = \frac{a^4 z}{1+cz} + a^2 q + q \tag{18}$$

and

$$gh(z) = \frac{a^2(a^2 z + q)}{1+c(a^2 z+q)} + q. \tag{19}$$

Define a comparison function $F(z)$ as $F(z) := hg(z) - gh(z)$, thus

$$F(z) = a^2 c \frac{a^2(a^2+cq-1)z^2 + (2a^2 q + cq^2)z + q^2}{(1+cz)(1+c(a^2 z+q))} \tag{20}$$

and

$$F(0) > 0. \tag{21}$$

Then $F(z)$ has the following property.

Lemma 1. When $|a| < 1$, $F(z)$ is a piecewise monotone function that strictly increases on $z \in [0, \bar{p})$ and strictly decreases on $z \in (\bar{p}, \infty)$.

Proof. To begin with, taking the derivative of $F(z)$ with respect to z yields

$$\begin{aligned} \frac{\mathrm{d}F(z)}{\mathrm{d}z} &= \frac{\mathrm{d}hg(z)}{\mathrm{d}z} - \frac{\mathrm{d}gh(z)}{\mathrm{d}z} \\ &= \frac{a^4}{(1+cz)^2} - \frac{a^4}{(1+ch(z))^2} \\ &= \frac{a^4 c((a^2-1)z+q)(2+cz+ch(z))}{(1+cz)(1+ch(z))}. \end{aligned}$$

Furthermore, let $\frac{\mathrm{d}F(z)}{\mathrm{d}z} = 0$, then we get $z = q/(1-a^2) = \bar{p}$. Thus, we have $\frac{\mathrm{d}F(z)}{\mathrm{d}z} > 0$ if $0 \leqslant z < \bar{p}$, and $\frac{\mathrm{d}F(z)}{\mathrm{d}z} < 0$ if $z > \bar{p}$, i.e., $F(z)$ strictly increases in $z \in [0, \bar{p})$ and strictly decreases in $z \in (\bar{p}, \infty)$.

Then we have the following result.

Theorem 1. The necessary and sufficient condition for $F(z) > 0, z \in \mathbb{R}^+$ is given as follows:

(1) $|a| \geqslant 1$, $z \geqslant 0$;

(2) $|a| < 1$ with $\rho = a^2(a^2+cq-1) \geqslant 0$, $z \geqslant 0$;

(3) $|a| < 1$ with $\rho = a^2(a^2+cq-1) < 0$, $0 \leqslant z < \tau$, where $\tau = \frac{-\sigma - \sqrt{\sigma^2 - 4q^2\rho}}{2\rho}$ and $\sigma = 2a^2 q + cq^2$.

Proof. "$\Leftarrow$": To prove the above condition is a necessary condition, we ought to obtain the solution set for $F(z) > 0$. The strategy for obtaining the solution set is through discussing the value of a.

First, we consider the case $|a| \geqslant 1$. Obviously, from (20) straightforward computation shows that $F(z) > 0$ for any $z \geqslant 0$. Next, we consider the case $|a| < 1$. With the observation that

$$\begin{aligned} \lim_{z\to\infty} F(z) &= \lim_{z\to\infty} hg(z) - \lim_{z\to\infty} gh(z) \\ &= \frac{a^4}{c} + a^2 q + q - \frac{a^2}{c} - q \\ &= \frac{\rho}{c}, \end{aligned}$$

we know $\lim_{z\to\infty} F(z)$ and ρ have the same sign. Then consider following two cases.

Case 1. $\rho \geqslant 0$. Combining (21) and the piecewise monotonicity of $F(z)$ from Lemma 1, we can conclude that $F(z) > 0$ for any $z \geqslant 0$.

Case 2. $\rho < 0$. Combining (21) and the piecewise monotonicity of $F(z)$ from Lemma 1, one can conclude that there exists a unique $\bar{z} \in \mathbb{R}^+$ making

$$F(\bar{z}) = 0. \tag{22}$$

From (20) and (22),

$$\bar{z} = \frac{-\sigma - \sqrt{\sigma^2 - 4q^2\rho}}{2\rho} = \tau, \tag{23}$$

then we have $F(z) > 0$ for $0 \leqslant z < \tau$.

"$\Rightarrow$": When $|a| \geqslant 1$, from (20), it is clear that $z \geqslant 0$ makes $F(z) > 0$. Now consider the case $|a| < 1$. Since $F(0) > 0$, combining Lemma 1 and $\lim_{z\to\infty} F(z) = \frac{\rho}{c}$, it easy to know that when $\rho \leqslant 0$, $z \geqslant 0$ makes $F(z) > 0$ and when $\rho > 0$, $0 \leqslant z < \tau$ makes $F(z) > 0$.

From this proof, it is clear that $F(z) = 0$ only arises when $|a| < 1$, $\rho < 0$ and $z = \tau$. When $|a| < 1$, $\rho < 0$, we have $\tau > \bar{p}$ since $F(\bar{p}) > 0$ and $F(\tau) = 0$.

Lemma 2. If $|a| < 1$ and $q_T < \bar{p}$ then $g(q_T) < h(q_T) < \bar{p}$. And if $|a| < 1$ and $q_T \geqslant \bar{p}$ then $g(q_T) < h(q_T) \leqslant q_T$ with equality iff $q_T = \bar{p}$.

Proof. We can verify this lemma straightforwardly from the definitions of h, g and $\bar{p}$.

As a consequence of Theorem 1 and Lemma 2, we are ready to present our main result for single scalar system.

Theorem 2 (The optimal schedule)**.** For the following three cases:

Case 1. $|a| \geqslant 1$, $q_T \geqslant 0$,

Case 2. $|a| < 1$, $\rho = a^2(a^2 + cq - 1) \geqslant 0$, $q_T \geqslant 0$,

Case 3. $|a| < 1$, $\rho = a^2(a^2 + cq - 1) < 0$, $0 \leqslant q_T < \tau$, where $\tau = \frac{-\sigma - \sqrt{\sigma^2 - 4q^2\rho}}{2\rho}$ and $\sigma = 2a^2q + cq^2$, the optimal schedule θ^* to Problem 1 is given as follows:

$$\theta^* = \{k_1^*, k_2^*, \ldots, k_d^*\}, \quad k_i^* = i - 1, \quad i = 1, 2, \ldots, d, \tag{24}$$

i.e., the system should be controlled as first as possible.

Proof. Notice that $J_\theta = p(0)x^2(0)$. So to prove $J_{\theta^*} < J_\theta$ for any $\theta \neq \theta^*$, we need to establish that $p_{\theta^*}(0) < p_\theta(0)$. First, we consider Cases 1 and 2. By Theorem 1 and the fact that $h(z) > 0$ and $g(z) > 0$ for any $z \geqslant 0$ and g is increasing function, we have

$$\begin{aligned} p_\theta(0) &= h^{k_1} g h^{k_2-k_1-1} g h^{k_3-k_2-1} \cdots g h^{k_d-k_{d-1}-1} g h^{T-k_d-1}(q_T) \\ &\geqslant g h^{k_2-1} g h^{k_3-k_2-1} \cdots g h^{k_d-k_{d-1}-1} g h^{T-k_d-1}(q_T) \\ &\geqslant g^2 h^{k_3-2} \cdots g h^{k_d-k_{d-1}-1} g h^{T-k_d-1}(q_T) \\ &\;\;\vdots \\ &\geqslant g^{d-1} h^{k_d-d+1} g h^{T-k_d-1}(q_T) \\ &\geqslant g^d h^{T-d}(q_T) = p_{\theta^*}(0). \end{aligned} \tag{25}$$

Since $\theta \neq \theta^*$, the above inequalities at least exists one strict inequality. Thus we have $p_{\theta^*}(0) < p_\theta(0)$. Next, consider Case 3. According to the fact that $\tau > \bar{p}$ and Lemma 2, we get

$$g(q_T) < h(q_T) < \tau, \quad 0 \leqslant q_T < \tau. \tag{26}$$

Therefore for any $\theta \in \Theta$, we obtain

$$p_\theta(k) < \tau, \quad k = 0, 1, \ldots, T - 1 \tag{27}$$

by utilizing (26) repeatedly. Thus by Theorem 1 again, each inequality in (25) also holds for the case $|a| < 1$, $\rho < 0$, $0 \leqslant q_T < \tau$. Again we have $p_{\theta^*}(0) < p_\theta(0)$.

This theorem implies that θ^* presented by Theorem 2 is the optimal schedule for the three cases which cover both stable and unstable systems. Note that such explicit schedule is independent of the terminal state weight q_T and is quite easy to to be implemented in the controller.

Remark 1. It is worth mentioning that for Case 3 in Theorem 2, when $q_T = \tau$, there may exist non-unique optimal schedules since $F(q_T) = 0$ in this case.

Remark 2. Notice that for Case 3 in Theorem 2, if the initial condition is violated, an explicit form of the optimal schedule is difficult obtained for a general time-horizon T and is depend on q_T. However, if d is deterministic but $T \to \infty$, θ^* is still optimal schedule since $\lim_{k\to\infty} h^k(z) = \bar{p} < \tau$, $z \geqslant 0$.

Remark 3. Consider a general vector discrete linear time-invariant system

$$x(k+1) = Ax(k) + \gamma(k)Bu(k), \quad k = 0, 1, \ldots, T-1, \tag{28}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$. The cost function is

$$\sum_{k=0}^{T-1} (x'(k)Qx(k) + \gamma(k)u'(k)Ru(k)) + x'(T)Q_Tx(T). \tag{29}$$

In this context, Ref. [27] provided a condition $(BR^{-1}B')^{-1} < Q$ such that θ^* from (24) is also optimal. Note that this condition only meets the case of $Q > 0$. However, if $Q = 0$, i.e., we only concern the error of the terminal state, θ^* may not be optimal. For example, if there exists the relationship of Loewner partial order [28] between AXA' and X for any $X > 0$, i.e., $AXA' \geqslant X$ or $AXA' < X$, then it is difficult to prove that when $AXA' > X$ θ^* is optimal, when $AXA' = X$ the optimal schedule is any a schedule that satisfies (14), and when $AXA' < X$ the optimal schedule is contrary to θ^*, i.e., the system should be controlled as late as possible. We will provide an example in Section 5 to illustrate the results.

4 Optimal control data scheduling for multiple systems

Here we consider the scenario of multiple systems that evolve independently. We assume that the considered multiple systems which are deployed different places are identical, i.e., $a^{(1)} = a^{(2)} = \cdots = a^{(N)} = a$, $b^{(1)} = b^{(2)} = \cdots = b^{(N)} = b$ and $x^{(1)}(0) = x^{(2)}(0) = \cdots = x^{(N)}(0) = x(0)$. Similar assumptions can be found in [19,23–26]. In particular, we consider the case where $B = 1$ and $d^{(i)} = 1$, i.e., at most one system can be controlled at each time instant and each system can be controlled no more than once within a time-horizon T. Thus the optimal scheduling problem becomes "find the time $k^{(i)}$ when system i should be controlled so that the total cost function V defined by (16) is minimized".

As we all know, solving this optimal problem by enumerating all possible schedules is not practical unless T and N are small. However, we are able to construct an explicit optimal scheduling policy for certain classes of problems, each of which exists a element of the tuple $\chi^{(i)}$ varying. Our main interest is to understand the mathematics of these particular problems so that guide research on more general problems.

First, we derive an analytical expression for V when there are multiple systems, each of which can be controlled no more than once. For system i, when no control data are taken from time $k = k^{(i)} + 1, k^{(i)} + 2, \ldots, T-1$, we have

$$\begin{aligned} p^{(i)}(k^{(i)}+1) &= m^{T-k^{(i)}-1} q_T^{(i)} + \sum_{n=0}^{T-k^{(i)}-2} m^n q^{(i)} \\ &= m^{T-k^{(i)}-1} q_T^{(i)} + \frac{q^{(i)}(1 - m^{T-k^{(i)}-1})}{1-m}, \end{aligned} \tag{30}$$

where $m = a^2$. In the special case where $m = 1$, this becomes

$$p^{(i)}(k^{(i)}+1) = q_T^{(i)} + (T - k^{(i)} - 1)q^{(i)}. \tag{31}$$

Therefore if system i is controlled at time $k^{(i)}$, combining (9) we have

$$p^{(i)}(0)=\frac{m^{k^{(i)}+1}\left(m^{T-k^{(i)}-1}q_T^{(i)}+\frac{q^{(i)}(1-m^{T-k^{(i)}-1})}{1-m}\right)}{1+c^{(i)}\left(m^{T-k^{(i)}-1}q_T^{(i)}+\frac{q^{(i)}(1-m^{T-k^{(i)}-1})}{1-m}\right)}+\frac{q^{(i)}(1-m^{k^{(i)}+1})}{1-m}, \tag{32}$$

where $c^{(i)}=b^2/r^{(i)}$. And if $m=1$, this becomes

$$p^{(i)}(0)=\frac{q_T^{(i)}+(T-k^{(i)}-1)q^{(i)}}{1+c^{(i)}(q_T^{(i)}+(T-k^{(i)}-1)q^{(i)})}+q^{(i)}(k^{(i)}+1). \tag{33}$$

There may exists the case $N>T$, thus system i maybe can not be controlled within a time-horizon T. In this case, the value of $c^{(i)}$ in (32) and (33) is deemed to be zero. Therefore the total cost, V, is given by

$$V=x^2(0)\sum_{i=1}^{N}p^{(i)}(0), \tag{34}$$

where $p^{(i)}(0)$ is given by (32) if $m\neq 1$ and (33) if $m=1$. Define $\phi:=\sum_{i=1}^{N}p^{(i)}(0)$. Then Problem 2 degrades to following problem.

Problem 3.

$$\begin{aligned}\min_{\theta\in\Theta}\ & V_\theta=x^2(0)\phi_\theta\\ \text{s.t.}\ & \sum_{k=0}^{T-1}\gamma^{(i)}(k)\leqslant 1,\quad i=1,2,\ldots,N,\\ & \sum_{i=1}^{N}\gamma^{(i)}(k)\leqslant 1,\quad k=0,1,\ldots,T-1.\end{aligned}$$

In what follows, we give the optimal scheduling policy for each of the three specific cases.

Theorem 3 (Different terminal state weights)**.** Consider a set of N scalar systems, each defined by (15), where $|a|\geqslant 1$, $q^{(i)}=q$ and $r^{(i)}=r$ for $i=1,2,\ldots,N$. Without loss generality, assume $q_T^{(1)}<q_T^{(2)}<\cdots<q_T^{(N)}$. Then the optimal schedule θ^* to Problem 3 should obey the following three rules:

Rule 1. Systems should be controlled as once as possible and as early as possible.

Rule 2. If system j is not controlled within a time-horizon T, then system $i<j$ should not be controlled, too.

Rule 3. If system i is controlled at time k, then system $j>i$ should be controlled at time $k'<k$.

Proof. Notice that $V=x^2(0)\phi$. So to prove θ^* is optimal, we just need to show that ϕ_{θ^*} is minimum. We will prove Theorem 3 through following three steps, each of which proves one of three rules.

Step 1. Since $g(z)<h(z)$ and the fact that both h and g are increasing functions, systems being controlled is always beneficial for decreasing the total cost function. Note that from Case 1 in Theorem 2, we know that for any system i, the optimal schedule is to take all $d^{(i)}$ control data at the start of a time-horizon when $|a|\geqslant 1$. Thus Rule 1 holds.

Step 2. Assume that θ is an optimal schedule but it violates Rule 2. Hence within θ, there exist two systems i and j such that $q_T^{(i)}<q_T^{(j)}$ and where system i controlled at $k_0\in\{0,1,\ldots,T-1\}$ but system j is not controlled within a time-horizon T. Now consider the interchanging case, where system j is controlled at time k_0 while system i is not controlled, and denote this schedule as θ'. The difference in the interchanging result is given by

$$\phi_{\theta'}-\phi_\theta=em^T(q_T^{(j)}-q_T^{(i)}), \tag{35}$$

where

$$e=\frac{1}{\mu_{k_0}^{i}\mu_{k_0}^{j}}-1 \tag{36}$$

and $\mu^n_{k_0} = 1 + c(m^{T-k_0-1}q^{(n)}_T + \frac{q(1-m^{T-k_0-1})}{1-m})$ if $m \neq 1$ and $\mu^n_{k_0} = 1 + c(q^{(n)}_T + (T-k_0-1)q)$ if $m = 1$, for $n \in i, j$. Notice that $e < 0$ for both $m = 1$ and $m \neq 1$. Therefore,

$$\phi_{\theta'} - \phi_\theta < 0. \tag{37}$$

So the interchanging result decreases the total cost, which produces a contradiction about the optimality of θ.

Step 3. Assume that θ is an optimal schedule but it violates Rule 3. Accordingly, within θ, there exist two systems i and j such that $q^{(i)}_T < q^{(j)}_T$ and where the control times of these two systems are $k^{(i)} < k^{(j)}$. Now interchanging these times so that system i is controlled at time $k^{(j)}$ and system j is controlled at time $k^{(i)}$, and denote this schedule as θ'. The difference of the interchanging result is given by

$$\phi_{\theta'} - \phi_\theta = wm^T(q^{(j)}_T - q^{(i)}_T), \tag{38}$$

where

$$w = \frac{\nu^i_{k^{(j)}}\nu^j_{k^{(j)}} - \nu^i_{k^{(i)}}\nu^j_{k^{(i)}}}{\nu^i_{k^{(i)}}\nu^i_{k^{(j)}}\nu^j_{k^{(i)}}\nu^j_{k^{(j)}}} \tag{39}$$

and $\nu^n_{k^{(l)}} = 1 + c(m^{T-k^{(l)}-1}q^{(n)}_T + \frac{q(1-m^{T-k^{(l)}-1})}{1-m})$ if $m \neq 1$ and $\nu^n_{k^{(l)}} = 1 + c(q^{(n)}_T + (T - k^{(l)} - 1)q)$ if $m = 1$, for $l, n \in i, j$. Notice that $\nu^n_{k^{(i)}} > \nu^n_{k^{(j)}}$ for $m \geqslant 1$ since $k^{(i)} < k^{(j)}$. Therefore it is clear that $w < 0$ for $m \geqslant 1$. Hence when $m \geqslant 1$,

$$\phi_{\theta'} - \phi_\theta < 0. \tag{40}$$

So interchanging the control times decreases the total cost, which produces a contradiction about the optimality of θ again.

Rule 1 implies that taking control data is always beneficial and if $T > N$, the N systems should be controlled at the first N time instants. Rule 2 shows that if $T < N$, the $N - T$ systems whose terminal state weights are smaller should be not controlled. Rule 3 means that systems should be controlled in decreasing order of terminal state weights.

Theorem 4 (Different state weights)**.** Consider a set of N scalar systems, each defined by (15), where $|a| \geqslant 1$, $q^{(i)}_T = q_T$ and $r^{(i)} = r$ for $i = 1, 2, \ldots, N$. Without loss generality, assume $q^{(1)} < q^{(2)} < \cdots < q^{(N)}$. Then the optimal schedule θ^* to problem 3 should obey the rules of Theorem 3.

Proof. As before, we will prove Theorem 4 through following three steps.

Step 1. See Step 1 in the proof of Theorem 3.

Step 2. As Step 2 in Theorem 3, assume that θ which violates Rule 2 is an optimal schedule where system i is controlled at $k_0 \in \{0, 1, \ldots, T-1\}$ and system j is not controlled within T and $q^{(i)} < q^{(j)}$. The difference of the interchanging result is given by

$$\Delta\phi_\theta = e(q^{(j)} - q^{(i)}), \tag{41}$$

where

$$e = s\left(\frac{1}{(1+\mu^i_{k_0})(1+\mu^j_{k_0})} - 1\right) < 0 \tag{42}$$

and $s = \frac{m^{k_0+1} - m^T}{1-m}$, $\mu^n_{k_0} = 1 + c(m^{T-k_0-1}q^{(n)}_T + \frac{q(1-m^{T-k_0-1})}{1-m})$, $n \in i, j$ if $m \neq 1$ and $s = T - k_0 - 1$, $\mu^n_{k_0} = 1 + c(q^{(n)}_T + (T-k_0-1)q)$, $n \in i, j$ if $m = 1$. Therefore, $\Delta\phi_\theta < 0$, which produces a contradiction about the optimality of θ.

Step 3. As Step 3 in Theorem 2, assume that θ which violates Rule 3 is optimal where the control times of two systems are $k^{(i)} < k^{(j)}$ and $q^{(i)} < q^{(j)}$. The difference of the interchanging result is given by

$$\Delta\phi_\theta = w(q^{(j)} - q^{(i)}), \tag{43}$$

where

$$w = \frac{1}{\nu^i_{k^{(i)}}\nu^i_{k^{(j)}}\nu^j_{k^{(i)}}\nu^j_{k^{(j)}}}[s^{(i)}(-\nu^i_{k^{(i)}}\nu^i_{k^{(j)}}\nu^j_{k^{(i)}}\nu^j_{k^{(j)}} + \nu^i_{k^{(j)}}\nu^j_{k^{(j)}}) + s^{(j)}(\nu^i_{k^{(i)}}\nu^i_{k^{(j)}}\nu^j_{k^{(i)}}\nu^j_{k^{(j)}} - \nu^i_{k^{(i)}}\nu^j_{k^{(i)}})] \tag{44}$$

and $s^{(i)} = \frac{m^{k^{(i)}+1}-m^T}{1-m}$, $s^{(j)} = \frac{m^{k^{(j)}+1}-m^T}{1-m}$, $\nu^n_{k^{(l)}} = 1 + c(m^{T-k^{(l)}-1}q_T + \frac{q^{(n)}(1-m^{T-k^{(l)}-1})}{1-m})$, $l, n \in i, j$ if $m \neq 1$ and $s^{(i)} = T - k^{(i)} - 1$, $s^{(j)} = T - k^{(j)} - 1$, $\nu^n_{k^{(l)}} = 1 + c(q_T + (T - k^{(l)} - 1)q^{(n)})$, $l, n \in i, j$ if $m = 1$. From $k^{(i)} < k^{(j)}$, we have $\nu^n_{k^{(i)}} > \nu^n_{k^{(j)}}$ for $m \geqslant 1$. Therefore we can conclude that

$$w < 0 \tag{45}$$

for $m \geqslant 1$. Further, $\Delta\phi_\theta < 0$, again producing a contradiction about the optimality of θ.

Theorem 5 (Different control weights). Consider a set of N scalar systems, each defined by (15), where $|a| \leqslant 1$, $q^{(i)}_T = q_T$ and $q^{(i)} = q$ for $i = 1, 2, \ldots, N$. Without loss generality, assume $r^1 < r^2 < \cdots < r^N$. Then the optimal schedule θ^* to Problem 3 should obey the following three rules:

Rule 1. See the Rule 1 in Theorem 3, but when $|a| < 1$, the other parameters of each system i should satisfy Cases 2 and 3 from Theorem 2.

Rule 2. If system i is not controlled within a time-horizon T, then system $j > i$ should not be controlled, too.

Rule 3. If system i is controlled at time k, then system $j > i$ should be controlled at time $k' > k$.

Proof. Again, we will prove Theorem 5 through following three steps.

Step 1. See Step 1 in the proof of Theorem 3.

Step 2. Again we proceed as for Step 2 in Theorems 3 and 4. Assume that θ which violates Rule 2 is an optimal schedule where system i is controlled at $k_0 \in \{0, 1, \ldots, T-1\}$ and system j is not controlled within T and $r^{(i)} < r^{(j)}$. The difference of the interchanging result is given by

$$\Delta\phi_\theta = -e(c^{(i)} - c^{(j)}), \tag{46}$$

where

$$e = \frac{m^{k_0+1}(\mu_{k_0})^2}{(1 + c^{(i)}\mu_{k_0})(1 + c^{(j)}\mu_{k_0})} > 0 \tag{47}$$

and $\mu_{k_0} = m^{T-k_0-1}q_T + \frac{q(1-m^{T-k_0-1})}{1-m}$ if $m \neq 1$ and $\mu_{k_0} = (q_T + (T - k_0 - 1)q)$ if $m = 1$. Therefore, $\Delta\phi_\theta < 0$, which produces a contradiction.

Step 3. Again we proceed as for Step 3 in Theorems 3 and 4. Assume that θ which violates Rule 3 is optimal where the control times of two systems are $k^{(i)} > k^{(j)}$ and $r^{(i)} < r^{(j)}$. The difference of the interchanging result is given by

$$\Delta\phi_\theta = \left(c^{(i)} - c^{(j)}\right)\left[wqs^{(i,j)} + y\left(m^{k^{(i)}+1} - m^{k^{(j)}+1}\right)\right], \tag{48}$$

where

$$w = \frac{\nu_{k^{(i)}} + \nu_{k^{(j)}} + (c^{(i)} + c^{(j)})\nu_{k^{(i)}}\nu_{k^{(j)}}}{(1 + c^{(i)}\nu_{k^{(i)}})(1 + c^{(i)}\nu_{k^{(j)}})(1 + c^{(j)}\nu_{k^{(i)}})(1 + c^{(j)}\nu_{k^{(j)}})}, \tag{49}$$

$$y = \nu^2_{k^{(j)}}\left(1 + c^{(i)}\nu_{k^{(i)}}\right)\left(1 + c^{(j)}\nu_{k^{(i)}}\right), \tag{50}$$

and $s^{(i,j)} = \frac{m^{T+k^{(i)}-k^{(j)}}-m^T}{1-m}$ if $m \neq 1$ and $s^{(i,j)} = k^{(j)} - k^{(i)}$ if $m = 1$. Here if $m \neq 1$, then $\nu_{k^{(l)}} = m^{T-k^{(l)}-1}q_T + \frac{q(1-m^{T-k^{(l)}-1})}{1-m}$ and if $m = 1$, then $\nu_{k^{(l)}} = q_T + (T - k^{(l)} - 1)q$ for $l \in i, j$. Notice that $w > 0$, $y > 0$, $s^{(i,j)} < 0$, $c^{(i)} > c^{(j)}$, $k^{(i)} > k^{(j)}$ for $m \leqslant 1$. Therefore, $\Delta\phi_\theta < 0$, which produces a contradiction.

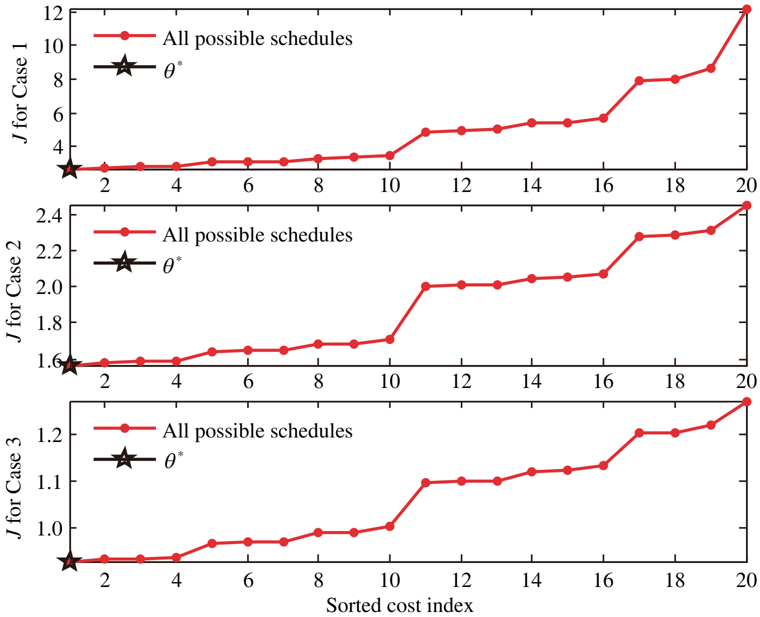

Figure 3 (Color online) Cost comparison for Cases 1–3.

Remark 4. Dual results in the context of state estimation are obtained by [19], where the authors construct optimal sensor measurement schedules in terms of the terminal error covariance for different initial variances, different process noise and different measurement noise respectively. Note that they only consider the case of $a = 1$ and $T = N$ for proposed results.

5 Examples

Though the optimality of the proposed schedules in the previous sections has been rigorously proved, we still provide concrete examples to illustrate the results.

5.1 Single system

Consider the optimal control data schedule for a single system. The time-horizon is $T = 6$, and we only can take $d = 3$ control data. Aiming to Cases 1–3 in Theorem 2, we use the following parameters respectively:

(1) $a = 1.2$, $b = 0.5$, $q = q_T = r = 1$, $x(0) = 1$;

(2) $a = 0.8$, $b = 0.5$, $q = q_T = r = 1$, $\rho = 0.0896 > 0$, $x(0) = 1$;

(3) $a = 0.8$, $b = 0.5$, $q = 0.5$, $r = 1$, $\rho = -0.0704 < 0$, $q_T = 10 < \tau = 11.1840$, $x(0) = 1$.

Figure 3 plots the cost function J under all possible schedules and J incurred by the schedule $\theta^* = \{0, 1, 2\}$ from Theorem 2 is marked by the star specially. To make the comparison result clear, the values of J for all possible schedules are sorted. Obviously, J under θ^* is the smallest among all the possible schedules for Cases 1–3.

According to Remark 2, θ^* is not necessarily optimal when $q_T > \tau$ in Case 3, which can be verified from the upper part of Figure 4 where $q_T = 40$ and the values of other parameters remain the same. On the other hand, again according to Remark 2, let T take a larger number such as $T = 10$, J under θ^* still can be the smallest, which is shown in the lower part of Figure 4.

We also run a simulation for Remark 3 when $A = \text{diag}\{\lambda, \lambda\}$, $B = [1\ 0.5]'$, $Q = \text{diag}\{0, 0\}$, $R = 2$, $Q_T = \text{diag}\{3, 3\}$, $T = 6$ and $d = 3$. For this system, the relationship of Loewner partial order between AXA' and X for any $X > 0$ holds. Let λ equals 1.2, 1 and 0.8 respectively, then the corresponding optimal schedule is $\theta^* = \{0, 1, 2\}$, $\theta^* = \{k_1, k_2, k_3\}$ with $0 \leqslant k_1 < k_2 < k_3 \leqslant 5$ and $\theta^* = \{3, 4, 5\}$. Figure 5 shows that the proposed optimal schedules are indeed optimal.

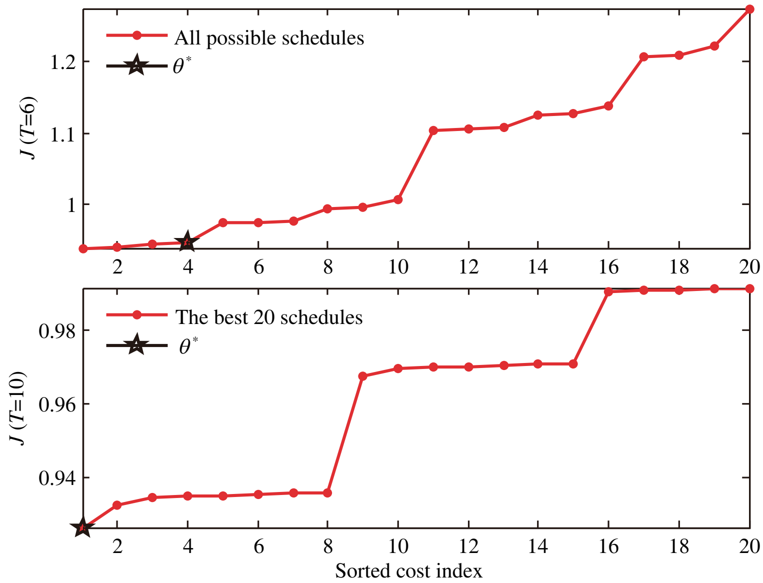

Figure 4 (Color online) Cost comparison for Case 3 where $q_T > \tau$.

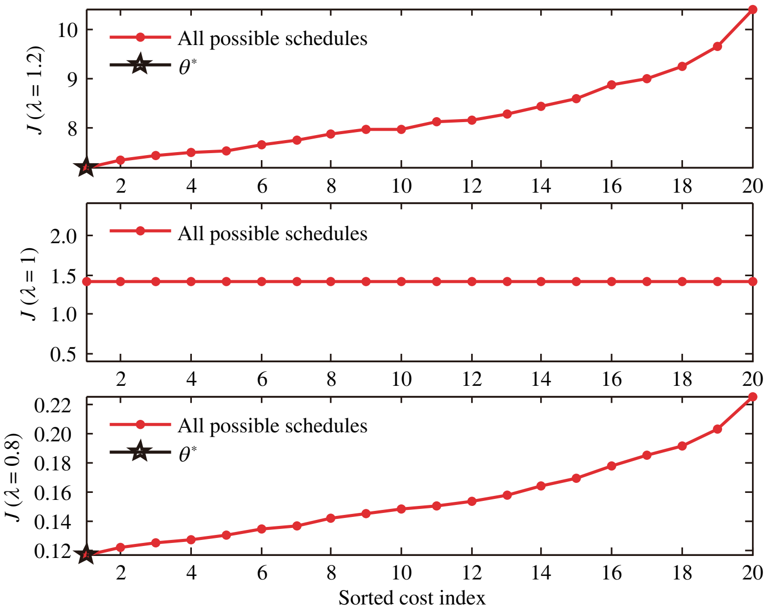

Figure 5 (Color online) Cost comparison for a class of vector systems.

5.2 Multiple systems

Here we provide simulation results for Theorem 3. Consider $N = 4$ identical systems, with $a = 1.5, b = 1, x(0) = 1$. The weights are given as follows: $q^{(i)} = 1, r^{(i)} = 1, q_T^{(i)} = i$. Assume that at most one system can be controlled at each time instant and each system can be controlled no more than once. Let $T = 5 > N$. According to Theorem 3, the optimal schedule θ^* from time 0 to 4 is $\gamma^{(1)}(k)$: 0 0 0 1 0, $\gamma^{(2)}(k)$: 0 0 1 0 0, $\gamma^{(3)}(k)$: 0 1 0 0 0 and $\gamma^{(4)}(k)$): 1 0 0 0 0. Let $T = 2 < N$. According to Theorem 3 again, the optimal schedule θ^* from time 0 to 1 is $\gamma^{(1)}(k)$: 0 0, $\gamma^{(2)}(k)$: 0 0, $\gamma^{(3)}(k)$: 0 1 and $\gamma^{(4)}(k)$): 1 0. For both the cases $T = 5$ and $T = 2$, a enumeration of the total cost function V across all possible schedules except the schedules that do not make full use of control data is shown in Figure 6. The values of V for all possible schedules are sorted again. Obviously, the total cost function V under θ^* is smallest.

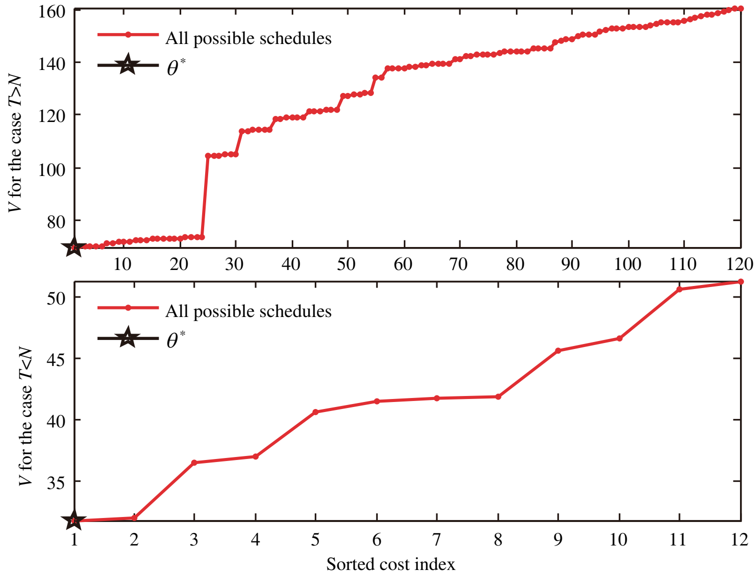

Figure 6 (Color online) Cost comparison for multiple systems.

6 Conclusion

In this work, optimal control data scheduling problems for LQR control with limited controller-plant communication are studied. We have provided a collection of optimal control data scheduling policies for scalar systems. For the single-system scenario, we extend the optimality of an explicit schedule from unstable systems in the previous work to general systems. For the multiple-system scenario, we give the explicit optimal scheduling policy for each of the three specific problems.

A future problem of significant interest is to relax the assumption that the controller communicates with the plant only once for the multiple-system scenario. Future work also includes investigating the optimal control data schedule for general higher-order systems for the single-system and multiple-system scenarios respectively.

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Conflict of interest The authors declare that they have no conflict of interest.

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