SCIENCE CHINA
Information Sciences

• REVIEW •

Special Focus on Analysis and Control of Finite-Valued Network Systems

January 2018, Vol. 61 010202:1–010202:17
https://doi.org/10.1007/s11432-017-9238-1

A survey on applications of semi-tensor product method in engineering

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Received 7 August 2017/Accepted 7 September 2017/Published online 12 December 2017

Abstract Semi-tensor product (STP) of matrices has attracted more and more attention from both control theory and engineering in the last two decades. This paper presents a comprehensive survey on the applications of STP method in engineering. Firstly, some preliminary results on STP method are recalled. Secondly, some applications of STP method in engineering, including gene regulation, power system, wireless communication, smart grid, information security, combustion engine and vehicle control, are reviewed. Finally, some potential applications of STP method are predicted.

Keywords semi-tensor product of matrices, gene regulation, power system, smart grid, information security, vehicle control

Citation Li H T, Zhao G D, Meng M, et al. A survey on applications of semi-tensor product method in engineering. Sci China Inf Sci, 2018, 61(1): 010202, https://doi.org/10.1007/s11432-017-9238-1

1 Introduction

Semi-tensor product (STP) of matrices was proposed by Cheng around twenty years ago [1]. Given two real matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$, the semi-tensor product of A and B, denoted by $A \ltimes B$, is defined as follows [2]:

$$A \ltimes B = (A \otimes I_{\frac{\alpha}{n}})(B \otimes I_{\frac{\alpha}{p}}), \tag{1}$$

where $\alpha = \mathrm{lcm}(n, p)$ represents the least common multiple of n and p, and $\otimes$ is the Kronecker product of matrices. Obviously, when $n = p$, STP method becomes the conventional matrix product. Thus, it is a generalization of the conventional matrix product. STP method not only keeps all the properties of the conventional matrix product, but also has its own special properties such as pseudo-communicative law. Due to these beautiful properties of STP method, it has attracted many scholars' research interests from both control theory and engineering in the last decade [2–4].

In the control theory field, STP method has two main applications. One is the analysis and control of nonlinear systems such as Morgan's problem [1], stability region [5], feedback linearization [6], symmetry of control systems [7], and so on [8]. The other one is the analysis and control of logical dynamic systems [4]. Using STP method, an algebraic state space representation approach is established for logical dynamic systems [9–11]. In the last decade, many excellent results have been proposed for the analysis and control of logical dynamic systems based on the algebraic state space representation approach, which

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include stability [12–23], controllability [24–44], observability [45–53], stabilization [54–69], synchronization [70–81], disturbance decoupling [82–89], optimal control [90–98], output tracking [99–105], and other related problems [23,106–117]. Some comprehensive surveys on the applications of STP method in control theory can be found in [118–123].

On the other hand, STP method has wide applications in the engineering related field. These applications include gene regulation [124, 125], power system [126–129], wireless communication [130–132], smart grid [133–139], finite automata [140–144], information security [145–147], vehicle control [148–150], indoor thermal comfort [151–154], fault detection of circuits [155, 156], spacecraft [157], epidemic vaccination [158], mobile robot [159]. The aim of this paper is to present a comprehensive survey on these applications.

The rest of this paper is structured as follows. Section 2 recalls some basic results on STP method. Section 3 reviews the application of STP method in gene regulation. Section 4 reviews the application of STP method in power systems. Section 5 reviews the application of STP method in vehicle control. Section 6 reviews the application of STP method in smart grid. The applications of STP method in finite automata, information security, wireless communication, spacecraft and mobile robot are reviewed in Section 7. Section 8 presents some concluding remarks.

Notations. "$\neg$", "$\wedge$" and "$\vee$" represents "negation", "conjunction" and "disjunction", respectively. $\mathcal{D}_k := \{0, 1, \ldots, k-1\}$, and $\mathcal{D}_k^n := \underbrace{\mathcal{D}_k \times \cdots \times \mathcal{D}_k}_{n}$. $\Delta_n := \{\delta_n^k : 1 \leqslant k \leqslant n\}$, where δ_n^k represents the k-th column of the identity matrix I_n. An $n \times t$ logical matrix $M = [\delta_n^{i_1}\ \delta_n^{i_2}\ \cdots\ \delta_n^{i_t}]$ is briefly denoted by $M = \delta_n[i_1\ i_2\ \cdots\ i_t]$. $\mathcal{L}_{n\times t}$ represents the set of $n \times t$ logical matrices. $\mathrm{Blk}_i(A)$ denotes the i-th $n \times n$ block of an $n \times mn$ matrix A. For a real matrix $A \in \mathbb{R}^{n\times m}$, $(A)_{i,j}$, $\mathrm{Col}_i(A)$ and $\mathrm{Row}_i(A)$ denote the (i, j)-th element of A, the i-th column of A, and the i-th row of A, respectively. We call $A > 0$, if $(A)_{i,j} > 0$ holds for any i and j. The Khatri-Rao product of $A \in \mathbb{R}^{p\times n}$ and $B \in \mathbb{R}^{q\times n}$ is

$$A * B = [\mathrm{Col}_1(A) \otimes \mathrm{Col}_1(B)\ \ \mathrm{Col}_2(A) \otimes \mathrm{Col}_2(B)\ \ \cdots\ \ \mathrm{Col}_n(A) \otimes \mathrm{Col}_n(B)] \in \mathbb{R}^{pq\times n}.$$

2 Preliminaries

Firstly, we recall some properties of STP method. For details, please refer to [2–4].

Lemma 1 (Associative law). Let $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{p\times q}$ and $C \in \mathbb{R}^{r\times s}$. Then, $(A \ltimes B) \ltimes C = A \ltimes (B \ltimes C)$.

Lemma 2 (Pseudo-commutative law). Let $X \in \mathbb{R}^{t\times 1}$ and $A \in \mathbb{R}^{m\times n}$. Then

$$X \ltimes A = (I_t \otimes A) \ltimes X. \tag{2}$$

Lemma 3 (Swap matrix). Let $X \in \mathbb{R}^{m\times 1}$ and $Y \in \mathbb{R}^{n\times 1}$. Then

$$Y \ltimes X = W_{[m,n]} \ltimes X \ltimes Y, \tag{3}$$

where $W_{[m,n]} \in \mathcal{L}_{mn\times mn}$ is the so-called swap matrix, which is defined as

$$\begin{aligned} W_{[m,n]} = \delta_{mn}[1\ \ & m+1\ \ \cdots\ \ (n-1)m+1 \\ 2\ \ & m+2\ \ \cdots\ \ (n-1)m+2 \\ & \vdots \\ m\ \ & m+m\ \ \cdots\ \ (n-1)m+m]. \end{aligned}$$

Lemma 4 (Dummy matrices). Let $X \in \Delta_m$ and $Y \in \Delta_n$. Define two dummy matrices $D_{\mathrm{f}}[m, n]$ and $D_{\mathrm{r}}[m, n]$ as follows:

$$D_{\mathrm{f}}[m, n] = I_m \otimes \mathbf{1}_n^{\mathrm{T}}, \tag{4}$$

$$D_{\mathrm{r}}[m, n] = \mathbf{1}_m^{\mathrm{T}} \otimes I_n. \tag{5}$$

Then, $D_{\mathrm{f}}[m, n] \ltimes X \ltimes Y = X$ and $D_{\mathrm{r}}[m, n] \ltimes X \ltimes Y = Y$.

Lemma 5 (Power-reducing matrix)**.** Let $X \in \mathbb{R}^{n\times 1}$ be a column vector. Then

$$M_{r,n}X = X^2,$$

where $M_{r,n} = \mathrm{diag}\{\delta_n^1, \delta_n^2, \ldots, \delta_n^n\}$.

One feature of STP method is to convert a logical function into an algebraic form. For $i \in \mathcal{D}_k$, identify i as a vector form δ_k^{i+1}. The following result shows this feature.

Lemma 6. Given a logical function $f(x_1, x_2, \ldots, x_s) : \mathcal{D}_k^s \mapsto \mathcal{D}_k$. Then

$$f(x_1, x_2, \ldots, x_s) = M_f \ltimes_{i=1}^{s} x_i, \quad x_i \in \Delta_k, \tag{6}$$

where $M_f \in \mathcal{L}_{k\times k^s}$ is called the structural matrix of f.

Finally, we recall some results on the row/column stacking form of matrices, which are also the special properties of STP method.

Definition 1. Consider a matrix $A = (a_{ij}) \in \mathbb{R}^{m\times n}$. The row stacking form of A, denoted by $V_{\mathrm{r}}(A)$, is given as follows:

$$V_{\mathrm{r}}(A) = (a_{11}, a_{12}, \ldots, a_{1n}, \ldots, a_{m1}, a_{m2}, \ldots, a_{mn})^{\mathrm{T}}. \tag{7}$$

The column stacking form of A, denoted by $V_{\mathrm{c}}(A)$, is given as follows:

$$V_{\mathrm{c}}(A) = (a_{11}, a_{21}, \ldots, a_{m1}, \ldots, a_{1n}, a_{2n}, \ldots, a_{mn})^{\mathrm{T}}. \tag{8}$$

Lemma 7. (1) Let $A \in \mathbb{R}^{m\times n}$. Then $V_{\mathrm{c}}(A) = V_{\mathrm{r}}(A^{\mathrm{T}})$.

(2) Let $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{n\times p}$ and $C \in \mathbb{R}^{p\times q}$. Then

$$V_{\mathrm{c}}(ABC) = (C^{\mathrm{T}} \otimes A)V_{\mathrm{c}}(B). \tag{9}$$

(3) Let $X \in \mathbb{R}^{m\times n}$ and $A \in \mathbb{R}^{n\times m}$. Then

$$V_{\mathrm{c}}(XAX) = (AX)^{\mathrm{T}}V_{\mathrm{c}}(X). \tag{10}$$

(4) Let $X \in \mathbb{R}^{m\times n}$ and $A \in \mathbb{R}^{n\times s}$. Then

$$XA = \pi_s^{\mathrm{r}}(I_m)(I_m \otimes A^{\mathrm{T}})V_{\mathrm{r}}(X), \tag{11}$$

where $\pi_s^{\mathrm{r}}(I_m) = I_m(I_m \otimes V_{\mathrm{r}}^{\mathrm{T}}(I_s))$.

3 Gene regulation

In 1960s, Jacob and Monod found that any cell contains a great deal of “regulatory” genes which can turn one another “on” and “off”. Then, Kauffman [160] firstly introduced Boolean networks to describe, analyze and simulate gene regulatory networks. Since then, the study of Boolean networks has attracted many scholars’ research interests. Particularly, Akutsu et al. [161] pointed out that the control problems of Boolean networks are NP-hard.

In 2009, using the STP method, Cheng et al. [4] firstly converted a Boolean network in the form of

$$\begin{cases} x_1(t+1) = f_1(X(t), U(t)), \\ x_2(t+1) = f_2(X(t), U(t)), \\ \quad\vdots \\ x_n(t+1) = f_n(X(t), U(t)), \\ y_j(t) = h_j(X(t)), \quad j = 1, \ldots, p \end{cases} \tag{12}$$

into an algebraic form as follows:

$$\begin{cases} x(t+1) = Lu(t)x(t), \\ y(t) = Hx(t), \end{cases} \tag{13}$$

Table 1 Biological meaning of each x_i in (14)

x_i	Biological meaning
x_1	Nitric oxide synthase (NOS)
x_2	Nitric oxide (NO)
x_3	Guanyl cyclase (GC)
x_4	Phospholipase C (PLC)
x_5	Inositol-1,4,5-trisphosphate (InsP3)
x_6	Ca^{2+} influx to the cytosol from intracellular stores (CIS)
x_7	Ca^{2+}ATPase
x_8	Cytosolic Ca^{2+} increase (Ca_c^{2+})

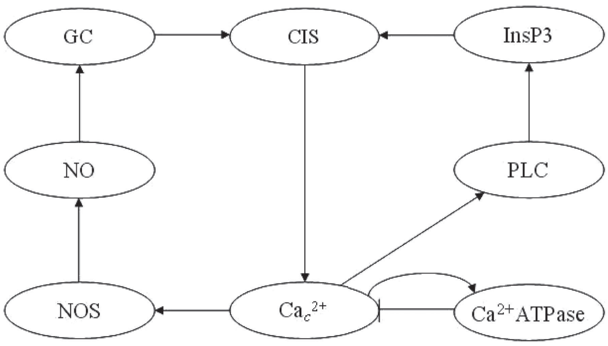

Figure 1 The network graph of system (14).

where $L \in \mathcal{L}_{2^n \times 2^{m+n}}$ and $H \in \mathcal{L}_{2^p \times 2^n}$ are called state transition matrix and output matrix, respectively. Eq. (13) is called the algebraic state space representation (ASSR) of the Boolean network (12).

Using the ASSR, many scholars studied the regulation of some gene regulatory networks such as D. melanogaster segmentation polarity gene network, the core network regulating the mammalian cell cycle, the lactose operon in Escherichia coli, signal transduction network, and apoptosis network. Zhang et al. [162] proved that one cannot arbitrarily control mammalian cell cycles. Meng et al. [125] studied how to identify the function perturbation in D. melanogaster segmentation polarity gene network. Li et al. [163] found all the attractors of signal transduction networks by using the logical matrix factorization method.

As an example, we consider the following sub-network of signal transduction networks:

$$\begin{cases} x_1(t+1) = x_8(t), \\ x_2(t+1) = x_1(t), \\ x_3(t+1) = x_2(t), \\ x_4(t+1) = x_8(t), \\ x_5(t+1) = x_4(t), \\ x_6(t+1) = x_3(t) \vee x_5(t), \\ x_7(t+1) = x_8(t), \\ x_8(t+1) = x_6(t) \wedge \neg x_7(t), \end{cases} \tag{14}$$

where the biological meaning of each x_i is given in Table 1, and the network graph of system (14) is shown in Figure 1.

Using the ASSR, one can convert system (14) into the following algebraic form:

$$z(t+1) = Lz(t), \tag{15}$$

where

$$
\begin{array}{l}
L=\delta_{256}[\begin{array}{cccccccccccccccc}
2 & 148 & 1 & 147 & 2 & 148 & 2 & 148 & 2 & 148 & 1 & 147 & 2 & 148 & 2 & 148 \\
10 & 156 & 9 & 155 & 10 & 156 & 10 & 156 & 10 & 156 & 9 & 155 & 10 & 156 & 10 & 156 \\
2 & 148 & 1 & 147 & 2 & 148 & 2 & 148 & 6 & 152 & 5 & 151 & 6 & 152 & 6 & 152 \\
10 & 156 & 9 & 155 & 10 & 156 & 10 & 156 & 14 & 160 & 13 & 159 & 14 & 160 & 14 & 160 \\
34 & 180 & 33 & 179 & 34 & 180 & 34 & 180 & 34 & 180 & 33 & 179 & 34 & 180 & 34 & 180 \\
42 & 188 & 41 & 187 & 42 & 188 & 42 & 188 & 42 & 188 & 41 & 187 & 42 & 188 & 42 & 188 \\
34 & 180 & 33 & 179 & 34 & 180 & 34 & 180 & 38 & 184 & 37 & 183 & 38 & 184 & 38 & 184 \\
42 & 188 & 41 & 187 & 42 & 188 & 42 & 188 & 46 & 192 & 45 & 191 & 46 & 192 & 46 & 192 \\
66 & 212 & 65 & 211 & 66 & 212 & 66 & 212 & 66 & 212 & 65 & 211 & 66 & 212 & 66 & 212 \\
74 & 220 & 73 & 219 & 74 & 220 & 74 & 220 & 74 & 220 & 73 & 219 & 74 & 220 & 74 & 220 \\
66 & 212 & 65 & 211 & 66 & 212 & 66 & 212 & 70 & 216 & 69 & 215 & 70 & 216 & 70 & 216 \\
74 & 220 & 73 & 219 & 74 & 220 & 74 & 220 & 78 & 224 & 77 & 223 & 78 & 224 & 78 & 224 \\
98 & 244 & 97 & 243 & 98 & 244 & 98 & 244 & 98 & 244 & 97 & 243 & 98 & 244 & 98 & 244 \\
106 & 252 & 105 & 251 & 106 & 252 & 106 & 252 & 106 & 252 & 105 & 251 & 106 & 252 & 106 & 252 \\
98 & 244 & 97 & 243 & 98 & 244 & 98 & 244 & 102 & 248 & 101 & 247 & 102 & 248 & 102 & 248 \\
106 & 252 & 105 & 251 & 106 & 252 & 106 & 252 & 110 & 256 & 109 & 255 & 110 & 256 & 110 & 256
\end{array}].
\end{array}
$$

Obviously, the state transition matrix L is so large that it is not easy to calculate all the attractors of system (14). Li and Wang [163] proposed a logical matrix factorization method to reduce the dimension of system (14). They obtained the following attractor characteristic matrix of system (14):

$$L^{*}=\delta_{5}[3\ 1\ 4\ 2\ 5]. \tag{16}$$

One can easily see from L^{*} that system (14) has a fixed point and a cycle with length 4.

It is believed that STP method will be applied to other gene regulatory networks in the future work.

4 Power system

In this section, we review the application of STP method in power system. Power system is a kind of high-order nonlinear systems [164–168]. Transient stability analysis is an important topic in the study of power system. A classic power system has the following form [5, 126]:

$$\dot{x}=f(x), \tag{17}$$

where $x\in\mathbb{R}^{n}$, and $f:\mathbb{R}^{n}\mapsto\mathbb{R}^{n}$ is a nonlinear function.

Suppose that x_{u} is an unstable equilibrium point of system (17). Denote the stable and unstable sub-manifolds of x_{u} by

$$W^{\mathrm{s}}(x_{\mathrm{u}})=\{p\in\mathbb{R}^{n}:\lim_{t\to+\infty}x(t,p)\to x_{\mathrm{u}}\} \tag{18}$$

and

$$W^{\mathrm{u}}(x_{\mathrm{u}})=\{p\in\mathbb{R}^{n}:\lim_{t\to-\infty}x(t,p)\to x_{\mathrm{u}}\}, \tag{19}$$

respectively.

Using STP method, Cheng et al. [5] obtained the following results on the stable sub-manifold of type-1 equilibrium points.

Theorem 1. Assume that $x_{\rm u} = 0$ is a type-1 equilibrium point of system (17). Denote $W^{\rm s}(x_{\rm u}) = \{x : h(x) = 0\}$. Then the following necessary and sufficient conditions uniquely determine $h(x)$:

$$\begin{cases} h(0) = 0, \\ h(x) = \eta^{\rm T} x + O(\|x\|^2), \\ L_f h(x) = \mu h(x), \end{cases} \tag{20}$$

where $L_f h(x)$ denotes the Lie derivative of $h(x)$ with respect to f, and η is an eigenvector of $J_f^{\rm T}(0)$ with respect to its only positive eigenvalue μ.

Theorem 2. The stable sub-manifold of $x_{\rm u}$, denoted by $h(x) = 0$, can be expressed as

$$h(x) = K_1(x) + \frac{1}{2} x^{\rm T} \Lambda x + O(\|x\|^3), \tag{21}$$

where

$$\begin{cases} K_1 = \eta^{\rm T}, \\ \Lambda = V_{\rm c}^{-1} \left\{ \left[\left(\frac{\mu}{2} I_n - J^{\rm T} \right) \otimes I_n + I_n \otimes \left(\frac{\mu}{2} I_n - J^{\rm T} \right) \right]^{-1} V_{\rm c} \left(\sum_{i=1}^{n} \eta_i {\rm Hess}(f_i(0)) \right) \right\}, \end{cases} \tag{22}$$

and ${\rm Hess}(f_i)$ denotes the Hessian matrix of f_i.

The main advantage of Theorem 2 is that it makes the analysis of power system easily verify with the help of digital computer. Therefore, Theorem 2 has been applied to the transient stability analysis of power system by many scholars in the last decade.

Ye et al. [167] studied the transient voltage stability of power system by using Theorem 2 and proposed a criterion to estimate the transient voltage stability based on the second-order approximation of stability boundary. The proposed criterion has some advantages, such as rapid convergence, high accuracy and high practicability.

Ma et al. [128] investigated how to approximate the boundary of attraction region of power system based on Theorem 2. A novel boundary approximation algorithm was derived from the topological characteristics of the stability boundary. The main feature of this algorithm is that it does not need any nonlinear transformation of the power system.

Using STP method, Sun et al. [126] studied how to calculate the polynomial approximation of a general nonlinear system, and showed that the equilibrium points of the polynomial approximate system can be arbitrarily close to that of the original system if the approximation order is high enough. They applied the theoretical results to the approximate boundary of the stability region of power system by using its corresponding polynomial approximate system [127].

Based on STP method and the quasi-steady state time domain simulation, Wang and Mei [168] proposed a new method to judge medium- and long-term voltage stability of power system. The main advantage is that it turns the short-term dynamic balance equations into algebraic equations, and thus can reduce the complexity of computing Jacobian matrix and Hessian matrix during solving stability margin index.

5 Vehicle control

In this section, we review the application of STP method in vehicle control. Taking vehicle motion's the longitudinal direction in consideration, we can get an expression of the vehicle dynamics in the following:

$$\sigma m \frac{{\rm d}v}{{\rm d}t} = F_{\rm d}(\tau_{\rm e}, i_{\rm g}) - F_{\rm a}(v) - F_{\rm i}(\theta) - F_{\rm r}(\theta), \tag{23}$$

in which m is the vehicle mass, σ is the inertial coefficient for the vehicle dynamics, the gear raito of the vehicle dynamics is $i_{\rm g}$, the engine torque is represented by $\tau_{\rm e}$, the road gradient is denoted by θ and the vehicle speed in m/s is v. Respectively, rolling resistance, gradient resistance, air resistance and the driving force on the driving wheel are denoted by $F_{\rm r}$, $F_{\rm i}$, $F_{\rm a}$ and $F_{\rm d}$, which is delineated as

$$\begin{cases} F_{\rm d}(\tau_{\rm e}, i_{\rm g}) = \dfrac{\tau_{\rm e} i_{\rm g} i_0 \eta}{R}, \\ F_{\rm a}(v) = \dfrac{1}{2}\rho_{\rm a} C_{\rm d} A v^2, \\ F_{\rm i}(\theta) = mg\sin\theta, \\ F_{\rm r}(\theta) = mgf\cos\theta, \end{cases} \tag{24}$$

where f, R, g, $C_{\rm d}$, A, $\rho_{\rm a}$, η and i_0 represent rolling resistance coefficient of the tyre, radius of wheels, gravity coefficient, air resistance coefficient, frontal area of the vehicle, air density, utilization efficiency coefficient, and the final drive ratio, respectively.

Generally, many results, [150] for example, transfer vehicle dynamics (23) to the following stochastic dynamic system:

$$v(k+1) = f(v(k), i_{\rm g}(k), \phi(k)), \tag{25}$$

where $\phi(i) \sim P_{N_R}^{\gamma}$, $\gamma = 1, 2, \ldots, N_R$, is the probability distribution for the accelerator pedal position.

Furthermore, in order to convert the vehicle dynamics (25) into the expression as logical networks, we should discrete the gear ratio and vehicle speed in the first place. Let V be an proper range of the vehicle speed and divide this range into finite disjoint intervals S^1, S^2, $\ldots$, S^m which is satisfied by

$$\begin{cases} S^i \cap S^j = \emptyset,\ i \neq j,\ i, j = 1, 2, \ldots, m, \\ \bigcup_{i=1}^{m} S^i = V. \end{cases}$$

Then the vehicle speed can be quantified as follows:

$$v(k) \in S^i \to x_k = \delta_m^i, \quad i = 1, 2, \ldots, m. \tag{26}$$

Eq. (26) implies that, for any given value of vehicle speed $v(k)$, we can found a sole corresponding logic value x_k. At the same time, the gear ratio $i_{\rm g}(k)$ contains finite values when the given vehicle is equipped with stepwise gearbox, i.e.,

$$i_{\rm g}(k) = i_{\rm g}^r \in IG := \{i_{\rm g}^1, i_{\rm g}^2, \ldots, i_{\rm g}^n\},\ r = 1, 2, \ldots, n,$$

where $i_{\rm g}^r$ is the gear ratio in the r-th gear position. Generally speaking, we can write it as logical variable

$$i_{\rm g}(k) = i_{\rm g}^r \leftrightarrow u_k = \delta_u^r,\ r = 1, 2, \ldots, n. \tag{27}$$

Thus, with (26) and (27) in hands, we can rewrite (25) as a typical logical dynamical system with stochastic property as follows:

$$x_{k+1} = \mathcal{L}(x_k, u_k, w_k), \tag{28}$$

where $k = 0, 1, 2, \ldots$ denotes the step index, the state $x_k \in X$ with $X = \{\delta_x^1, \delta_x^2, \ldots, \delta_x^m\}$ is the logic state space with finite states, U is the control state space with finite logic control input δ_u^r $(r = 1, 2, \ldots, n)$, and w_k is the external stochastic disturbance characterized by conditional probabilities $P_W(\cdot \mid x_k, u_k)$.

Actually, by STP method, [97] rewrited (28) as

$$x(t+1) = L^* u(t) x(t), \tag{29}$$

where $x(t)$ represetns the probability distribution of state, and $L^* u(t)$ denotes the Markovian transition matrix. Furthermore, define $L^* u := A(u) = (a_{i,j}(u))$. Then, we have

$$P(x(t+1) = i \mid x(t) = j, u(t) = u) = a_{i,j}(u).$$

Therefore, one can investigate control problems in vehicle dynamics (23) via (29) by using STP method. Based on (29), [96] designed a finite horizon optimal control algorithm for stochastic logical dynamical systems with an algebraic expression of, when minimizing the fuel consumption without loss the acceleration capability

$$\min J_\pi(x_0) = \sum_{\gamma=1}^{N_R} E_{\phi_k \sim P_{N_R}^{\gamma}, k=0,1,\ldots,N_\gamma-1} \left\{ \sum_{k=0}^{N_\gamma-1} g(x_k, u_k) \right\}. \tag{30}$$

Optimal control problem about cyclic variation of residual gas fraction in combustion engines was studies by [148]. Ref. [149] proposed the policy iteration method to solve the control problem about residual gas fraction in IC engines. Ref. [150] solved the fuel efficiency optimization problem for commuting vehicles. In addition, Ref. [169] studied the fuzzy logic controller design for multi-variable fuzzy systems based on STP method and applied the theoretical results to the design of fuzzy controller for energy management and control strategy of parallel hybrid electric vehicles.

6 Smart grid

In this section, we review the application of STP method in smart grid. Ref. [138] investigated demand-side management of some kind of smart grid. It adopted STP method to tackle with the problems in smart grid within the framework of networked evolutionary game.

We use the following example to demonstrate this application.

A networked evolutionary game (NEG) is evolving among many remote rural communities, in which there exists a newly constructed networked power grid. Before we build the power grid, the communities used power generated by diesel generators. In order to cover the cost, if there are less users, the price of grid power is high. The price of grid power would decrease along with the number of users grows. Besides, when the number of users grows extravagant large, because of the shortage of supply, the price would increase again.

The key problem of the price policy is that, no single community want to be the first user of the power grid, therefore its price would be high from the start. In addition to that, we may reach an unstable optimal common benefit.

We assume that there exists a power grid which is connecting 4 communities. Every community has two choices: local diesel power or grid power. Let $p_{\rm d} = 7.2$ be the diesel power price, and $p_{\rm g}(t)$ be the grid power price, where $p_{\rm g}(0) = 8$, $p_{\rm g}(1) = 7$, $p_{\rm g}(2) = 7$, $p_{\rm g}(3) = 6.5$, $p_{\rm g}(4) = 7.5$. Define the strategy space of community i as $X_i = \{1, 2\}$, where 1 represents grid power, and 2 represents local diesel power. The topological structure of the network is defined by an undirected graph, whose adjacent matrix is

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

The real-time prices is not available for communities, instead, they fully know the spends of their neighbors. Let p_i be the cost of community i. The cost function is defined by

$$c_i(x_i(t), x_{-i}(t)) = p_i(t) + \alpha \left(p_i(t) - \min_{j \in \mathcal{N}_i} p_j(t) \right), \tag{31}$$

where $\alpha > 0$ is a constant weight coefficient and $\mathcal{N}_i$ is community i's neighbours. The updating law is given by unconditional imitation with fixed priority

$$x_i(t+1) = x_{j^*}(t), \quad j^* = \arg\min_{j \in \mathcal{N}_i} c_j(x_j(t), x_{-j}(t)). \tag{32}$$

If j^* is non-unique, then select the minimal j^* as priority. Eq. (32) implies that one community could adjust its strategy to the strategy of its neighbor with the lowest cost. Define the common benefit at time t as $C(t) = \sum_{i=1}^{4} p_i(t)$. It is worthy noting that, the optimal common benefit exists when there are three communities using grid power, and the left one uses diesel power.

Let community 4 be the controller, and suppose the updating law is given by (32). The objective is to design $u = x_4 \in X_4$, so that the total cost $\sum_{i=1}^{4} p_i(x_i, x_{-i})$ is minimized.

Based on the updating law given by (32) and STP method, the controlled NEG can be described by

$$x_i(t+1) = f(x_i(t), x_{-i}(t), c_i(t)) = M_i x(t), \tag{33}$$

where $x(t) = \ltimes_{i=1}^{3} x_i(t)$, and

$$M_1 = \begin{cases} \delta_2[1\ 1\ 1\ 1\ 1\ 1\ 1\ 2], & \text{for } u = 1, \\ \delta_2[1\ 1\ 1\ 1\ 1\ 1\ 1\ 2], & \text{for } u = 2, \end{cases}$$

$$M_2 = \begin{cases} \delta_2[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1], & \text{for } u = 1, \\ \delta_2[1\ 1\ 1\ 1\ 1\ 1\ 2\ 2], & \text{for } u = 2, \end{cases}$$

$$M_3 = \begin{cases} \delta_2[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1], & \text{for } u = 1, \\ \delta_2[1\ 1\ 1\ 1\ 1\ 2\ 1\ 2], & \text{for } u = 2. \end{cases}$$

From (33), the overall controlled logical dynamics can be rewritten as $x(t+1) = M_f(u(t))x(t)$, where

$$\begin{cases} M_f(\delta_2^1) & = M_1(\delta_2^1) * M_2(\delta_2^1) * M_3(\delta_2^1) \\ & = \delta_8[1\ 1\ 1\ 1\ 1\ 1\ 1\ 5], \\ M_f(\delta_2^2) & = M_1(\delta_2^2) * M_2(\delta_2^2) * M_3(\delta_2^2) \\ & = \delta_8[1\ 1\ 1\ 1\ 1\ 2\ 3\ 8]. \end{cases} \tag{34}$$

From (34), it is easy to see that for $x(0) \in \{\delta_8^8\} \sim \{2, 2, 2\}$, $M_f(\delta_2^1) \ltimes M_f(\delta_2^1) = \delta_8[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$. Consequently, letting $u(0) = \delta_2^1$ and $u(1) = \delta_2^1$, it follows that $x(2) = \delta_8^1$. Then, setting $u(t) = \delta_2^2$ for $t \geqslant 2$, one can see that the optimal Nash equilibrium $(1, 1, 1, 2)$ will be maintained.

The above example describes that, via STP method, we can convert the problems in smart grid into the problems in networked evolutionary game, and formulate them in the framework of logical networks. Then, one can solve these problems by using the classic control theory. For example, Ref. [138] used this method to deal with pricing problem in smart grid.

7 Other applications

Besides the aforementioned applications, STP method can also be utilized to some more areas. In this section, we briefly review some other applications of STP method to engineering, such as finite automata, information security, graph theory and formation control, spacecraft and mobile robot.

7.1 Finite automata

First, automata can be used to well model discrete-event systems and hybrid systems [170–173]. The specific definition of finite automata is given as follows.

Definition 2 ([174])**.** A finite automaton is a three-tuple $A = (X, \Sigma, f)$, where X is a finite set of states, Σ is a finite set of input symbols, leading to transitions between states, and $f : X \times \Sigma \to 2^X$ with 2^X denoting the power set of X.

Assume that $X = \{x_1, x_2, \ldots, x_n\}$, $\Sigma = \{u_1, u_2, \ldots, u_m\}$. Considering the vector form expression and by defining a transition structure matrix F, one can obtain the algebraic form of finite automaton A in the following theorem [141, 174].

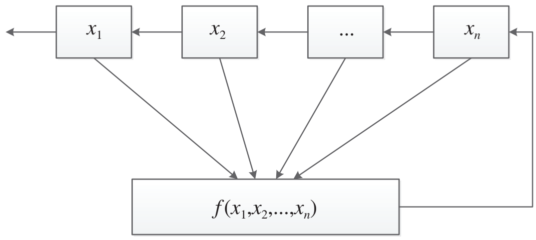

Figure 2 Shift register.

Theorem 3. Given an input string $\ltimes_{j=1}^{s} u(j)$ for finite automaton A, the dynamics of A can be described as

$$x(t+1) = Qu(t)x(t), \quad 1 \leqslant t \leqslant s. \tag{35}$$

Subsequently, considering the vector forms of finite automata, by resorting to Theorem 3 and STP method, some interesting problems, including reachability, observability, controllability and stabilization, were studied [141,143,144,174,175]. In detail, Xu et al. [141] proposed a necessary and sufficient condition for reachability of finite automata by using the introduced matrix-based approach, from which the vector form of the required input string can be easily constructed. The main advantage is that this method can be performed uniformly for analyzing reachability of deterministic and nondeterministic automata. Then in [143], reachability of finite automata was reconsidered with its application to language recognition, and an algorithm was designed to discover all the paths between two states. Xu and Hong [175] uniformly analyzed observability of partial observed non-deterministic automata either with or without input information by the matrix expression method, and also established a constructive algorithm to design the required observers. The proposed algorithm can avoid the symbolic manipulated and be easy to implement in some softwares. As interesting and important topics, controllability and stabilization analysis of finite automata were studied based on the algebraic expression in [144]. These aforementioned references gave constructive analysis and proof, and established concrete algorithms.

7.2 Information security

As we know, a feedback shifter register depicted in Figure 2 can be utilized to produce random sequences of numbers in different areas, such as cryptographic systems [176], error detecting and correcting codes [177], cell phones and digital cable [178], to name just a few. Recently, many researchers have paid great concern on nonlinear feedback shift register in designing cryptographic algorithms. Without using STP method, there are still no general and effective research methods to tackle various remaining problems.

It should be pointed out that a (linear or nonlinear) feedback shift register can be viewed as a special class of Boolean networks, whose nodes are the memory cells. Therefore, numerous approaches based on STP method, which were applied to investigate Boolean networks, can also be used to study feedback shift registers. Let $X_t = (x_1(t), x_2(t), \ldots, x_n(t))$, where $x_i(t) \in \mathcal{D}_2$, $i = 1, 2, \ldots, n$, be the state of a nonlinear feedback shift register at time t in Figure 2, and $f(x_1(t), x_2(t), \ldots, x_n(t))$ is the feedback function. Consider the vector form, and assume that the structure matrix of f is $M_f \in \mathcal{L}_{2\times 2^n}$. Then an algebraic expression of the considered nonlinear shift register can be given as [145]

$$\begin{cases} x_1(t+1) = x_2(t), \\ x_2(t+1) = x_3(t), \\ \qquad \vdots \\ x_{n-1}(t+1) = x_n(t), \\ x_n(t+1) = M_f x(t), \end{cases} \tag{36}$$

where $x(t) = \ltimes_{i=1}^{n} x_i(t)$. By some definitions and properties in Section 2, another algebraic form was derived as [145]

$$x(t+1) = Lx(t), \tag{37}$$

where $L = D_{\rm r}[2, 2^{n-1}](I_{2^n} \otimes M_f)M_{r,2^n} \in \mathcal{L}_{2^n \times 2^n}$. Based on the obtained transition matrix L and results about Boolean networks in [4], an open problem in feedback shift registers, that is, how to determine the number of fixed points and cycles of different lengths, were easily solved in [145]. Besides, Zhao et al. [145] analyzed the synthesis of nonlinear feedback shift registers, that is, constructions of the shortest nonlinear feedback shift register for a given sequence. It has been pointed out that even though STP method has some limitations in computational complexity, it can solve the existing open problems to some extent. With the consideration that a stable nonlinear feedback shift register can avoid an error-propagation, Zhong et al. [146] studied the global stability, as well as local stability of nonlinear feedback shift registers by using a novel method, a Boolean network method, which reduced the time complexity of computations compared with the exhaustive search and Lyapunov directed approach. Due to that a nonlinear feedback shift register is a Boolean network with a special form, some concise and deeper results were obtained by Zhong et al. [146], making advantage to analyze a given nonlinear feedback shift register and to design stable nonlinear feedback shift registers.

7.3 Graph theory and formation control

An important issue in wireless communication networks is the frequency assignment problem [179–181]. In general, the frequency assignment problem is to find an efficient way to allocate the frequency with some compatibility constraints, which can be viewed as a classical problem in graph theory, that is, coloring problem. The transmitters in a wireless communication network can be regarded as the vertexes of a graph or hybergraph and a set constituted by the possible intercommunicated transmitters can be considered as an edge of a graph or a hypergraph. The coloring problem was first solved by STP method in [130], in which necessary and sufficient conditions were derived in the form of matrix conditions. By applying T-coloring and conflict-free coloring problem [131, 132], the frequency assignment with certain constraints was settled. The main advantage of this method to study wireless communication problem is the precise analysis, reflected in the algebraic equivalent conditions obtained by rigorous deduction. In addition, the formation control problem was investigated via a mix-valued logic-based approach in [182], and a new algorithm was established for the feedback formation control.

7.4 Spacecraft

In spacecraft control system design, accurate and reliable attitude stabilization is one of the most significant issues, which has been investigated by some existing methods, such as sliding-mode control [183], optimal control [184], and adaptive control [185]. Jia et al. [157] first applied STP method to analyze the attitude tracking control problem of rigid spacecraft involving uncertainty in inertia matrix parameters. The main advantage of using STP method is that STP method has pseudo-commutative property while the conventional matrix product does not have.

7.5 Mobile robot

Researchers have paid great attention to building mobile robots with onboard odor sensor or/and winds sensor to realize the odor source localization task. Jiang et al. [159] used a multi-input multi-output fuzzy control system based on STP method to model robot odor source localization in order to fully apply the multisensor information. With the proposed fuzzy control system, relative searching strategies can be activated dependent on the timely multisensor information got from mobile robot. This method, compared to the classical approaches, can avoid some random searching without odor information. In [159], an in-depth study was given from the perspective of mathematics, which enriches the theory of the mobile robot odor source localization.

8 Conclusion

In this survey, we have reviewed a number of applications of STP method to engineering, including gene regulation, power system, wireless communication, smart grid, information security, combustion engine and vehicle control. By utilizing STP method, constructive and precise analysis from the perspective of mathematics has been reflected in these applications. With the rapid development of science and technology, we are confident that STP method will receive more attention and wider applications in engineering in the future.

Based on the literature review, some related topics for future potential applications in engineering are given as follows. First, as an important applied field of Boolean calculus, which has been well studied based on STP method, circuit fault detection may be investigated from a different angle. Furthermore, greater efforts should be made towards the control problems in economics since the growing interests in game theory by using STP method. In addition, considering that graph theory is of great significance in multi-agent systems, future of interest is to do some research on graph theory based on STP method in order to solve some problems in agent networks, including networked robot manipulators and unmanned aerial vehicle. Finally, an important task of using STP method is still to reduce the computational complexity of the obtained results.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61374065, 61374025, 61503225), Natural Science Foundation of Shandong Province (Grant No. ZR2015FQ003), and Natural Science Fund for Distinguished Young Scholars of Shandong Province (Grant No. JQ201613).

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