SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •
Special Focus on Analysis and Synthesis for Stochastic Systems

December 2017, Vol. 60 120208:1–120208:18
doi: 10.1007/s11432-017-9163-4

Delay-dependent dissipative filtering for nonlinear stochastic singular systems with time-varying delays

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Received May 10, 2017; accepted July 3, 2017; published online November 9, 2017

Abstract This paper concentrates on studying the delay-dependent dissipative filtering problem for nonlinear stochastic singular systems with time-varying delays via a Takagi-Sugeno (T-S) fuzzy control approach. The T-S fuzzy model is employed to represent a nonlinear stochastic singular system with unknown or partially unknown membership functions. Firstly, based on an auxiliary vector function, by utilizing an integral inequality and the free-weighting-matrix approach, a delay-dependent sufficient condition is derived to enable the considered filtering error system with time-varying delays to be stochastically admissible and dissipative. Furthermore, on the basis of the derived condition, by using a new type of candidate Lyapunov-Krasovskii function, the solvability conditions of the dissipative filter are addressed, and the corresponding fuzzy filter parameters can be obtained by solving a set of linear matrix inequalities. And then, we deduce the solving method for the H_∞ filter. The delay-dependent sufficient conditions are proposed to guarantee the systems to be regular, impulse-free, stochastically stable and to achieve a prescribed performance index $\hat{\gamma}$. Finally, some simulation examples are proposed to manifest the effectiveness and merits of the filter design methodology developed in the paper.

Keywords nonlinear stochastic singular systems, dissipative filtering, unknown membership functions, stochastic admissibility, time-varying delays

Citation Xing S Y, Deng F Q. Delay-dependent dissipative filtering for nonlinear stochastic singular systems with time-varying delays. Sci China Inf Sci, 2017, 60(12): 120208, doi: 10.1007/s11432-017-9163-4

1 Introduction

As we all known, stochastic systems have played a significant role in the branches of nature science and engineering in the past few years. A multitude of paramount contributions in this direction have appeared in numerous literature (see [1–8] and references therein). Recently, a wider class of stochastic systems described by the stochastic singular form is considered in [9,10], and the research of such systems has also made some rapid progress. For instance, Xing et al. [11] investigated the stochastic stability and exact observability for a class of discrete stochastic singular systems by a type of new generalised Lyapunov equation. In [12], Gao et al. gave the controller design method for the stochastic singular systems through a complete separation design technique. Boukas [13] considered the problem of the stabilization for the nonlinear stochastic singular hybrid systems under some appropriate assumptions.

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In the past few years, the T-S fuzzy method for controller and filter designs has served as an innovative and efficient approach in the study of the complex nonlinear dynamic systems. The stochastic T-S fuzzy model can properly describe the nonlinear stochastic system when the model parameters are affected by the statistical characteristics of the known noise. Based on this, Tseng [14] discussed the robust fuzzy filter design problem for nonlinear stochastic systems, and the corresponding H_∞ fuzzy filter was designed. Balasubramanium et al. [15] considered the problem of the delay dependent robust asymptotic state estimation for fuzzy neural networks with mixed random time-varying delay. Muralisankar et al. [16] studied the robust stability problem for the uncertain neutral stochastic system with the T-S fuzzy model and Markovian jumping parameters, and then some novel sufficient conditions were derived to guarantee the asymptotic stability of the equilibrium point in the mean square. The mean-square admissibility problem for a kind of stochastic T-S fuzzy singular systems was discussed in [17], and some novel relaxed mean-square admissibility conditions were obtained via an extended quadratic Lyapunov function. Zhao et al. [18] presented the H_∞ filter design method for the singular fuzzy systems with random disturbance and state time-varying delays, and then the stochastic admissibility conditions of such systems were presented. It is pointed that the results as reported previously were established under the membership functions that were assumed to be completely known. Nevertheless, due to uncertainty in engineering practice, the membership functions maybe contain a lot of uncertainties such as immeasurable premise variables, uncertain parameters or unknown perturbations. Therefore, the membership functions are not always known. So far, some fruitful results have been conducted on the H_∞ filtering and control synthesis for T-S fuzzy systems with uncertain membership functions (see [19–24] and references therein). However, up to date no related theory has been studied for the delay-dependent dissipative filtering of nonlinear singular systems with random disturbance and time-varying delays via fuzzy models, especially when the membership functions are unknown or partially unknown.

This paper presents a delay-dependent dissipative filtering analysis approach for a type of nonlinear stochastic singular system which the membership functions may be unknown or partially unknown via T-S fuzzy models. By employing an auxiliary vector function, an integral inequality, and the free-weighting-matrix approach as well as the linear matrix inequalities (LMIs) technique, sets of sufficient conditions are proposed for the filtering error system to be stochastically admissible and dissipative. Furthermore, the desired fuzzy filter is got by solving a set of linear matrix inequalities. As a special form of dissipativity, the H_∞ strategy is even paramount and considered to be an efficient tool for studying stochastic T-S fuzzy singular systems. Consequently, as a corollary, the design method for the H_∞ filter in the same design process is elicited, the corresponding sufficient conditions are stated and proved for the solvability of the H_∞ filtering problem. The main contributions of the paper are as follows.

(1) A nonlinear stochastic singular system with time-varying delays is modeled as a stochastic fuzzy singular time-delay system with unknown or partially unknown membership functions. Based on this model, stochastic admissibility and the filter design methods can be derived.

(2) It should be stressed that the coefficient matrices of state variables are different in the measurement output equation, then the proposed method reduces the conservatism compared to the analysis method of the reference [20]. To sum up, our research results are innovative in the sense.

(3) We employ an auxiliary vector function and the new free-weighting-matrix approach to reduce the conservatism of the solution for the systems. Moreover, it is more convenient for the solution of the dissipative filter.

(4) In order to show the application for the proposed method, we consider a continuous stirred tank reactor (CSTR) simulation example. The simulation results demonstrate that the method stabilizes the filtering error system and achieves dissipative performance. This ensures the stable operation of a stirred tank reactor.

Notations. Throughout this paper, the notations used are quite standard. $\mathbb{R}^n$ represents the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. The symbol * in a matrix block implies that it can be induced by symmetric position. A^{T} represents the transpose of matrix A. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalues of real square matrix A, respectively. The notation $X \geqslant 0$ $(X > 0)$ means that X is a real symmetric and positive semi-definite (positive definite) matrix. Symbol $\varepsilon\{\cdot\}$ stands for the mathematical expectation operator. C^1 represents an one order differentiable function. $\|\cdot\|$ denotes the Euclidean norm for a given vector or the induced norm of a matrix. C_{n,d_0} represents the family of the continuous functions for $[-d_0,0]$ to $\mathbb{R}^n$, where $d_0>0$ denotes the upper bound for the time delays.

2 Problem formulation and preliminaries

For a fundamental probability space $(\Omega,\mathcal{F},\mathrm{Pr})$, we consider the dissipative filtering problem for a kind of nonlinear stochastic singular system with time-varying delays via a T-S fuzzy model. The stochastic T-S fuzzy model is in the following form.

Rule *i*. If $\zeta_1(t)$ is $\hat{M}_{i1}$ and $\zeta_2(t)$ is $\hat{M}_{i2}$ ...and $\zeta_p(t)$ is $\hat{M}_{ip}$, then

$$\begin{aligned} &E\mathrm{d}x(t)=(A_ix(t)+A_{id}x(t-d(t))+B_iv(t))\mathrm{d}t+J_ix(t)\mathrm{d}\omega(t),\\ &\mathrm{d}y(t)=(C_{1i}x(t)+D_iv(t))\mathrm{d}t,\\ &z(t)=C_{2i}x(t),\quad i=1,2,\ldots,s,\\ &x(t)=\phi(t),\quad t\in[-d_0,0], \end{aligned}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector of the plant, $y(t)\in\mathbb{R}^m$ is the measurable output vector, $v(t)\in L_2[0,\infty)$ stands for the external disturbance signal and $z(t)\in\mathbb{R}^q$ is the output vector to be estimated. $\zeta(t)=[\zeta_1(t),\zeta_2(t),\ldots,\zeta_p(t)]^{\mathrm{T}}$ are the premise variables, $\hat{M}_{ij}$ $(i=1,2,\ldots,s,\ j=1,2,\ldots,p)$ are the fuzzy sets, s is the number of fuzzy rules. The matrix $E\in\mathbb{R}^{n\times n}$ may be singular and we assume $\mathrm{rank}(E)=r\leqslant n$. $A_i,\ A_{id},\ B_i,\ J_i,\ C_{1i},\ C_{2i},\ D_i$ are known constant matrices with appropriate dimensions. $\omega(t)$ is one-dimensional standard Brownian motion defined on the probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geqslant0},\mathrm{Pr})$, where Ω is the sample space, $\mathcal{F}_t\subset\mathcal{F}$ is the σ-algebra of subsets of the sample space, and Pr is the probability measured on $\mathcal{F}$. $\phi(t)$ is a compatible vector valued initial function. $d(t)$ is a time-varying delay and it satisfies for all $t\geqslant0$, $0\leqslant d(t)\leqslant d_0$, $\dot{d}(t)\leqslant\bar{d}$ $(0<\bar{d}<1)$, where d_0 and $\bar{d}$ are scalars.

Based on the standard fuzzy singleton inference method and weighted center-average defuzzifier process, the dynamic fuzzy model of (1) can be expressed by

$$\begin{aligned} E\mathrm{d}x(t)&=\sum_{i=1}^{s}(h_i(\zeta(t))+\theta_i(\zeta(t)))\{(A_ix(t)+A_{id}x(t-d(t))\\ &\quad+B_iv(t))\mathrm{d}t+J_ix(t)\mathrm{d}\omega(t)\},\\ \mathrm{d}y(t)&=\sum_{i=1}^{s}(h_i(\zeta(t))+\theta_i(\zeta(t)))(C_{1i}x(t)+D_iv(t))\mathrm{d}t,\\ z(t)&=\sum_{i=1}^{s}(h_i(\zeta(t))+\theta_i(\zeta(t)))C_{2i}x(t),\\ x(t)&=\phi(t),\quad t\in[-d_0,0], \end{aligned}\tag{2}$$

where

$$h_i(\zeta(t))+\theta_i(\zeta(t))=\frac{\sigma_i(\zeta(t))}{\sum_{i=1}^{s}\sigma_i(\zeta(t))},\quad \sigma_i(\zeta(t))=\prod_{j=1}^{p}M_{ij}(\zeta_j(t)),\tag{3}$$

in which $\hat{M}_{ij}(\zeta_j(t))$ is the grade of membership of $\zeta_j(t)$ in the fuzzy set $\hat{M}_{ij}$. For any $\zeta(t)$, we assume

$$\sigma_i(\zeta(t))\geqslant0,\quad i=1,2,\ldots,s,\quad \sum_{i=1}^{s}\sigma_i(\zeta(t))>0.$$

Therefore, for all t, one has

$$h_i(\zeta(t))+\theta_i(\zeta(t))\geqslant0,\quad \sum_{i=1}^{s}(h_i(\zeta(t))+\theta_i(\zeta(t)))=1.\tag{4}$$

In this paper, $h_i(\zeta(t))$ and $\theta_i(\zeta(t))$ represent the unknown and known parts of the membership functions, respectively. In addition, suppose that $\underline{h}_i \leqslant h_i(\zeta(t)) \leqslant \overline{h}_i$, where $\overline{h}_i$ and $\underline{h}_i$ are known upper and lower bounds of $h_i(\zeta(t))$.

Consider the following fuzzy filter.

Rule *i*. If $\zeta_1(t)$ is $\hat{M}_{i1}$ and $\zeta_2(t)$ is $\hat{M}_{i2}$ ... and $\zeta_p(t)$ is $\hat{M}_{ip}$, then

$$\begin{aligned} \mathrm{d}x_f(t) &= A_{fi}x_f(t)\mathrm{d}t + B_{fi}\mathrm{d}y(t), \\ z_f(t) &= C_{fi}x_f(t), \end{aligned}$$

where $x_f(t) \in \mathbb{R}^n$ is the state of the filter and $z_f(t) \in \mathbb{R}^q$ is the corresponding output. A_{fi}, B_{fi} and C_{fi} are matrices to be determined. Then, the global fuzzy filter is

$$\begin{aligned} \mathrm{d}x_f(t) &= \sum_{i=1}^{s} \tilde{\theta}_i(\zeta(t))(A_{fi}x_f(t)\mathrm{d}t + B_{fi}\mathrm{d}y(t)), \\ z_f(t) &= \sum_{i=1}^{s} \tilde{\theta}_i(\zeta(t))C_{fi}x_f(t), \end{aligned} \tag{5}$$

where $\tilde{\theta}_i(\zeta(t))$, $i = 1, 2, \ldots, s$, are the known membership functions, and they satisfy

$$\tilde{\theta}_i(\zeta(t)) \geqslant 0, \quad \sum_{i=1}^{s} \tilde{\theta}_i(\zeta(t)) = 1.$$

From (2) and (5), we can obtain the following filtering error system:

$$\begin{aligned} E_e\mathrm{d}x_e(t) &= \sum_{i=1}^{s}(h_i(\zeta(t)) + \theta_i(\zeta(t)))\sum_{j=1}^{s}\tilde{\theta}_j(\zeta(t)) \times \{(A_{eij}x_e(t) + A_{eid}x_e(t-d(t)) \\ &\quad + B_{eij}v(t))\mathrm{d}t + J_{ei}x_e(t)\mathrm{d}\omega(t)\} \\ &:= (\tilde{A}_h x_e(t) + \tilde{A}_{hd}x_e(t-d(t)) + \tilde{B}_h v(t))\mathrm{d}t + \tilde{J}_h x_e(t)\mathrm{d}\omega(t), \\ z_e(t) &= \sum_{i=1}^{s}(h_i(\zeta(t)) + \theta_i(\zeta(t)))\sum_{j=1}^{s}\tilde{\theta}_j(\zeta(t)) \times C_{ij}x_e(t) := \tilde{C}_h x_e(t), \\ x_e(t) &= \phi_e(t), \quad t \in [-d_0, 0], \end{aligned} \tag{6}$$

where

$$\begin{aligned} &E_e = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \quad A_{eij} = \begin{bmatrix} A_i & 0 \\ B_{fj}C_{1i} & A_{fj} \end{bmatrix}, \quad A_{eid} = \begin{bmatrix} A_{id} & 0 \\ 0 & 0 \end{bmatrix}, \\ &B_{eij} = \begin{bmatrix} B_i \\ B_{fj}D_i \end{bmatrix}, \quad J_{ei} = \begin{bmatrix} J_i & 0 \\ 0 & 0 \end{bmatrix}, \quad C_{eij} = \begin{bmatrix} C_{2i} & -C_{fj} \end{bmatrix}, \\ &x_e(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x_f^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}, \quad z_e(t) = z(t) - z_f(t), \quad \phi_e(t) = \begin{bmatrix} \phi^{\mathrm{T}}(t) & x_{f0}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \end{aligned} \tag{7}$$

Remark 1. In order to simplify the structure of the filtering error system, the coefficient matrices of state variables in the measurement output equation were assumed the same as in the refs. [20, 22]. In this case, it is easy to deal with the H_∞ filter design problem for considered systems no matter whether the membership functions are unknown or partially unknown, but a large conservatism will be unavoidable. In our work, the coefficient matrices C_{1i} $(i = 1, 2, \ldots, s)$ of state variables in the measurement output equation are different. Furthermore, the conservatism will be reduced.

In what follows, we introduce some related definitions and necessary lemmas in the paper. Consider the following system:

$$E_e\mathrm{d}x_e(t) = (\tilde{A}_h x_e(t) + \tilde{A}_{hd}x_e(t-d(t)))\mathrm{d}t + \tilde{J}_h x_e(t)\mathrm{d}\omega(t). \tag{8}$$

Definition 1. (1) The pair $(E_e, \tilde{A}_h)$ is said to be regular if $\det(sE_e - \tilde{A}_h)$ is not identically zero.

(2) The pair $(E_e, \tilde{A}_h)$ is said to be impulse-free if $\deg(\det(sE_e - \tilde{A}_h)) = \mathrm{rank}(E_e)$.

Definition 2 ([25])**.** For a given scalar $d_0 > 0$ and any time-varying delay $d(t)$ satisfying $0 \leqslant d(t) \leqslant d_0$, if the pairs $(E_e, \tilde{A}_h)$ and $(E_e, \tilde{A}_h + \tilde{A}_{hd})$ are regular and impulse-free, then system (8) is said to be regular and impulse-free.

Lemma 1 ([26])**.** Consider the stochastic singular linear system is given by

$$E\mathrm{d}x(t) = Ax(t)\mathrm{d}t + Jx(t)\mathrm{d}\omega(t).$$

Let $V(x(t)) = x^{\mathrm{T}}(t)E^{\mathrm{T}}Xx(t)$, where X is an invertible matrix satisfying $E^{\mathrm{T}}X = X^{\mathrm{T}}E \geqslant 0$. Define a weak infinitesimal operator $\mathcal{L}$. Then the stochastic derivative of $V(x(t))$ is given by

$$\mathrm{d}V(x(t)) = \mathcal{L}V(x(t))\mathrm{d}t + 2x^{\mathrm{T}}(t)X^{\mathrm{T}}Jx(t)\mathrm{d}\omega(t), \tag{9}$$

where E^{+} is the generalized inverse of the matrix E,

$$\mathcal{L}V(x(t)) = x^{\mathrm{T}}(t)(A^{\mathrm{T}}X + X^{\mathrm{T}}A + J^{\mathrm{T}}(E^{+})^{\mathrm{T}}E^{\mathrm{T}}XE^{+}J)x(t).$$

The following assumption will be applied in the next text.

Assumption 1 ([13])**.** The pair $(E_e, \tilde{A}_h)$ is regular and $\mathrm{rank}([\, E_e \;\; \tilde{J}_h \,]) = \mathrm{rank}(E_e)$.

Under Assumption 1, if $\mathrm{rank}(E_e) = r + n$, the coefficient matrices in (8) can be decomposed into

$$E_e = \begin{bmatrix} I_{r+n} & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{A}_h = \begin{bmatrix} \tilde{A}_{h11} & \tilde{A}_{h12} \\ \tilde{A}_{h21} & \tilde{A}_{h22} \end{bmatrix}, \quad \tilde{A}_{hd} = \begin{bmatrix} \tilde{A}_{hd11} & \tilde{A}_{hd12} \\ \tilde{A}_{hd21} & \tilde{A}_{hd22} \end{bmatrix}, \quad \tilde{J}_h = \begin{bmatrix} \tilde{J}_{h11} & \tilde{J}_{h12} \\ 0 & 0 \end{bmatrix}, \tag{10}$$

and denote $x_e(t) = [\, x_{1e}^{\mathrm{T}}(t) \;\; x_{2e}^{\mathrm{T}}(t) \,]^{\mathrm{T}}$ with $\tilde{A}_{h11} \in \mathbb{R}^{2(r+n)\times 2(r+n)}$, $\tilde{A}_{hd11} \in \mathbb{R}^{2r\times 2r}$, $\tilde{J}_{h11} \in \mathbb{R}^{2r\times 2r}$, $x_{1e}(t) \in \mathbb{R}^{2r}$.

Furthermore, by expression (10), system (8) is equivalent to

$$\begin{aligned} \mathrm{d}x_{1e}(t) = {} & \{(\tilde{A}_{h11}x_{1e}(t) + \tilde{A}_{h12}x_{2e}(t) + \tilde{A}_{hd11}x_{1e}(t-d(t)) \\ & + \tilde{A}_{hd12}x_{2e}(t-d(t))\}\mathrm{d}t + (\tilde{J}_{h11}x_{1e}(t) + \tilde{J}_{h12}x_{2e}(t))\mathrm{d}\omega(t), \end{aligned} \tag{11a}$$

$$0 = \{(\tilde{A}_{h21}x_{1e}(t) + \tilde{A}_{h22}x_{2e}(t) + \tilde{A}_{hd21}x_{1e}(t-d(t)) + \tilde{A}_{hd22}x_{2e}(t-d(t))\}\mathrm{d}t, \tag{11b}$$

and $\phi_e(t) = [\, \phi_{1e}^{\mathrm{T}}(t) \;\; \phi_{2e}^{\mathrm{T}}(t) \,]^{\mathrm{T}}$.

Remark 2. Through the above-mentioned assumption, we know that the diffusive term does not affect the inner structure of systems. It is worth noting that the pair $(E_e, \tilde{A}_h)$ is regular and impulse-free, which can ensure the existence and uniqueness of the impulse-free solution to system (8). Since the proof of the existence and uniqueness of the solutions for system (8) is similar to that of Lemma 2.2 in ref. [27], the proof is omitted here.

Lemma 2 ([28])**.** The pair $(E_e, \tilde{A}_h)$ is impulse-free if and only if $\tilde{A}_{h22}$ is nonsingular.

The Lyapunov functionals are usually defined based on a Lyapunov function. Actually the Lyapunov functionals can be regarded as Lyapunov functions. Thus, we can easily compute $\{\mathcal{L}V(t, \phi_e)\}$ for a Lyapunov functional $V(t, \phi_e)$, where $\phi_e \in C_{n,d_0}$.

Lemma 3. Assume $\tilde{A}_{h22}$ is nonsingular. Then, system (8) is stochastic asymptotically stable if there exist positive numbers α, β, γ and a continuous functional $V : C_{n,d_0} \to \mathbb{R}^{+}$ such that

$$\beta\|\phi_{1e}(0)\|^2 \leqslant \varepsilon\{V(t, \phi_e)\} \leqslant \gamma\|\phi_e\|^2, \tag{12a}$$

$$\varepsilon\{\mathcal{L}V(t, \phi_e)\} \leqslant -\alpha\|\phi_e\|^2, \quad \phi_e \in C_{n,d_0}. \tag{12b}$$

Proof. Since the proof is completely similar to those in [29], [30] for their Theorem 3.1, we omit the details. We need only to notice that the Lipschitz condition is naturally satisfied here. The proof is completed.

Lemma 4 ([31]). Given any matrices X and Y with appropriate dimensions, for any real matrix $W > 0$, the following inequality holds

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leqslant X^{\mathrm{T}}WX + Y^{\mathrm{T}}W^{-1}Y.$$

Now, we define the following quadratic energy supply rate:

$$\Gamma(v(t), z(t)) = z^{\mathrm{T}}(t)\hat{Q}z(t) + 2z^{\mathrm{T}}(t)\hat{S}v(t) + v^{\mathrm{T}}(t)\hat{R}v(t), \tag{13}$$

where $\hat{S}$ is a real matrix, $\hat{Q}$ and $\hat{R}$ are real symmetric matrices. Let $\hat{Q} = \hat{Q}^{\mathrm{T}} < 0$.

Definition 3. System (2) with supply rate function (13) is said to be stochastically dissipative if the following inequality is true for a nonnegative real function $V(\cdot)$ with $V(0) = 0$, called the storage,

$$\varepsilon\left\{\int_0^{t_1} \mathcal{L}V(x(t))\mathrm{d}t\right\} \leqslant \varepsilon\left\{\int_0^{t_1} \Gamma(v(t), z(t))\mathrm{d}t\right\}, \quad t_1 > 0. \tag{14}$$

Inequality (14) is called the stochastic dissipation inequality. Furthermore, system (2) is said to be stochastic strictly dissipative, if inequality (14) holds strictly.

The so-called dissipative filtering problem in this paper is stated as, for a scalar $\delta > 0$ and given the system (2), design a filter in the form (5) satisfying:

(1) The system (6) is impulse-free and stochastically admissible.

(2) Under the zero initial compatible condition, for external disturbance $v(t) \neq 0$, system (6) satisfies inequality (14).

3 Main results

The purpose of this section is to derive the stochastic admissibility condition for system (6). Then, we solve the designed dissipative filter by the LMI technology.

An auxiliary vector function $\eta(t)$ is introduced:

$$\eta(t) = \tilde{A}_h x_e(t) + \tilde{A}_{hd} x_e(t - d(t)) + \tilde{B}_h v(t). \tag{15}$$

Taking into account expression (6), we can obtain the following equality:

$$E_e \mathrm{d}x_e(t) = \eta(t)\mathrm{d}t + \tilde{J}_h x_e(t)\mathrm{d}\omega(t).$$

By integrating the above equality over the period $[t - d(t), t]$, it is obtained that

$$E_e x_e(t) - E_e x_e(t - d(t)) = \int_{t-d(t)}^{t} \eta(\tau)\mathrm{d}\tau + \int_{t-d(t)}^{t} \tilde{J}_h x_e(\tau)\mathrm{d}\omega(\tau). \tag{16}$$

Denote

$$\nabla_i = \max\left\{\underline{h}_i^2, \overline{h}_i^2\right\}, \quad \Delta_{ij} = \{\theta_{ij}^L, \theta_{ij}^U\}, \tag{17}$$

where $\theta_{ij}^L = \min\{\theta_i(\zeta(t)) \cdot \tilde{\theta}_j(\zeta(t))\}$, $\theta_{ij}^U = \max\{\theta_i(\zeta(t)) \cdot \tilde{\theta}_j(\zeta(t))\}$.

Theorem 1. For a scalar $\bar{d} > 0$, the filtering error system (6) is stochastically admissible and dissipative if there exist matrices P_e, $Q_e > 0$, $Z_e > 0$, and matrices M, N, $R_{ij} > 0$ such that for all $\rho_{ij} \in \Delta_{ij}$, $(i, j = 1, 2, \ldots, s)$. The following matrix inequalities hold

$$E_e^{\mathrm{T}} P_e = P_e^{\mathrm{T}} E_e \geqslant 0, \tag{18}$$

$$\begin{bmatrix} \tilde{\Lambda}_1 & \bar{\Pi}_1 & \bar{\Pi}_2 & \cdots & \bar{\Pi}_s \\ * & -R_1 & 0 & \cdots & 0 \\ * & * & -R_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & -R_s \end{bmatrix} < 0, \tag{19}$$

where

$$\tilde{\Lambda}_1 = \sum_{i=1}^{s}\sum_{j=1}^{s}(\nabla_i R_{ij} + \rho_{ij}\Pi_{ij}), \quad \Psi_1 = \begin{bmatrix} \Lambda_1 & \Lambda_2 & E_e^{\mathrm{T}}M \\ * & \Lambda_3 & E_e^{\mathrm{T}}N \\ * & * & -Z_e \end{bmatrix}, \quad \Pi_{ij} = \begin{bmatrix} \Psi_1 & \Psi_2 \\ * & \Psi_3 \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} \Lambda_4 & d_0 A_{eij}^{\mathrm{T}} Z_e & C_{eij}^{\mathrm{T}}\hat{Q}_1^{\mathrm{T}} \\ 0 & d_0 A_{eid}^{\mathrm{T}} Z_e & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Psi_3 = \begin{bmatrix} -\hat{R} & d_0 B_{eij}^{\mathrm{T}} Z_e & 0 \\ * & -Z_e & 0 \\ * & * & -I \end{bmatrix}, \quad \Lambda_4 = P_e^{\mathrm{T}} B_{eij} - C_{eij}^{\mathrm{T}}\hat{S},$$

$$\Lambda_1 = A_{eij}^{\mathrm{T}} P_e + P_e^{\mathrm{T}} A_{eij} + J_{ei}^{\mathrm{T}}(E_e^{+})^{\mathrm{T}} E_e^{\mathrm{T}} P_e E_e^{+} J_{ei} + Q_e - E_e^{\mathrm{T}}(M + M^{\mathrm{T}})E_e,$$

$$\Lambda_2 = P_e^{\mathrm{T}} A_{eid} + E_e^{\mathrm{T}}(M - N^{\mathrm{T}})E_e, \quad \Lambda_3 = -(1-\bar{d})Q_e + E_e^{\mathrm{T}}(N + N^{\mathrm{T}})E_e, \quad \hat{Q}_1 = (-\hat{Q})^{\frac{1}{2}},$$

$$\bar{\Pi}_i = \begin{bmatrix} \frac{1}{2}\Pi_{i1}^{\mathrm{T}} & \frac{1}{2}\Pi_{i2}^{\mathrm{T}} & \cdots & \frac{1}{2}\Pi_{is}^{\mathrm{T}} \end{bmatrix}, \quad R_i = \mathrm{diag}\left\{R_{i1}, R_{i2}, \ldots, R_{is}\right\}.$$

Proof. We divide the proof of this theorem into two parts. At first, we analyze the stochastic admissibility for system (6) with $v(t) = 0$ and then discuss the dissipativity of the system.

To derive the stochastic admissibility of system (6) ($v(t) = 0$), we prove first that the system is impulse-free. Under Assumption 1, there are nonsingular matrices U_e and V_e such that

$$\begin{aligned} U_e E_e V_e &= \begin{bmatrix} I_{r+n} & 0 \\ 0 & 0 \end{bmatrix}, \quad U_e \tilde{A}_{hd} V_e = \begin{bmatrix} \tilde{A}_{hd11} & \tilde{A}_{hd12} \\ \tilde{A}_{hd21} & \tilde{A}_{hd22} \end{bmatrix}, \\ U_e \tilde{A}_h V_e &= \begin{bmatrix} \tilde{A}_{h11} & \tilde{A}_{h12} \\ \tilde{A}_{h21} & \tilde{A}_{h22} \end{bmatrix}, \quad U_e^{-\mathrm{T}} P_e V_e = \begin{bmatrix} P_{e11} & P_{e12} \\ P_{e21} & P_{e22} \end{bmatrix}. \end{aligned} \tag{20}$$

From (18), it follows that $V_e^{\mathrm{T}} E_e^{\mathrm{T}} U_e^{\mathrm{T}} U_e^{-\mathrm{T}} P_e V_e = V_e^{\mathrm{T}} P_e^{\mathrm{T}} U_e^{-1} U_e E_e V_e \geqslant 0$, that is

$$\begin{bmatrix} I_{r+n} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{e11} & P_{e12} \\ P_{e21} & P_{e22} \end{bmatrix} = \begin{bmatrix} P_{e11}^{\mathrm{T}} & P_{e21}^{\mathrm{T}} \\ P_{e12}^{\mathrm{T}} & P_{e22}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} I_{r+n} & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0.$$

Then we have $P_{e11} = P_{e11}^{\mathrm{T}},\ P_{e12} = 0$.

From (19), the following matrix inequality holds

$$\begin{aligned} &\sum_{i=1}^{s}(h_i(\zeta(t)) + \theta_i(\zeta(t)))\sum_{j=1}^{s}\tilde{\theta}_j(\zeta(t))\{A_{eij}^{\mathrm{T}} P_e + P_e^{\mathrm{T}} A_{eij} \\ &\quad + J_{ei}^{\mathrm{T}}(E_e^{+})^{\mathrm{T}} E_e^{\mathrm{T}} P_e E_e^{+} J_{ei} + Q_e - E_e^{\mathrm{T}}(M + M^{\mathrm{T}})E_e\} < 0. \end{aligned} \tag{21}$$

It implies that

$$\sum_{i=1}^{s}(h_i(\zeta(t)) + \theta_i(\zeta(t)))\sum_{j=1}^{s}\tilde{\theta}_j(\zeta(t))\{A_{eij}^{\mathrm{T}} P_e + P_e^{\mathrm{T}} A_{eij} - E_e^{\mathrm{T}}(M + M^{\mathrm{T}})E_e\} < 0. \tag{22}$$

Thus

$$\begin{aligned} &\sum_{i=1}^{s}(h_i(\zeta(t)) + \theta_i(\zeta(t)))\sum_{j=1}^{s}\tilde{\theta}_j(\zeta(t))\{V_e^{\mathrm{T}} A_{eij}^{\mathrm{T}} U_e^{\mathrm{T}} U_e^{-\mathrm{T}} P_e V_e + V_e^{\mathrm{T}} P_e^{\mathrm{T}} U_e^{-1} U_e A_{eij} V_e \\ &\quad - V_e^{\mathrm{T}} E_e^{\mathrm{T}} U_e^{\mathrm{T}} U_e^{-\mathrm{T}}(M + M^{\mathrm{T}})U_e^{-1} U_e E_e V_e\} < 0, \end{aligned} \tag{23}$$

that is $V_e^{\mathrm{T}} \tilde{A}_h^{\mathrm{T}} U_e^{\mathrm{T}} U_e^{-\mathrm{T}} P_e V_e + V_e^{\mathrm{T}} P_e^{\mathrm{T}} U_e^{-1} U_e \tilde{A}_h V_e - V_e^{\mathrm{T}} E_e^{\mathrm{T}} U_e^{\mathrm{T}} U_e^{-\mathrm{T}}(M + M^{\mathrm{T}})U_e^{-1} U_e E_e V_e < 0$.

Taking into account expression (20) and the above inequality, we have

$$\begin{bmatrix} \otimes & \tilde{\otimes} \\ * & (\tilde{A}_{h22}^{\mathrm{T}} P_{e22} + P_{e22}^{\mathrm{T}} \tilde{A}_{h22}) \end{bmatrix} < 0. \tag{24}$$

Since $\otimes$ and $\tilde{\otimes}$ are irrelevant to the results of the following discussion, the real expression of these two variables is omitted here. From (24), it is easy to see that $\tilde{A}_{h22}^{\mathrm{T}}P_{e22}+P_{e22}^{\mathrm{T}}\tilde{A}_{h22}<0$, which means that $\tilde{A}_{h22}$ is nonsingular. Thus the pair $(E_e,\tilde{A}_h)$ is impulse-free by Lemma 2.

Additionally, from (19), we can obtain the following matrix inequality:

$$\sum_{i=1}^{s}(h_i(\zeta(t))+\theta_i(\zeta(t)))\sum_{j=1}^{s}\tilde{\theta}_j(\zeta(t))\begin{bmatrix}\Lambda_1 & \Lambda_2\\ * & \Lambda_3\end{bmatrix}<0. \tag{25}$$

Before and after multiplying (25) $[\,I\ \ I\,]$ by $[\,I\ \ I\,]^{\mathrm{T}}$, we can easily obtain

$$\begin{aligned}\sum_{i=1}^{s}(h_i(\zeta(t))+\theta_i(\zeta(t)))\sum_{j=1}^{s}\tilde{\theta}_j(\zeta(t))\{&(A_{eij}^{\mathrm{T}}+A_{eid}^{\mathrm{T}})P_e\\ &+P_e^{\mathrm{T}}(A_{eij}+A_{eid})+J_{ei}^{\mathrm{T}}(E_e^{+})^{\mathrm{T}}E_e^{\mathrm{T}}P_eE_e^{+}J_{ei}+\bar{d}Q_e\}<0.\end{aligned}$$

Using the same approach as above, we have

$$\begin{bmatrix}\star & \tilde{\star}\\ * & (\tilde{A}_{h22}^{\mathrm{T}}+\tilde{A}_{hd22}^{\mathrm{T}})P_{e22}+P_{e22}^{\mathrm{T}}(\tilde{A}_{h22}+\tilde{A}_{hd22})\end{bmatrix}<0. \tag{26}$$

Because $\star$ and $\tilde{\star}$ are irrelevant to the results of the following discussion, the real expression of these two variables is omitted here as well. From (26), we can easily see that $(\tilde{A}_{h22}^{\mathrm{T}}+\tilde{A}_{hd22}^{\mathrm{T}})P_{e22}+P_{e22}^{\mathrm{T}}(\tilde{A}_{h22}+\tilde{A}_{hd22})<0$, which implies that the pair $(E_e,\tilde{A}_h+\tilde{A}_{hd})$ is regular and impulse-free. Thus, according to Definition 2, system (6) $(v(t)=0)$ is regular and impulse-free.

Next, we prove system (6) with $v(t)=0$ is stochastically stable. Construct a candidate Lyapunov-Krasovskii function

$$V(x_{et})=V_1(x_{et})+V_2(x_{et})+V_3(x_{et}), \tag{27}$$

where

$$\begin{aligned}&V_1(x_{et})=x_e^{\mathrm{T}}(t)E_e^{\mathrm{T}}P_ex_e(t),\quad V_2(x_{et})=\int_{t-d(t)}^{t}x_e^{\mathrm{T}}(\tau)Q_ex_e(\tau)\mathrm{d}\tau,\\ &V_3(x_{et})=d_0\int_{-d_0}^{0}\int_{t+\vartheta}^{t}\eta^{\mathrm{T}}(\tau)Z_e\eta(\tau)\mathrm{d}\tau\mathrm{d}\vartheta.\end{aligned}$$

By Lemma 1, the stochastic derivative of $V(x_{et})$ along the trajectory of system (6) can be obtained as follows:

$$\mathrm{d}V(x_{et})=\mathcal{L}V(x_{et})\mathrm{d}t+2x_e^{\mathrm{T}}(t)P_e^{\mathrm{T}}\tilde{J}_hx_e(t)\mathrm{d}\omega(t), \tag{28}$$

where

$$\mathcal{L}V(x_{et})=\mathcal{L}V_1(x_{et})+\mathcal{L}V_2(x_{et})+\mathcal{L}V_3(x_{et}),$$

and

$$\begin{aligned}\mathcal{L}V_1(x_{et})&=2x_e^{\mathrm{T}}(t)P_e^{\mathrm{T}}(\tilde{A}_hx_e(t)+\tilde{A}_{hd}x_e(t-d(t)))+x_e^{\mathrm{T}}(t)\tilde{J}_h^{\mathrm{T}}(E_e^{+})^{\mathrm{T}}E_e^{\mathrm{T}}P_eE_e^{+}\tilde{J}_hx_e(t)\\ &=x_e^{\mathrm{T}}(t)(P_e^{\mathrm{T}}\tilde{A}_h+\tilde{A}_h^{\mathrm{T}}P_e+\tilde{J}_h^{\mathrm{T}}(E_e^{+})^{\mathrm{T}}E_e^{\mathrm{T}}P_eE_e^{+}\tilde{J}_h)x_e(t)\\ &\quad+x_e^{\mathrm{T}}(t)P_e^{\mathrm{T}}\tilde{A}_{hd}x_e(t-d(t))+x_e^{\mathrm{T}}(t-d(t))\tilde{A}_{hd}^{\mathrm{T}}P_ex_e(t),\end{aligned} \tag{29}$$

$$\begin{aligned}\mathcal{L}V_2(x_{et})&=x_e^{\mathrm{T}}(t)Q_ex_e(t)-(1-\dot{d}(t))x_e^{\mathrm{T}}(t-d(t))Q_ex_e(t-d(t))\\ &\leqslant x_e^{\mathrm{T}}(t)Q_ex_e(t)-(1-\bar{d})x_e^{\mathrm{T}}(t-d(t))Q_ex_e(t-d(t)),\end{aligned} \tag{30}$$

$$\begin{aligned}\mathcal{L}V_3(x_{et})&=d_0^2\eta^{\mathrm{T}}(t)Z_e\eta(t)-d_0\int_{t-d_0}^{t}\eta^{\mathrm{T}}(\tau)Z_e\eta(\tau)\mathrm{d}\tau\\ &\leqslant d_0^2\eta^{\mathrm{T}}(t)Z_e\eta(t)-d_0\int_{t-d(t)}^{t}\eta^{\mathrm{T}}(\tau)Z_e\eta(\tau)\mathrm{d}\tau.\end{aligned} \tag{31}$$

Because of $0 \leqslant d(t) \leqslant d_0$, using Jensen inequality to (31), we can get

$$\mathcal{L}V_3(x_{et}) \leqslant d_0^2\eta^{\mathrm{T}}(t)Z_e\eta(t) - \int_{t-d(t)}^{t}\eta^{\mathrm{T}}(\tau)\mathrm{d}\tau Z_e \int_{t-d(t)}^{t}\eta(\tau)\mathrm{d}\tau. \tag{32}$$

From (16), for any matrices M, N, we have

$$\begin{aligned} 0 = 2\left[x_e^{\mathrm{T}}(t)E_e^{\mathrm{T}}M + x_e^{\mathrm{T}}(t-d(t))E_e^{\mathrm{T}}N\right]&\left[\int_{t-d(t)}^{t}\eta(\tau)\mathrm{d}\tau\right.\\ &\left.+\int_{t-d(t)}^{t}\tilde{J}_h x_e(\tau)\mathrm{d}\omega(\tau) - E_e x_e(t) + E_e x_e(t-d(t))\right]. \end{aligned} \tag{33}$$

Furthermore, from expressions (28) and (33), we can obtain

$$\begin{aligned} \mathrm{d}V(x_{et}) = {}& \mathcal{L}\tilde{V}(x_{et})\mathrm{d}t + 2x_e^{\mathrm{T}}(t)P_e^{\mathrm{T}}\tilde{J}_h x_e(t)\mathrm{d}\omega(t) + 2[x_e^{\mathrm{T}}(t)E_e^{\mathrm{T}}M + x_e^{\mathrm{T}}(t-d(t))E_e^{\mathrm{T}}N]\\ &\times \int_{t-d(t)}^{t}\tilde{J}_h x_e(\tau)\mathrm{d}\omega(\tau), \end{aligned} \tag{34}$$

where

$$\begin{aligned} \mathcal{L}\tilde{V}(x_{et}) = {}& \mathcal{L}V(x_{et}) + 2\left[x_e^{\mathrm{T}}(t)E_e^{\mathrm{T}}M + x_e^{\mathrm{T}}(t-d(t))E_e^{\mathrm{T}}N\right]\\ &\times\left[-E_e x_e(t) + E_e x_e(t-d(t)) + \int_{t-d(t)}^{t}\eta(\tau)\mathrm{d}\tau\right] \leqslant \xi^{\mathrm{T}}(t)\Theta\xi(t), \end{aligned} \tag{35}$$

and

$$\xi^{\mathrm{T}}(t) = \left[x_e^{\mathrm{T}}(t)\ \ x_e^{\mathrm{T}}(t-d(t))\ \ \int_{t-d(t)}^{t}\eta^{\mathrm{T}}(\tau)\mathrm{d}\tau\right], \quad \Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & E_e^{\mathrm{T}}M \\ * & \Theta_{22} & E_e^{\mathrm{T}}N \\ * & * & -Z_e \end{bmatrix},$$

$$\Theta_{11} = \tilde{A}_h^{\mathrm{T}}P_e + P_e^{\mathrm{T}}\tilde{A}_h + \tilde{J}_h^{\mathrm{T}}(E_e^{+})^{\mathrm{T}}E_e^{\mathrm{T}}P_e E_e^{+}\tilde{J}_h + d_0^2\tilde{A}_h^{\mathrm{T}}Z_e\tilde{A}_h + Q_e - E_e^{\mathrm{T}}(M+M^{\mathrm{T}})E_e,$$
$$\Theta_{12} = P_e^{\mathrm{T}}\tilde{A}_{hd} + E_e^{\mathrm{T}}(M - N^{\mathrm{T}})E_e + d_0^2\tilde{A}_h^{\mathrm{T}}Z_e\tilde{A}_{hd},$$
$$\Theta_{22} = -(1-\bar{d})Q_e + E_e^{\mathrm{T}}(N+N^{\mathrm{T}})E_e + d_0^2\tilde{A}_{hd}^{\mathrm{T}}Z_e\tilde{A}_{hd}.$$

For condition (19), by the Schur complement lemma, we have

$$\sum_{i=1}^{s}\sum_{j=1}^{s}\left(\nabla_i R_{ij} + \frac{1}{4}\Pi_{ij}^{\mathrm{T}}R_{ij}^{-1}\Pi_{ij}\right) + \sum_{i=1}^{s}\sum_{j=1}^{s}\rho_{ij}\Pi_{ij} < 0.$$

Using the convexity principle (see [32] for more details), we deduce that

$$\sum_{i=1}^{s}\sum_{j=1}^{s}\left(\nabla_i R_{ij} + \frac{1}{4}\Pi_{ij}^{\mathrm{T}}R_{ij}^{-1}\Pi_{ij}\right) + \sum_{i=1}^{s}\sum_{j=1}^{s}\theta_i(\zeta(t))\tilde{\theta}_j(\zeta(t))\Pi_{ij} < 0. \tag{36}$$

Therefore, by Lemma 4, we have

$$\begin{aligned} &\sum_{i=1}^{s}(h_i(\zeta(t)) + \theta_i(\zeta(t)))\sum_{j=1}^{s}\tilde{\theta}_j(\zeta(t))\Pi_{ij}\\ &\quad= \sum_{i=1}^{s}\sum_{j=1}^{s}h_i(\zeta(t))\tilde{\theta}_j(\zeta(t))\left(\frac{1}{2}\Pi_{ij} + \frac{1}{2}\Pi_{ij}^{\mathrm{T}}\right) + \sum_{i=1}^{s}\sum_{j=1}^{s}\theta_i(\zeta(t))\tilde{\theta}_j(\zeta(t))\Pi_{ij}\\ &\quad\leqslant \sum_{i=1}^{s}\sum_{j=1}^{s}\left(h_i^2(\zeta(t))\tilde{\theta}_j^2(\zeta(t))R_{ij} + \frac{1}{4}\Pi_{ij}^{\mathrm{T}}R_{ij}^{-1}\Pi_{ij}\right) + \sum_{i=1}^{s}\sum_{j=1}^{s}\theta_i(\zeta(t))\tilde{\theta}_j(\zeta(t))\Pi_{ij}\\ &\quad\leqslant \sum_{i=1}^{s}\sum_{j=1}^{s}\left(\nabla_i R_{ij} + \frac{1}{4}\Pi_{ij}^{\mathrm{T}}R_{ij}^{-1}\Pi_{ij}\right) + \sum_{i=1}^{s}\sum_{j=1}^{s}\theta_i(\zeta(t))\tilde{\theta}_j(\zeta(t))\Pi_{ij}. \end{aligned} \tag{37}$$

By inequality (36) and the above expression (37), it is obtained that

$$\Pi_h = \sum_{i=1}^{s} (h_i(\zeta(t)) + \theta_i(\zeta(t))) \sum_{j=1}^{s} \tilde{\theta}_j(\zeta(t))\Pi_{ij} < 0,$$

that is,

$$\Pi_h = \begin{bmatrix} \Psi_{1h} & \Psi_{2h} \\ * & \Psi_{3h} \end{bmatrix} < 0, \tag{38}$$

where

$$\Psi_{1h} = \begin{bmatrix} \Lambda_{1h} & \Lambda_{2h} & E_e^{\mathrm{T}} M \\ * & \Lambda_3 & E_e^{\mathrm{T}} N \\ * & * & -Z_e \end{bmatrix}, \quad \Psi_{2h} = \begin{bmatrix} \Lambda_{4h} & d_0 \tilde{A}_h^{\mathrm{T}} Z_e & \tilde{C}_h^{\mathrm{T}} \hat{Q}_1^{\mathrm{T}} \\ 0 & d_0 \tilde{A}_{hd}^{\mathrm{T}} Z_e & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Psi_{3h} = \begin{bmatrix} -\hat{R} & d_0 \tilde{B}_h^{\mathrm{T}} Z_e & 0 \\ * & -Z_e & 0 \\ * & * & -I \end{bmatrix},$$

$$\Lambda_{1h} = \tilde{A}_h^{\mathrm{T}} P_e + P_e^{\mathrm{T}} \tilde{A}_h + \tilde{J}_h^{\mathrm{T}} (E_e^{+})^{\mathrm{T}} E_e^{\mathrm{T}} P_e E_e^{+} \tilde{J}_h + Q_e - E_e^{\mathrm{T}} (M + M^{\mathrm{T}}) E_e,$$

$$\Lambda_{2h} = P_e^{\mathrm{T}} \tilde{A}_{hd} + E_e^{\mathrm{T}} (M - N^{\mathrm{T}}) E_e, \quad \Lambda_{4h} = P_e^{\mathrm{T}} \tilde{B}_h - \tilde{C}_h^{\mathrm{T}} \hat{S}.$$

For inequality (38), by Schur complement Lemma, we have

$$\tilde{\Pi}_h = \begin{bmatrix} \Theta & \Theta_1 \\ * & \Theta_2 \end{bmatrix} < 0, \tag{39}$$

where

$$\Theta_1 = \begin{bmatrix} \begin{pmatrix} d_0^2 \tilde{A}_h^{\mathrm{T}} Z_e \tilde{B}_h + \\ P_e^{\mathrm{T}} \tilde{B}_h - \tilde{C}_h^{\mathrm{T}} \hat{S} \end{pmatrix} & \tilde{C}_h^{\mathrm{T}} \hat{Q}_1^{\mathrm{T}} \\ d_0^2 \tilde{A}_{hd} Z_e \tilde{B}_h & 0 \\ 0 & 0 \end{bmatrix}, \quad \Theta_2 = \mathrm{diag}\{d_0^2 \tilde{A}_h^{\mathrm{T}} Z_e \tilde{B}_h - \hat{R}, 0\}.$$

Inequality (39) implies that

$$\Theta < 0. \tag{40}$$

The above inequality (40) can be arranged as

$$\sum_{i=1}^{s} (h_i(\zeta(t)) + \theta_i(\zeta(t))) \sum_{j=1}^{s} \tilde{\theta}_j(\zeta(t))\Theta_{ij} < 0, \tag{41}$$

where

$$\Theta_{ij} = \begin{bmatrix} \tilde{\Theta}_{11} & \tilde{\Theta}_{12} & E_e^{\mathrm{T}} M \\ * & \tilde{\Theta}_{22} & E_e^{\mathrm{T}} N \\ * & * & -Z_e \end{bmatrix}, \quad \tilde{\Theta}_{12} = P_e^{\mathrm{T}} A_{eid} + E_e^{\mathrm{T}} (M - N^{\mathrm{T}}) E_e + d_0^2 A_{eij}^{\mathrm{T}} Z_e A_{eid},$$

$$\tilde{\Theta}_{11} = A_{eij}^{\mathrm{T}} P_e + P_e^{\mathrm{T}} A_{eij} + J_{ei}^{\mathrm{T}} (E_e^{+})^{\mathrm{T}} E_e^{\mathrm{T}} P_e E_e^{+} J_{ei} + d_0^2 A_{eij}^{\mathrm{T}} Z_e A_{eij} + Q_e - E_e^{\mathrm{T}} (M + M^{\mathrm{T}}) E_e,$$

$$\tilde{\Theta}_{22} = -(1 - \bar{d}) Q_e + E_e^{\mathrm{T}} (N + N^{\mathrm{T}}) E_e + d_0^2 A_{eid}^{\mathrm{T}} Z_e A_{eid}.$$

Furthermore, we have $\Theta_{ij} < 0$. Thus, we have

$$\varepsilon\{\mathcal{L}V(x_{et})\} \leqslant \varepsilon\{\mathcal{L}\tilde{V}(x_{et})\} \leqslant - \lambda_{\max_{1 \leqslant i,j \leqslant s}} (\Theta_{ij}) \|\xi(t)\|^2 \leqslant - \lambda_{\max_{1 \leqslant i,j \leqslant s}} (\Theta_{ij}) \|x_e(t)\|^2.$$

On the other hand, we can easily derive that

$$\lambda_{\min}(P_{e11}) \|x_{1e}(t)\|^2 \leqslant \varepsilon\{V(x_{et})\}$$

$$\leqslant \Bigg\{ d_0^2 \sum_{i=1}^{s} (h_i(\zeta(t)) + \theta_i(\zeta(t))) \sum_{j=1}^{s} \tilde{\theta}_j(\zeta(t)) \left[\|A_{eij}^{\mathrm{T}} Z_e A_{eij}\| + 2\|A_{eij}^{\mathrm{T}} Z_e A_{eid}\| \right.$$

$$\left. + \|A_{eid}^{\mathrm{T}} Z_e A_{eid}\| \right] + \lambda_{\max}(P_{e11}) + d_0^2 \|Q_e\| \Bigg\} \sup_{\vartheta \in [-2d_0, 0]} \|x(t+\vartheta)\|^2$$

$$\leqslant \left[d_0^2 (\bar{\lambda}_1 + 2\bar{\lambda}_2 + \bar{\lambda}_3) + \lambda_{\max}(P_{e11}) + d_0^2 \|Q_e\| \right] \sup_{\vartheta \in [-2d_0, 0]} \|x(t+\vartheta)\|^2,$$

where $\bar{\lambda}_1 = \max_{1\leqslant i,j\leqslant s} \|A_{eij}^{\mathrm{T}} Z_e A_{eij}\|$, $\bar{\lambda}_2 = \max_{1\leqslant i,j\leqslant s} \|A_{eij}^{\mathrm{T}} Z_e A_{eid}\|$, $\bar{\lambda}_3 = \max_{1\leqslant i,j\leqslant s} \|A_{eid}^{\mathrm{T}} Z_e A_{eid}\|$.

We can conclude that system (6) is stochastically stable via Lemma 3. Therefore, system (6) is stochastically admissible.

Next, we will show that the filtering error system (6) is dissipative. Taking into account inequality (35) and supply rate function (13), we have

$$\mathcal{L}\tilde{V}(x_{et}) - \Gamma(v(t), z_e(t)) \leqslant \begin{pmatrix} \xi(t) \\ v(t) \end{pmatrix}^{\mathrm{T}} \tilde{\Theta}_h \begin{pmatrix} \xi(t) \\ v(t) \end{pmatrix},$$

where

$$\tilde{\Theta}_h = \begin{bmatrix} \begin{pmatrix} \Theta_{11} - \\ \tilde{C}_h^{\mathrm{T}} \hat{Q} \tilde{C}_h \end{pmatrix} & \Theta_{12} & E_e^{\mathrm{T}} M & \begin{pmatrix} d_0^2 \tilde{A}_h^{\mathrm{T}} Z_e \tilde{B}_h + \\ P_e^{\mathrm{T}} \tilde{B}_h - \tilde{C}_h^{\mathrm{T}} \hat{S} \end{pmatrix} \\ * & \Theta_{22} & E_e^{\mathrm{T}} N & d_0^2 \tilde{A}_{hd} Z_e \tilde{B}_h \\ * & * & -Z_e & 0 \\ * & * & * & d_0^2 \tilde{A}_h^{\mathrm{T}} Z_e \tilde{B}_h - \hat{R} \end{bmatrix}.$$

For inequality (39), by Schur complement lemma and setting $\hat{Q}_1 = (-\hat{Q})^{\frac{1}{2}}$, we then get

$$\mathcal{L}\tilde{V}(x_{et}) - \Gamma(v(t), z_e(t)) \leqslant 0. \tag{42}$$

By integrating (42) from 0 to t, then taking expectations on both sides of (42), we finally get

$$\varepsilon \left\{ \int_0^t \mathcal{L}V(x_{e\tau}) \mathrm{d}\tau \right\} \leqslant \varepsilon \left\{ \int_0^t \Gamma(v(\tau), z_e(\tau)) \mathrm{d}\tau \right\}.$$

Through Definition 3, it is obvious that system (6) is dissipative. This completes the proof.

Remark 3. It should be noted that Theorem 1 provides a set of sufficient conditions under which the filtering error system (6) is stochastically admissible based on the auxiliary vector function. When the diffusive term is zero, system (6) reduces to a deterministic T-S fuzzy singular system with time-varying delays.

Remark 4. Theorem 1 gives a strict proof of the stochastic stability for system (6) based on the stochastic Lyapunov-Krasovskii candidate function, some appropriate free-weighting matrices and the stochastic functional theory.

In what follows, we will present the solvability of the considered fuzzy filtering problem.

Theorem 2. For given positive numbers $\bar{d}$, δ, the filtering error system (6) is stochastically admissible and dissipative if there exist matrices P, $\overline{P}$, positive definite matrices Q, $\overline{Q}$, Z and matrices M_{11}, M_{12}, M_{21}, M_{22}, N_{11}, N_{12}, N_{21}, N_{22}, $K_{A_{fj}}$, $K_{B_{fj}}$, C_{fj}, $\bar{R}_{ij} > 0$ such that for all $\rho_{ij} \in \Delta_{ij}$, $(i, j = 1, 2, \ldots, s)$, the following matrix inequalities hold

$$E^{\mathrm{T}} P = P^{\mathrm{T}} E \geqslant 0, \tag{43}$$

$$\begin{bmatrix} \tilde{\Lambda}_2 & \bar{\Xi}_1 & \bar{\Xi}_2 & \cdots & \bar{\Xi}_s \\ * & -\bar{R}_1 & 0 & \cdots & 0 \\ * & * & -\bar{R}_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & -\bar{R}_s \end{bmatrix} < 0, \tag{44}$$

where

$$\bar{\Xi}_i=\left[\frac{1}{2}\Xi_{i1}^{\mathrm{T}}\ \frac{1}{2}\Xi_{i2}^{\mathrm{T}}\ \cdots\ \frac{1}{2}\Xi_{is}^{\mathrm{T}}\right],\ \Xi_{ij}=\begin{bmatrix}\Phi_1 & \Phi_2 & \Phi_3\\ * & \Phi_4 & \Phi_5\\ * & * & \Phi_6\end{bmatrix},\ \Phi_1=\begin{bmatrix}\Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14}\\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24}\\ * & * & \Upsilon_{33} & \Upsilon_{34}\\ * & * & * & \Upsilon_{44}\end{bmatrix},\ \Upsilon_{14}=E^{\mathrm{T}}(M_{12}-N_{21}^{\mathrm{T}}),$$

$$\Phi_2=\begin{bmatrix}E^{\mathrm{T}}M_{11} & E^{\mathrm{T}}M_{12} & \Upsilon_{15}\\ M_{21} & M_{22} & \Upsilon_{25}\\ E^{\mathrm{T}}N_{11} & E^{\mathrm{T}}N_{12} & 0\\ N_{21} & N_{22} & 0\end{bmatrix},\ \Phi_4=\begin{bmatrix}-Z & 0 & 0\\ * & -\delta\bar{P} & 0\\ * & * & -\hat{R}\end{bmatrix},\ \Phi_5=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ d_0B_i^{\mathrm{T}}Z & \Upsilon_{71} & 0\end{bmatrix},\ \Upsilon_{15}=P^{\mathrm{T}}B_i-C_{2i}^{\mathrm{T}}\hat{S},$$

$$\Phi_3=\begin{bmatrix}d_0A_i^{\mathrm{T}}Z & \Upsilon_{16} & \Upsilon_{17}\\ 0 & d_0\delta K_{A_{fj}}^{\mathrm{T}} & -C_{fj}^{\mathrm{T}}\hat{Q}_1^{\mathrm{T}}\\ d_0A_{id}^{\mathrm{T}}Z & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \Phi_6=\begin{bmatrix}-Z & 0 & 0\\ * & -\delta\bar{P} & 0\\ * & * & -I\end{bmatrix},\ \Upsilon_{12}=C_{1i}^{\mathrm{T}}K_{B_{fj}}^{\mathrm{T}}-E^{\mathrm{T}}(M_{12}+M_{21}^{\mathrm{T}}),$$

$$\Upsilon_{11}=A_i^{\mathrm{T}}P+P^{\mathrm{T}}A_i+J_i^{\mathrm{T}}(E^{+})^{\mathrm{T}}E^{\mathrm{T}}PE^{+}J_i+Q-E^{\mathrm{T}}\times(M_{11}+M_{11}^{\mathrm{T}})E,\ \Upsilon_{16}=d_0\delta C_{1i}^{\mathrm{T}}K_{B_{fj}}^{\mathrm{T}},$$

$$\Upsilon_{13}=P^{\mathrm{T}}A_{id}+E^{\mathrm{T}}(M_{11}-N_{11}^{\mathrm{T}})E,\ \Upsilon_{22}=K_{A_{fj}}^{\mathrm{T}}+K_{A_{fj}}+\bar{Q}-(M_{22}+M_{22}^{\mathrm{T}}),\ \Upsilon_{25}=K_{B_{fj}}D_i+C_{fj}^{\mathrm{T}}\hat{S},$$

$$\Upsilon_{23}=(M_{21}-N_{12}^{\mathrm{T}})E,\ \Upsilon_{33}=-(1-\bar{d})Q+E^{\mathrm{T}}(N_{11}+N_{11}^{\mathrm{T}})E,\ \Upsilon_{34}=E^{\mathrm{T}}(N_{12}+N_{21}^{\mathrm{T}}),$$

$$\Upsilon_{17}=C_{2i}^{\mathrm{T}}\hat{Q}_1,\ \Upsilon_{24}=(M_{22}-N_{22}^{\mathrm{T}}),\ \Upsilon_{44}=-(1-\bar{d})\bar{Q}+(N_{22}+N_{22}^{\mathrm{T}}),\ \Upsilon_{71}=d_0\delta D_i^{\mathrm{T}}K_{B_{fj}}^{\mathrm{T}},$$

$$\tilde{\Lambda}_2=\sum_{i=1}^{s}\sum_{j=1}^{s}(\nabla_i\bar{R}_{ij}+\rho_{ij}\Xi_{ij}),\ \bar{R}_i=\mathrm{diag}\left\{\bar{R}_{i1},\bar{R}_{i2},\ldots,\bar{R}_{is}\right\}.$$

In this case, a desired dissipative filter is obtained in the form (5) with parameters (A_{fj},B_{fj},C_{fj}) as $A_{fj}=\bar{P}^{-\mathrm{T}}K_{A_{fj}},B_{fj}=\bar{P}^{-\mathrm{T}}K_{B_{fj}}$.

Proof. Define

$$P_e=\begin{bmatrix}P & 0\\ 0 & \bar{P}\end{bmatrix},\quad Q_e=\begin{bmatrix}Q & 0\\ 0 & \bar{Q}\end{bmatrix},\quad Z_e=\begin{bmatrix}Z & 0\\ 0 & \delta\bar{P}\end{bmatrix},\quad M=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix},\quad N=\begin{bmatrix}N_{11} & N_{12}\\ N_{21} & N_{22}\end{bmatrix}. \tag{45}$$

Substituting (7) and (45) into (18) and (19) in Theorem 1, then the proof of this result is quite similar to that of Theorem 1. Let $A_{fj}^{\mathrm{T}}\bar{P}=K_{A_{fj}}^{\mathrm{T}},\ B_{fj}^{\mathrm{T}}\bar{P}=K_{B_{fj}}^{\mathrm{T}}$. Then it can be seen that Eqs. (43) and (44) hold. This completes the proof.

Remark 5. It is well known that the H_∞ filtering is an important branch of the dissipative dynamical systems theory. So we can propose the following corollary based on Theorem 2, which reveals the inner relationship among them.

Corollary 1. For given positive numbers $\bar{d}$, δ, the filtering error system (6) with H_∞ performance index $\hat{\gamma}>0$ is stochastically admissible, if there exist matrices P, $\overline{P}$, positive definite matrices Q, $\overline{Q}$, Z and matrices M_{11}, M_{12}, M_{21}, M_{22}, N_{11}, N_{12}, N_{21}, N_{22}, $K_{A_{fj}}$, $K_{B_{fj}}$, C_{fj}, $\bar{R}_{ij}>0$ such that for all $\rho_{ij}\in\Delta_{ij}$, $(i,j=1,2,\ldots,s)$, the matrix inequality (43) and the following matrix inequality hold

$$\begin{bmatrix}\tilde{\Lambda}_3 & \widehat{\Xi}_1 & \widehat{\Xi}_2 & \cdots & \widehat{\Xi}_s\\ * & -\bar{R}_1 & 0 & \cdots & 0\\ * & * & -\bar{R}_2 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ * & * & * & * & -\bar{R}_s\end{bmatrix}<0, \tag{46}$$

where

$$\widehat{\Xi}_i = \left[\ \frac{1}{2}\breve{\Xi}_{i1}^{\mathrm{T}}\ \frac{1}{2}\breve{\Xi}_{i2}^{\mathrm{T}}\ \cdots\ \frac{1}{2}\breve{\Xi}_{is}^{\mathrm{T}}\ \right],\ \tilde{\Lambda}_3 = \sum_{i=1}^{s}\sum_{j=1}^{s}(\nabla_i\bar{R}_{ij} + \rho_{ij}\breve{\Xi}_{ij}),\ \breve{\Xi}_{ij} = \begin{bmatrix} \Phi_1 & \tilde{\Phi}_2 & \tilde{\Phi}_3 \\ * & \tilde{\Phi}_4 & \Phi_5 \\ * & * & \Phi_6 \end{bmatrix},$$

$$\tilde{\Phi}_2 = \begin{bmatrix} E^{\mathrm{T}}M_{11} & E^{\mathrm{T}}M_{12} & P^{\mathrm{T}}B_i \\ M_{21} & M_{22} & K_{B_{fj}}D_i \\ E^{\mathrm{T}}N_{11} & E^{\mathrm{T}}N_{12} & 0 \\ N_{21} & N_{22} & 0 \end{bmatrix},\ \tilde{\Phi}_3 = \begin{bmatrix} d_0A_i^{\mathrm{T}}Z & \Upsilon_{16} & C_{2i}^{\mathrm{T}} \\ 0 & d_0\delta K_{A_{fj}}^{\mathrm{T}} & -C_{fj}^{\mathrm{T}} \\ d_0A_{id}^{\mathrm{T}}Z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \tilde{\Phi}_4 = \begin{bmatrix} -Z & 0 & 0 \\ * & -\delta\bar{P} & 0 \\ * & * & -\hat{\gamma}^2I \end{bmatrix},$$

and all other symbols are given in Theorem 2. In this case, a desired H_∞ filter is obtained in the form (5) with parameters $A_{fj} = \bar{P}^{-\mathrm{T}}K_{A_{fj}},\ B_{fj} = \bar{P}^{-\mathrm{T}}K_{B_{fj}}$, and C_{fj}.

Proof. Letting $\hat{Q} = -I,\ \hat{S} = 0,\ \hat{R} = \hat{\gamma}^2I$, we can derive Corollary 1 from Theorem 2. The proof is completed.

4 Illustrative examples

In this section, we will illustrate the usage of our method by a numerical example and a practical example, respectively.

Example 1. The nonlinear stochastic singular time-delay system is considered as follows:

$$E\mathrm{d}x(t) = (Ax(t) + A_dx(t-d(t)) + Bv(t))\mathrm{d}t + Jx(t)\mathrm{d}\omega(t). \tag{47}$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} \begin{pmatrix} -1-0.5x_1^2(t) \\ -0.5\Delta(t)x_1^2(t) \end{pmatrix} & 2 & 0 \\ -1 & -10 & 0 \\ -1 & 0 & \begin{pmatrix} -2-x_1^2(t) \\ -\Delta(t)x_1^2(t) \end{pmatrix} \end{bmatrix},$$

$$A_d = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix},\ J = \begin{bmatrix} 0.3 & 0.1 & 0 \\ 0.1 & 0 & 0.2 \\ 0 & 0 & 0 \end{bmatrix}.$$

The membership functions are chosen as

$$h_1(x_1(t),\Delta(t)) = 1 - \frac{x_1^2(t)}{a^2} - \frac{\Delta(t)x_1^2(t)}{a^2},\ \ h_2(x_1(t),\Delta(t)) = \frac{x_1^2(t)}{a^2} + \frac{\Delta(t)x_1^2(t)}{a^2},$$

with $x_1(t) \in [-a, a],\ a = 1,\ \Delta(t) \in [0, 0.1]$ is the uncertain parameter.

System (47) can be represented by the following stochastic fuzzy singular time-delay system.

Rule 1. If $x_1(t)$ is about 0, then

$$\begin{aligned} &E_1\mathrm{d}x(t) = (A_1x(t) + A_{1d}x(t-d(t)) + B_1v(t))\,\mathrm{d}t + J_1x(t)\mathrm{d}\omega(t), \\ &\mathrm{d}y(t) = (C_{11}x(t) + D_1v(t))\mathrm{d}t, \\ &z(t) = C_{21}x(t), \\ &x(t) = \phi(t), \quad t \in [-d_0, 0]. \end{aligned}$$

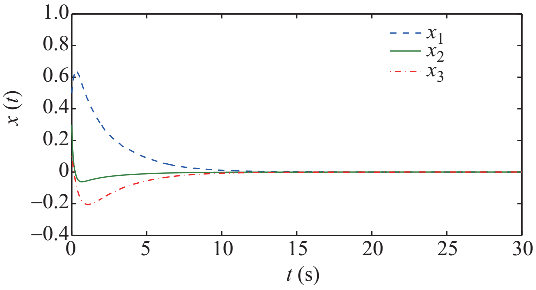

Figure 1 (Color online) State responses of system (2).

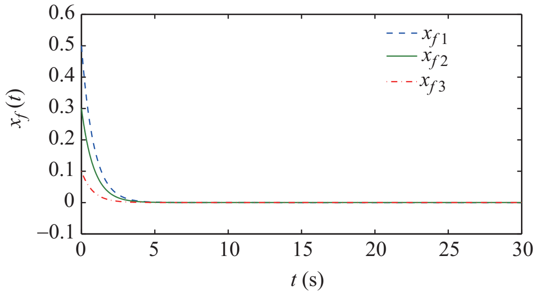

Figure 2 (Color online) State responses of filter (5).

Rule 2. If $x_1(t)$ is about $\pm a$, then

$$\begin{aligned}
&E_2\mathrm{d}x(t) = (A_2x(t) + A_{2d}x(t-d(t)) + B_2v(t))\,\mathrm{d}t + J_2x(t)\mathrm{d}\omega(t),\\
&\mathrm{d}y(t) = (C_{12}x(t) + D_2v(t))\mathrm{d}t,\\
&z(t) = C_{22}x(t),\\
&x(t) = \phi(t), \quad t \in [-d_0, 0],
\end{aligned}$$

where

$$E_1 = E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 2 & 0 \\ -1 & -10 & 0 \\ -1 & 0 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1-0.5a^2 & 2 & 0 \\ -1 & -10 & 0 \\ -1 & 0 & -2-a^2 \end{bmatrix},$$

$$A_{1d} = A_{2d} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \quad J_1 = J_2 = \begin{bmatrix} 0.3 & 0.1 & 0 \\ 0.1 & 0 & 0.2 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_1 = D_2 = [1],$$

$$C_{11} = C_{12} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad C_{21} = C_{22} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

Let $d_0 = 0.2$, $\bar{d} = 0.5$, $\hat{Q} = -I$, $\hat{S} = 1$, $\hat{R} = 10$. For any delay $0 \leqslant d(t) \leqslant d_0 = 0.2$, by solving (43) and (44) in Theorem 2, the corresponding filter parameters are

$$A_{f1} = \begin{bmatrix} -1.1897 & 0.0006 & 0.0022 \\ 0.0004 & -1.1879 & -0.0000 \\ -0.0009 & -0.0001 & -1.2846 \end{bmatrix}, \quad B_{f1} = 10^{-4}\cdot\begin{bmatrix} -0.1435 \\ 0.7347 \\ 0.4143 \end{bmatrix},$$

$$A_{f2} = \begin{bmatrix} -1.1901 & -0.0001 & 0.0025 \\ 0.0008 & -1.1882 & 0.0006 \\ -0.0014 & -0.0008 & -1.2853 \end{bmatrix}, \quad B_{f2} = 10^{-4}\cdot\begin{bmatrix} -0.2224 \\ 0.8433 \\ 0.3267 \end{bmatrix},$$

$$C_{f1} = 10^{-3}\cdot\begin{bmatrix} -0.1307 & 0.4275 & 0.0965 \end{bmatrix}, \quad C_{f2} = 10^{-3}\cdot\begin{bmatrix} -0.2234 & 0.5085 & -0.0395 \end{bmatrix}.$$

Figures 1 and 2 describe the state trajectories of system (2) and filter (5), respectively, which means that the filtering error system is stochastically stable. Figure 3 shows the trajectories of the filtering error $z_e(t) = z(t) - z_f(t)$. It can be seen that the designed filter satisfies the requirements in Theorem 2.

Next, we furthermore discuss the design of the H_∞ filter by applying Corollary 1. Here, let $\hat{Q} = -I$, $\hat{S} = 0$, $\hat{R} = 0.96^2 I$. By solving (43) and (46), we can obtain the following feasible solutions of a H_∞ filter with upper bound $\hat{\gamma} = 0.96$.

$$A_{f1} = \begin{bmatrix} -1.2160 & 0.0001 & 0.0001 \\ -0.0001 & -1.2152 & -0.0004 \\ 0.0004 & 0.0003 & -1.2946 \end{bmatrix}, \quad B_{f1} = 10^{-4}\cdot\begin{bmatrix} 0.0765 \\ 0.0024 \\ 0.1262 \end{bmatrix},$$

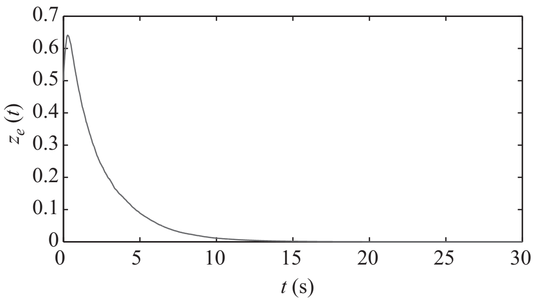

Figure 3 The trajectory of the dissipative filtering error.

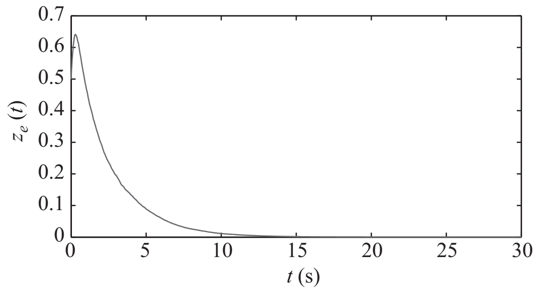

Figure 4 The trajectory of the H_∞ filtering error.

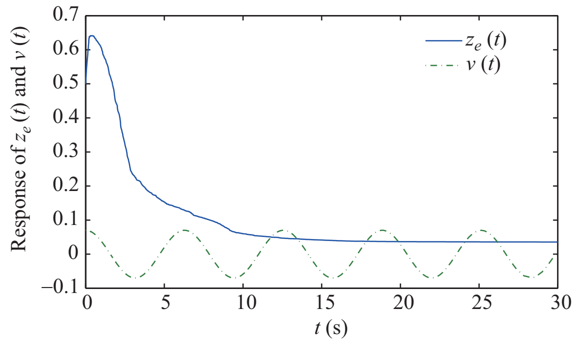

Figure 5 (Color online) The measured output and external disturbance of the filtering error system.

$$A_{f2} = \begin{bmatrix} -1.2158 & 0.0002 & -0.0001 \\ -0.0004 & -1.2152 & 0.0000 \\ 0.0004 & -0.0001 & -1.2949 \end{bmatrix}, \quad B_{f2} = 10^{-4} \cdot \begin{bmatrix} 0.0996 \\ 0.1508 \\ 0.1784 \end{bmatrix},$$

$$C_{f1} = 10^{-4} \cdot \begin{bmatrix} 0.1447 & -0.1533 & -0.6941 \end{bmatrix}, \quad C_{f2} = 10^{-4} \cdot \begin{bmatrix} 0.1964 & -0.1452 & -0.7776 \end{bmatrix}.$$

The trajectories of the designed fuzzy H_∞ filter as shown in Figures 4 and 5 show that the considered error system can suppress the external disturbance signal to a certain extent.

Example 2. Let us consider a CSTR, where the first-order irreversible exothermic reaction **A**→**B** happens. **A** and **B** are two irreversible exothermic reactors. Similar to the discussion to [33], the model of CSTR can be described by

$$\widehat{V}\frac{\mathrm{d}\widehat{A}}{\mathrm{d}t} = \lambda q\widehat{A}_0 + q(1-\lambda)\widehat{A}(t-\alpha) - q\widehat{A}(t) - \widehat{V}K_0 \exp\left[\frac{-\widehat{E}}{\widehat{R}T(t)}\right]\widehat{A}(t),$$

$$\widehat{V}\widehat{C}\widehat{\rho}\frac{\mathrm{d}T}{\mathrm{d}t} = q\widehat{C}\widehat{\rho}[\lambda T_0 + (1-\lambda)T(t-\alpha) - T(t)] + \widehat{V}(-\Delta\widehat{H}(t))K_0 \exp\left[\frac{-\widehat{E}}{\widehat{R}T(t)}\right]\widehat{A}(t) - U(T(t) - T_\omega),$$

where the meanings of system parameters can be found in [33]. In engineering practice, the environment is always affected by the white noise. We measure the heat of reaction by using a calorimeter, and suppose that the change of the environment directly affects the measurement process. In this case, similar to the model in [18], the state equation of this model is described by

$$\begin{aligned}
\mathrm{d}x_1(t) &= \frac{-1}{\lambda}x_1(t) + D_\alpha(1-x_1(t))\exp\left(\frac{x_2(t)}{\frac{1+x_2(t)}{\gamma_0}}\right) + \left(\frac{1}{\lambda}-1\right)x_1(t-\alpha)\mathrm{d}t,\\
\mathrm{d}x_2(t) &= \left\{\left(\frac{1}{\lambda}+\beta\right)x_2(t) + \left(\frac{1}{\lambda}-1\right)x_2(t-\alpha) + \beta v(t)\right.\\
&\quad \left. + H_0 D_\alpha(1-x_1(t))\exp\left(\frac{x_2(t)}{\frac{1+x_2(t)}{\gamma_0}}\right)\right\}\mathrm{d}t + Jx(t)\mathrm{d}\omega(t),
\end{aligned}$$

where $\omega(t)$ is assumed to be a standard one-dimensional Wiener process.

Here, we assume only the temperature can be measured on line. Let $\mathrm{d}y(t) = [\,0\ 1\,]x(t)\mathrm{d}t$. Now, take the fuzzy rules as follows.

Rule *i*. If $x_2(t)$ is about $\hat{M}_{i2},\ i=1,2,3$, then

$$\begin{aligned}
&\mathrm{d}x(t) = (A_i x(t) + A_{id}x(t-\alpha) + B_i v(t))\,\mathrm{d}t + J_i x(t)\mathrm{d}\omega(t),\\
&\mathrm{d}y(t) = C_{1i}x(t)\mathrm{d}t,\\
&z(t) = C_{2i}x(t),
\end{aligned}$$

where

$$A_1 = \begin{bmatrix} -1.4274 & 0.0757\\ -1.4189 & -0.9442\end{bmatrix},\quad A_{1d} = \begin{bmatrix} 0.25 & 0\\ 0 & 0.25\end{bmatrix},\quad B_1 = \begin{bmatrix} 0\\ 0.3\end{bmatrix},\quad J_1 = \begin{bmatrix} 0 & 0\\ 0.0350 & 0.0150\end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2.0508 & 0.3958\\ -6.4066 & -1.6268\end{bmatrix},\quad A_{2d} = \begin{bmatrix} 0.25 & 0\\ 0 & 0.25\end{bmatrix},\quad B_2 = \begin{bmatrix} 0\\ 0.3\end{bmatrix},\quad J_2 = \begin{bmatrix} 0 & 0\\ -0.0316 & 0.0131\end{bmatrix},$$

$$A_3 = \begin{bmatrix} -4.5279 & 0.3167\\ -26.2228 & -0.9387\end{bmatrix},\quad A_{3d} = \begin{bmatrix} 0.25 & 0\\ 0 & 0.25\end{bmatrix},\quad B_3 = \begin{bmatrix} 0\\ 0.3\end{bmatrix},\quad J_3 = \begin{bmatrix} 0 & 0\\ -0.6556 & 0.0633\end{bmatrix},$$

$$C_{11} = C_{12} = C_{13} = [\,0\ 1\,],\quad C_{21} = C_{22} = C_{23} = [\,1\ 0\,].$$

$\hat{M}_{i2}$ is the fuzzy set. There may exist errors when the temperature $x_2(t)$ is measured in the practical system model, therefore, the membership functions h_1, h_2 and h_3 contain the unknown parameter denoted by $\Delta(t)$. Three corresponding membership functions are

$$h_1(x_2(t),\Delta(t)) = \begin{cases} 1, & x_2(t) \leqslant 0.8862,\\ 1 - \frac{x_2(t)-0.8862}{2.7520-0.8862} + \Delta(t), & 0.8862 < x_2(t) < 2.7520,\\ 0, & x_2(t) \geqslant 2.7520,\end{cases}$$

$$h_2(x_2(t),\Delta(t)) = \begin{cases} 1 - h_1(x_2(t)), & x_2(t) < 2.7520,\\ 1 - h_3(x_2(t)), & x_2(t) \geqslant 2.7520,\end{cases}$$

$$h_3(x_2(t),\Delta(t)) = \begin{cases} 0, & x_2(t) \leqslant 2.7520,\\ 1 - \frac{x_2(t)-2.7520}{4.7052-2.7520} - \Delta(t), & 2.7520 < x_2(t) < 4.7052,\\ 0, & x_2(t) \geqslant 4.7052,\end{cases}$$

where $\Delta(t) \in [-0.01,\ \ 0.01]$.

Let $d_0 = 0.5$, $\hat{Q} = -I$, $\hat{S} = 1$, $\hat{R} = 20$. By solving the matrix inequalities (43) and (44) in Theorem 2,

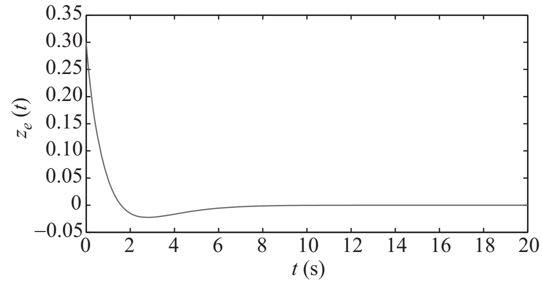

Figure 6 The trajectory of the dissipative filtering error.

we can obtain the following filter parameters:

$$A_{f1} = \begin{bmatrix} -0.8153 & -0.0004 \\ -0.0005 & -0.8168 \end{bmatrix}, \quad B_{f1} = 10^{-5} \cdot \begin{bmatrix} -0.6338 \\ -0.3485 \end{bmatrix}, \quad C_{f1} = 10^{-4} \cdot \begin{bmatrix} -0.1249 & -0.5926 \end{bmatrix},$$

$$A_{f2} = \begin{bmatrix} -0.8144 & -0.0002 \\ 0.0002 & -0.8176 \end{bmatrix}, \quad B_{f2} = 10^{-5} \cdot \begin{bmatrix} -0.5906 \\ -0.5933 \end{bmatrix}, \quad C_{f2} = C_{f3} = 10^{-3} \cdot \begin{bmatrix} 0.0628 & 0.1106 \end{bmatrix},$$

$$A_{f3} = \begin{bmatrix} -0.8155 & -0.0001 \\ 0.0004 & -0.8190 \end{bmatrix}, \quad B_{f3} = 10^{-4} \cdot \begin{bmatrix} -0.1445 \\ -0.0040 \end{bmatrix}.$$

The trajectories of the filtering error are shown in Figure 6. That is to say, the filtering error system is stochastically stable. This example gives the application of dissipative filtering for a CSTR and shows the effectiveness of the proposed approach.

5 Conclusion

The delay-dependent dissipative filtering problem for nonlinear stochastic singular systems with time-varying delays via a T-S fuzzy control approach has been addressed. When the membership functions are partially unknown, based on an auxiliary vector function, by using an integral inequality and a novel free-weighting-matrix approach, a delay-dependent sufficient condition has been derived for the filtering error system to be stochastically admissible and dissipative. With these conditions, the corresponding dissipative filter parameters are solved based on the linear matrix inequalities technology. In addition, we also have presented the H_∞ filter design method in the same design process. Finally, two simulation examples are given to demonstrate the validity of the proposed theoretical results. In this paper, the proposed stochastic admissibility and dissipativity results are developed only for the impulse-free case. In the future work, we will explore the stochastic admissibility for nonlinear stochastic singular systems possessing impulsive behavior.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61573156, 61733008, 61503142), Natural Science Foundation of Guangdong Province (Grant No. 2017A030313332), Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20130172110027), Natural Science Foundation of Liaoning Province (Grant No. 201602604), Science Research Fund of Education Department of Liaoning Province (Grant No. L2014237), Ministry of Housing and Urban-Rural Development of the People's Republic of China (Grant No. 2016K8-062) and Shenyang Jianzhu University Discipline Content Education Project (Grant No. XKHY2-107).

Conflict of interest The authors declare that they have no conflict of interest.

References

1 Wang Z D, Ho D W C. Filtering on nonlinear time-delay stochastic systems. Automatica, 2003, 39: 101–109

2 Deng F Q, Mao W H, Wan A H. A novel result on stability analysis for uncertain neutral stochastic time-varying delay systems. Appl Math Comput, 2013, 221: 132–143

3 Zhao X Y, Deng F Q. Moment stability of nonlinear discrete stochastic systems with time-delays based on H-representation technique. Automatica, 2014, 50: 530–536

4 Xu S, Chen T. Robust H_∞ control for uncertain stochastic systems with state delay. IEEE Trans Automat Control, 2002, 47: 2089–2094

5 Mao X R. Stochastic Differential Equations and Their Applications. Chichester: Horwood, 1997

6 Hu J, Wang Z D, Shen B, et al. Quantized recursive filtering for a class of nonlinear systems with multiplicative noises and missing measurements. Int J Control, 2013, 86: 650–663

7 Hu J, Wang Z D, Liu S, et al. A variance-constrained approach to recursive state estimation for time-varying complex networks with missing measurements. Automatica, 2016, 64: 155–162

8 Zhang J Y, Zhang Q L, Wang Y Y. A new design of sliding mode control for Markovian jump systems based on stochastic sliding surface. Inf Sci Int J, 2017, 391: 9–27

9 Dai L. Filtering and LQG problems for discrete-time stochastic singular systems. IEEE Trans Autom Control, 1989, 34: 1105–1081

10 Darouach M, Boutayeb M, Zasadzinski M. Kalman filtering for continuous descriptor systems. Am Control Conf, 1997, 3: 2108–2112

11 Xing S Y, Zhang Q L. Stability and exact observability of discrete stochastic singular systems based on generalised Lyapunov equations. IET Control Theory Appl, 2016, 10: 971–980

12 Gao Z W, Shi X Y. Observer-based controller design for stochastic descriptor systems with Brownian motions. Automatica, 2013, 49: 2229–2235

13 Boukas E K. Stabilization of stochastic singular nonlinear hybrid systems. Nonlinear Anal, 2006, 64: 217–228

14 Tseng C S. Robust fuzzy filter design for a class of nonlinear stochastic systems. IEEE Trans Fuzzy Syst, 2007, 15: 261–274

15 Balasubramanium P, Vembarasan V, Rakkiyappan R. Delay-dependent robust asymptotic state estimation of Takagi-Sugeno fuzzy Hopfield neural networks with mixed random time-varying delays. Commun Nonlinear Sci Number Simul, 2011, 16: 2109–2129

16 Muralisankar S, Manivannan A, Balasubramaniam P. Robust stability criteria for uncertain neutral type stochastic system with Takagi-Sugeno fuzzy model and Markovian jumping parameters. Commun Nonlinear Sci Number Simul, 2012, 17: 3876–3893

17 Xing S Y, Zhang Q L, Zhu B Y. Mean-square admissibility for stochastic T-S fuzzy singular systems based on extended quadratic Lyapunov function equations. Fuzzy Sets Syst, 2017, 307: 99–114

18 Zhao F, Zhang Q L. H_∞ filtering for stochastic singular fuzzy systems with time-varying delay. Nonlinear Dyn, 2015, 79: 215–228

19 Yoneyama J. H_∞ output feedback control for fuzzy systems with immeasurable premise variables: discrete-time case. Appl Soft Comput, 2008, 160: 949–958

20 Li X J, Yang G H. Switching-type H_∞ filter design for T-S fuzzy systems with unknown or partially unknown membership functions. IEEE Trans Fuzzy Syst, 2013, 21: 385–392

21 Wu L G, Ho D W C. Fuzzy filter design for Itô stochastic systems with application to sensor fault detection. IEEE Trans Fuzzy Syst, 2009, 17: 233–242

22 Li X J, Yang G H. Fault detection for T-S fuzzy systems with unknown membership functions. IEEE Trans Fuzzy Syst, 2014, 22: 139–152

23 Zhu B Y, Zhang Q L, Zhao E L. Robust dissipative filtering for a kind of T-S fuzzy descriptor system with immeasurable premise variables. Int J Syst Sci, 2015, 47: 1–18

24 Dong J X, Yang G H. H_∞ filtering for continuous-time T-S fuzzy systems with partly immeasurable premise varibles. IEEE Trans Syst Man Cybern Syst, 2017, 47: 1931–1940

25 Su H Y, Ji X F, Chu J. Delay-dependent robust control for uncertain singular time-delay systems. Asian J Control, 2006, 8: 180–189

26 Ho D, Shi X, Wang Z, et al. Filtering for a class of stochastic descriptor systems. Dynam Cont Discrete Impulsive Syst, 2005, 2: 848–853

27 Huang L R, Mao X R. Stability of singular stochastic systems with Markovian switching. IEEE Trans Autom Control, 2011, 56: 273–277

28 Xu S Y, Lam J. Robust Control and Filtering of Singular Systems. Berlin: Springer, 2006

29 Fridman E. Stability of linear descriptor systems with delay: a Lyapunov-based approach. J Math Anal Appl, 2002, 273: 24–44

30 Zhao X Y, Deng F Q. New-type stability theorem for stochastic functional differential equations with application to SFDSs with distributed delays. Int J Syst Sci, 2013, 45: 1156–1169

31 Marx B, Ragot J. Stability and L_2-norm bound conditions for Takagi-Sugeno descriptor systems. In: Proceedings of the 17th IFAC World Congress, Seoul, 2008. 9976–9981

32 Boyd S, Vandenberghe L. Convex optimization with engineering applications. In: Lecture Notes. Stanford: Stanford University, 2001

33 Cao Y, Frank P M. Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach. IEEE Trans Fuzzy Syst, 2000, 2: 200–211