

# Logical control scheme with real-time statistical learning for residual gas fraction in IC engines

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Received 3 September 2017/Accepted 17 October 2017/Published online 12 December 2017

**Abstract** In this paper, an optimal control scheme for reducing the fluctuation of residual gas fraction (RGF) under variational operating condition is developed by combining stochastic logical system approach with statistical learning method. The method estimating RGF from measured in-cylinder pressure is introduced firstly. Then, the stochastic properties of the RGF are analyzed according to statistical data captured by conducting experiments on a test bench equipped with a L4 internal combustion engine. The influences to the probability distribution of the RGF from both control input and environment parameters are also analyzed. Based on the statistical analysis, a stochastic logical transient model is adopted for describing cyclic behavior of the RGF. Optimal control policy maps for different fixed operating conditions are calculated then. Besides, a statistical learning-based method is applied to learn the probability density function (PDF) of RGF in the real-time which is used to adjust the control MAP based on logical optimization. The whole optimal control policy map is obtained based on Gaussian process regression with consideration of statistical information of RGF. Finally, the performance of the proposed method is experimentally validated.

**Keywords** combustion engine, statistical learning, residual gas fraction, variable valve timing, logical control

**Citation** Shen X, Wu Y H, Shen T L. Logical control scheme with real-time statistical learning for residual gas fraction in IC engines. *Sci China Inf Sci*, 2018, 61(1): 010203, <https://doi.org/10.1007/s11432-017-9268-2>

## 1 Introduction

It is a fundamental issue to improve fuel economy and decrease emissions in internal combustion (IC) engine control which is dramatically related to the engine's cyclic combustion quality [1]. Nowadays, we are able to guarantee high quality cyclic combustion by applying the theory of dynamical control systems due to the development of the progressed electric control technologies [2, 3].

The residual gas fraction (RGF) is a critical index for evaluating combustion quality since the combustion process of the current cycle is strongly effected by previous cycles' residual gas [4]. In the past several decades, the detection of residual gas mass or RGF have been investigated in a lot of studies. For instance, in-cylinder pressure-based detection methods are presented in [5] and a predictive model for RGF is proposed in [6]. Moreover, mean value model approach and the wavelet-based technique are applied to address the cyclic transient behavior in [7] and [8], respectively.

Most of these studies target on the deterministic state-space model based on which is tractable to design feedback control law. However, rather than a deterministic process, the combustion in engine cylinder is

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more suitable to be regarded as a stochastic event. Thus, it is challenging to model the combustion event in the deterministic sense [9].

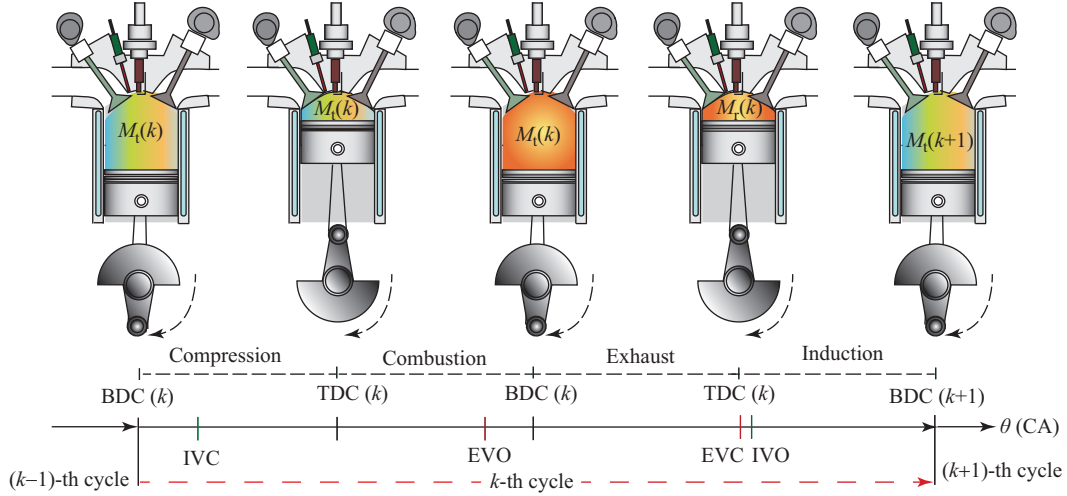
In the last two decades, logical dynamical systems have attracted much attention in dynamical systems theory. In [10,11], the state variables are defined in the logic domain which includes finite or countably infinite states. Then, a simple logical control law can be derived after analysis and synthesis. Also, several studies, such as [12,13], have addressed stochastic logical systems which considers the random characteristic. Ref. [14] investigates the pinning control for the disturbance decoupling problem of Boolean networks with disturbances. The problems that enumerates siphons and minimal siphons in ordinary Petri nets is address in [15]. Based on the semi-tensor product method, the output feedback of Boolean control networks is stabilized in [16]. More development on logical dynamical system theory are introduced in [17].

The framework of logical dynamic system with uncertainty is suitable for RGF control problem. Theoretically, the control performance should be better if we take continuous RGF value as feedback. While, in the engine system, we should firstly consider the existence of uncertainty in combustion process which causes that RGF value is a random variable rather than a deterministic one. RGF value at any time is more weakly correlated to the prior history compared to the case in deterministic system. It is not possible to predict or control RGF value of the forthcoming cycle exactly. However, the trend is possible to be predicted and controlled. Moreover, the actuator for changing variable valve timing is not continuous, therefore, it cannot give precise response to the continuous feedback signal. In [18,19], the control problem of decreasing RGF's cyclic variation in IC engines is addressed in the framework of logical dynamic system with uncertainty. These studies consider the problem under an identical steady operating condition. However, when operating condition changes, such as water temperature or manifold pressure is different, the stochastic property of RGF varies even with constant control inputs. The methods proposed in [18,19] are not effective any more and need to be improved.

In this paper, the control problem under variational operating condition is addressed by combining logical optimization approach with real-time probability density function learning. Firstly, the stochastic properties of RGF, which is measured based on in-cylinder pressure, is discussed. The influences to the stochastic property of RGF from both control input and environment parameter are analyzed then. A logic variable is defined to represent the RGF in the process of quantization. Then, a logical dynamical system is established to model the RGF cyclic transition characteristic under the framework of transient probability. Afterwards, the optimal control policy maps for different fixed operating conditions are calculated offline based on the adopted logical dynamical model in order to decrease the variation of the RGF. Then, Gaussian process regression method is applied to obtain the whole optimal control policy map considering current RGF state and RGF probability density function which is learned in real time by statistical learning method. The proposed approach is experimentally validated in a four-cylinder gasoline engine.

## 2 Background and control problem

Four strokes, induction, compression, combustion and exhaust, consist in the physical process of the combustion cycle in four-stroke internal combustion engines [9]. The engine cyclic process is illustrated in Figure 1, where EVO stands for exhaust valve open, EVC for exhaust valve close, IVC for intake valve close, and IVO for intake valve open. The bottom dead center ( $BDC(k)$ ) of the cylinder position is the beginning of the  $k$ -th cycle while it ends at  $BDC(k+1)$  which denotes the bottom dead center. Also, the  $(k+1)$ -th cycle starts from  $BDC(k+1)$ . During the induction stroke, the fresh air-fuel mixture is charged in the cylinder and then it is compressed in the compression stroke. The in-cylinder temperature increases as the air-fuel mixture is compressed smaller and smaller, at the end of compression stroke, an external spark plug is used to generate a spark which ignites the mixture and initiates the combustion process. Along with the combustion, the piston and crankshaft produce the mechanical work transformed from the heat energy. At the end of the combustion stroke, exhaust gas is expelled after the exhaust



**Figure 1** (Color online) Representation of the engine cyclic gas exchange phenomena.

valve opens since the pressure of the gas is higher. While, the existence of the overlapping of the exhaust and intake valves causes some gas flow back to cylinder. The gas flowing back to cylinder is residual gas. The residual gas contains unburned fuel-air mixture as well as the combustion products [20].

The RGF can be defined as

$$\text{RGF}(k) = \frac{M_r(k)}{M_t(k)}, \quad (1)$$

where  $M_r(k)$  and  $M_t(k)$  denote the mass of residual gas at  $\text{EVC}(k)$  and the mass of total gas at  $\text{IVC}(k)$ , respectively.  $\text{EVC}(k)$  is the end of the exhaust valve closing timing while and  $\text{IVC}(k)$  is the end of the intake valve closed timing.

## 2.1 Measurement of RGF

The combustion performance can be dramatically improved through meliorating the cyclic behavior of the RGF. However, it is difficult to measure the RGF directly cycle-by-cycle. Cylinder pressure data can be used to estimate the RGF indirectly [5].

Supposing the ideal gas equation is tenable on the in-cylinder gas while the exhaust valve is about to closing, we have

$$M_r(k) = M_{\text{EVC}}(k) = \frac{P_{\text{EVC}}(k)V_{\text{EVC}}(k)}{T_{\text{EVC}}(k)R}, \quad (2)$$

where  $P_{\text{EVC}}(k)$  is the pressure,  $V_{\text{EVC}}(k)$  the volume, and  $T_{\text{EVC}}(k)$  the temperature at  $k$ -th cycle's EVC, respectively. Moreover the ideal gas constant is denoted as  $R$ . Similarly, we get

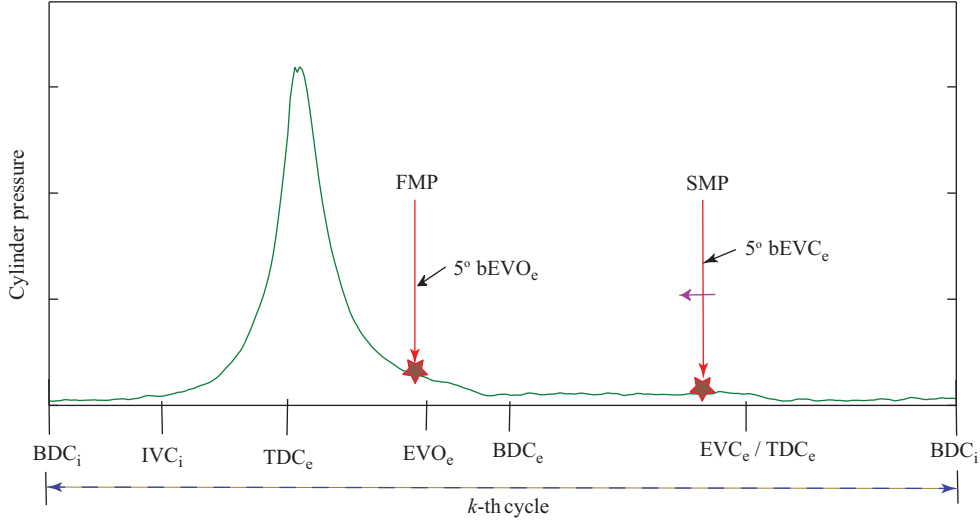
$$M_t(k) = M_{\text{EVO}}(k) = \frac{P_{\text{EVO}}(k)V_{\text{EVO}}(k)}{T_{\text{EVO}}(k)R}. \quad (3)$$

From (2) and (3), the RGF can be calculated by

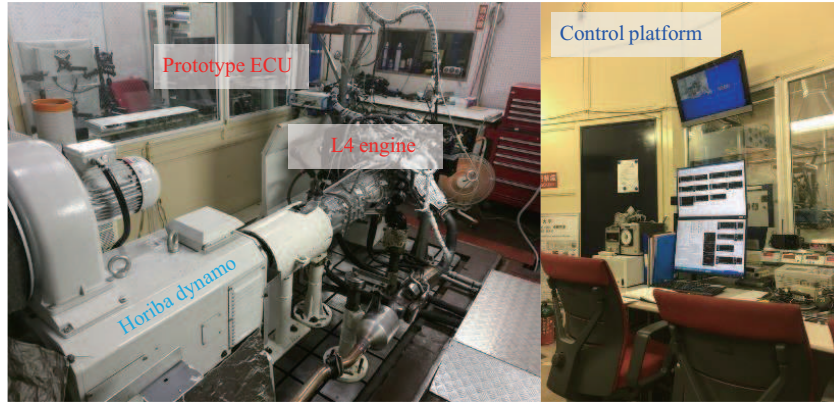
$$\text{RGF}(k) = \frac{P_{\text{EVC}}(k)V_{\text{EVC}}(k)T_{\text{EVO}}(k)}{T_{\text{EVC}}(k)P_{\text{EVO}}(k)V_{\text{EVO}}(k)}. \quad (4)$$

Since the exhaust process is assumed to be an adiabatic polytropic process, we can express the relationship between  $\text{EVO}(k)$  and  $\text{EVC}(k)$  as

$$T_{\text{EVC}}(k) = T_{\text{EVO}}(k) \left( \frac{P_{\text{EVC}}(k)}{P_{\text{EVO}}(k)} \right)^{\frac{n-1}{n}}, \quad (5)$$



**Figure 2** (Color online) Pressure measurement points.



**Figure 3** (Color online) Test bench.

where  $n$  is a constant value (1.32) denoting the polytropic coefficient. Combining (4) and (5), the RGF of the  $k$ -th cycle is written as

$$\text{RGF}(k) = \frac{M_r(k)}{M_t(k)} = \left( \frac{V_{\text{EVC}}(k)}{V_{\text{EVO}}(k)} \right) \left( \frac{P_{\text{EVC}}(k)}{P_{\text{EVO}}(k)} \right)^{\frac{1}{n}}, \quad (6)$$

where  $P_{\text{EVO}}$  and  $V_{\text{EVO}}$  are the in-cylinder pressure and volumes at EVO while  $P_{\text{EVC}}$  and  $V_{\text{EVC}}$  are the ones at EVC. Volumes are calculated from crank angle and the engine parameters as introduced in [9]

$$V(\theta) = V_c + \frac{\pi B^2}{4} \left( l + a(1 - \cos \theta) - \sqrt{l^2 - a^2 \sin^2 \theta} \right), \quad (7)$$

where  $V_c$  denotes the clearance volume,  $l$  the connecting rod length,  $B$  the cylinder bore diameter,  $\theta$  the crank angle and  $a$  the crank radius. Two-point in-cylinder pressure is used to detect the RGF in (6). Figure 2 shows the measured points. The first one written as FMP is 5 degree crank angle before  $EVO_e$  which is for avoiding the delayed response in the mechanical valve system as well as signal delay. The second measuring point (SMP) is taken 5 degree crank angle before  $EVC_e$  in case of the high fluctuations in exhaust gas flow when the exhaust valve is about to close.

## 2.2 Statistical analysis for RGF distribution

The data for statistical analysis is measured from the experiment conducted on a test bench equipped with a full-scale gasoline engine with four cylinders. The low inertia dynamometer is connected to the engine

**Table 1** Specifications of the engine test bench

Engine specification		Dynamo technical data	
Item	Value	Item	Value
Cylinder type	L-type 4 cylinders	Rated power (Absorbing/Driving) (kW)	250/225
Ignition system	DIS	Rated torque (Absorbing/Driving) (Nm)	480/442
Compression ratio	13:1	Rated speed (Absorbing/Driving) (rpm)	4980/4860
Displacement	1797 ml	Maximum speed (rpm)	10000
Maximum torque	142 N.m	Moment of inertia ( $\text{kg} \cdot \text{m}^2$ )	0.36

for providing the load. Moreover, a rapid prototyping electronic control unit which includes DS1006 (dSPACE) and a fully opened production-ECU is constructed. The test bench environment is as shown in Figure 3. The specifications of the engine test bench are listed in Table 1.

The cyclic variations in air-fuel mixture's temperature before ignition and the air fuel ratio causes the variations in combustion processes even under the invariable control inputs [21]. Thus, the combustion event is a stochastic event. Even in steady operating conditions, the RGF evolves with cyclic variations as shown in Figure 4. When engine is operated under the fixed operating condition with engine speed as 1200 rpm and manifold pressure  $p_m$  as 0.53 bar, the RGF has different stochastic behaviours as variable valve timing (VVT) is changed from 27 to 36 as red and blue lines. On the other hand, when the operating conditions changes, such as manifold pressure varies, the stochastic behavior of RGF also is different. Blue and green lines show the RGF under different manifold pressures as 0.53 bar and 0.56 bar with the same VVT as 36.

The time return maps can be obtained from the cycle-to-cycle RGF data series as shown in Figure 4 and gives more information about the stochastic properties of RGF. Actually, RGF does not have a clearly deterministic relationship with previous cycle which is concluded from the randomly changed shape of the time return maps. Then, based on the maximum likelihood estimation (MLE), the mean and the standard deviation of RGF can be calculated and then obtain the probability density maps of RGF as in Figure 4. The RGF distribution is a Gaussian distribution, and both VVT and manifold pressure influence the distribution of RGF evidently as illustrated in Figure 5. Thus, the variation in the probability distribution of RGF under the constant VVT can be regarded as the indication of operating condition variation.

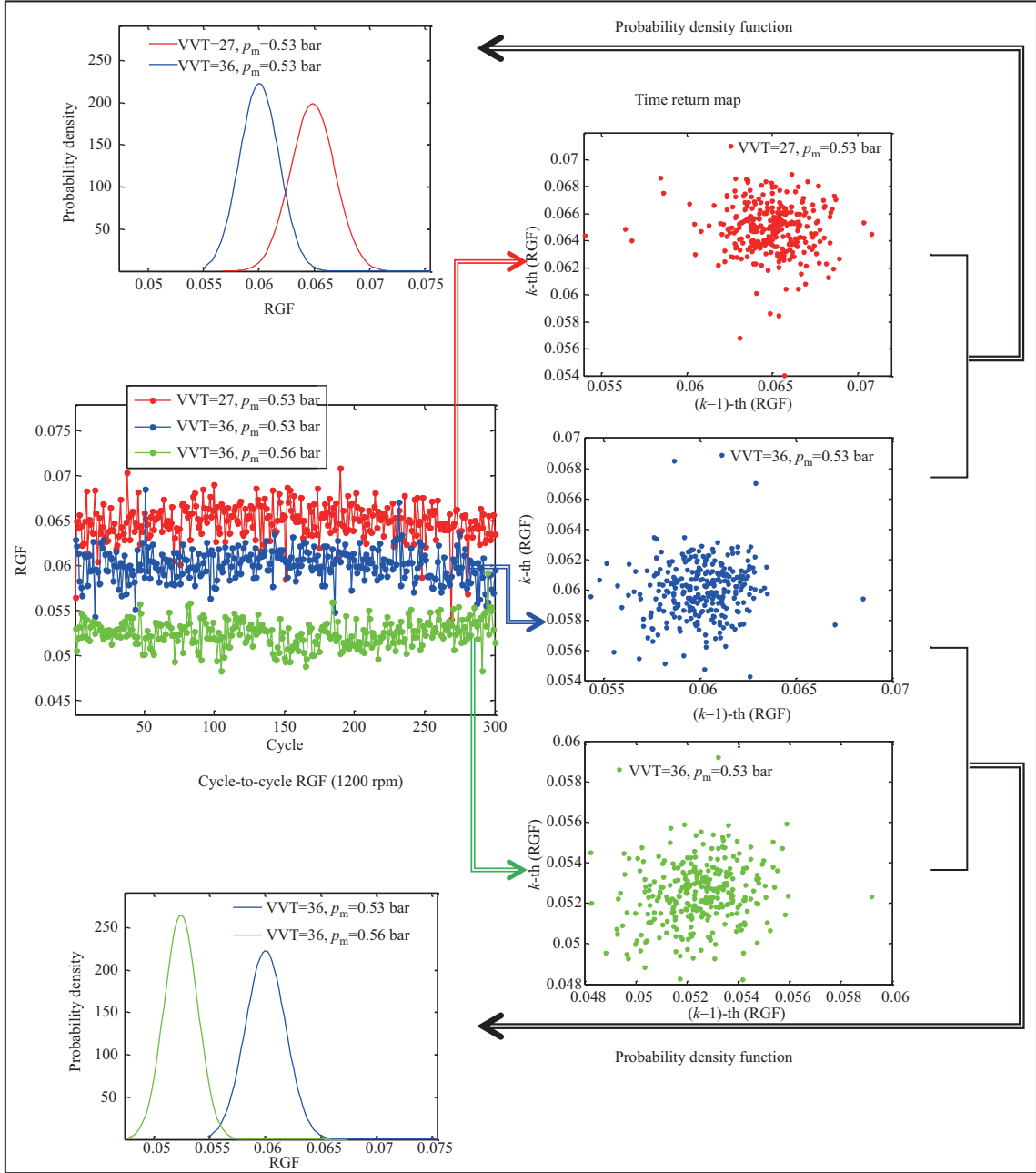
### 2.3 RGF control problem

According to the experimental analysis in previous part, it is obvious that the RGF exhibits stochastic properties which varies as control input VVT or environment parameters changes. The control problem addressed here is to design an optimal feedback control law of control input VVT to realize the reduction in the variance of RGF with as little change in VVT as possible under variational operating condition. Indication for variations in the operating condition is necessary for adjusting the feedback control law due to the existence of the uncertain variations in thermal environment, load and combustion.

## 3 Control design

### 3.1 Proposed control scheme

The proposed control scheme is shown in Figure 6. At each cycle, the real value of RGF is calculated from the in-cylinder pressure and the crank angle. Then, the corresponding logical state of RGF is obtained based on the quantization process. Simultaneously, the mean and variance of RGF for current operating condition are given by the block of statistical learning based on current RGF and operated VVT. The mean and variance value of RGF are used to adjust the optimal VVT MAP which regards logic state of RGF as feedback. The MAP outputs the optimal VVT as VVT command to gasoline engine. The MAP is fitted based on gaussian process regression from the MAPs which take RGF logical state as input and VVT as output and are calculated based on logical optimization.



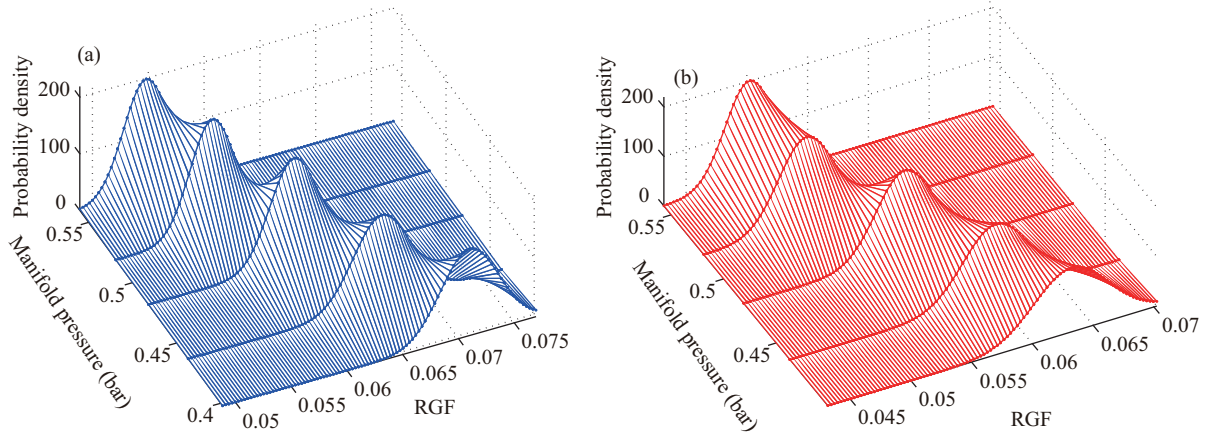
**Figure 4** (Color online) Stochastic properties of RGF.

### 3.2 Feedback MAP design based on logical optimization

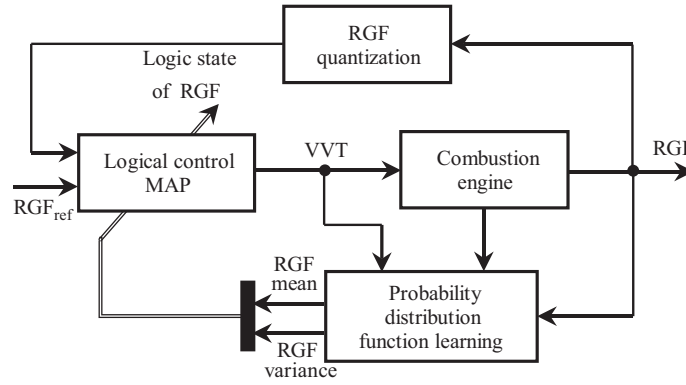
Before applying logical optimization, RGF quantization process should be implemented to transform the continuous RGF value into logic state by firstly dividing the whole range of RGF into nine intervals as

$$\begin{cases} A^1 := (0, \sigma - 7/6\mu]; A^2 := (\sigma - 7/6\mu, \sigma - 5/6\mu], \\ A^3 := (\sigma - 5/6\mu, \sigma - 3/6\mu]; A^4 := (\sigma - 3/6\mu, \sigma - 1/6\mu], \\ A^5 := (\sigma - 1/6\mu, \sigma + 1/6\mu]; A^6 := (\sigma + 1/6\mu, \sigma + 3/6\mu], \\ A^7 := (\sigma + 3/6\mu, \sigma + 5/6\mu]; A^8 := (\sigma + 5/6\mu, \sigma + 7/6\mu], \\ A^9 := (\sigma + 7/6\mu, 1], \end{cases} \quad (8)$$

where  $\mu$  denotes the mean of the RGF and  $\sigma$  the standard deviation which is calculated from the total



**Figure 5** (Color online) Influence of environment to RGF probability distribution. (a) VVT = 33; (b) VVT = 39.



**Figure 6** Block diagram of the control scheme.

measurements of the RGF. Then, the quantization process is implemented as follows:

$$y_k \in A^i \rightarrow x_k = \delta_s^i, \quad i = 1, 2, \dots, 9. \quad (9)$$

Here,  $\delta_s^i$  is the  $i$ -column of  $I_s$  which is an identify matrix. Then, any measurement of the RGF  $y_k$  matches along with  $x_k$  which is a logical value.

For the value of intake VVT which is the control variable, nine values: 21, 24, 27, 30, 33, 36, 39, 42, and 45, are available. Then, we establish the logical framework for intake VVT as

$$21 \leftrightarrow \delta_r^1, \quad 24 \leftrightarrow \delta_r^2, \quad \dots, \quad 45 \leftrightarrow \delta_r^9. \quad (10)$$

Afterwards, the logic space for state variable RGF can be written as  $S = \{\delta_s^1, \delta_s^2, \dots, \delta_s^9\}$ , and intake VVT as  $U = \{\delta_r^1, \delta_r^2, \dots, \delta_r^9\}$ . Consequently, the logical dynamical system with uncertainty for modelling cyclic RGF evolution is expressed as [18, 22]

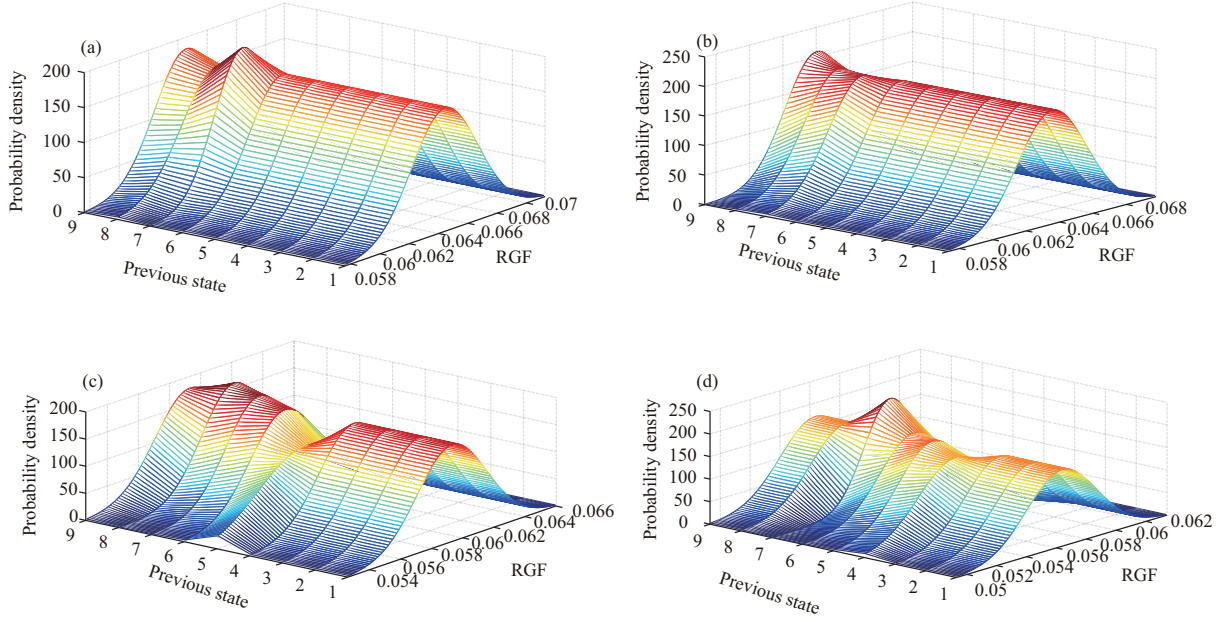
$$x_{k+1} = f(x_k, u_k, w_k). \quad (11)$$

Here,  $x_k \in S$  is state variable,  $u_k \in U$ , the control input. Moreover,  $w_k$  denotes uncertain disturbances at  $k$ -th cycle which is for the combustion uncertainty.

For the calculation of conditional probability density function (CPDF),  $Y_{i,j}$ , all the samples of  $y_{t+1}$  corresponding to  $u_t$  as  $\delta_r^j$ , when previous state  $y_t$  is  $A^i$ , is denoted as

$$Y_{i,j} = \{y_{t+1} | y_t \in A^i, u_t = \delta_r^j\}, \quad \forall i = 1, \dots, 9, \quad j = 1, \dots, 9. \quad (12)$$

$Y_{i,j}$ ,  $\forall i = 1, \dots, 9, j = 1, \dots, 9$  is supposed to obey Gaussian distribution. And expectation is  $\mu_{i,j}$  while standard deviation is  $\sigma_{i,j}$ , namely,  $Y_{i,j} \sim N(\mu_{i,j}, \sigma_{i,j}^2)$ .  $\mu_{i,j}$  and  $\sigma_{i,j}$  can be estimated based on the data



**Figure 7** (Color online) Conditional probability density for  $y_{k+1}$  given  $y_k \in A^i$ ,  $i = 1, 2, \dots, 9$ , under different VVT values. (a) VVT = 27; (b) VVT = 30; (c) VVT = 36; (d) VVT = 39.

set by MLE method. Figure 7 illustrates the CPDFs of  $Y_{i,j}$  under several different VVT values with engine speed as 1200 rpm and load torque as 45 Nm. In Figure 7, the previous logical state of the RGF is written as the number  $i$  ( $i = 1, \dots, 9$ ).

Then, after defining  $\delta_s^{\gamma_1}$  and  $\delta_r^{\gamma_2}$  as the reference RGF and VVT logical states respectively, the cost function describing the variance of RGF and VVT per-cycle with weight coefficients  $\lambda_1, \lambda_2$ , for trade-off is as follows:

$$g(x, u) := \lambda_1 \tilde{d}(x, \delta_s^{\gamma_1}) + \lambda_2 \tilde{d}(u, \delta_r^{\gamma_2}). \quad (13)$$

Besides,  $\tilde{d}(\delta_s^i, \delta_s^j) := |i - j|$ ,  $\tilde{d}(\delta_r^i, \delta_r^j) := |i - j|$ ,  $\forall i, j = 1, 2, \dots, 9$  is defined for the discrete distance function.

For given initial state  $x_0$  and admissible policy  $\pi = \{\mu_0, \mu_1, \dots\}$ , the total expected cost

$$J_\pi(x_0) = \lim_{N \rightarrow \infty} \mathop{E}_{w_k} \sum_{k=0}^{N-1} \alpha^k g(x_k, u_k), \quad (14)$$

subject to (11) with the discount factor  $0 < \alpha < 1$  is considered here.  $\Pi$  is denoted as the set of admissible policies  $\pi = \{\mu_0, \mu_1, \dots\}$ . Then,  $J^*$ , the infinite horizon cost function, is defined by

$$J^*(x) = \inf_{\pi \in \Pi} J_\pi(x), \quad x \in S. \quad (15)$$

An optimal policy  $\pi^* \in \Pi$  should be determined to minimize  $J^*$ , which is expressed as

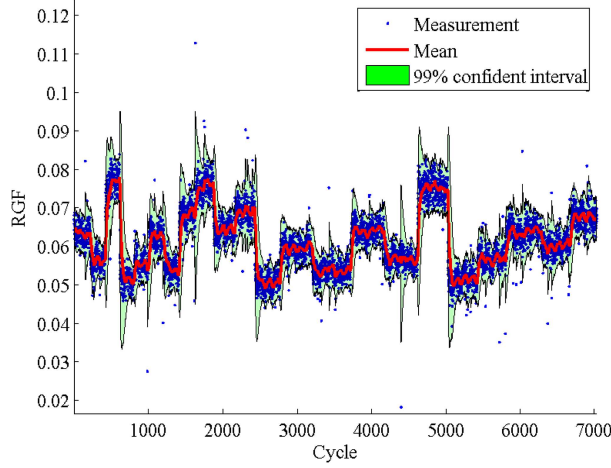
$$J_{\pi^*}(x_0) = J^*(x_0), \quad \text{for all } x_0 \in S.$$

The problem can be solved by policy iteration approach which can be referred to [19, 23, 24].

### 3.3 On-line learning for RGF probability distribution

As mentioned in Subsection 3.2, in certain operation condition where the control input VVT and manifold pressure are constant, the probability distribution of RGF is regarded as identically normal distribution and cycle-to-cycle independent. Namely, the series of RGF can be defined as  $r(1), r(2), \dots, r(i), \dots$ , also





**Figure 8** (Color online) Statistical learning for RGF.

written as  $r(\cdot)$ , which is regarded as gaussian process with identically normal probability distribution. Then, the estimated mean value of  $r$  can be calculated based on exponential moving average as

$$r_m(i) = r_m(i-1) + \beta(r(i) - r_m(i-1)), \quad (16)$$

the variance is calculated as

$$d_r^2(i) = (1 - \beta)(d_r^2(i-1) + \beta(r(i) - r_m(i-1))^2), \quad (17)$$

where  $i = 1, 2, 3, \dots, n$ . The proof for the convergency of (16) and (17) is omitted here and can be referred to [25, 26].

## 4 Experimental validation

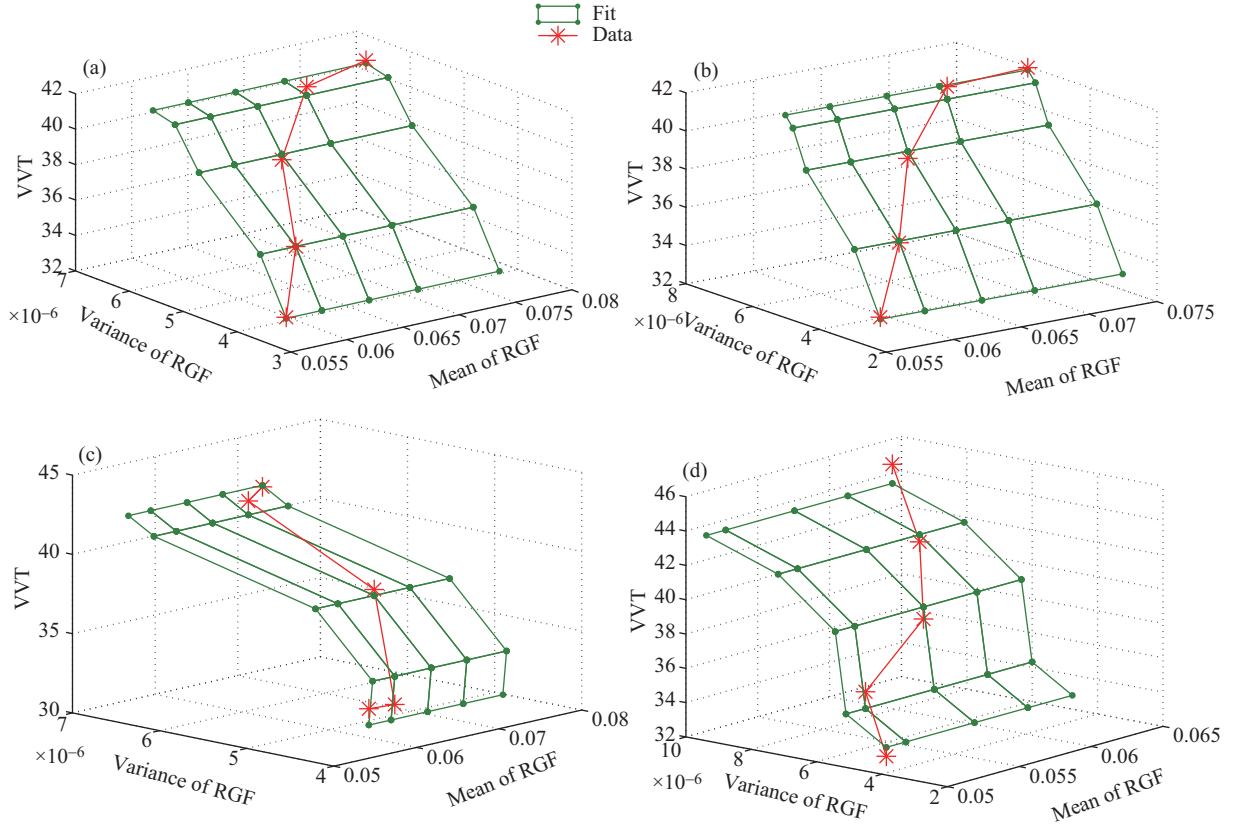
It is mentioned in Section 3 that the main purpose of the research is to design an optimal feedback control law of control input VVT to realize the reduction in the variance of RGF with as little change in VVT as possible under variational operating condition. The experiment was conducted on the cylinder 1 of the L4 gasoline engine described in Section 2. During the experiment, the engine speed is constantly 1200 rpm. The water temperature is kept as 353 K. The throttle angle follows a sinusoidal shape to cause the variation in manifold pressure.

### 4.1 Statistical learning validation

Figure 8 illustrates the performance of statistical learning algorithm presented in Section 3. The blue points are the measurement of RGF. The red line shows the estimated mean value given by statistical learning algorithm. The green area is the 99% confident interval which means the measurement will locate in the interval with the probability of 99%. The interval is  $[r_m - 2.33d_r, r_m + 2.33d_r]$  where  $r_m$  and  $d_r$  represent mean and standard deviation values of RGF respectively since the probability distribution of RGF is supposed to be normal distribution. It can be observed that the algorithm gives accurate estimation.

### 4.2 Optimal policy calibration

The optimal policy is calculated through the stochastic logical system approach presented in Subection 3.3. For instance, the optimal policy for the operating condition with mean and variance



**Figure 9** (Color online) Optimal feedback control law for VVT. (a) RGF  $A^1$ ; (b) RGF  $A^3$ ; (c) RGF  $A^5$ ; (d) RGF  $A^7$ .

of RGF as 0.0619 and  $4.6623E-6$  when VVT is 33 can be expressed as

$$\text{VVT}(k) = \begin{cases} 33, & \text{if } \text{RGF}(k) \in A^1, \\ 36, & \text{if } \text{RGF}(k) \in A^2, \\ 39, & \text{if } \text{RGF}(k) \in A^3 \cup A^4 \cup A^5 \cup A^6 \cup A^7, \\ 42, & \text{if } \text{RGF}(k) \in A^8, \\ 45, & \text{if } \text{RGF}(k) \in A^9. \end{cases} \quad (18)$$

However, for feedback signal RGF in different intervals, the optimal policy is different, Figure 9 shows the optimal VVT for four different feedback RGF states under different conditions with different steady mean and variance of RGF. The redline is the data from every map for every fixed environment. The green line is fitted from the data based gaussian process regression which is a probabilistic, non-parametric Bayesian approach.

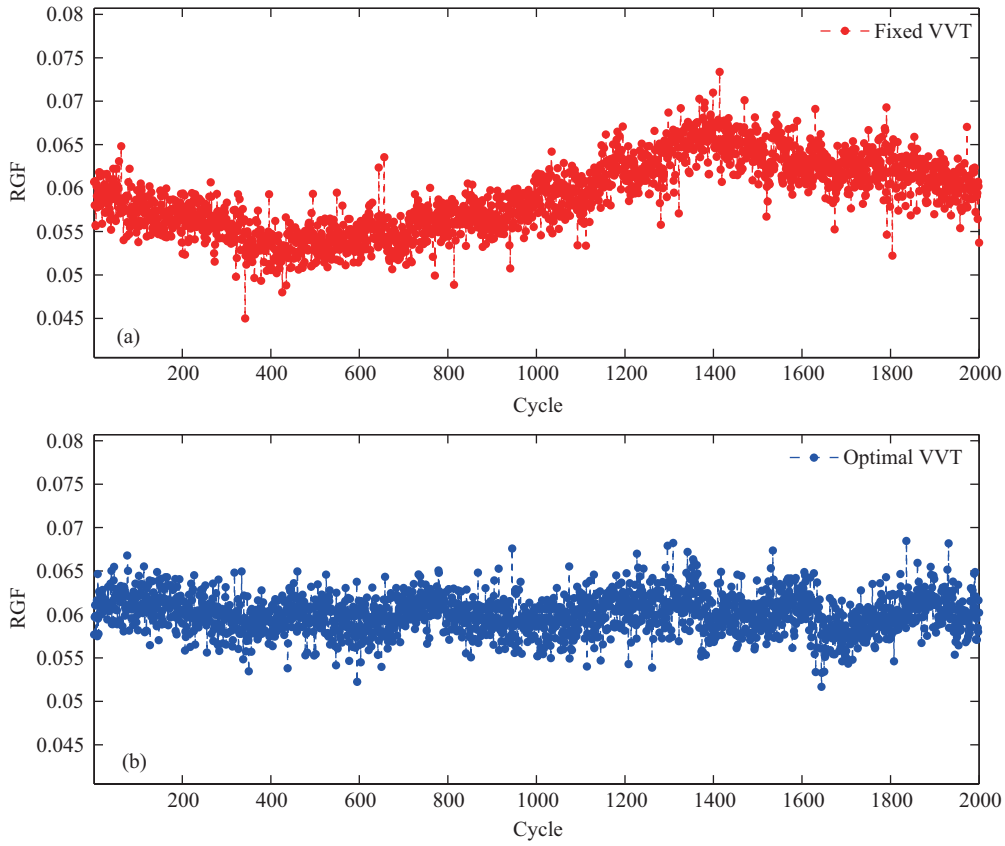
Denote D-dimensional input as  $x$  and the scalar output as  $y$ , the function from  $x$  to  $y$  is

$$y = f(x), \quad (19)$$

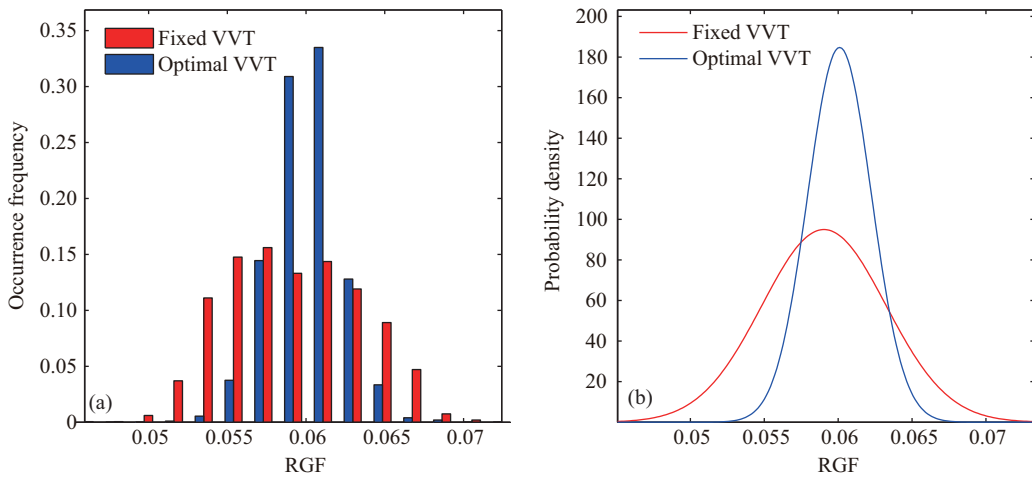
the regression task boils down to making inference about  $f(x)$  based on the given training data set  $\mathcal{D} \equiv \{x_j, y_j | j = 1, \dots, n\}$ . Here, the redlines with stars in Figure 9 are training data set. Input is  $x_j = (r_m(j), d_r(j))$  and output is  $y_j = \text{VVT}^*$ . Denote  $Y = [y_1, \dots, y_n]^T$  as the output training data set and  $y^*$  as the estimated output when input is  $x^*$ . Then, the maximum likelihood estimation for output is calculated as

$$y^* = k_{*f} K_{ff}^{-1} Y, \quad (20)$$

where  $K_{ff}$  is an  $n \times n$  covariance function matrix and  $k_{*f}$  is a  $1 \times n$  vector.  $K_{ff}$  is calculated from the input data set  $\mathcal{X} = x_1, \dots, x_n$  according to the chosen kernel function  $k(i, j), i, j \in 1, \dots, n$  while  $k_{*f}$  is



**Figure 10** (Color online) Cycle-to-cycle RGF response with (a) fixed and (b) optimal control VVT input.



**Figure 11** (Color online) Comparison of (a) the occupancy frequency distributions and (b) probability density functions of RGF.

calculated from  $x_*$  and  $\mathcal{X}$ . The details for calculation of  $K_{ff}$  and  $k_{*f}$  are omitted here but can be referred to [27].

### 4.3 Result and analysis

Figure 10(a) shows evolution of RGF with a fixed VVT and Figure 10(b) shows the one with optimal control VVT. Compare to the case with a fixed VVT, the maximal variation of RGF in the case with optimal control VVT is smaller. Figure 11(a) and (b) shows the frequency occurrence distributions and

probability densities of both real RGF response to the fixed VVT and optimal VVT in the experiment, respectively. The standard deviation of RGF under the control of fixed VVT is 0.0042 while the one with optimal control VVT is 0.0022. The relative decrement of standard deviation is 47.6%. From all these comparisons, we can see the proposed method effectively control the RGF to the reference value.

## 5 Conclusion

It is a critical issue to decrease the variance of the RGF to ensure the stable and high quality combustion. The control problem of the RGF under variational operating condition is addressed here after providing an cylinder-pressure-based detection method for the RGF. By considering the transient distribution of the RGF, logical control scheme with real-time PDF learning is proposed. The optimal control policy maps for different operating conditions are calculated offline after modelling the RGF transient behavior. The gaussian process regression method is applied to form the whole optimal control policy map which takes current RGF state as feedback signal and adjusts the output optimal VVT according to RGF mean and variance value learned in the real-time by statistical learning method. The effectiveness of the proposed method is shown in the experimental evaluation with a full-scale gasoline engine test bench.

**Acknowledgements** The authors gratefully acknowledge the support and generosity of Toyota Motor Corporation, without which the present study could not have been completed.

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