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Integral sliding mode control design for nonlinear stochastic systems under imperfect quantization

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Abstract This paper presents a sliding mode control (SMC) scheme via output-feedback approach for Itô stochastic systems under a quantization mechanism. The quantization process is formulated with the imperfection that random packet loss occurs at the logarithmic quantizer. A Luenberger observer is designed, based on the packet loss rate and the imperfect quantized measurement. A novel SMC law is synthesized by utilization of an integral sliding surface. The stochastic stability of the resulting closed-loop system is analyzed in terms of Lyapunov stability, and a set of solvable matrix inequalities are established for practical application requirements. Finally, a simulation example is employed for the illustration of the effectiveness of the presented control scheme.

Keywords stochastic systems, quantized control, sliding mode control, observer design, packet loss

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1 Introduction

Over the decades, networked-control systems (NCSs) have been undergoing dramatic changes. Compared with analog implementation, NCSs whose signal (data) transmission operates via a digital communication channel have certain advantages, such as increased system flexibility, high reliability, and low costs of installation and maintenance [1]. Representative results on the analysis and synthesis of the complex characteristics of NCSs can be found in [2–8] and the references therein. With growing interest in NCSs, however, the stabilization of the systems under saturating quantized measurements has raised significant concern [9–12], because a quantization process is required before data is transmitted to a controller (actuator) unit or a filter unit. To achieve this, a real-valued signal is mapped into a piecewise constant signal, with its value taken from a finite set [13]. Based on this concept, advanced quantization strategies are important for sensor measurements or control commands transmitted over networks. Recently, some studies have focused on this field [14–19]. For instance, the work in [20] dealt with the full- and reduced-order dynamic output feedback control problems of semi-Markovian jump systems based on a quantized mechanism. Hayakawa et al. presented a direct adaptive control using a developed logarithmic quantizer for linear uncertain systems [12]. Wang et al. [17] studied the quantized control of stochastic systems under a network environment with missing probabilistic data.

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As a control methodology that can effectively manage nonlinearity and uncertainties such as system perturbation and exogenous disturbances, sliding mode control (SMC) [21] has been widely adopted in uncertain systems [22–25], nonlinear systems [26], stochastic systems [27], switched systems [28], jumping systems [29], and singular systems [30]. The essential strategy of SMC is to employ a discontinuous control to drive the desired trajectories towards, and/or to stay in, a predesigned subspace in which the system has required attributes such as stability, tracking ability, and disturbance rejection capability [21,31]. To mention a few systems that have benefitted from the strategy of SMC for quantized control systems, Shi et al. [29] synthesized a sliding mode observer to compensate for the quantization effects in Markovian jump systems with actuator faults. Hao et al. [32] dealt with the robust fault-tolerant integral SMC by using only quantized signals for uncertain linear systems over networks. However, few studies have focused on quantized SMC for nonlinear stochastic systems, especially with external disturbances. It is noteworthy that the main challenges to be dealt with while using quantized SMC for uncertain nonlinear stochastic systems under network environments are:

(1) random communication packet loss may occur in each communication channel, particularly during the quantization process, which makes it difficult to design the observer and the SMC;

(2) even with a well-designed sliding mode observer, choosing a suitable sliding variable and analyzing the sliding motion stability remain a challenge;

(3) there are technical obstacles in deriving the stability criteria for the overall control system in terms of the external disturbances and stochastic nonlinearity.

In this study, an observer-based SMC under a logarithmic quantization mechanism is developed for nonlinear stochastic systems over networks. The control design aims to (1) synthesize an SMC law for the stochastic plant when the output measurements are quantized with random packet losses; (2) design a state observer to estimate the unavailable plant states under the network environment with imperfect quantization; (3) establish a stability criterion and performance index for the overall quantized NCS.

The rest of the paper is organized as follows. Section 2 describes the NCS formulated with a nonlinear stochastic plant. In Section 3, an observer-based SMC with imperfect quantization is presented. Section 4 shows the simulations. Section 5 summarizes the paper.

Notation. The space of a square-integrable vector is denoted by $\mathcal{L}_2 \in [0,+\infty)$. $V(x(t),t) \in \mathbb{C}^{2,1}$ $(\mathbb{R}^n \times \mathbb{R}^+, \mathbb{R}^+)$ stands for the set of real-valued functions $V(x(t),t)$ being continuously twice differentiable in x and t. $\mathrm{E}\{x\}$ is used to denote the expectation of x. $\|\cdot\|$ and $\|\cdot\|_1$ represent the Euclidean norm and 1-norm (sum of absolute values), respectively. $\|x(t)\|_{\mathrm{E}_2} \triangleq \left(\mathrm{E}\{\int_0^t |x(t)|\,\mathrm{d}t\}\right)^{\frac{1}{2}}$.

2 Problem formulations

2.1 Stochastic plant

Consider the following plant formulated by an Itô stochastic system in the probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\mathrm{d}x(t) = (Ax(t) + Bu(t) + Dw(t))\mathrm{d}t + Ef(x(t), y(t))\mathrm{d}\varpi(t), \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^h$ are the state vector and the measurable output vector, respectively; $f(x(t), y(t)) \in \mathbb{R}^v$ denotes a family of nonlinear function vectors such that

$$f^{\mathrm{T}}(x(t), y(t))f(x(t), y(t)) \leqslant \alpha x^{\mathrm{T}}(t)x(t) + \beta y^{\mathrm{T}}(t)y(t), \tag{3}$$

where α and β are known scalars, and only one of them is allowed to be zero, $u(t) \in \mathbb{R}^m$ and $w(t) \in \mathbb{R}^l$ denote, respectively, a control input to be designed and an exogenous disturbance input belonging to $\mathcal{L}_2 \in [0,+\infty)$; and $\varpi(t) \in \mathbb{R}$ stands for a one-dimensional Brownian motion on $(\Omega, \mathcal{F}, \mathcal{P})$ satisfying $\mathrm{E}\{\mathrm{d}\varpi^2(t)\} = \mathrm{d}t$ and $\mathrm{E}\{\mathrm{d}\varpi(t)\} = 0$. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{h\times n}$, $D \in \mathbb{R}^{n\times l}$, and $E \in \mathbb{R}^{n\times v}$ are known system matrices, where matrix B is assumed to be of full column rank, in this study. The structural diagram of the whole SMC strategy with an imperfect quantization is shown in Figure 1.

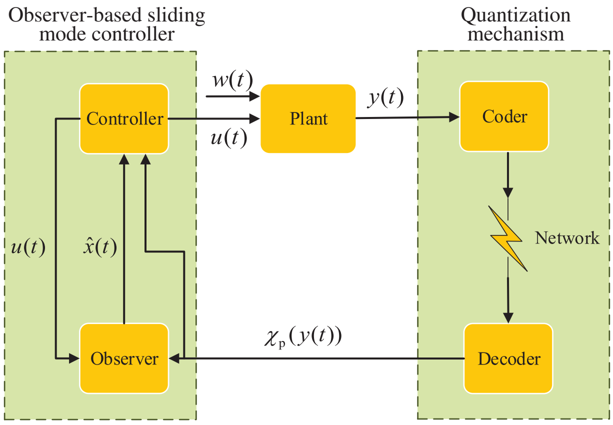

Figure 1 (Color online) Structural diagram of the quantized sliding mode control system.

2.2 Imperfect quantizer

In this study, the measurement $y(t)$ is quantized by the following logarithmic quantizer:

$$\chi_i(y_i(t)) = \begin{cases} \xi_i^{(k)}, & \text{if } \frac{1}{1+\delta_i}\xi_i^{(k)} < y_i(t) \leqslant \frac{1}{1-\delta_i}\xi_i^{(k)},\ y_i(t) > 0, \\ 0, & \text{if } y_i(t) = 0, \\ -\chi_i(-y_i(t)), & \text{otherwise,} \end{cases}$$

where $\delta_i \triangleq (1-\varrho_i)/(1+\varrho_i)$. The set of quantized levels of $\chi_i(y_i(t))$ is described by

$$\Psi_i = \left\{ \pm\, \xi_i^{(k)} \mid \xi_i^{(k)} = \varrho_i^{(k)} \cdot \xi_i^{[0]},\ k \in \{\pm 1, \pm 2, \dots\} \right\} \cup \left\{ \pm\, \xi_i^{[0]} \right\} \cup \left\{ 0 \right\},$$

where $\xi_i^{(k)}$ stands for the quantization value, $\xi_i^{[0]}$ is the initial quantization value, and ϱ_i is the quantizer density for each sub-quantizer $\chi_i(y_i(t))$. The i-th quantization level $\xi_i^{(k)}$ corresponds to a segment with the quantizer $\chi_i(y_i(t))$. Thus, all segments that are disjointed from each other form a partition of the space $\mathbb{R}^h$. By utilization of the sector-bound approaches presented in [15, 33], $\chi(y(t))$ can be written as

$$\chi(y(t)) = (I_h + \Delta(t))y(t),$$

in which the time-varying matrix $\Delta(t) \triangleq \mathrm{diag}\{\hat{\delta}_1(t), \hat{\delta}_2(t), \dots, \hat{\delta}_h(t)\}$, of which the elements belong to $[-\delta, \delta]$, denotes the unknown uncertainty. Considering a random communication packet loss over the network, the final imperfect measurement receives

$$\chi_{\mathrm{p}}(y(t)) = \varepsilon(t)(I + \Delta(t))y(t), \tag{4}$$

where $\varepsilon(t)$ denotes a Bernoulli distributed white sequence such that

$$\begin{aligned} \mathrm{Prob}\{\varepsilon(t) = 1\} &= \mathrm{E}\{\varepsilon(t)\} = \hat{\varepsilon}, \\ \mathrm{Prob}\{\varepsilon(t) = 0\} &= 1 - \mathrm{E}\{\varepsilon(t)\} = 1 - \hat{\varepsilon}, \end{aligned}$$

where $\hat{\varepsilon}$ is the packet loss probability.

2.3 Luenberger observer

Based on the quantized measurement $\chi_{\mathrm{p}}(y(t))$, we adopt the following Luenberger observer:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + BL\big(\chi_{\mathrm{p}}(y(t)) - \hat{\varepsilon}C\hat{x}(t)\big), \tag{5}$$

where $L \in \mathbb{R}^{m\times v}$ is the observer gain to be determined.

The objectives of this study are (a) to synthesize an SMC based on an observer (5) under the presented imperfect quantization mechanism for a plant (1); and (b) to establish the stability criterion of the overall quantized NCS. The following definition and lemma are provided as the basis for the theoretical results in the next section.

Definition 1. The stochastic systems (1) and (2) are said to be stochastically stable if the dynamical system ($u(t) = 0$, $w(t) = 0$) is mean-square asymptotically stable. Furthermore, the unforced stochastic system ($u(t) = 0$, $w(t) \neq 0$) in (1) and (2) is said to be stochastically stable with an $\mathcal{H}_\infty$ performance γ if it is mean-square asymptotically stable and

$$\|y(t)\|_{\mathrm{E}_2} \leqslant \gamma \|w(t)\|_{\mathrm{E}_2},$$

for any $0 \neq w(t) \in \mathcal{L}_2[0, +\infty)$ under zero initial condition.

Lemma 1 ([34])**.** For an n-dimensional Itô process $x(t)$ on $t \geqslant 0$ with

$$\mathrm{d}x(t) = g(t)\mathrm{d}t + h(t)\mathrm{d}\varpi(t),$$

where $g(t) \in \mathbb{R}^n$ and $h(t) \in \mathbb{R}^n$, the stochastic differential of the real-valued Itô process $V(x(t), t) \in \mathbb{C}^{2,1}$ $(\mathbb{R}^n \times \mathbb{R}^+,\ \mathbb{R}^+)$ can be given by

$$\mathrm{d}V(x(t), t) = \mathcal{L}V(x(t), t)\mathrm{d}t + V_x(x(t), t)h(t)\mathrm{d}\varpi(t),$$

where

$$\begin{aligned}
&\mathcal{L}V(x(t), t) \triangleq V_t(x(t), t) + V_x(x(t), t)g(t) + \frac{1}{2}\mathrm{trace}\left(h^{\mathrm{T}}(t)V_{xx}(x(t), t)h(t)\right),\\
&V_t(x(t), t) \triangleq \frac{\partial V(x(t), t)}{\partial t},\\
&V_x(x(t), t) \triangleq \left(\frac{\partial V(x(t), t)}{\partial x_1}, \ldots, \frac{\partial V(x(t), t)}{\partial x_n}\right),\\
&V_{xx}(x(t), t) \triangleq \left(\frac{\partial^2 V(x(t), t)}{\partial x_i x_j}\right)_{n\times n}, \quad i, j = 1, 2, \ldots, n.
\end{aligned}$$

3 SMC design and stability analysis

First, an integral sliding surface is designed using the observer state (5). Then, the SMC synthesis is presented for the resulting NCS. On the basis of the Lyapunov stability theory, the stability for the overall NCS forced by the SMC is analyzed.

3.1 Sliding variable and SMC design

We use the sliding variable

$$s(t) = G\hat{x}(t) - \int_0^t G(A + BK)\hat{x}(v)\mathrm{d}v, \tag{6}$$

where the matrix $G \in \mathbb{R}^{m\times n}$ is chosen such that GB is nonsingular, and $K \in \mathbb{R}^{m\times n}$ is a pre-defined matrix such that matrix $A + BK$ is a Hurwitz matrix. The so-called equivalent control is obtained from $\dot{s}(t) = s(t) = 0$ as follows:

$$u_e(t) = (GB)^{-1}\left[K\hat{x}(t) - L\left(\chi_{\mathrm{p}}(y(t)) - \hat{\varepsilon}C\hat{x}(t)\right)\right]. \tag{7}$$

The sliding motion dynamics can be obtained from (4), (5) and (7)as follows:

$$\begin{aligned}
\dot{\hat{x}}(t) &= A\hat{x}(t) + B(GB)^{-1}\left[K\hat{x}(t) - L\left(\chi_{\mathrm{p}}(y(t)) - \hat{\varepsilon}C\hat{x}(t)\right)\right]\\
&\quad + BL\left[\varepsilon(t)(I + \Delta(t))Cx(t) - \hat{\varepsilon}C\hat{x}(t)\right]\\
&= \left[A + B(GB)^{-1}(K - \hat{\varepsilon}LC)\right]\hat{x}(t) + \varepsilon(t)B\left[I - (GB)^{-1}\right]L(I + \Delta(t))Cx(t).
\end{aligned} \tag{8}$$

Remark 1. Since the sliding motion stability of (8) is relative to the dynamics of observer state $\hat{x}(t)$, we can analyze the stability of the overall closed-loop system, instead of the stability of only the observer (5). The resultant overall closed-loop system is presented in (12), based on which the stability of the overall dynamics system is analyzed in Theorem 2.

3.2 Reachability and stability analysis

This subsection will present the procedure for reachability analysis under the designed controller (9) according to the dynamics of a sliding surface (11), and then give the stability criterion of the overall dynamics system.

Based on a constant-rate reaching law $u_s(t) = -\varphi(GB)^{-1}\text{sgn}(s(t))$, the following SMC is given for stochastic system (1):

$$u(t) = u_e(t) + u_s(t), \tag{9}$$

where φ is a known positive scalar. Then the corresponding reachability analysis can be described using the following theorem.

Theorem 1. Consider sliding surface (6). The SMC synthesized in (9) can force the trajectories of observer (5) onto $s(t) = 0$ in a finite time $T_f(t_0)$ as follows:

$$T_f(t) \leqslant \varphi^{-1} \|s(t_0)\| . \tag{10}$$

Proof. Choose the Lyapunov functional $V_r(t) = 0.5s^{\mathrm{T}}(t)s(t)$. The derivative of sliding variable $s(t)$ is calculated as

$$\begin{aligned} \dot{s}(t) &= GA\hat{x}(t) + GBu(t) + GBL\big(\chi_{\mathrm{p}}(y(t)) - \hat{\varepsilon}C\hat{x}(t)\big) - G(A+BK)\hat{x}(t) \\ &= GB\big[u(t) + L\big(\chi_{\mathrm{p}}(y(t)) - \hat{\varepsilon}C\hat{x}(t)\big) - K\hat{x}(t)\big]. \end{aligned} \tag{11}$$

Thus, from (9) and (11), it follows that

$$\begin{aligned} \dot{V}_r(t) &= s^{\mathrm{T}}(t)\dot{s}(t) \\ &= -s^{\mathrm{T}}(t)GB\Big\{(GB)^{-1}\big[K\hat{x}(t) - L(\chi_{\mathrm{p}}(y(t)) - \hat{\varepsilon}C\hat{x}(t))\big] - \varphi(GB)^{-1}\text{sgn}(s(t)) \\ &\quad + L\big(\chi_{\mathrm{p}}(y(t)) - \hat{\varepsilon}C\hat{x}(t)\big) - K\hat{x}(t)\Big\} \\ &= -\varphi s^{\mathrm{T}}(t)\text{sgn}(s(t)) = -\varphi\|s(t)\|_1 \leqslant -\varphi\|s(t)\| . \end{aligned}$$

It is obvious that convergence of the sliding surface can be guaranteed for all $s(t) \neq 0$. Since

$$\dot{V}_r(t) \leqslant -\varphi\|s(t)\| = -\varphi(2V_r(t))^{\frac{1}{2}}, \quad \forall\, t \geqslant 0,$$

it can be solved as follows:

$$V_r^{1-\frac{1}{2}}(t) \leqslant V_r^{1-\frac{1}{2}}(t_0) - \sqrt{2}\varphi\left(1 - \frac{1}{2}\right)t, \quad 0 \leqslant t \leqslant T_f(t_0).$$

That is to say, when $t \geqslant T_f(t_0)$, $V_r(t) = 0$. Thus, the observer state trajectories $\hat{x}(t)$ can be forced onto the surface $s(t) = 0$ in a finite time $T_f(t) \leqslant \frac{1}{\varphi}\sqrt{2}V_r^{\frac{1}{2}}(t_0) = \frac{1}{\varphi}\|s(t_0)\|$. The proof is completed.

Next, we present the stability analysis of the overall NCS including $x(t)$ and $\hat{x}(t)$ as stated in Remark 1. Considering the dynamics in (1) and (8) and denoting $\zeta(t) \triangleq [\,\tilde{x}^{\mathrm{T}}(t)\ \hat{x}^{\mathrm{T}}(t)\,]^{\mathrm{T}}$ with $\tilde{x}(t) = x(t) - \hat{x}(t)$, the resultant augmented system is formulated as

$$\mathrm{d}\zeta(t) = \Big\{\big[A_a + \varepsilon(t)\big(\bar{A}_a + \tilde{A}_a(t)\big)\big]\zeta(t) + D_a w(t)\Big\}\mathrm{d}t + J_a f(x(t), y(t))\mathrm{d}\varpi(t), \tag{12}$$

where

$$A_a \triangleq \begin{bmatrix} A & B(GB)^{-1}(K - \hat{\varepsilon}LC) \\ 0_{n\times n} & A + B(GB)^{-1}(K - \hat{\varepsilon}LC) \end{bmatrix}, \qquad \tilde{A}_a(t) \triangleq F_a\Delta(t)H_a,$$

$$\bar{A}_a \triangleq \begin{bmatrix} -B(GB)^{-1}LC & -B(GB)^{-1}LC \\ B\left[I-(GB)^{-1}\right]LC & B\left[I-(GB)^{-1}\right]LC \end{bmatrix}, \quad H_a \triangleq \begin{bmatrix} C & C \end{bmatrix},$$

$$F_a \triangleq \begin{bmatrix} -B(GB)^{-1}L \\ B\left[I-(GB)^{-1}\right]L \end{bmatrix}, \quad D_a \triangleq \begin{bmatrix} D \\ 0_{n\times l} \end{bmatrix}, \quad J_a \triangleq \begin{bmatrix} E \\ 0_{n\times v} \end{bmatrix}.$$

The augmented system (12) represents the overall dynamics of NCS in Figure 1. Using Lyapunov stability theory, the following theorem gives a stochastic stability criterion for system (12).

Theorem 2. Consider the SMC in (9) with given scalar $\varphi > 0$, for pre-defined matrices $L \in \mathbb{R}^{m\times h}$ in (5) and $K \in \mathbb{R}^{m\times n}$. The system (12) under the SMC in (9) is stochastically stable in $\mathcal{H}_\infty$ sense if there exist scalars $\sigma > 0$ and $\theta > 0$, and positive definite matrices $Q \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{n\times n}$, satisfying

$$\Gamma < 0, \tag{13}$$

$$E^{\mathrm{T}}QE - \sigma I \leqslant 0, \tag{14}$$

where

$$\Gamma \triangleq \begin{bmatrix} \left[P_a(A_a+\hat{\varepsilon}\bar{A}_a)\right]_s + C_a & P_aD_a & \hat{\varepsilon}P_aF_a & \theta H_a^{\mathrm{T}} \\ * & -\gamma^2 I_l & 0 & 0 \\ * & * & -\theta I & 0 \\ * & * & * & -\theta I \end{bmatrix}, \quad C_a \triangleq \begin{bmatrix} \bar{C} & \bar{C} \\ \bar{C} & \bar{C} \end{bmatrix}, \quad \bar{C} \triangleq \sigma\alpha I_n + (\sigma\beta+1)C^{\mathrm{T}}C.$$

Proof. Select the candidate Lyapunov function

$$V_s(\zeta(t),t) = \zeta^{\mathrm{T}}(t)P_a\zeta(t),$$

with $P_a \triangleq \mathrm{diag}\{Q,R\} > 0$ for system (12). According to Lemma 1, we have

$$\begin{aligned} \mathrm{d}V_s(\zeta(t),t) &= \mathcal{L}V_s(\zeta(t),t)\mathrm{d}t + 2\zeta^{\mathrm{T}}(t)P_aJ_af(x(t),y(t))\mathrm{d}\varpi(t), \\ \mathrm{E}\big\{V_s(\zeta(t),t)\big\} &= \mathrm{E}\left\{\int_0^t \mathcal{L}V_s(\zeta(v),v)\mathrm{d}v\right\} + V_s(\zeta(0),0), \end{aligned}$$

with

$$\mathcal{L}V_s(\zeta(t),t) = 2\zeta^{\mathrm{T}}(t)P_a\Big\{\big[A_a+\varepsilon(t)(\bar{A}_a+\tilde{A}_a(t))\big]\zeta(t)+D_aw(t)\Big\} + f^{\mathrm{T}}(x(t),y(t))J_a^{\mathrm{T}}P_aJ_af(x(t),y(t)).$$

Recalling condition (3) and (14), we have

$$\begin{aligned} f^{\mathrm{T}}(x(t),y(t))J_a^{\mathrm{T}}P_aJ_af(x(t),y(t)) &= f^{\mathrm{T}}(x(t),y(t))E^{\mathrm{T}}QEf(x(t),y(t)) \\ &\leqslant \sigma\big(\alpha x^{\mathrm{T}}(t)x(t) + \beta y^{\mathrm{T}}(t)y(t)\big) \\ &= \sigma\big[\alpha(\tilde{x}(t)+\hat{x}(t))\big]^{\mathrm{T}}\big[\alpha(\tilde{x}(t)+\hat{x}(t))\big] + \beta\big[\alpha(\tilde{x}(t)+\hat{x}(t))\big]^{\mathrm{T}} \\ &\quad \times C^{\mathrm{T}}C\big[\alpha(\tilde{x}(t)+\hat{x}(t))\big]. \end{aligned} \tag{15}$$

Then we take the expectation that

$$\begin{aligned} &\mathrm{E}\Big\{\int_0^t \Big(\mathrm{E}\big\{\mathcal{L}V_s(\zeta(v),v)\mid\zeta(v)\big\} + y^{\mathrm{T}}(v)y(v) - \gamma^2w^{\mathrm{T}}(v)w(v)\Big)\mathrm{d}v\Big\} \\ &\leqslant \mathrm{E}\Big\{\int_0^t \Big\{2\zeta^{\mathrm{T}}(v)P_a\big\{\big[A_a+\hat{\varepsilon}(\bar{A}_a+\tilde{A}_a(v))\big]\zeta(v)+D_aw(v)\big\} \\ &\quad +\sigma(\alpha x^{\mathrm{T}}(v)x(v)+\beta y^{\mathrm{T}}(v)y(v)) + y^{\mathrm{T}}(v)y(v) - \gamma^2w^{\mathrm{T}}(v)w(v)\Big\}\mathrm{d}v\Big\} \\ &= \mathrm{E}\Big\{\int_0^t \big[\,\zeta^{\mathrm{T}}(v)\ w^{\mathrm{T}}(v)\,\big]\tilde{\Gamma}\big[\,\zeta^{\mathrm{T}}(v)\ w^{\mathrm{T}}(v)\,\big]^{\mathrm{T}}\mathrm{d}v\Big\}, \end{aligned}$$

where

$$\tilde{\Gamma} \triangleq \begin{bmatrix} \left[P_a[A_a + \hat{\varepsilon}(\bar{A}_a + \tilde{A}_a(v))]\right]_s + C_a & P_a D_a \\ * & -\gamma^2 I_l \end{bmatrix}.$$

Applying S-procedure [35, 36] on condition $\Gamma < 0$ with $\Delta^{\mathrm{T}}(t)\Delta(t) < I$, one can obtain

$$\tilde{\Gamma} = \bar{\Gamma} + [\,\hat{\varepsilon}(P_a F_a)^{\mathrm{T}} \;\; 0\,]^{\mathrm{T}} \Delta(t) [\, H_a \;\; 0\,] + [\, H_a \;\; 0\,]^{\mathrm{T}} \Delta^{\mathrm{T}}(t) [\,\hat{\varepsilon}(P_a F_a)^{\mathrm{T}} \;\; 0\,] < 0,$$

where

$$\bar{\Gamma} \triangleq \begin{bmatrix} \left[P_a(A_a + \hat{\varepsilon}\bar{A}_a)\right]_s + C_a & P_a D_a \\ * & -\gamma^2 I_l \end{bmatrix}.$$

Then it yields

$$\mathrm{E}\{\mathcal{L}V_s(\zeta(t), t) + y^{\mathrm{T}}(t)y(t) - \gamma^2 w^{\mathrm{T}}(t)w(t)\} < 0.$$

Further, by utilization of the $\mathcal{H}_\infty$ disturbance attenuation approach to handling the uncertainty $w(t)$ in output $y(t)$, the following inequality under zero initial condition can be obtained by integrating from 0 to $+\infty$:

$$\begin{aligned} &\mathrm{E}\left\{\int_0^{+\infty} y^{\mathrm{T}}(t)y(t)\mathrm{d}t - \int_0^{+\infty} \gamma^2 w^{\mathrm{T}}(t)w(t)\mathrm{d}t\right\} \\ &= \mathrm{E}\left\{\int_0^{+\infty} \Big(\mathrm{E}\left\{\mathcal{L}V_s(\zeta(v), v) \mid \zeta(v)\right\} + y^{\mathrm{T}}(v)y(v) - \gamma^2 w^{\mathrm{T}}(v)w(v)\Big)\mathrm{d}v\right\} - \mathrm{E}\left\{\int_0^{t} \mathcal{L}V_s(\zeta(v), v)\mathrm{d}v\right\} \\ &\leqslant \mathrm{E}\left\{\int_0^{+\infty} \Big(\mathrm{E}\left\{\mathcal{L}V_s(\zeta(v), v) \mid \zeta(v)\right\} + y^{\mathrm{T}}(v)y(v) - \gamma^2 w^{\mathrm{T}}(v)w(v)\Big)\mathrm{d}v\right\} \\ &< \mathrm{E}\big\{V_s(\zeta(0), 0) - V_s(\zeta(\infty), \infty)\big\} \leqslant 0. \end{aligned}$$

Furthermore, under the disturbance-free case, one can easily obtain $\mathrm{E}\left\{\mathcal{L}V_s(\zeta(t), t)\right\} < 0$ from the conditions (13) and (14) by setting $w(t) = 0$ for system (12). Consequently, according to Definition 1, the stochastic stability in an $\mathcal{H}_\infty$ sense can be guaranteed for system (12) under the SMC (9). The proof is completed.

Remark 2. Based on the above results, we know that the pre-defined parametric matrices G, K, and L affect the feasibility of the conditions in Theorem 2. Therefore, choosing proper values is important for the feasible solution of the conditions. Future work will deal with the discussion on the decoupling problem of these matrices with parametric matrices Q and R.

4 Simulation

This section describes an air vehicle system [37] with stochastic process simulated to illustrate the effectiveness of the proposed controller. The modified dynamics equations of the air vehicle system are as follows:

$$\begin{cases} \mathrm{d}x_1(t) = x_2(t)\mathrm{d}t, \\ \mathrm{d}x_2(t) = \vartheta(-x_2(t) + u)\mathrm{d}t, \\ \mathrm{d}x_3(t) = \vartheta(x_1(t) - x_3(t))\mathrm{d}t + w(t)\mathrm{d}t + 0.1x_1(t)\sin(0.2y(t))\mathrm{d}\varpi(t), \\ \mathrm{d}x_4(t) = 5(x_2(t) - x_4(t))\mathrm{d}t + w(t)\mathrm{d}t, \\ y(t) = x_3(t) + x_4(t), \end{cases}$$

where $x_1(t)$ (m) denotes the range, $x_2(t)$ (m/s) is the range rate, and $x_3(t)$ (m/s) and $x_4(t)$ (m/s) are their low-pass filtered versions, respectively. $x(t) = \left[\, x_1(t) \;\, x_2(t) \;\, x_3(t) \;\, x_4(t) \,\right]^{\mathrm{T}}$. ϑ, assumed to be 5, denotes the bandwidth of the velocity control loop. We assume that the $x_3(t)$ of the target range is corrupted with $0.1x_1(t)\sin(0.2y(t))\mathrm{d}\varpi(t)$. Accordingly, the aim in simulation is to implement the regulation of output

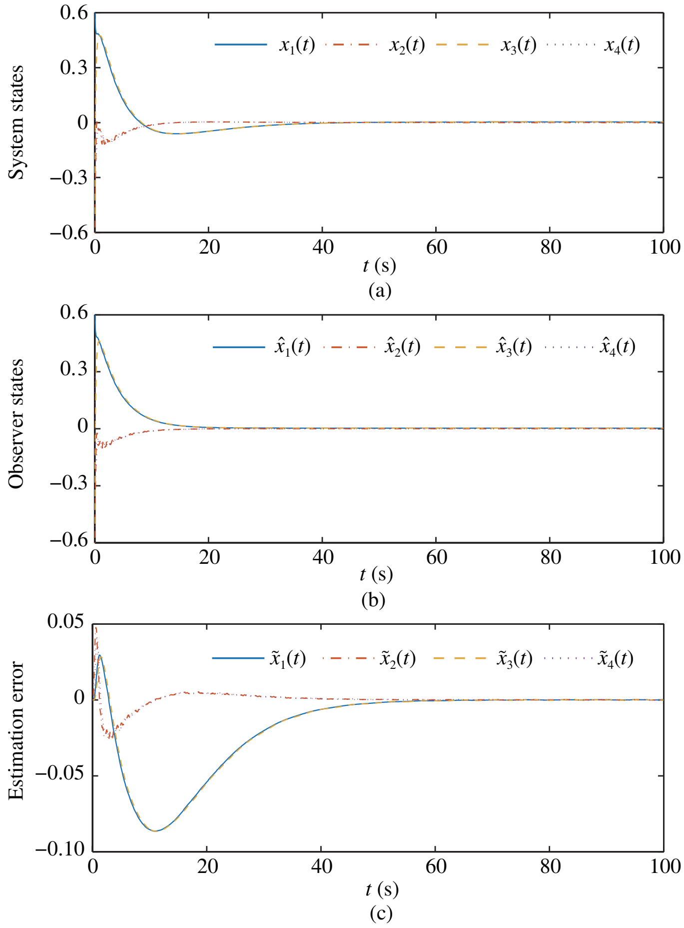

Figure 2 (Color online) Trajectories of (a) the plant states $x_i(t)$, (b) the observer states $\hat{x}_i(t)$, and (c) their error $\tilde{x}_i(t)$ $(i = 1, 2, 3, 4)$.

$y(t)$ utilizing only $x_3(t)$ and $x_4(t)$ despite the presence of various sources of disturbances, including $w(t)$ and $\varpi(t)$. Thus, the system parameters in the form system (1) are as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 5 & 0 & -5 & 0 \\ 0 & 5 & 0 & -5 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \ D = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \ C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}^{\mathrm{T}},$$

$$E = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix}, \ F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.02 \end{bmatrix}, \ H = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.01 \end{bmatrix}^{\mathrm{T}}.$$

In the simulation procedure, we set exogenous disturbances $w(t) = 0.01\sin(t)$, and $\varpi(t)$ with $f(x(t), y(t))$ characterizes a zero-mean Brownian motion with a nonlinear variation vector

$$f(x(t), y(t)) = [\, 0 \ \ 0 \ \ 0.1x_1(t)\sin(0.2y(t)) \ \ 0\,]^{\mathrm{T}},$$

during the actual operation. According to the presented structure of the overall control system in (1), we set the initial quantizer point $\xi^{[0]} = 40$, and the quantizer density $\varrho = 0.6667$, for the logarithmic quantizer. Thus, we have $\delta = 0.2$. Furthermore, we assume an imperfect quantization process with a

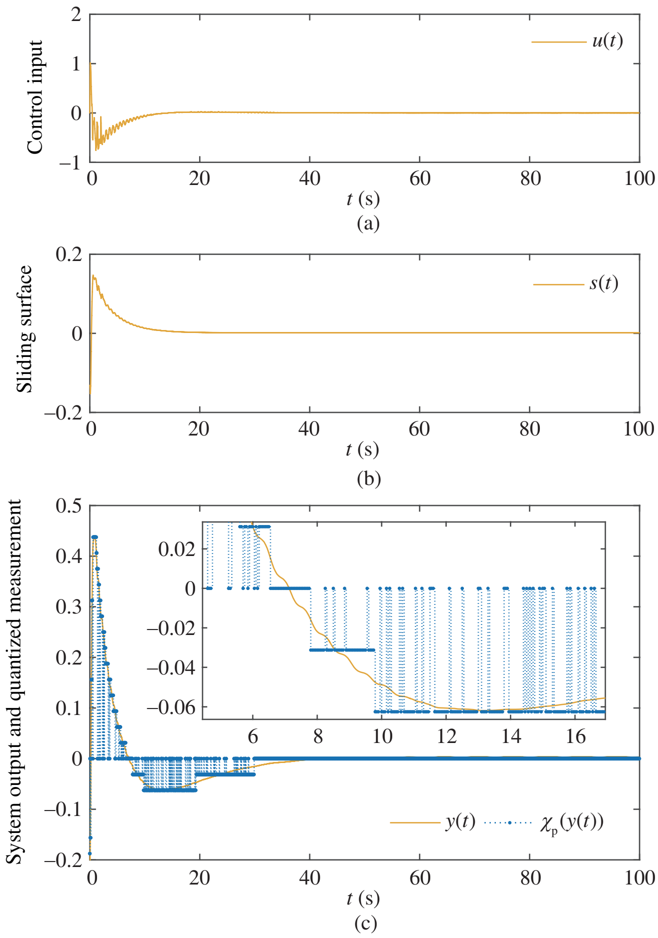

Figure 3 (Color online) Trajectories of (a) the control law $u(t)$, (b) sliding surface $s(t)$, (c) output $y(t)$, and imperfectly quantized measurement $\chi_{\rm p}(y(t))$.

random communication packet loss rate $\hat{\varepsilon} = 0.2$. Considering the feasible solution of the conditions in Theorem 2, we give the observer gain $L = 1$ and

$$G = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix}, \quad K = \begin{bmatrix} -3 & -2 & -4 & -5 \end{bmatrix},$$

which results in a minimum index $\gamma_{\min} = 0.6209$ by solving the minimization optimization problem:

$$\min \quad \varrho \quad \text{s.t. Eqs. (13) and (14) with } \varrho = \gamma^2.$$

For the SMC (9), we set $\varphi = 0.001$, under the zero initial condition with exogenous disturbance and Brownian motion. The responses of the NCS are depicted in Figures 2 and 3.

Figure 2(a) depicts the state trajectories x_i of the air vehicle system. Figure 2(b) depicts the observer state trajectories $\hat{x}_i$. Figure 2(c) depicts the error $\tilde{x}_i = x_i - \hat{x}_i$. Figure 3(a) gives the SMC input $u(t)$. Figure 3(b) shows the trajectory of the designed integral sliding surface $s(t)$. Figure 3(c) provides the output measurement $y(t)$ and its imperfectly quantized measurement $\chi_{\rm p}(y(t))$ under random communication packet loss. Obviously, the overall NCS was effectively controlled and the output was well-regulated against various sources of disturbances and imperfect quantization.

5 Conclusion

A solution has been provided for the issues of integral SMC and Luenberger observer design of stochastic systems under imperfect quantization mechanism. The quantization process considered involved a random communication packet loss. Based on the imperfect quantized measurement, a state observer has been designed. A sliding surface is designed using the observer state. The integral SMC is synthesized to guarantee the finite-time reachability of the designed sliding surface. The $\mathcal{H}_\infty$ stochastic stability criterion is established for the overall closed-loop system based on Lyapunov stability theory. Furthermore, the simulation example provided illustrates the effectiveness of the proposed control method.

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Conflict of interest The authors declare that they have no conflict of interest.

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