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Decentralized backstepping adaptive output tracking of large-scale stochastic nonlinear systems

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Abstract The problem of decentralized adaptive output tracking control of a class of interconnected stochastic nonlinear systems is considered. In the control design, decentralized state observers and backstepping techniques are applied. To eliminate the influences of interactions with other subsystems, a differentiable function is employed. It is shown that the designed local adaptive controllers can ensure that all the signals in the closed-loop system are bounded in probability. Furthermore, the tracking errors can be limited to a small residual set around the origin in the fourth moment sense and can be adjusted by choosing suitable design parameters.

Keywords stochastic nonlinear systems, adaptive control, backstepping, interconnected subsystems, tracking control

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1 Introduction

Interconnected systems consisting of subsystem interactions are a special class of large-scale systems. Decentralized control using only local measurements is a practicable and effective way to control interconnected systems. In the last few decades, much effort has been made toward decentralized control of interconnected systems (for a review, see [1] and the references therein). With the development of the adaptive backstepping technique [2], many results for decentralized adaptive control have been obtained for interconnected systems with uncertainties [3–14]. Since Pan and Başar first extended the backstepping technique to the control design of stochastic nonlinear systems in [15], research on interconnected stochastic nonlinear systems has received considerable attention.

In [12], global decentralized stabilization controllers based on both state feedback and output feedback are designed for a class of interconnected stochastic nonlinear systems. When considering output feedback control design, it is noteworthy that nonlinear interactions depending upon the outputs exist only in the drift terms. A decentralized risk-sensitive control scheme is presented for a class of interconnected stochastic nonlinear systems in [3], where nonlinear interactions depending only on the subsystem outputs exist in both the drift and diffusion terms. However, one should know all of the diffusion terms during the design procedure. Further, in [13], a decentralized adaptive output feedback stabilization controller is developed for a large class of interconnected stochastic nonlinear systems with inverse dynamics, parametric

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uncertainties, and unknown nonlinear interactions in both the diffusion and drift terms. On the basis of the work in [13], the nonlinear dynamic interactions are considered in [16]. However, most of these papers are concerned with the decentralized adaptive stabilization problem, and very few of them focus on tracking nonzero local reference signals with interconnected stochastic nonlinear systems [17–19]. A decentralized adaptive state feedback tracking control procedure is proposed for a class of interconnected stochastic nonlinear systems with interactions only in the drift terms in [17]. By using a state estimation filter, decentralized adaptive output feedback tracking control for interconnected stochastic systems is considered in [18], and the developed technique is generalized to an enlarged class of interconnected stochastic systems in [19]. However, the results of [19] are applicable only to systems in which the diffusion terms are bounded by the products of the system outputs and bounded functions. Clearly, this matching condition is very strict. Actually, the study of the tracking problem for interconnected stochastic systems is relatively complicated compared to that for the stabilization problem. The techniques developed for the stabilization case cannot be directly generalized to the tracking case. The main reason is that the tracking errors of subsystems are affected by nonzero reference signals through interactions. It is not easy to manage these effects in the adaptive backstepping design procedure; further, one should also consider the effect of the white noise on the design of tracking control. These considerations motivate us to conduct the current study.

In this paper, inspired by [13, 14], we consider the problem of decentralized backstepping adaptive tracking control design for a class of interconnected stochastic nonlinear systems consisting of unknown nonlinear interactions and uncertainties. Unlike the case in [19], the unknown nonlinear interactions do not need to be bounded by the products of the system outputs and known bounded output-dependent functions, but need only to be bounded by the products of parameter uncertainties and known output-dependent functions. Because only the system outputs can be used in controller design, the K-filter proposed in [20] is generalized to decentralized stochastic cases to estimate the local system states. To counteract the effects of the reference signals and unknown interactions in the backstepping design procedure, a differentiable function is constructed. The designed decentralized backstepping tracking controllers can guarantee the boundedness of all signals in the closed-loop system in probability. Moreover, the tracking error can be limited to a small region near the origin in the fourth moment sense. In addition, the proper design parameters can be chosen to make the region arbitrarily small.

The paper is organized as follows. Section 2 gives some preliminary results and notation. Section 3 describes the problem to be investigated and presents the design of decentralized state estimators. The decentralized adaptive tracking control design procedure is presented in Section 4. In Section 5, the stability of the decentralized tracking scheme is analyzed. A numerical example is given in Section 6 to illustrate the efficiency of our procedure. Concluding remarks are presented in Section 7.

2 Notation and preliminary results

Notation. I_n represents the identity matrix of order n. The transpose of a matrix or vector Y is denoted by Y^{T}. $\mathrm{Tr}(Y)$ represents the trace of a square matrix Y. $\|Y\|$ stands for the induced norm of a matrix Y. The Euclidean norm of a vector Y is denoted by $|Y|$. For any matrix Y, $\|Y\|_F := \sqrt{\mathrm{Tr}(Y^{\mathrm{T}}Y)}$. $\lambda_{\max}(Y)$ and $\lambda_{\min}(Y)$ are the maximal and minimal eigenvalues of a symmetric real matrix X, respectively.

Consider the following stochastic nonlinear system:

$$\mathrm{d}x = f(t,x)\mathrm{d}t + g(t,x)\mathrm{d}w, \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state, $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are continuously differentiable in their arguments, $f(t,0)$ and $g(t,0)$ are bounded uniformly in t, and w is an r-dimensional standard Brownian motion.

Definition 1. For any given $\mathcal{C}^2$ function $V(t,x)$ associated with the stochastic differential equation (1), we define the differential operator $\mathcal{L}$ as follows:

$$\mathcal{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x) + \frac{1}{2}\mathrm{Tr}\left\{\frac{\partial^2 V}{\partial x^2} g g^{\mathrm{T}}\right\}. \tag{2}$$

Definition 2. A stochastic process $\{x(t),\ t \geqslant 0\}$ is said to be bounded in probability if $\lim_{c\to\infty}\sup_{t\geqslant 0} P\{|x(t)| > c\} = 0$ [21].

The result of our technique for the solution of system (1) is given by the following theorem.

Theorem 1. Consider the stochastic nonlinear system (1). If there exists a $\mathcal{C}^2$ function $V(x)$, class $\mathcal{K}_\infty$ functions λ_1, λ_2, and constants $m_1 > 0, m_2 \geqslant 0$ such that for any $x \in \mathbb{R}^n$ and $t > 0$,

$$\lambda_1(|x|) \leqslant V(x) \leqslant \lambda_2(|x|), \quad \mathcal{L}V(x) \leqslant -m_1 V(x) + m_2, \tag{3}$$

then for any given initial value x_0 there exists a global unique strong solution $x(t)$ of system (1), and it is bounded in probability and satisfies

$$\mathrm{E}[V(x)] \leqslant \mathrm{e}^{-m_1 t} V(x_0) + m_1^{-1} m_2. \tag{4}$$

This theorem can be derived directly from [13, Theorem 1] and [22, Theorem 4.1].

3 Problem formulation

In this paper, we consider the following interconnected stochastic nonlinear systems of the form:

$$\begin{cases} \mathrm{d}x_i = A_{0,i}x_i\mathrm{d}t + F_i(t,y)\mathrm{d}t + \Phi_i(y_i)a_i\mathrm{d}t + b_i u_i \mathrm{d}t + G_i^{\mathrm{T}}(t,y)\mathrm{d}w_i, \\ y_i = x_{i,1}, \quad y = (y_1, y_2, \ldots, y_N), \quad i = 1, \ldots, N, \end{cases} \tag{5}$$

where

$$A_{0,i} = \begin{pmatrix} 0 & & \\ \vdots & I_{n_i-1} & \\ 0 & \cdots & 0 \end{pmatrix}, \quad \Phi_i = \begin{pmatrix} \varphi_{i,1,1} & \cdots & \varphi_{i,1,m_i} \\ \vdots & \ddots & \vdots \\ \varphi_{i,n_i,1} & \cdots & \varphi_{i,n_i,m_i} \end{pmatrix}, \quad F_i = \begin{pmatrix} F_{i,1} \\ \vdots \\ F_{i,n_i} \end{pmatrix}, \quad a_i = \begin{pmatrix} a_{i,1} \\ \vdots \\ a_{i,m_i} \end{pmatrix},$$

$x_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,n_i})^{\mathrm{T}} \in \mathbb{R}^{n_i}$; $u_i \in \mathbb{R}^1$ and $y_i \in \mathbb{R}^1$ are the input and output states of the i-th subsystem, respectively; $a_i \in \mathbb{R}^{m_i}$ and $b_i = (0, 0, \ldots, b_{i,s_i}, \ldots, b_{i,0})^{\mathrm{T}} \in \mathbb{R}^{n_i}$ are unknown vectors; $\Phi_i(y_i) \in \mathbb{R}^{n_i \times m_i}$ is a known smooth function; $G_i(t,y) = (G_{i,1}, G_{i,2}, \ldots, G_{i,n_i}) \in \mathbb{R}^{r_i \times n_i}$ and $F_i(t,y) \in \mathbb{R}^{n_i}$ are uncertain $\mathcal{C}^1$ functions; w_i is an r_i-dimensional standard Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, P)$; and $F_i(t,y)$ represents the unmodeled parts of the i-th subsystem and the interactions with other subsystems. The following assumptions are made for each subsystem.

Assumption 1. The relative degree $\rho_i (= n_i - s_i)$ and the sign of b_{i,s_i} are known.

Assumption 2. The polynomial $b_{i,s_i}s^{s_i} + \cdots + b_{i,1}s + b_{i,0}$ is Hurwitz.

Assumption 3. The reference signal $y_{r_i}(t)$ and its derivatives $y_{r_i}^{(k)}(t),\ k = 1, \ldots, \rho_i$ are bounded, known, and piecewise continuous.

Assumption 4. For each $1 \leqslant i \leqslant N$ and $1 \leqslant j \leqslant n_i$, there exist known smooth functions $f_{i,j,l} \geqslant 0$, $g_{i,j,l} \geqslant 0$ and unknown constants $l_{i,j}^1 > 0,\ l_{i,j}^2 > 0$ such that for $\forall (t,y) \in \mathbb{R}_+ \times \mathbb{R}^N$,

$$|F_{i,j}(t,y)| \leqslant l_{i,j}^1 \sum_{l=1}^{N} f_{i,j,l}(|y_l|), \quad |G_{i,j}(t,y)| \leqslant l_{i,j}^2 \sum_{l=1}^{N} g_{i,j,l}(|y_l|).$$

The control object is to design a decentralized adaptive controller with K-filters for the stochastic nonlinear system (5) satisfying Assumptions 1–4 such that all signals of the closed-loop system are bounded in probability, and the given reference $y_{r_i}(t)$ can be tracked by the system output as closely as possible in the fourth moment sense.

In the following, using only the local output and input, we will design decentralized K-filters to estimate the states of each local system. The filters for the i-th subsystem are designed as

$$\begin{aligned} \dot{\xi}_i &= A_i\xi_i + (I_{n_i,n_i})^{\mathrm{T}}u_i, \ \xi_i \in \mathbb{R}^{n_i}, \\ \dot{\eta}_i &= A_i\eta_i + K_iy_i, \qquad \eta_i \in \mathbb{R}^{n_i}, \\ \dot{\Xi}_i &= A_i\Xi_i + \Phi_i(y_i), \qquad \Xi_i \in \mathbb{R}^{n_i\times m_i}, \end{aligned} \tag{6}$$

where $K_i = [k_{i,1}, \ldots, k_{i,n_i}]^{\mathrm{T}} \in \mathbb{R}^{n_i}$ is selected such that the matrix $A_i = A_{0,i} - K_iI_{n_i,1}$ is Hurwitz. Then, there exists a matrix Q_i such that $Q_iA_i + A_i^{\mathrm{T}}Q_i = -I_{n_i},\ Q_i = Q_i^{\mathrm{T}} > 0$.

Define

$$v_{i,j} := A_i^j\xi_i, \quad j = 0, 1, \ldots, s_i. \tag{7}$$

Clearly, $v_{i,s_i,j} = v_{i,s_i,j}(\xi_{i,1}, \ldots, \xi_{i,s_i+j})$. Using the equation $A_i^j(I_{n_i,n_i})^{\mathrm{T}} = (I_{n_i,n_i-j})^{\mathrm{T}},\ j = 1, \ldots, n_i$, we obtain $\dot{v}_{i,j} = A_iv_{i,j} + (I_{n_i,n_i-j})^{\mathrm{T}}u_i,\ j = 0, \ldots, s_i$. Let $\theta_i := [b_{i,s_i}, \ldots, b_{i,0}, a_{i,1}, \ldots, a_{i,m_i}]^{\mathrm{T}}$ and $T_i := [v_{i,s_i}, \ldots, v_{i,0}, \Xi_i]$. By using the above designed filters, we define the state estimate as

$$\hat{x}_i(t) = \eta_i + T_i\theta_i. \tag{8}$$

Hence, the state estimation error is given by

$$\epsilon_i = x_i - \hat{x}_i, \tag{9}$$

which satisfies

$$\mathrm{d}\epsilon_i = (A_i\epsilon_i + F_i(t, y))\mathrm{d}t + G_i^{\mathrm{T}}(t, y)\mathrm{d}w_i. \tag{10}$$

4 Design of decentralized adaptive controllers

A decentralized adaptive output tracking control scheme will be developed by means of a backstepping technique in this section. First, a new state transformation is given by

$$\begin{aligned} z_{i,1} &= y_i - y_{r_i}, \\ z_{i,j} &= v_{i,s_i,j} - \alpha_{i,j-1}, \quad j = 2, \ldots, \rho_i, \end{aligned} \tag{11}$$

where $\alpha_{i,j-1} = \alpha_{i,j-1}(y_i, \xi_{i,1}, \ldots, \xi_{i,s_i+j-1}, \eta_i, \Xi_i, \hat{\theta}_i, \hat{\zeta}_i, y_{r_i}, \dot{y}_{r_i}, \ldots, y_{r_i}^{(j-1)})$ is the virtual control law. For notational simplicity, let $z_{i,\rho_i+1} = 0,\ \alpha_{i,0} = 0$, and $\alpha_{i,\rho_i} = v_{i,s_i,\rho_i+1} + u_i$. To counteract the influence of interactions in the control design, the following auxiliary function $s_i(\cdot)$ is introduced:

$$s_i(z_{i,1}) = \begin{cases} \frac{1}{z_{i,1}^4}, & |z_{i,1}| \geqslant \sigma_i, \\ \frac{1}{(\sigma_i^4 - z_{i,1}^4)^{\rho_i} + z_{i,1}^4}, & |z_{i,1}| < \sigma_i, \end{cases} \tag{12}$$

where σ_i is a positive design parameter. It is evident that $s_i(z_{i,1})$ is $(\rho_i - 1)$-th order differentiable.

According to Itô's formula, we have

$$\begin{aligned} \mathrm{d}z_{i,1} &= (\chi_i - \dot{y}_{r_i})\mathrm{d}t + G_{i,1}^{\mathrm{T}}\mathrm{d}w_i, \\ \mathrm{d}z_{i,j} &= \dot{v}_{i,s_i,j}\mathrm{d}t - \mathrm{d}\alpha_{i,j-1} \\ &= \Bigg[v_{i,s_i,j+1} - k_{i,j}v_{i,s_i,1} + I_{n_i,j,\rho_i}u_i - \frac{\partial\alpha_{i,j-1}}{\partial y_i}\chi_i\mathrm{d}t - \beta_{i,j-1} - \frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i - \frac{\partial\alpha_{i,j-1}}{\partial\hat{\zeta}_i}\dot{\hat{\zeta}}_i \\ &\quad - \frac{1}{2}\frac{\partial^2\alpha_{i,j-1}}{\partial y_i^2}G_{i,1}^{\mathrm{T}}G_{i,1}\Bigg]\mathrm{d}t - \frac{\partial\alpha_{i,j-1}}{\partial y_i}G_{i,1}^{\mathrm{T}}\mathrm{d}w_i, \quad j = 2, \ldots, \rho_i, \end{aligned} \tag{13}$$

where

$$W_i = [v_{i,s_i,2}, \ldots, v_{i,0,2}, \Xi_{i,2} + \Phi_{i,1}]^{\mathrm{T}}, \quad \bar{W}_i = [0, v_{i,s_i-1,2}, \ldots, v_{i,0,2}, \Xi_{i,2} + \Phi_{i,1}]^{\mathrm{T}},$$
$$\chi_i = \eta_{i,2} + F_{i,1} + \bar{W}_i^{\mathrm{T}}\theta_i + b_{i,s_i}(z_{i,2} + \alpha_{i,1}) + \epsilon_{i,2},$$
$$\beta_{i,j-1} = \sum_{l=1}^{s_i+j-1} \frac{\partial \alpha_{i,j-1}}{\partial \xi_{i,l}}(\xi_{i,l+1} - k_{i,l}\xi_{i,1}) + \frac{\partial \alpha_{i,j-1}}{\partial \eta_i}(A_i\eta_i + K_i y_i) + \frac{\partial \alpha_{i,j-1}}{\partial \Xi_i}(A_i\Xi_i + \Phi_i) + \sum_{l=1}^{j} \frac{\partial \alpha_{i,j-1}}{\partial y_{r_i}^{(l-1)}} y_{r_i}^{(l)}, \quad j = 2, \ldots, \rho_i.$$

We now consider the following Lyapunov function candidate:

$$V_i = \frac{1}{4}\sum_{j=1}^{\rho_i} z_{i,j}^4 + \frac{1}{2}(\epsilon_i^{\mathrm{T}} Q_i \epsilon_i)^2 + \frac{1}{2}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\tilde{\theta}_i + \frac{|b_{i,s_i}|}{2\gamma_i}\tilde{\zeta}_i^2, \tag{14}$$

where Γ_i is a positive definite matrix, γ_i is a designed positive number, and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{\zeta}_i = \zeta_i - \hat{\zeta}_i$ are the parameter estimation errors.

It follows from (10) and (13) that

$$\begin{aligned} \mathcal{L}V_i =& z_{i,1}^3(\chi_i - \dot{y}_{r_i}) + \sum_{j=2}^{\rho_i} z_{i,j}^3\Bigg[\alpha_{i,j} + z_{i,j+1} - k_{i,j}v_{i,s_i,1} - \frac{\partial \alpha_{i,j-1}}{\partial y_i}\chi_i - \frac{1}{2}\frac{\partial^2 \alpha_{i,j-1}}{\partial y_i^2}G_{i,1}^{\mathrm{T}}G_{i,1} - \beta_{i,j-1} \\ & - \frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i - \frac{\partial \alpha_{i,j-1}}{\partial \hat{\zeta}_i}\dot{\hat{\zeta}}_i\Bigg] + \frac{3}{2}z_{i,1}^2 G_{i,1}^{\mathrm{T}}G_{i,1} + \frac{3}{2}\sum_{j=2}^{\rho_i} z_{i,j}^2\left(\frac{\partial \alpha_{i,j-1}}{\partial y_i}\right)^2 G_{i,1}^{\mathrm{T}}G_{i,1} - (\epsilon_i^{\mathrm{T}}Q_i\epsilon_i)\epsilon_i^{\mathrm{T}}\epsilon_i \\ & + 2(\epsilon_i^{\mathrm{T}}Q_i\epsilon_i)F_i^{\mathrm{T}}Q_i\epsilon_i + \mathrm{Tr}[G_i(\epsilon_i^{\mathrm{T}}Q_i\epsilon_i Q_i + 2Q_i\epsilon_i\epsilon_i^{\mathrm{T}}Q_i)G_i^{\mathrm{T}}] - \tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\hat{\theta}}_i - \frac{|b_{i,s_i}|}{\gamma_i}\tilde{\zeta}_i\dot{\hat{\zeta}}_i. \end{aligned} \tag{15}$$

Because $f_{i,j,l},\ g_{i,j,l},\ i,\ l = 1, \ldots, N,\ j = 1, \ldots, n_i$, are smooth, there exist smooth nonnegative functions $p_{i,j,l}(y_l),\ q_{i,j,l}(y_l)$ such that

$$\begin{aligned} \left(\sum_{l=1}^{N} f_{i,j,l}(|y_l|)\right)^4 \leqslant \sum_{l=1}^{N} p_{i,j,l}y^4 + 8\left(\sum_{l=1}^{N} f_{i,j,l}(0)\right)^4, \\ \left(\sum_{l=1}^{N} g_{i,j,l}(|y_l|)\right)^4 \leqslant \sum_{l=1}^{N} q_{i,j,l}y^4 + 8\left(\sum_{l=1}^{N} g_{i,j,l}(0)\right)^4. \end{aligned} \tag{16}$$

From Assumption 4, Young's inequality, and the norm inequalities, we have

$$\begin{aligned} 2(\epsilon_i^{\mathrm{T}}Q_i\epsilon_i)F_i^{\mathrm{T}}Q_i\epsilon_i \leqslant 2||Q_i||^2|\epsilon_i|^3|F_i| &\leqslant 2||Q_i||\left(\frac{3}{4}\delta_{i,8}^{4/3}|\epsilon_i|^4 + \frac{1}{4\delta_{i,8}^4}|F_i|^4\right) \\ &\leqslant \frac{3}{2}||Q_i||^2\delta_{i,8}^{4/3}|\epsilon_i|^4 + \frac{||Q_i||^2 n_i}{2\delta_{i,8}^4}\left(|F_{i,1}|^4 + \cdots + |F_{i,n_i}|^4\right) \\ &\leqslant \frac{3}{2}||Q_i||^2\delta_{i,8}^{4/3}|\epsilon_i|^4 + \frac{||Q_i||^2 n_i l_i^4}{2\delta_{i,8}^4}\sum_{j=1}^{n_i}\left(\sum_{l=1}^{N} p_{i,j,l}y_l^4 + 8\left(\sum_{l=1}^{N} f_{i,j,l}(0)\right)^4\right), \end{aligned}$$

$$z_{i,1}^3 F_{i,1} \leqslant \frac{3}{4}\delta_{i,9}^{4/3}z_{i,1}^4 + \frac{l_i^4}{4\delta_{i,9}^4}\left(\sum_{l=1}^{N} p_{i,1,l}y_l^4 + 8\left(\sum_{l=1}^{N} f_{i,1,l}(0)\right)^4\right),$$

$$\begin{aligned} \sum_{j=2}^{\rho_i} z_{i,j}^3\frac{\partial \alpha_{i,j-1}}{\partial y_i}F_{i,1} &\leqslant \sum_{j=2}^{\rho_i}\left(\frac{3}{4}\mu_{i,1,j}^{4/3}\left(\frac{\partial \alpha_{i,j-1}}{\partial y_i}\right)^{4/3} z_{i,j}^4 + \frac{1}{4\mu_{i,1,j}^4}|F_{i,1}|^4\right) \\ &\leqslant \sum_{j=2}^{\rho_i}\frac{3}{4}\mu_{i,1,j}^{4/3}\left(\frac{\partial \alpha_{i,j-1}}{\partial y_i}\right)^{4/3} z_{i,j}^4 + \sum_{j=2}^{\rho_i}\frac{l_i^4}{4\mu_{i,1,j}^4}\left(p_{i,1,l}y_l^4 + 8\left(\sum_{l=1}^{N} f_{i,1,l}(0)\right)^4\right), \end{aligned}$$

$$\begin{aligned}
\mathrm{Tr}[G_i(\epsilon_i^{\mathrm{T}}Q_i\epsilon_iQ_i+2Q_i\epsilon_i\epsilon_i^{\mathrm{T}}Q_i)G_i^{\mathrm{T}}] &= 2\mathrm{Tr}[G_iQ_i\epsilon_i(G_iQ_i\epsilon_i)^{\mathrm{T}}]+\epsilon_i^{\mathrm{T}}Q_i\epsilon_i\mathrm{Tr}[G_i^{\mathrm{T}}Q_iG_i]\\
&\leqslant 2||G_iQ_i\epsilon_i||_F^2+|\epsilon_i^{\mathrm{T}}Q_i\epsilon_i|\lambda_{\max}(Q_i)||G_i||_F^2\\
&\leqslant 2||Q_i||^2|\epsilon_i|^2||G_i||_F^2+|\epsilon_i|^2||Q_i||\lambda_{\max}(Q_i)||G_i||_F^2\\
&\leqslant \frac{1}{\mu_{i,2}}|\epsilon_i|^4+\mu_{i,2}||Q_i||^2||G_i||_F^4+\frac{\lambda_{\max}^2(Q_i)}{2\mu_{i,2}}|\epsilon_i|^4+\frac{\mu_{i,2}}{2}||Q_i||^2||G_i||_F^4\\
&\leqslant \frac{2+\lambda_{\max}^2(Q_i)}{2\mu_{i,2}}|\epsilon_i|^4+\frac{3n_il_i^4||Q_i||^2\mu_{i,2}}{2}\sum_{j=1}^{n_i}\left(\sum_{l=1}^{N}q_{i,j,l}y_l^4\right.\\
&\quad\left.+8\left(\sum_{l=1}^{N}g_{i,j,l}(0)\right)^4\right),
\end{aligned}$$

where the variables δ and μ and $l_i \geqslant \max_{1\leqslant j\leqslant n_i}\{l_{i,j}^1, l_{i,j}^2\}$ are positive design parameters. The above inequalities, together with (15) and Assumption 4, give that

$$\begin{aligned}
\mathcal{L}V_i \leqslant& z_{i,1}^3\left(\eta_{i,2}+\bar{W}_i^{\mathrm{T}}\hat{\theta}_i+b_{i,s_i}\alpha_{i,1}-\dot{y}_{r_i}+\frac{3}{4}\delta_{i,1}^{4/3}+\frac{3}{4}|b_{i,s_i}|\delta_{i,2}^{4/3}z_{i,1}+\frac{3}{4}\delta_{i,6}^2z_{i,1}+\frac{3}{4}\delta_{i,9}^{4/3}z_{i,1}\right)+\frac{|b_{i,s_i}|}{4\delta_{i,2}^4}z_{i,2}^4\\
&+\sum_{j=2}^{\rho_i}z_{i,j}^3\left[\alpha_{i,j}-k_{i,j}v_{i,s_i,1}-\frac{\partial\alpha_{i,j-1}}{\partial y_i}(\eta_{i,2}+\bar{W}_i^{\mathrm{T}}\hat{\theta}_i+\hat{b}_{i,s_i}v_{i,s_i,2})+\left(\frac{3}{4}\delta_{i,3,j}^{4/3}+\frac{1}{4\delta_{i,3,j-1}^4}\right)z_{i,j}\right.\\
&+\left(\frac{3}{4}\delta_{i,4,j}^{4/3}+\frac{3}{4}\mu_{i,1,j}^{4/3}\right)\left(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\right)^{4/3}z_{i,j}+\frac{1}{4}\delta_{i,5,j}^2\left(\frac{\partial^2\alpha_{i,j-1}}{\partial y_i^2}\right)^2z_{i,j}^3+\frac{3}{4}\delta_{i,7,j}^2\left(\frac{\partial\alpha_{i,j-1}}{\partial y_i}\right)^4z_{i,j}\\
&\left.-\beta_{i,j-1}\right]-\sum_{j=2}^{\rho_i}z_{i,j}^3\left[\frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i+\frac{\partial\alpha_{i,j-1}}{\partial\hat{\zeta}_i}\dot{\hat{\zeta}}_i+\frac{\partial\alpha_{i,j-1}}{\partial y_i}(\bar{W}_i\tilde{\theta}_i+\tilde{b}_{i,s_i}v_{i,s_i,2})\right]-\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\dot{\hat{\theta}}_i-\frac{|b_{i,s_i}|}{\gamma_i}\tilde{\zeta}_i\dot{\hat{\zeta}}_i\\
&+\left(\frac{3}{4\delta_{i,6}^2}+\sum_{j=2}^{\rho_i}\left(\frac{1}{4\delta_{i,5,j}}+\frac{3}{4\delta_{i,7,j}^2}\right)+\frac{3}{2}\mu_{i,2}||Q_i||^2\right)n_il_i^4\sum_{j=1}^{n_i}\left(\sum_{l=1}^{N}q_{i,j,l}y_l^4+8\left(\sum_{l=1}^{N}g_{i,j,l}(0)\right)^4\right)\\
&+\left(\frac{1}{4\delta_{i,9}^4}+\frac{||Q_i||^2}{2\delta_{i,8}^4}+\sum_{j=2}^{\rho_i}\frac{1}{4\mu_{i,1,j}^4}\right)n_il_i^4\sum_{j=1}^{n_i}\left(\sum_{l=1}^{N}p_{i,j,l}y_l^4+8\left(\sum_{l=1}^{N}f_{i,j,l}(0)\right)^4\right)-\left[\lambda_{\min}(Q_i)-\frac{1}{4\delta_{i,1}^4}\right.\\
&\left.-\sum_{j=2}^{\rho_i}\frac{1}{4\delta_{i,4,j}^4}-\frac{3\delta_{i,8}^{4/3}||Q_i||}{2}+\frac{2+\lambda_{\max}^2(Q_i)}{2\mu_{i,2}}\right]|\epsilon_i|^4, \qquad (17)
\end{aligned}$$

where the variables μ and δ are positive design parameters.

By using (17), the local controller and parameter update laws can be designed step by step for each subsystem as follows:

$$\alpha_{i,1}=-\hat{\zeta}_i\bar{\chi}_i-\frac{3}{4}\mathrm{sign}(b_{i,s_i})\delta_{i,2}^{4/3}z_{i,1}, \qquad (18)$$

$$\alpha_{i,j}=-c_{i,j}z_{i,j}+\pi_{i,j}+\frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_i}\dot{\hat{\theta}}_i+\frac{\partial\alpha_{i,j-1}}{\partial\hat{\zeta}_i}\dot{\hat{\zeta}}_i-I_{n_i,j,2}\hat{b}_{i,s_i}\frac{\mathrm{sign}(b_{i,s_i})}{4\delta_{i,2}^4}z_{i,2},\quad j=2,\ldots,\rho_i, \qquad (19)$$

$$u_i=\alpha_{i,\rho_i}-v_{i,s_i,\rho_i+1}, \qquad (20)$$

$$\dot{\hat{\zeta}}_i=\gamma_i\mathrm{sign}(b_{i,s_i})\bar{\chi}_iz_{i,1}^3-2c_{\zeta_i}\hat{\zeta}_i, \qquad (21)$$

$$\dot{\hat{\theta}}_i=\Gamma_i\tau_{i,\rho_i}-2c_{\theta_i}\hat{\theta}_i, \qquad (22)$$

where $c_{i,j},\ j=1,\ldots,s_i, c_{\zeta_i},$ and c_{θ_i} are positive design constants,

$$\pi_{i,j}=k_{i,j}v_{i,s_i1}+\frac{\partial(\alpha_{i,j-1})}{\partial y_i}(\eta_{i,2}+\bar{W}_i^{\mathrm{T}}\hat{\theta}_i+\hat{b}_{i,s_i}v_{i,s_i,2})-\left(\frac{3}{4}\delta_{i,3,j}^{4/3}+\frac{1}{4\delta_{i,3,j-1}^4}\right)z_{i,j}$$

$$- \left(\frac{3}{4}\delta_{i,4,j}^{4/3} + \frac{3}{4}\delta_{i,1,j}^{4/3}\right)\left(\frac{\partial \alpha_{i,j-1}}{\partial y_i}\right)^{4/3} z_{i,j} - \frac{1}{4}\delta_{i,5,j}^{2}\left(\frac{\partial^2 \alpha_{i,j-1}}{\partial y_i^2}\right)^{2} z_{i,j}^{3}$$
$$- \frac{3}{4}\delta_{i,7,j}^{2}\left(\frac{\partial \alpha_{i,j-1}}{\partial y_i}\right)^{4} z_{i,j} + \beta_{i,j-1},$$
$$\bar{\chi}_i = c_{i,1}z_{i,1} + \eta_{i,2} + \bar{W}_i^{\mathrm{T}}\hat{\theta}_i - \dot{y}_{r_i} + \frac{3}{4}(\delta_{i,1}^{4/3} + \delta_{i,9}^{4/3} + \delta_{i,6}^{2})z_{i,1} + z_{i,1}s_i(z_{i,1})\sum_{m=1}^{N} C_m\Psi_m(y_i),$$
$$C_i = \left[\frac{1}{4\delta_{i,9}^4} + \frac{||Q_i||^2}{2\delta_{i,8}^4} + \sum_{j=2}^{\rho_i}\frac{1}{4\mu_{i,1,j}^4} + \frac{3}{4\delta_{i,6}^2} + \sum_{j=2}^{\rho_i}\left(\frac{1}{4\delta_{i,5,j}} + \frac{3}{4\delta_{i,7,j}^2}\right) + \frac{3}{2}\mu_{i,2}||Q_i||^2\right] n_i l_i^4,$$
$$\Psi_i(y_l) = \sum_{j=1}^{n_i}(p_{i,j,l}y_l^4 + q_{i,j,l}y_l^4),$$
$$\Psi_{i0} = \sum_{j=1}^{n_i}\left(\left(\sum_{l=1}^{N} g_{i,j,l}(0)\right)^4 + \left(\sum_{l=1}^{N} f_{i,j,l}(0)\right)^4\right),$$

and the tuning functions are chosen as follows:

$$\begin{aligned}
\tau_{i,1} &= \bar{W}_i z_{i,1}^3, \\
\tau_{i,2} &= \tau_{i,1} + \frac{\mathrm{sign}(b_{i,s_i})}{4\delta_{i,2}^4} z_{i,2}^4 (I_{n_i,1})^{\mathrm{T}} - \frac{\partial \alpha_{i,1}}{\partial y_i} z_{i,2}^3 W_i, \\
\tau_{i,j} &= \tau_{i,j-1} - \frac{\partial \alpha_{i,j-1}}{\partial y_i} z_{i,j}^3 W_i, \quad j = 3, \ldots, \rho_i.
\end{aligned} \tag{23}$$

5 Stability analysis

The analysis of the tracking errors and stability of the overall closed-loop system consisting of local systems, decentralized filters, controllers, and parameter update laws are presented in this section. First, a Lyapunov function candidate for the overall closed-loop system is defined as

$$V = \sum_{i=1}^{N} V_i, \tag{24}$$

which satisfies

$$\mathcal{L}V = \sum_{i=1}^{N} \mathcal{L}V_i.$$

Hence, substituting (18)–(22) into (17) yields

$$\begin{aligned}
\mathcal{L}V \leqslant{}& \sum_{i=1}^{N}\left(-c_i\sum_{j=1}^{\rho_i} z_{i,j}^4 - \bar{c}_i|\epsilon_i|^4 + 2c_{\theta_i}\tilde{\theta}_i^{\mathrm{T}}\Gamma_i^{-1}\hat{\theta}_i + 2c_{\zeta_i}\frac{b_{i,s_i}}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i\right) \\
&+ \sum_{i=1}^{N}(-z_{i,1}^4 s_i(z_{i,1})D_i(y_i) + D_i(y_i)) + \sum_{i=1}^{N} E_i,
\end{aligned} \tag{25}$$

where $c_i = \min\{c_{i,j}|1 \leqslant j \leqslant \rho_i\}$, $\bar{c}_i = \lambda_{\min}(Q_i) - \frac{1}{4\delta_{i,1}^4} - \sum_{j=2}^{\rho_i}\frac{1}{4\delta_{i,4,j}^4} - \frac{3\delta_{i,8}^{4/3}||Q_i||}{2} + \frac{2+\lambda_{\max}^2(Q_i)}{2\mu_{i,2}}$, $D_i = \sum_{m=1}^{N} C_m\Psi_m(y_i)$, and $E_i = 8C_i\Psi_i$. Here, the positive parameters δ and μ are chosen such that $\bar{c}_i > 0$. Let $h_i = -z_{i,1}^4 s_i(z_{i,1})D_i(y_i) + D_i(y_i)$. It is evident that h_i is bounded. Hence, we obtain that

$$\sum_{i=1}^{N}(-z_{i,1}^4 s_i(z_{i,1})D_i(y_i)) + \sum_{i=1}^{N} E_i \leqslant \sum_{i=1}^{N} H_i + \sum_{i=1}^{N}|E_i| := M, \tag{26}$$

where H_i is the bound of h_i. In conclusion, the main results are presented in the following theorem.

Theorem 2. Under Assumptions 1–4, consider the closed-loop adaptive stochastic system consisting of the plant (5), the filters (6), control laws (18)–(20), and parameter update laws (21) and (22). If a prior bound on the unknown constants $l_{i,j}^1$ and $l_{i,j}^2$ is available, then all the signals of the closed-loop system are bounded in probability, and the tracking error can be limited to a small region near the origin in the fourth moment sense. In addition, the adaptive control laws can be adjusted to make the region arbitrarily small.

Proof. By adding two positive terms, $\sum_{i=1}^N c_{\theta_i}\hat{\theta}_i^{\rm T}\Gamma_i^{-1}\hat{\theta}_i$ and $\sum_{i=1}^N(|b_{i,s_i}|/\gamma_i)c_{\zeta_i}\hat{\zeta}_i^2$, to the right-hand side of (25) and using inequality (26), we obtain

$$\begin{aligned}\mathcal{L}V \leqslant & \sum_{i=1}^N\left(-c_i\sum_{j=1}^{\rho_i}z_{i,j}^4-\frac{\bar{c}_i}{\lambda_{\max}^2(Q_i)}(\epsilon_i^{\rm T}Q_i\epsilon_i)^2-c_{\theta_i}\tilde{\theta}_i^{\rm T}\Gamma_i^{-1}\tilde{\theta}_i-c_{\zeta_i}\frac{|b_{i,s_i}|}{\gamma_i}\tilde{\zeta}_i^2\right)+\sum_{i=1}^N\left(c_{\theta_i}|\Gamma_i^{-1}||\theta_i|^2\right.\\ &\left.+\frac{|b_{i,s_i}|}{\gamma_i}c_{\zeta_i}|\zeta_i|^2\right)+M\\ \leqslant & -CV+T,\end{aligned}\tag{27}$$

where

$$C:=\min_{1\leqslant i\leqslant N}\left\{4c_i,\frac{2\bar{c}_i}{\lambda_{\max}^2(Q_i)},2c_{\theta_i},2c_{\zeta_i}\right\},$$
$$T:=\sum_{i=1}^N\left(c_{\theta_i}|\Gamma_i^{-1}||\theta_i|^2+\frac{|b_{is_i}|}{\gamma_i}c_{\zeta_i}|\zeta_i|^2\right)+M.$$

It follows from Theorem 1 that z_i, ϵ_i, $\tilde{\theta}_i$, and $\tilde{\zeta}_i$ are bounded in probability. Because z_{i1} and y_{r_i} are bounded in probability, y_i is also bounded in probability. Because y_i and u_i are bounded and A_i is Hurwitz, the variables v_{ij}, ξ_i, η_i, and Ξ_i are bounded in probability. Finally, x_i is bounded by (7)–(9). Hence, all signals are bounded in probability. Furthermore, also by Theorem 1, we have

$$\mathrm{E}[V(t)]\leqslant \mathrm{e}^{-Ct}V(0)+C^{-1}T,\quad \forall t\geqslant 0.\tag{28}$$

From the structure of the Lyapunov function $V(x)$, we can easily obtain

$$\begin{aligned}\mathrm{E}(|y-y_{\rm r}|^4)&=\mathrm{E}\left(\left(\sum_{i=1}^N|y_i-y_{r_i}|^2\right)^2\right)\leqslant \mathrm{E}\left(N\left(\sum_{i=1}^N|z_{i,1}|^4\right)\right)\\ &\leqslant 4N\mathrm{E}(V(t))\leqslant 4N\mathrm{e}^{-Ct}V(0)+C^{-1}NT.\end{aligned}\tag{29}$$

The above inequality shows that one can choose suitable parameters δ, μ, c, Γ_i, and γ_i such that C is sufficiently large to guarantee that the tracking error converges to an arbitrarily small region near the origin in the fourth moment sense.

6 Numerical example

A numerical example is given to illustrate the efficiency of our procedure in this section. Consider the following interconnected systems:

$$\mathrm{d}\begin{pmatrix}x_{1,1}\\x_{1,2}\end{pmatrix}=\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}x_{1,1}\\x_{1,2}\end{pmatrix}\mathrm{d}t+\begin{pmatrix}2y_1&y_1^2\\0&y_1\end{pmatrix}\begin{pmatrix}a_{1,1}\\a_{1,2}\end{pmatrix}\mathrm{d}t+\begin{pmatrix}0\\b_1\end{pmatrix}u_1\mathrm{d}t+\begin{pmatrix}0\\0.1y_1\sin(y_2)\end{pmatrix}\mathrm{d}w_1,\tag{30}$$
$$y_1=x_{1,1},\qquad y_{\rm r1}=\sin(2t),$$

$$\mathrm{d}\begin{pmatrix}x_{2,1}\\x_{2,2}\end{pmatrix}=\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}x_{2,1}\\x_{2,2}\end{pmatrix}\mathrm{d}t+\begin{pmatrix}0&0\\y_2&1+y_1\end{pmatrix}\begin{pmatrix}a_{2,1}\\a_{2,2}\end{pmatrix}\mathrm{d}t+\begin{pmatrix}0\\b_1\end{pmatrix}u_2\mathrm{d}t+\begin{pmatrix}0\\0.1y_2\sin(y_1)\end{pmatrix}\mathrm{d}w,\tag{31}$$
$$y_2=x_{2,1},\qquad y_{\rm r2}=\sin t,$$

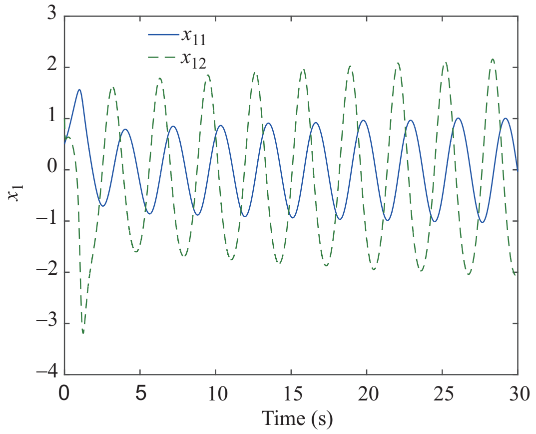

Figure 1 (Color online) System state x_1.

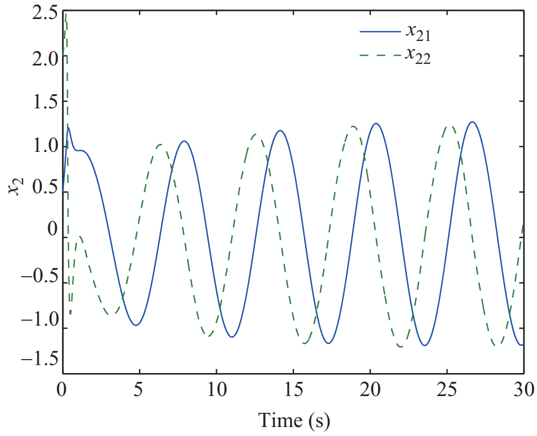

Figure 2 (Color online) System state x_2.

where $(a_{1,1}, a_{1,2})^{\mathrm{T}} = (0.2, 0.3)^{\mathrm{T}},\ (a_{2,1}, a_{2,2})^{\mathrm{T}} = (0.6, 0.9)^{\mathrm{T}},\ b_1 = 0.4,$ and $b_2 = 0.7$ are unknown parameters. The decentralized K-filters are designed as follows:

$$\begin{aligned} \dot{v}_{i,0} &= A_i v_{i,0} + (I_{2,2})^{\mathrm{T}} u_i, \\ \dot{\eta}_i &= A_i \eta_i + k_i y_i, \\ \dot{\Xi}_i &= A_i \Xi_i + \Phi_i, \quad i = 1, 2, \end{aligned} \tag{32}$$

where $k_i = (k_{i,1}, k_{i,2})^{\mathrm{T}}$, and $A_i = A_{0,i} - k_i I_{2,1}$. The local parameter update and control laws are given by

$$\begin{aligned} &\begin{pmatrix} \dot{\hat{b}}_i \\ \dot{\hat{a}}_{i,1} \\ \dot{\hat{a}}_{i,2} \end{pmatrix} = \Gamma_i \begin{pmatrix} \tau_{i,1} \\ \tau_{i,2} \\ \tau_{i,3} \end{pmatrix} - 2c_{\theta_i} \begin{pmatrix} \hat{b}_i \\ \hat{a}_{i,1} \\ \hat{a}_{i,2} \end{pmatrix}, \\ &\begin{pmatrix} \tau_{i,1} \\ \tau_{i,2} \\ \tau_{i,3} \end{pmatrix} = z_{i1}^3 \begin{pmatrix} 0 \\ \Xi_{i,2,1} + \Phi_{i,1,1} \\ \Xi_{i,2,2} + \Phi_{i,1,2} \end{pmatrix} + \frac{z_{i2}^3}{4\delta_{i4}^4}(I_{3,1})^{\mathrm{T}} - \frac{\partial \alpha_{i1}}{\partial y_i} z_{i2}^3 \begin{pmatrix} v_{i02} \\ \Xi_{i,2,1} + \Phi_{i,1,1} \\ \Xi_{i,2,2} + \Phi_{i,1,2} \end{pmatrix}, \\ &\dot{\hat{\zeta}}_i = \gamma_i \hat{\chi}_i z_{i,1}^3 - 2c_{\zeta_i} \hat{\zeta}_i, \\ &\alpha_{i,1} = -\hat{\zeta}_i \hat{\chi}_i - \frac{3}{4} \delta_{i,2}^{4/3} z_{i,1}, \\ &u_i = \alpha_{i,2} = c_{i,2} z_{i,2} + \pi_{i,2} + \dot{\hat{\theta}}_i^{\mathrm{T}} \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i} + \dot{\hat{\zeta}}_i^{\mathrm{T}} \frac{\partial \alpha_{i,1}}{\partial \hat{\zeta}_i} - \frac{\hat{b}_i}{4\delta_{i,2}^4} z_{i,2}, \end{aligned}$$

where the design parameters are chosen as $c_{1,1} = c_{2,1} = 2, c_{1,2} = c_{2,2} = 3,$ all $\delta_{ij} = \delta_{i,j,l} = \mu_{i,j} = \mu_{i,j,l} = 1$ except that $\delta_{1,8} = \delta_{2,8} = 0.1,\ \mu_{1,2} = \mu_{2,2} = 10,$ and $\gamma_1 = \gamma_2 = 1, \Gamma_1 = \Gamma_2 = I_2$. The initial values are set to $x_1 = (0.5, 1)^{\mathrm{T}}, x_2 = (0.5, 2)^{\mathrm{T}}, v_{10} = v_{20} = (0.2, 0.3)^{\mathrm{T}}, \eta_1 = (0.1, 0.1)^{\mathrm{T}}, \eta_2 = (0.2, 0.4)^{\mathrm{T}}, \Xi_{1,i,j}(0) = 0.2,$ and $\Xi_{2,i,j}(0) = 0.1$. Figures 1 and 2 show the system states x_1 and x_2. The outputs y_1, y_2 and the reference signals y_{r_1}, y_{r_2} are shown in Figures 3 and 4. It can be seen that the simulation results verify the efficiency of our proposed scheme.

7 Concluding remarks

A decentralized adaptive output tracking controller is proposed for a class of interconnected stochastic nonlinear systems with parametric uncertainties. The design procedure is backstepping-based and constructive. A new differentiable function is introduced to counteract the nonlinear interactions in the

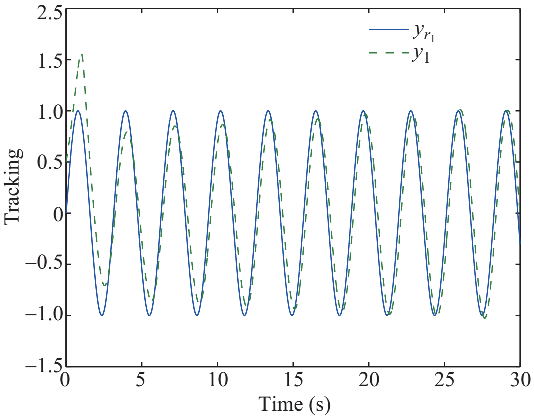

Figure 3 (Color online) Output y_1 and trajectory y_{r_1}.

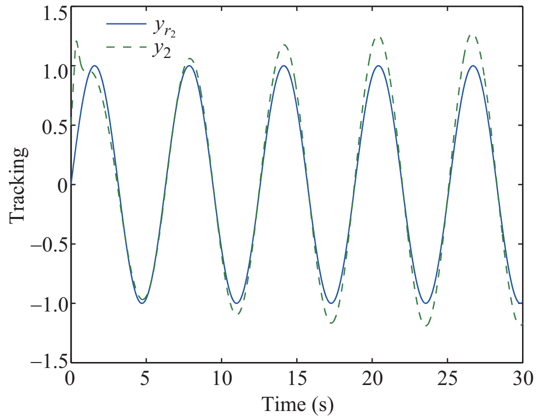

Figure 4 (Color online) Output y_2 and trajectory y_{r_2}.

control design. It is proved that the designed decentralized output tracking controllers can guarantee the boundedness of all the signals of the closed-loop system in probability. Further, the tracking error can be limited to a small residual set around the origin in the fourth moment sense. Moreover, the residual set can be adjusted by choosing suitable design parameters.

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Conflict of interest The authors declare that they have no conflict of interest.

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