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Robust neural output-feedback stabilization for stochastic nonlinear process with time-varying delay and unknown dead zone

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Abstract This article investigates the output-feedback control of a class of stochastic nonlinear system with time-varying delay and unknown dead zone. A robust neural stabilizing algorithm is proposed by using the circle criterion, the NNs approximation and the MLP (minimum learning parameter) technique. In the scheme, the nonlinear observer is first designed to estimate the unmeasurable states and the assumption "linear growth" of the nonlinear function is released. Furthermore, the uncertainty of the whole system (including the perturbation of time-varying delay) is lumped and compensated by employing one RBF NNs (radial basis function neural networks). Though, only two weight-norm related parameters are required to be updated online for the merit of the MLP technique. And the gain-inversion related adaptive law is targetly designed to mitigate the adverse effect of unknown dead zone. Comparing with the previous work, the proposed algorithm obtains the advantage: a concise form and easy to implementation due to its less computational burden. The theoretical analysis and comparison example demonstrate the substantial effectiveness of the proposed scheme.

Keywords stochastic system, robust neural control, time delay, dead zone, output-feedback

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1 Introduction

Stochastic nonlinear process is frequently encountered in the practical industry. And most industrial plants, involving material (or energy) transportation and mechanical actuated servo, are characterized by time delay and unknown dead zone [1]. Stabilization of the stochastic nonlinear system with time delay and unknown dead zone has attracted considerable attention, due to its theoretical difficulties and important applications, e.g., mechanical servo motor, hearth negative pressure system, and the fresh water cooling system on ships [2].

In the recent years, many stochastic nonlinear control schemes have been reported in the existed references, e.g., adaptive fuzzy control [3,4] and robust control [5,6]. In [5], the Backstepping scheme is applied to address the stabilization of stochastic nonlinear plant, which is meaningful and significant in the early

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work. Furthermore, the related stabilizing results have been extended to the output-feedback control of stochastic nonlinear system [7, 8], the nonlinear large-scale system [9, 10]. In [11], to tackle the system uncertainty, an adaptive output-feedback algorithm is developed by combining the Backstepping method and the neural networks (NNs) approximation, which is an efficient way in the control of uncertain nonlinear systems [12]. However, as described in [13, 14], the basis of the approximation-based algorithms is the Backstepping scheme and the essential constraints are unavoidable. In the conventional Backstepping method, the repeated differentiations of virtual control are impossible to be implemented in practice, and that may lead to a complicated algorithm with heavy computational burden for the high-order nonlinear system [15]. That is the first constraint "explosion of complexity". The second constraint is "curse of dimensionality", which is inherent in increasing the node number of NNs to guarantee the approximation quality, especially for the high-order uncertain nonlinear plant. Then, the adequate weights need to be online trained in the algorithm [16, 17], and the updating time tends to be unacceptably long in the engineering application. Subsequently, a series of interesting results has been done to relax these constraints, e.g., the dynamic surface control (DSC) [18] and the minimum learning parameter (MLP) technique [19–21]. In [21, 22], the MLP technique is applied to derive an adaptive NN control for the stochastic nonlinear systems with unknown control direction and time delay, respectively. In the scheme, the online learning parameters are the compressed NNs variables, instead of the weights themselves. That could relax the above constraints and decrease the online training time dramatically. Moreover, the DSC and MLP techniques are also with the effective application in the periodically time-varying disturbed nonlinear system [23] and the marine control process [20].

In many industrial processes, e.g., valves, and mechanical servo motor, dead zone and unknown time delays are common factors that degrade the static and dynamic system performances. That may result in the undesirable inaccuracy, even instability. As for the time delay system, some valuable studies were done by virtue of the Lyapunov-Krasovaskii functionals [21, 22]. In [22], the nonlinear time delay system was with more general form, including the delay terms in both the drift fields and the diffusion terms. And the proposed adaptive neural controller contained fewer adaptive parameters for the merit of MLP technique. Chen et al. [24] further dealt with the time-varying delay problem for stochastic nonlinear system. And only one NNs was employed to compensate for the perturbed terms depending on the delayed output. In [25], a combined homogeneous and sign function approach is proposed to deal with the stochastic nonlinear systems with time-varying delay, and the global asymptotical stability is obtained in probability. Further with more practical case, Xue et al. [26] investigated the tracking problem of stochastic delayed plant by fusing of Lyapunov-Krasovaskii functional method and power integrator technique, where the power order relaxed to a certain interval rather than a precisely known point. Although, the aforementioned schemes may be failed if the practical plant is with unknown dead zone input. For the purpose, Li et al. [27] developed a neural network control algorithm for the stochastic nonlinear system with unknown dead zone, which required no prior knowledge on bounds of dead zone parameters. That is only for the state feedback case. In [28], one investigated an adaptive fuzzy output-feedback tracking control for the multi-input and multi-output stochastic nonlinear systems, where the adverse effects of unknown dead-zone, unknown control direction could be eliminated by the fuzzy logic approximator and a linear state transformation.

Motivated by the above observations, this article concerns on the practical output-feedback control of stochastic nonlinear process with time-varying delay and unknown dead zone. A novel robust neural output-feedback scheme is developed by employing the NNs approximation, the MLP technique and the circle criterion. For merits of the robust neural terms, both the stabilizing performance and the mean-square semi-globally uniformly ultimately bounded stability (MSUUB in probability) are guaranteed for the stochastic closed-loop system. The main contributions of this article can be summarized as follows.

(1) By virtue of the nonlinear observer design, the uncertainty of the whole system is lumped, including the perturbation of time-varying delay. Different from the existed work, only one NNs is introduced to approximate the lumped uncertainties and the adaptive variables are compressed as two weight-norm-related parameters due to the merit of the MLP algorithm. Thus, the control law requires much less knowledge of the process and is easy to be implemented in practice due to its lower computational burden.

(2) As to the dead zone nonlinearity in the output-feedback case, the negative effect is dealt with by the gain-inversion related adaptive law designed by the robust damping technique, without constructing the dead zone inverse or requiring the bounds information of dead zone slopes. By using the algorithm, the possible control singularity problem could be avoided.

2 Problem formulation and preliminaries

Thought this paper, $|\cdot|$ is the absolute operator os a scalar, $\|\cdot\|$ denotes the Euclidean norm of a vector and $\|\cdot\|_{\mathrm{F}}$ is the Frobenius norm. $\mathrm{Tr}\left\{\cdot\right\}$ indicates the trace of a square matrix. For a matrix $\boldsymbol{A}=[a_{i,j}]\in\mathbb{R}^{m\times n}$, $\|\boldsymbol{A}\|_{\mathrm{F}}^2=\sum_{i=1}^m\sum_{j=1}^n a_{i,j}^2=\mathrm{Tr}\left\{\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\right\}$. $\lambda_{\min}\left(\cdot\right)$ denotes the minimal eigenvalue of the symmetric matrix. $\hat{(\cdot)}$ is the estimate of the variable $(\cdot)$, and the estimation error $\tilde{(\cdot)}=(\cdot)-\hat{(\cdot)}$.

2.1 Problem formulation

Consider the following stochastic nonlinear process with time-varying delay and unknown dead zone:

$$\begin{cases} \mathrm{d}x_i=\left[\sum_{j=1}^{i+1}a_{i,j}x_j+f_i\left(\bar{\boldsymbol{x}}_i\right)+h_i\left(y,y\left(t-\tau(t)\right)\right)\right]\mathrm{d}t+g_i\left(y,y\left(t-\tau(t)\right)\right)\mathrm{d}\omega, \\ \qquad i=1,2,\ldots,n-1, \\ \mathrm{d}x_n=\left[u+\sum_{j=1}^{n}a_{i,j}x_j+f_n\left(\boldsymbol{x}\right)+h_n\left(y,y\left(t-\tau(t)\right)\right)\right]\mathrm{d}t+g_n\left(y,y\left(t-\tau(t)\right)\right)\mathrm{d}\omega, \\ y=x_1, \end{cases} \tag{1}$$

with

$$u=D\left(v\right)=\begin{cases} \ell_l(v), & v\leqslant b_l, \\ 0, & b_l<v<b_r, \\ \ell_r(v), & v\geqslant b_r, \end{cases} \tag{2}$$

where $\boldsymbol{x}=\left[x_1,\ldots,x_n\right]^{\mathrm{T}}\in\mathbb{R}^n$, $y(t)\in\mathbb{R}$ are the system state vector and system output, respectively. $\bar{\boldsymbol{x}}_i=\left[x_1,\ldots,x_i\right]^{\mathrm{T}}\in\mathbb{R}^i$. $a_{i,j}$ are known constants, and $\boldsymbol{\omega}$ is an r-dimensional standard Brownian motion defined on the complete probability space $\{\Omega,F,P\}$. In addition, $f_i\left(\cdot\right):\mathbb{R}^i\to\mathbb{R}$ denotes the known drifting nonlinearity with $f_i\left(0\right)=0$, which would increase the difficulty of the observer design. Both $h_i\left(\cdot\right):\mathbb{R}^2\to\mathbb{R}$ and $g_i\left(\cdot\right):\mathbb{R}^2\to\mathbb{R}^r$ are all unknown locally Lipschitz time-delay functions with $h_i\left(y,0\right)=0, g_i\left(y,0\right)=0$. $\tau(t)\in[0,\bar{\tau}]$ is the unknown time-varying delay, satisfying $\dot{\tau}(t)\leqslant d<1$, where d is an unknown constant only for the analysis.

Eq. (2) gives the unknown dead zone with $u\in\mathbb{R}$ being the output and $v\in\mathbb{R}$ being the input. $b_l<0, b_r>0$ are the unknown bounds parameter.

Remark 1. In this article, one considers that only the output variable $y(t)$ of stochastic nonlinear process (1) is measurable. Actually, the time delay effect is inevitable in the process of signal measuring [24]. That is the actual source of the time-varying delay in the plant (1). Thus, it is reasonable and in accordance with the engineering practice that the time delay functions $h_i(\cdot,\cdot), g_i(\cdot,\cdot)$ contain the system output and its delayed variable.

Assumption 1. Around the dead zone model (2), the functions $\ell_l(v),\ell_r(v)$ are smooth, and their virtual gradient gains are confined within a certain range such that

$$\begin{aligned} &0<\ell_{l0}\leqslant\ell_l^{'}(v)\leqslant\bar{\ell}_l, \quad \forall v\in(-\infty,b_l], \\ &0<\ell_{r0}\leqslant\ell_r^{'}(v)\leqslant\bar{\ell}_r, \quad \forall v\in[b_r,+\infty), \end{aligned} \tag{3}$$

where the lower and upper bounded parameters $\ell_{l0},\ell_{r0},\bar{\ell}_l,\bar{\ell}_r$ are unknown positive constants, and $\ell_l^{'}(v)=\frac{\mathrm{d}\ell_l(z)}{\mathrm{d}z}|_{z=v}>0,\ell_r^{'}(v)=\frac{\mathrm{d}\ell_r(z)}{\mathrm{d}z}|_{z=v}>0$. Furthermore, there exist unknown positive constants $\ell_0\leqslant\min\{\ell_{l0},\ell_{r0}\},\bar{\ell}\geqslant\max\{\bar{\ell}_l,\bar{\ell}_r\}$. It should be mentioned that the above bounds parameters are only required for analytical purposes, which are not necessarily known in the proposed algorithm.

Remark 2. Combining the mean value theorem and the separation principle [27, 28], one can rewritten the dead zone as (4) to facilitate the control design:

$$u = D\left(v\right) = \mathbf{\Theta}^{\mathrm{T}}(t)\mathbf{\Phi}(t)v + d(v), \tag{4}$$

where $\mathbf{\Theta}(t) = [m_r(v), m_l(v)]^{\mathrm{T}}, \mathbf{\Phi}\left(t\right) = [\phi_r(t), \phi_l(t)]^{\mathrm{T}}$,

$$m_r(v) = \begin{cases} 0, & v \leqslant b_l, \\ \ell_r^{'}(\varsigma_r(v)), & b_l < v < +\infty, \end{cases} \quad m_l(v) = \begin{cases} \ell_l^{'}(\varsigma_l(v)), & -\infty < v < b_r, \\ 0, & v \geqslant b_r, \end{cases}$$

$$\phi_r(t) = \begin{cases} 1, & v > b_l, \\ 0, & v \leqslant b_l, \end{cases} \quad \phi_l(t) = \begin{cases} 1, & v < b_r, \\ 0, & v \geqslant b_r, \end{cases} \quad d(v) = \begin{cases} -\ell_r^{'}\left(\varsigma_r(v)\right) b_r, & v \geqslant b_r, \\ -\left[\ell_l^{'}\left(\varsigma_l(v)\right) + \ell_r^{'}\left(\varsigma_r(v)\right)\right] v, & b_l < v < b_r, \\ -\ell_l^{'}\left(\varsigma_l(v)\right) b_l, & v \leqslant b_l, \end{cases}$$

$\varsigma_l \in (v, b_l)$, if $v < b_l$; $\varsigma_l \in (b_l, v)$, if $b_l \leqslant v < b_r$; $\varsigma_r \in (b_r, v)$, if $b_r < v$; $\varsigma_r \in (v, b_r)$, if $b_l \leqslant v < b_r$. It is obvious that $|d(v)| \leqslant \delta^*, \ell_0 \leqslant \mathbf{\Theta}^{\mathrm{T}}(t)\mathbf{\Phi}(t) \leqslant 2\bar{\ell}$, and $\delta^* = \left(\bar{\ell}_l + \bar{\ell}_r\right)$.

In following, one reformulates the stochastic process (1) into the matrix form to facilitate the observer design:

$$\begin{cases} \mathrm{d}\boldsymbol{x} = \left[\boldsymbol{A}\boldsymbol{x} + \boldsymbol{F}\left(\boldsymbol{x}\right) + \boldsymbol{H}\left(y, y\left(t - \tau(t)\right)\right) + \boldsymbol{B}u\right]\mathrm{d}t + \boldsymbol{G}\left(y, y\left(t - \tau(t)\right)\right)\mathrm{d}\boldsymbol{\omega}, \\ y = \boldsymbol{C}\boldsymbol{x}, \end{cases} \tag{5}$$

where $\boldsymbol{C} = [1, 0, \ldots, 0] \in \mathbb{R}^n, \boldsymbol{B} = [0, 0, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^n$ and

$$\boldsymbol{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & 0 & \cdots & 0 \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & a_{n,3} & \cdots & a_{n,n} \end{bmatrix}, \quad \boldsymbol{F}\left(\cdot\right) = \begin{bmatrix} f_1\left(\bar{x}_1\right) \\ f_2\left(\bar{x}_2\right) \\ \vdots \\ f_n\left(\bar{x}_n\right) \end{bmatrix}, \quad \boldsymbol{H}\left(\cdot, \cdot\right) = \begin{bmatrix} h_1\left(\cdot, \cdot\right) \\ h_2\left(\cdot, \cdot\right) \\ \vdots \\ h_n\left(\cdot, \cdot\right) \end{bmatrix}, \quad \boldsymbol{G}\left(\cdot, \cdot\right) = \begin{bmatrix} g_1\left(\cdot, \cdot\right) \\ g_2\left(\cdot, \cdot\right) \\ \vdots \\ g_n\left(\cdot, \cdot\right) \end{bmatrix}.$$

Furthermore, assumption (2) is useful for the developed algorithm.

Assumption 2. Around the known nonlinear function $\boldsymbol{F}\left(\boldsymbol{x}\right)$, it can be decomposed as $\boldsymbol{F}\left(\cdot\right) = \boldsymbol{C}_F \cdot \boldsymbol{J}\left(\boldsymbol{x}\right)$, where $\boldsymbol{C}_F$ is a constant matrix and $\boldsymbol{J}\left(\cdot\right)$ is a known vector function. Then, the relationship (6) holds [24]

$$\frac{\partial \boldsymbol{J}(\cdot)}{\partial \boldsymbol{x}} + \left(\frac{\partial \boldsymbol{J}(\cdot)}{\partial \boldsymbol{x}}\right)^{\mathrm{T}} \geqslant 0, \quad \forall \boldsymbol{x} \in \mathbb{R}^n. \tag{6}$$

The objective of this article is to develop a practical robust neural output-feedback controller for v such that the output y is stabilized to the origin in presence of stochastic disturbances, and all states in the closed-loop system are bounded in probability.

2.2 Some lemmas and preliminaries

In this article, one employs the radial basis function (RBF) NNs in the proposed control scheme to deal with the model uncertainty. To design the robust neural damping terms and analyse the stability of the stochastic nonlinear system, one briefly describes the stochastic stability theory and NNs-based function approximation as follows.

Lemma 1 ([27, 29])**.** For the stochastic system $\mathrm{d}\boldsymbol{x} = \boldsymbol{f}\left(\boldsymbol{x}\right)\mathrm{d}t + \boldsymbol{g}\left(\boldsymbol{x}\right)\mathrm{d}\boldsymbol{\omega}, \boldsymbol{f}\left(\mathbf{0}\right) = \mathbf{0}, \boldsymbol{g}\left(\mathbf{0}\right) = \mathbf{0}$, where $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector and $\boldsymbol{\omega}$ describes the standard Brownian motion. $\boldsymbol{f} : \mathbb{U} \to \mathbb{R}^n, \boldsymbol{g} : \mathbb{U} \to \mathbb{R}^r$ are continuous functions on the open sets $\mathbb{U} \subset \mathbb{R}^n$ and $\boldsymbol{x} = \mathbf{0} \in \mathbb{U}$. If there exists a positive definite continuous function $V\left(\boldsymbol{x}\right) : \mathbb{U} \to \mathbb{R}$, its infinitesimal gradient, defined as (7), satisfies the relationship (8) with $c_0 > 0$ and a small positive value ϱ. Then, there is a unique solution and all variables in the closed-loop system are mean-square uniformly ultimately bounded in probability $1 - p$, $0 < p < 1$.

$$\mathcal{L}V\left(\boldsymbol{x}\right) = \frac{\partial V}{\partial \boldsymbol{x}}\boldsymbol{f}\left(\boldsymbol{x}\right) + \frac{1}{2}\mathrm{Tr}\left\{\boldsymbol{g}\left(\boldsymbol{x}\right)^{\mathrm{T}}\frac{\partial^2 V}{\partial \boldsymbol{x}^2}\boldsymbol{g}\left(\boldsymbol{x}\right)\right\}, \tag{7}$$

$$\mathcal{L}V(\boldsymbol{x}) + c_0 V(\boldsymbol{x}) \leqslant \varrho. \tag{8}$$

Lemma 2 ([20,30])**.** For any given real continuous function $h(\boldsymbol{x})$ with $h(\boldsymbol{0}) = 0$ defined on a compact set $\mathbb{U} \subset \mathbb{R}^m$, one applies the continuous function separation technique and RBF NNs approximation theorem, then $h(\boldsymbol{x})$ can be remodelled as

$$h(\boldsymbol{x}) = \boldsymbol{S}(\boldsymbol{x}) \boldsymbol{W} \boldsymbol{x} + \sigma(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in \mathbb{U}, \tag{9}$$

where $\boldsymbol{S}(\boldsymbol{x}) = [s_1(\boldsymbol{x}), s_2(\boldsymbol{x}), \ldots, s_l(\boldsymbol{x})]$ is a vector of RBF basis functions with the Gaussian form (10), $\boldsymbol{\mu}_j = [\mu_{j1}, \mu_{j2}, \ldots, \mu_{jm}]^{\mathrm{T}}$ is the centre of the receptive fields and ς_j is the width of the Gaussian function. $\boldsymbol{W} \in \mathbb{R}^{l \times m}$ is the optimal weight matrix, l is the node number of NNs and m is the dimension number of the state vector $\boldsymbol{x}$. $\sigma(\boldsymbol{x})$ is the NNs inherent approximaiton error. In addition, there exists a known positive function $\psi(\boldsymbol{x})$ and an unknown positive constant θ such that $|\sigma(\cdot)| \leqslant \theta\psi(\cdot)$.

$$s_j(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi}\varsigma_j} \exp\left(-\frac{(\boldsymbol{x} - \boldsymbol{\mu}_j)^{\mathrm{T}} (\boldsymbol{x} - \boldsymbol{\mu}_j)}{2\varsigma_j^2} \right), \quad j = 1, 2, \ldots, l. \tag{10}$$

3 Robust neural output-feedback algorithm and stability analysis

In this section, the robust neural output-feedback controller is developed for stochastic nonlinear processes (1) and (2) with the time-varying delay and unknown dead zone, by employing the DSC and MLP techniques. First, the nonlinear observer (11) is incorporated by using the circle criterion [24], which is independent of the time delay terms,

$$\mathrm{d}\hat{\boldsymbol{x}} = [\boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{L}(\boldsymbol{C}\hat{\boldsymbol{x}} - y) + \boldsymbol{F}(\hat{\boldsymbol{x}} + \boldsymbol{K}(\boldsymbol{C}\hat{\boldsymbol{x}} - y)) + \boldsymbol{B}u]\,\mathrm{d}t, \tag{11}$$

where $\boldsymbol{K} \in \mathbb{R}^n, \boldsymbol{L} \in \mathbb{R}^n$ are the design parameters, satisfying (12). In addition, $\boldsymbol{P}, \boldsymbol{Q}$ are all positive-definite square matrix, $\boldsymbol{I}$ is the unit matrix, which are to derive the design parameters.

$$\begin{aligned} &(\boldsymbol{A} + \boldsymbol{LC})^{\mathrm{T}} \boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A} + \boldsymbol{LC}) \leqslant -\boldsymbol{Q}, \\ &\boldsymbol{C}_F^{\mathrm{T}} \boldsymbol{P} + (\boldsymbol{I} + \boldsymbol{KC}) = 0. \end{aligned} \tag{12}$$

Define the observer error $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ and one can obtain the error dynamics as

$$\mathrm{d}\tilde{\boldsymbol{x}} = [(\boldsymbol{A} + \boldsymbol{LC})\tilde{\boldsymbol{x}} + \boldsymbol{F}(\boldsymbol{x}) - \boldsymbol{F}(\hat{\boldsymbol{x}} - \boldsymbol{K}(\boldsymbol{C}\hat{\boldsymbol{x}} - y)) + \boldsymbol{H}(\cdot, \cdot)]\,\mathrm{d}t + \boldsymbol{G}(\cdot, \cdot)\,\mathrm{d}\boldsymbol{\omega}, \tag{13}$$

Consider the Lypunov function candidate $V_0 = \frac{1}{2}\left(\tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{x}}\right)^2$, one takes its infinitesimal gradient following (7). Combining with the (13), it is obtained

$$\begin{aligned} \mathcal{L}V_0 = {}& \left(\tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{x}}\right) \tilde{\boldsymbol{x}}^{\mathrm{T}} \left[(\boldsymbol{A} + \boldsymbol{LC})^{\mathrm{T}} \boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A} + \boldsymbol{LC})\right] \tilde{\boldsymbol{x}} + 2\left(\tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{x}}\right) \tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \left(\boldsymbol{F}(\boldsymbol{x}) - \boldsymbol{F}(\hat{\boldsymbol{x}} - \boldsymbol{K}(\boldsymbol{C}\hat{\boldsymbol{x}} - y))\right) \\ & + 2\left(\tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{x}}\right) \tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{H}(\cdot, \cdot) + 2\mathrm{Tr}\{\boldsymbol{G}^{\mathrm{T}}(\cdot, \cdot) \left(2\boldsymbol{P}\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} + \tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{x}} \boldsymbol{P}\right) \boldsymbol{G}(\cdot, \cdot)\}. \end{aligned} \tag{14}$$

Based on the characteristic of the time delay terms $\boldsymbol{H}(\cdot, \cdot), \boldsymbol{G}(\cdot, \cdot)$, one can apply the mean value theorem to obtain (15). $\boldsymbol{H}_\iota(y, y(t - \tau(t))) = \frac{\partial \boldsymbol{H}(y,s)}{\partial s}|_{s=\iota_H y(t-\tau(t))}, \boldsymbol{G}_\iota(y, y(t - \tau(t))) = \frac{\partial \boldsymbol{G}(y,s)}{\partial s}|_{s=\iota_G y(t-\tau(t))}$, $0 < \iota_H, \iota_G \leqslant 1$.

$$\begin{aligned} \boldsymbol{H}(y, y(t - \tau(t))) &= y(t - \tau(t)) \boldsymbol{H}_\iota(y, y(t - \tau(t))), \\ \boldsymbol{G}(y, y(t - \tau(t))) &= y(t - \tau(t)) \boldsymbol{G}_\iota(y, y(t - \tau(t))). \end{aligned} \tag{15}$$

From the Young's inequality [5,6], one has

$$\begin{aligned} &\left(\tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{x}}\right) \tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \boldsymbol{H}(\cdot, \cdot) \leqslant \frac{3\varepsilon_H^{4/3}}{4} \|\boldsymbol{P}\|_{\mathrm{F}}^{8/3} \|\tilde{\boldsymbol{x}}\|^4 + \frac{y^4(t - \tau(t))}{4\varepsilon_H^4} \|\boldsymbol{H}_\iota(y, y(t - \tau(t)))\|^4, \\ &\mathrm{Tr}\left\{\boldsymbol{G}^{\mathrm{T}}(\cdot) \left(2\boldsymbol{P}\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} + \tilde{\boldsymbol{x}}^{\mathrm{T}} \boldsymbol{P} \tilde{\boldsymbol{x}} \boldsymbol{P}\right) \boldsymbol{G}(\cdot)\right\} \end{aligned}$$

$$\leqslant \frac{3n\sqrt{n}\varepsilon_G^2}{2}\left\|\boldsymbol{P}\right\|_{\mathrm{F}}^4\left\|\tilde{\boldsymbol{x}}\right\|^4+\frac{3n\sqrt{n}y^4\left(t-\tau(t)\right)}{2\varepsilon_G^2}\|\boldsymbol{G}_\iota(y,y(t-\tau(t)))\|^4. \tag{16}$$

Define the variable $\boldsymbol{v}=\boldsymbol{x}-(\hat{\boldsymbol{x}}-\boldsymbol{K}\left(\boldsymbol{C}\hat{\boldsymbol{x}}-y\right))=\left(\boldsymbol{I}-\boldsymbol{K}\boldsymbol{C}\right)\tilde{\boldsymbol{x}}$, Eq. (17) holds when the design parameters are set appropriately, i.e., satisfying the relationship (12),

$$\begin{aligned}\tilde{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{P}\left(\boldsymbol{F}(\tilde{\boldsymbol{x}})-\boldsymbol{F}(\hat{\boldsymbol{x}}-\boldsymbol{K}\left(\boldsymbol{C}\hat{\boldsymbol{x}}-y\right))\right)&=\tilde{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{C}_F\left(\boldsymbol{J}(\boldsymbol{x})-\boldsymbol{J}(\boldsymbol{x}-\boldsymbol{v})\right)\\&=-\frac{1}{2}\boldsymbol{v}^{\mathrm{T}}\left[\int_0^1\left(\frac{\partial\boldsymbol{J}}{\partial\boldsymbol{s}}+\left(\frac{\partial\boldsymbol{J}}{\partial\boldsymbol{s}}\right)^{\mathrm{T}}\right)_{\boldsymbol{s}=\boldsymbol{x}-\vartheta\boldsymbol{v}}\mathrm{d}\vartheta\right]\boldsymbol{v}\leqslant 0.\end{aligned} \tag{17}$$

Combining with (16) and (17), the inequality below can be obtained easily:

$$\mathcal{L}V_0\leqslant -c_0\left\|\tilde{\boldsymbol{x}}\right\|^4+y^4\left(t-\tau(t)\right)\mathcal{F}_0(y,y(t-\tau(t))), \tag{18}$$

with $c_0=\lambda_{\min}(\boldsymbol{P})\lambda_{\min}(\boldsymbol{Q})-\frac{3\varepsilon_H^{4/3}}{2}\left\|\boldsymbol{P}\right\|_{\mathrm{F}}^{8/3}-3n\sqrt{n}\varepsilon_G^2\left\|\boldsymbol{P}\right\|_{\mathrm{F}}^4$, $\mathcal{F}_0\left(\cdot\right)=\frac{1}{2\varepsilon_H^4}\left\|\boldsymbol{H}_\iota\left(y,y(t-\tau(t))\right)\right\|^4+\frac{3n\sqrt{n}}{2\varepsilon_G^2}\left\|\boldsymbol{G}_\iota\left(y,y\left(t-\tau(t)\right)\right)\right\|^4$ is an unknown delayed function, which would be compensated by fuse of the NNs.

3.1 Control design

In this part, the Backstepping design procedure contains n steps. In the first step, the residual uncertainty in (18), including the time-varying delay effect, is delivered to the subsystem. The NNs approximation is introduced to compensate for the unknown nonlinearity of the whole system. In the final step, the adaptive law is targetedly designed to mitigate the effect of the dead zone nonlinearity, and the developed design can avoid the possible control singularity problem.

Step 1. Along with (1) and (11), one defines the error variables $z_1=y, z_2=\hat{x}_2-\bar{\alpha}_2$. α_2 denotes the virtual control of $\hat{x}_2$. To avoid redifferentiating the virtual control in the following step, which leads to the so-called "explosion of complexity", the first-order dynamic surface is introduced with the output $\bar{\alpha}_2$. $y_{f2}=\bar{\alpha}_2-\alpha_2$ and η_2 is the time constant,

$$\eta_2\dot{\bar{\alpha}}_2+\bar{\alpha}_2=\alpha_2,\quad \bar{\alpha}_2(0)=\alpha_2(0). \tag{19}$$

Furthermore, one obtains the following Ito's differentiations:

$$\mathrm{d}z_1=\left[a_{1,1}x_1+a_{1,2}\left(z_2+y_{f2}+\alpha_2\right)+a_{1,2}\tilde{x}_2+f_1\left(x_1\right)+h_1\left(y,y(t-\tau(t))\right)\right]\mathrm{d}t+g_1\left(y,y(t-\tau(t))\right)\mathrm{d}\omega, \tag{20}$$

$$\mathrm{d}y_{f2}=\left[-\frac{y_{f2}}{\eta_2}+B_2\left(x_1,y_{f2},\hat{\theta}_1,\hat{\lambda}_1\right)\right]\mathrm{d}t+C_2\left(x_1,y_{f2},\hat{\theta}_1,\hat{\lambda}_1\right)\mathrm{d}\omega, \tag{21}$$

where $B_2(\cdot),C_2\left(\cdot\right)$ are continuous functions defined in a compact set. In this compact set, there exist positive constants $\bar{B}_2,\bar{C}_2$ such that $\left|B_2\left(\cdot\right)\right|\leqslant\bar{B}_2,\left|C_2\left(\cdot\right)\right|\leqslant\bar{C}_2$ [16,31].

From Young's inequality [5,6], the following (22)–(25) are useful for the control design:

$$\begin{aligned}&z_1^3\left(a_{1,2}z_2+a_{1,2}y_{f2}+a_{1,2}\tilde{x}_2+h_1\left(y,y(t-\tau(t))\right)\right)\\&\quad\leqslant\frac{3}{4}\left(\left(\epsilon_{11}a_{1,2}\right)^{\frac{4}{3}}+\left(\epsilon_{12}a_{1,2}\right)^{\frac{4}{3}}+\left(\epsilon_{13}a_{1,2}\right)^{\frac{4}{3}}+\epsilon_{14}^{\frac{4}{3}}\right)z_1^4\\&\qquad+\frac{z_2^4}{4\epsilon_{11}^4}+\frac{y_{f2}^4}{4\epsilon_{12}^4}+\frac{\left|\tilde{x}_2\right|^4}{4\epsilon_{13}^4}+\frac{y^4(t-\tau(t))\left|h_{1\iota}\left(y,y(t-\tau(t))\right)\right|^4}{4\epsilon_{14}^4},\end{aligned} \tag{22}$$

$$z_1^2g_1\left(y,y(t-\tau(t))\right)g_1^{\mathrm{T}}\left(y,y(t-\tau(t))\right)\leqslant\frac{\epsilon_{15}z_1^4}{2}+\frac{y^4(t-\tau(t))\left|g_{1\iota}\left(y,y(t-\tau(t))\right)\right|^4}{2\epsilon_{15}}, \tag{23}$$

$$y_{f2}^3B_2\left(\cdot\right)\leqslant\frac{3\bar{B}_2^{\frac{4}{3}}}{4\epsilon_{16}}y_{f2}^4+\frac{\epsilon_{16}}{4}, \tag{24}$$

$$y_{f2}^2\mathrm{Tr}\left\{C_2\left(\cdot\right)C_2^{\mathrm{T}}\left(\cdot\right)\right\}\leqslant\frac{\bar{C}_2^4}{2\epsilon_{17}}y_{f2}^4+\frac{\epsilon_{17}}{2}. \tag{25}$$

Based on the above calculation, one extracts the time-varying delay terms $\mathcal{F}_1(\cdot) = \frac{|h_{1\iota}(y,y(t-\tau(t)))|^4}{4\epsilon_{14}^4} + \frac{3|g_{1\iota}(y,y(t-\tau(t)))|^4}{4\epsilon_{15}}$. Since the subsystems of the observer (11) are considered in the following steps to design the immediate control law, the delayed terms of the whole system can be described by $\mathcal{F}(y, y(t-\tau(t))) = \mathcal{F}_0(\cdot) + \mathcal{F}_1(\cdot)$. Based on Lemma 2, one employs the NNs to approximate the unknown smooth function $\frac{y}{1-\xi}\mathcal{F}(y,y)$, and the variance between the two functions $\frac{y(t-\tau(t))}{1-\xi}\mathcal{F}(y, y(t-\tau(t)))$ and $\frac{y}{1-\xi}\mathcal{F}(y,y)$ would be further analyzed by constructing the special Lyapunov candidate,

$$\frac{y}{1-\xi}\mathcal{F}(y,y) = \boldsymbol{S}(y)\boldsymbol{W}y + \sigma(y) = b_1\boldsymbol{S}(y)\boldsymbol{\varpi} + \sigma(y), \tag{26}$$

where $b_1 = \|\boldsymbol{W}\|, \boldsymbol{W}^m = (\boldsymbol{W}/b_1)$ and $\boldsymbol{\varpi} = \boldsymbol{W}^m y = (\boldsymbol{W}/b_1)y, |\sigma(y)| \leqslant \theta\psi(y)$. Thus, one can get $b_1\boldsymbol{\varpi} = \boldsymbol{W}y$.

For concise description, the immediate control law is presented in (27), which could be derived from the error dynamics (20) and (26). In addition, the detailed synthesis analysis will be given in Subsection 3.2.

$$\alpha_2 = \frac{1}{a_{1,2}}\left(-c_1 y - a_{1,1}x_1 - f_1(x_1) - \frac{\hat{\lambda}}{4\gamma_1}y^3\boldsymbol{S}(y)\boldsymbol{S}^{\mathrm{T}}(y) - \hat{\theta}\psi(y)\tanh\left(\frac{y^3\hat{\theta}\psi(y)}{\kappa}\right)\right), \tag{27}$$

where c_1 is positive design constant, $\kappa > 0$ is with small value. $\hat{\lambda}, \hat{\theta}$ are the estimated parameters, whose adaptive law are design as (28). The parameter $\lambda = b_1^2$,

$$\begin{aligned}\dot{\hat{\lambda}} &= \mathit{\Gamma}_\lambda\left(\frac{1}{4\gamma_1}y^6\boldsymbol{S}(y)\boldsymbol{S}^{\mathrm{T}}(y) - \rho_\lambda\left(\hat{\lambda} - \hat{\lambda}(0)\right)\right),\\ \dot{\hat{\theta}} &= \mathit{\Gamma}_\theta\left(|y|^3\psi(y) - \rho_\theta\left(\hat{\theta} - \hat{\theta}(0)\right)\right).\end{aligned} \tag{28}$$

In (28), $\mathit{\Gamma}_\lambda, \mathit{\Gamma}_\theta, \rho_\lambda, \rho_\theta$ are positive design parameters, $\hat{\lambda}(0), \hat{\theta}(0)$ are the corresponding initial values.

Step i $(2 \leqslant i \leqslant n-1)$. In this step, a similar procedure is employed recursively for the error dynamics $z_i = \hat{x}_i - \bar{\alpha}_i$, whose time derivative is given in (29). Since there exists no uncertainties in this subsystem, the design procedure is more simple. Eq. (30) gives the employed dynamic surface, where the error variable $y_{fi+1} = \bar{\alpha}_{i+1} - \alpha_{i+1}$,

$$\mathrm{d}z_i = \left[\sum_{j=1}^{i} a_{i,j}\hat{x}_j + a_{i,i+1}\left(z_{i+1} + y_{f_{i+1}} + \alpha_{i+1}\right) - l_i\tilde{x}_1 + f_i\left(\bar{\hat{\boldsymbol{x}}}_i - \boldsymbol{k}_i\tilde{x}_1\right) + \frac{y_{fi}}{\eta_i}\right]\mathrm{d}t, \tag{29}$$

$$\eta_{i+1}\dot{\bar{\alpha}}_{i+1} + \bar{\alpha}_{i+1} = \alpha_{i+1}, \quad \bar{\alpha}_{i+1}(0) = \alpha_{i+1}(0), \tag{30}$$

$$\dot{y}_{fi+1} = -\frac{y_{fi+1}}{\eta_{i+1}} + B_{i+1}(x_1, \hat{x}_2, \ldots, \hat{x}_i, y_{fi}), \tag{31}$$

where $B_{i+1}(\cdot)$ is the continuous function in a compact set, with the uppder bounds $|B_{i+1}(\cdot)| \leqslant \bar{B}_{i+1}$.

Then, one chooses the virtual control law as (32), $c_i > 0$ is the constant parameter,

$$\alpha_{i+1} = \frac{1}{a_{i,i+1}}\left(-c_i z_i - \sum_{j=1}^{i} a_{i,j}\hat{x}_j + l_i\tilde{x}_1 - f_i\left(\bar{\hat{\boldsymbol{x}}}_i - \boldsymbol{k}_i\tilde{x}_1\right) - \frac{y_{fi}}{\eta_i}\right). \tag{32}$$

Step n. Define the immediate variable $\alpha_v = \boldsymbol{\Theta}^{\mathrm{T}}(t)\boldsymbol{\Phi}(t)v, \lambda_{\mathrm{D}} = \left[\boldsymbol{\Theta}^{\mathrm{T}}(t)\boldsymbol{\Phi}(t)\right]^{-1}$, the actual control $v = \hat{\lambda}_{\mathrm{D}}\alpha_v$. Following (4) and (11), one can obtain the dynamics of $z_n = \hat{x}_n - \bar{\alpha}_n$ as follows:

$$\mathrm{d}z_n = \left[\sum_{j=1}^{n} a_{n,j}\hat{x}_j + \alpha_v - \boldsymbol{\Theta}^{\mathrm{T}}(t)\boldsymbol{\Phi}(t)\tilde{\lambda}_{\mathrm{D}}\alpha_v + d(v) - l_n\tilde{x}_1 + f_n\left(\hat{\boldsymbol{x}} - \boldsymbol{k}_n\tilde{x}_1\right) + \frac{y_{fn}}{\eta_n}\right]\mathrm{d}t. \tag{33}$$

The immediate control variable α_v is given in

$$\alpha_v = -c_n z_n - \sum_{j=1}^{n} a_{n,j}\hat{x}_j + l_n\tilde{x}_1 - f_n\left(\hat{\boldsymbol{x}} - \boldsymbol{k}_n\tilde{x}_1\right) - \frac{y_{fn}}{\eta_n}, \tag{34}$$

Eq. (35) is the corresponding gain-inversion related adaptive law, which could alleviate the impact of the unknown dead zone for merits of the robust damping technique,

$$\dot{\hat{\lambda}}_{\mathrm{D}} = \mathit{\Gamma}_{\mathrm{D}} \left(\alpha_v z_n^3 - \rho_{\mathrm{D}} \left(\hat{\lambda}_{\mathrm{D}} - \hat{\lambda}_{\mathrm{D}}(0)\right)\right), \tag{35}$$

where $c_n, \mathit{\Gamma}_{\mathrm{D}}, \rho_{\mathrm{D}}$ are all positive design constants, and ρ_{D} is meaningful for improving the robustness of the closed-loop system.

3.2 Main result

Now, the main result is summarized as follows.

Theorem 1. Considering the closed-loop system composed of the stochastic nonlinear process (1) with the unknown dead zone (2), satisfying Assumptions 1 and 2, the observer (11), the control laws (27), (32), (34) and the adaptive laws (28), (35). For all initial conditions in an appropriately chosen compact set $\Omega_0 := \{\tilde{\boldsymbol{x}}, z_i, y_{fi}, \tilde{\lambda}, \tilde{\theta}, \tilde{\lambda}_{\mathrm{D}} | \left(\tilde{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{P}\tilde{\boldsymbol{x}}\right)^2 + \sum_{i=1}^n z_i^4 + \sum_{i=2}^n y_{fi}^4 + \frac{2}{\mathit{\Gamma}_\lambda}\tilde{\lambda}^2 + \frac{2}{\mathit{\Gamma}_\theta}\tilde{\theta}^2 + \frac{4\bar{\ell}}{\mathit{\Gamma}_{\mathrm{D}}}\tilde{\lambda}_{\mathrm{D}}^2 \leqslant 4\Lambda\}$, $\Lambda > 0$ is an arbitrary constant, one can choose the related parameters $c_1, \ldots, c_n, \eta_2, \ldots, \eta_n, \mathit{\Gamma}_\lambda, \mathit{\Gamma}_\theta, \mathit{\Gamma}_D, \rho_\lambda, \rho_\theta, \rho_{\mathrm{D}}$ appropriately and $\boldsymbol{K}, \boldsymbol{L}$ satisfying (12), such that all states in the closed-loop system are MSUUB in probability.

Proof. One constructs the following positive definite functions as the Lypunov candidate for the closed-loop system:

$$V = V_0 + \frac{1}{4}\sum_{i=1}^n z_i^4 + \frac{1}{4}\sum_{i=2}^n y_{fi}^4 + \frac{\tilde{\lambda}^2}{2\mathit{\Gamma}_\lambda} + \frac{\tilde{\theta}^2}{2\mathit{\Gamma}_\theta} + \frac{\boldsymbol{\Theta}^{\mathrm{T}}(t)\boldsymbol{\Phi}(t)}{2\mathit{\Gamma}_{\mathrm{D}}}\tilde{\lambda}_{\mathrm{D}}^2 + \underbrace{\frac{1}{1-\xi}\int_{t-\tau(t)}^t y^4(s)\mathcal{F}(y, y(s))\mathrm{d}s}_{V_Q}. \tag{36}$$

Note, in (36), $\boldsymbol{\Theta}^{\mathrm{T}}(t)\boldsymbol{\Phi}(t)(t) \geqslant \ell_0 > 0$, V_Q is introduced to compensate for the time delay effect, and its time derivative can be calculated as

$$\mathcal{L}V_Q \leqslant \frac{1}{1-\xi}y^4\mathcal{F}(y, y) - y^4(t-\tau(t))\mathcal{F}(y, y(t-\tau(t))). \tag{37}$$

From the ith subsystem (29), it is worth to be mentioned that

$$z_i^3 a_{i,i+1}z_{i+1} + z_i^3 a_{i,i+1}y_{fi+1} \leqslant \frac{3}{4}\left((\epsilon_{i,1}a_{i,i+1})^{\frac{4}{3}} + (\epsilon_{i,2}a_{i,i+1})^{\frac{4}{3}}\right)z_i^4 + \frac{z_{i+1}^4}{4\epsilon_{i,1}^4} + \frac{y_{fi+1}^4}{4\epsilon_{i,2}^4}, \tag{38}$$

$$y_{fi+1}^3 B_{i+1}(\cdot) \leqslant \frac{3B_{i+1}^{\frac{4}{3}}(\cdot)y_{fi+1}^4\bar{B}_{i+1}^{\frac{4}{3}}}{4\epsilon_{i,6}\bar{B}_{i+1}^{\frac{4}{3}}} + \frac{\epsilon_{i,6}}{4}. \tag{39}$$

According to Lemma 1, the infinitesimal gradient of V along the solution of (20), (21), (29), (31), (33) and (38), (39) is obtained,

$$\begin{aligned}
\mathcal{L}V \leqslant & -c_0\|\tilde{\boldsymbol{x}}\|^4 + z_1^3\left(a_{1,1}x_1 + a_{1,2}\alpha_2 + \frac{3}{4}\left((\epsilon_{11}a_{1,2})^{\frac{4}{3}} + (\epsilon_{12}a_{1,2})^{\frac{4}{3}} + (\epsilon_{13}a_{1,2})^{\frac{4}{3}} + \epsilon_{14}^{\frac{4}{3}} + \epsilon_{15}\right)z_1 + f_1(x_1)\right. \\
& \left. + \frac{y\mathcal{F}(y,y)}{1-\xi}\right) - \left(\frac{1}{\eta_2} - \frac{3\bar{B}_2^{\frac{4}{3}}}{4\epsilon_{16}} - \frac{3\bar{C}_2^4}{4\epsilon_{17}} - \frac{1}{4\epsilon_{12}^4}\right)y_{f2}^4 \\
& - \sum_{i=2}^{n-1}\left(c_i - \frac{3(\epsilon_{i,1}a_{i,i+1})^{\frac{4}{3}}}{4} - \frac{3(\epsilon_{i,2}a_{i,i+1})^{\frac{4}{3}}}{4} - \frac{1}{4\epsilon_{i-1,1}^4}\right)z_i^4 - \sum_{i=2}^{n-1}\left(\frac{1}{\eta_{i+1}} - \frac{3\bar{B}_{i+1}^{\frac{4}{3}}}{4\epsilon_{i6}} - \frac{1}{4\epsilon_{i2}^4}\right)y_{fi+1}^4 \\
& - z_n^3\left(\sum_{j=1}^n a_{n,j}\hat{x}_j + \alpha_v - \boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{\Phi}\hat{\lambda}_{\mathrm{D}}\alpha_v + d(v) - l_n\tilde{x}_1 + f_n(\hat{\boldsymbol{x}} - \boldsymbol{k}_n\tilde{x}_1) + \frac{y_{fn}}{\eta_n}\right) \\
& + \frac{1}{4\epsilon_{13}^4}|\tilde{x}_2|^4 + \sum_{i=1}^{n-1}\frac{\epsilon_{i6}}{4} + \frac{3\epsilon_{17}}{4} - \mathit{\Gamma}_\lambda^{-1}\tilde{\lambda}\dot{\hat{\lambda}} - \mathit{\Gamma}_\theta^{-1}\tilde{\theta}\dot{\hat{\theta}} - \frac{\boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{\Phi}}{\mathit{\Gamma}_{\mathrm{D}}}\tilde{\lambda}_{\mathrm{D}}\dot{\hat{\lambda}}_{\mathrm{D}}.
\end{aligned} \tag{40}$$

Furthermore, the following (41) and (42) would facilitate the stability analysis and $\boldsymbol{\varpi}^{\mathrm{T}}\boldsymbol{\varpi} = \frac{\boldsymbol{W}^{\mathrm{T}}\boldsymbol{W}}{\|\boldsymbol{W}\|^2}y^2 = y^2$. Note, the relationship (42) is commonly used in the field of nonlinear control and has been detailed in [28].

$$z_1^3 b_1 \boldsymbol{S}(y)\boldsymbol{\varpi} \leqslant \frac{\lambda}{4\gamma_1}y^6\boldsymbol{S}(y)\boldsymbol{S}^{\mathrm{T}}(y) + \gamma_1\boldsymbol{\varpi}^{\mathrm{T}}\boldsymbol{\varpi}, \tag{41}$$

$$\left|y^3\hat{\theta}\psi(y)\right| - y^3\hat{\theta}\psi(y)\tanh\left(\frac{y^3\hat{\theta}\psi(y)}{\kappa}\right) \leqslant 0.2785\kappa. \tag{42}$$

Considering Remark 2, one has

$$z_n^3 d(v) \leqslant \frac{3\epsilon_n^{\frac{4}{3}}}{4}z_n^4 + \frac{1}{4\epsilon_n^4}\delta^{*4}. \tag{43}$$

Submitting (27), (28), (34), (35) and considering (41)–(43), Eq. (40) can be expressed as

$$\begin{aligned}
\mathcal{L}V \leqslant & -c_0\|\tilde{\boldsymbol{x}}\|^4 - \left(c_1 - \frac{3}{4}\left((\epsilon_{11}a_{1,2})^{\frac{4}{3}} + (\epsilon_{12}a_{1,2})^{\frac{4}{3}} + (\epsilon_{13}a_{1,2})^{\frac{4}{3}} + \epsilon_{14}^{\frac{4}{3}} + \epsilon_{15}\right)\right)z_1^4 \\
& - \left(\frac{1}{\eta_2} - \frac{3\bar{B}_2^{\frac{4}{3}}}{4\epsilon_{16}} - \frac{3\bar{C}_2^4}{4\epsilon_{17}} - \frac{1}{4\epsilon_{12}^4}\right)y_{f2}^4 - \sum_{i=2}^{n-1}\left(c_i - \frac{3(\epsilon_{i,1}a_{i,i+1})^{\frac{4}{3}}}{4} - \frac{3(\epsilon_{i,2}a_{i,i+1})^{\frac{4}{3}}}{4} - \frac{1}{4\epsilon_{i-1,1}^4}\right)z_i^4 \\
& - \sum_{i=2}^{n-1}\left(\frac{1}{\eta_{i+1}} - \frac{3\bar{B}_{i+1}^{\frac{4}{3}}}{4\epsilon_{i6}} - \frac{1}{4\epsilon_{i2}^4}\right)y_{fi+1}^4 - \left(c_n - \frac{3\epsilon_n^{\frac{4}{3}}}{4}\right)z_n^4 - \frac{\rho_\lambda}{2}\tilde{\lambda}^2 - \frac{\rho_\theta}{2}\tilde{\theta}^2 - \frac{\rho_{\mathrm{D}}\boldsymbol{\Theta}^{\mathrm{T}}\boldsymbol{\Phi}}{2}\tilde{\lambda}_{\mathrm{D}}^2 \\
& + \varrho + \frac{1}{4\epsilon_{13}^4}|\tilde{x}_2|^4,
\end{aligned} \tag{44}$$

where $\varrho = 0.2785\kappa + \sum_{i=1}^{n-1}\frac{\epsilon_{i6}}{4} + \frac{3\epsilon_{17}}{4} + \frac{\delta^{*4}}{4\epsilon_n^4} + \frac{\rho_\lambda}{2}(\lambda - \hat{\lambda}(0))^2 + \frac{\rho_\theta}{2}(\theta - \hat{\theta}(0))^2 + \rho_D\bar{\ell}(\lambda_{\mathrm{D}} - \hat{\lambda}_{\mathrm{D}}(0))^2$.

Letting

$$\begin{aligned}
& c = \min\left\{\frac{c_0}{\lambda_{\max}^2(\boldsymbol{P})}, \frac{\rho_\lambda\Gamma_\lambda}{2}, \frac{\rho_\theta\Gamma_\theta}{2}, \frac{\rho_D\Gamma_{\mathrm{D}}}{2}\right\}, \\
& c_1 = \frac{c}{2} + \frac{3}{4}\left((\epsilon_{11}a_{1,2})^{\frac{4}{3}} + (\epsilon_{12}a_{1,2})^{\frac{4}{3}} + (\epsilon_{13}a_{1,2})^{\frac{4}{3}} + \epsilon_{14}^{\frac{4}{3}} + \epsilon_{15}\right), \\
& \frac{1}{\eta_2} = \frac{c}{2} + \frac{3\bar{B}_2^{\frac{4}{3}}}{4\epsilon_{16}} + \frac{3\bar{C}_2^4}{4\epsilon_{17}} + \frac{1}{4\epsilon_{12}^4}, \\
& c_i = \frac{c}{2} + \frac{3(\epsilon_{i,1}a_{i,i+1})^{\frac{4}{3}}}{4} + \frac{3(\epsilon_{i,2}a_{i,i+1})^{\frac{4}{3}}}{4} + \frac{1}{4\epsilon_{i-1,1}^4}, \\
& \frac{1}{\eta_{i+1}} = \frac{c}{2} + \frac{3\bar{B}_{i+1}^{\frac{4}{3}}}{4\epsilon_{i6}} + \frac{1}{4\epsilon_{i2}^4}, i = 2, 3, \ldots, n-1, \quad c_n = \frac{c}{2} + \frac{3\epsilon_n^{\frac{4}{3}}}{4},
\end{aligned}$$

Eq. (44) can be further derived as follows and, it is obvious to note that the term $|\tilde{x}_2|^4$ in (44) can be eliminated by selecting the parameter c_0 appropriately:

$$\mathcal{L}V \leqslant -2c\left[V(t) - V_Q(t)\right] + \varrho. \tag{45}$$

From (45), the related error variables $\tilde{\boldsymbol{x}}, z_i, y_{fi}, \tilde{\lambda}, \tilde{\theta}$ and $\tilde{\lambda}_{\mathrm{D}}$ converge to the compact set $\mathbb{O}_e$ with a small bounds. If some error variables are outside the scope of $\mathbb{O}_e$, one has $\mathcal{L}V < 0$. Furthermore, the effect of $V_Q(t)$ is bounded and diminished along with $t \to +\infty$. Based on Lemma 1 and the above analysis, there must exist a positive constant $\bar{c}$ satisfying $0 \leqslant E\left[V(t)\right] \leqslant \left(V(0) - \frac{\varrho}{2\bar{c}}\right)\exp\left(-2\bar{c}t\right) + \frac{\varrho}{2\bar{c}}$. The conclusion is obvious that there exists a constant $T > 0$ such that $V(t) \leqslant \varrho/2\bar{c}$ for all $t \geqslant t_0 + T$. And the bound $\varrho/2\bar{c}$ can be made arbitrarily small if the design parameters are chosen appropriately. Hence, $y, \tilde{\boldsymbol{x}}, z_2, \ldots, z_n, \hat{\lambda}, \hat{\theta}, \hat{\lambda}_{\mathrm{D}}$ are all bounded. Thus, for the initial conditions in Theorem 1, no finite-time escape phenomenon may happen and the closed-loop system is mean-square semiglobally uniformly ultimately bounded in probability, i.e., there exists the rational and very small value $\varepsilon > 0$ such that $\lim_{t\to\infty}|y| < \varepsilon$.

Remark 3. In this article, since the approximation property of NNs can only be guaranteed in convex regions of interest, the stability established in this article is semiglobal. That is normal for NNs or fuzzy system based adaptive control scheme unless some other techniques are employed, e.g., the feedback linearization method [3, 12] and the sliding-mode control [28]. In addition, unlike the existed literatures, the approximation error satisfies $|\sigma(\boldsymbol{x})| \leqslant \theta\psi(\cdot)$, which meets more practical cases. Actually, the convex region cannot be identified exactly before the control law is implemented. Comparing with the previous references around "global stability", the plant addressed in this note includes the unknown time-varying delay, unknown control gain function and the dead zone. That could enhance the possibility to implement the algorithm in the industry practice.

4 Illustrative example

In this section, the proposed control scheme would be compared with the results in [24]. In [24], it is emphasized that the stochastic nonlinear process belongs to hardly controllable system under the output-feedback frame. For comparison purposes, one considers the stochastic nonlinear time delay plant:

$$\begin{cases} \mathrm{d}x_1 = \left[0.5x_1 + 1.5x_2 - x_1^3 + yy\left(t-\tau(t)\right)\right]\mathrm{d}t + \frac{y(t-\tau(t))}{1+y^2}\mathrm{d}\omega, \\ \mathrm{d}x_2 = \left[u + 2x_1 - 2x_2 + x_1^3 - x_2^5 + \sin\left(yy\left(t-\tau(t)\right)\right)\right]\mathrm{d}t + y\left(t-\tau(t)\right)\ln\left(1+y^2\right)\mathrm{d}\omega, \\ u = D(v), \quad y = x_1. \end{cases} \tag{46}$$

As for the system output, the time-varying delay is chosen as $\tau(t) = 1+0.8\sin(t)$. u is the output of the dead zone nonlinearity with the model (2), v is the actual control input. In details, $b_l = -0.2, b_r = 0.3$, $u = (1-0.1\sin(v))(v-0.3)$ for $v \geqslant 0.3$, $u = 0$ for $-0.2 < v < 0.3$ and $u = (0.8-0.1\cos(v))(v+0.2)$ for $v \leqslant -0.2$. It is obvious to note that the system (46) satisfies Assumptions 1 and 2.

In (46), ω is the 1-dimensional standard Brownian motion. It is simulated by integrating the Gauss white noise, which is with the mean value 0 and the variance 0.5. That has been verified in the existed references [3, 4, 13].

For the comparison purpose, one reproduces the control law in [24] to stabilize the plant (46) with or without the dead zone input. And it employed one NNs with 9 network nodes to tackle the system uncertainty and unknown time delays. If it is implemented in the engineering condition, that may lead to the burden-some problem in the practical embedded system. As to the detailed parameter setting, please refer to the result in [24]. For the algorithm developed in this article, one selects $\boldsymbol{K} = [-0.5, 1]^{\mathrm{T}}, \boldsymbol{L} = [-1, -2]^{\mathrm{T}}$ and the observer is given in (47). Eq. (48) presents the actual control law with the parameter setting $c_1 = 2.0, c_2 = 5.6, \gamma_1 = 0.42, \kappa = 0.3, \psi(y) = 1+y^2, \varGamma_\lambda = \varGamma_\theta = 5.0, \rho_\lambda = 0.3, \rho_\theta = 0.1, \varGamma_{\mathrm{D}} = 2.0, \rho_{\mathrm{D}} = 1.2$. For the NNs in (48), it is with centres μ_j spaced evenly in $[-1.0, 1.0]$ and widths $\varsigma_j = 0.23, j = 1, 2, \ldots, 11$.

$$\begin{cases} \dot{\hat{x}}_1 = 0.5\hat{x}_1 + 1.5\hat{x}_2 - (\hat{x}_1 - y) - \left[\hat{x}_1 - 0.5\left(\hat{x}_1 - y\right)^3\right], \\ \dot{\hat{x}}_2 = u + 2\hat{x}_1 - 2\hat{x}_2 - 2\left(\hat{x}_1 - y\right) + \left[\hat{x}_1 - 0.5\left(\hat{x}_1 - y\right)\right]^3 - \left[\hat{x}_2 + \left(\hat{x}_1 - y\right)\right]^5, \end{cases} \tag{47}$$

$$\begin{aligned} &\alpha_2 = \frac{2}{3}\left(-c_1 y - 0.5y + y^3 - \frac{\hat{\lambda}}{4\gamma_1}y^3\boldsymbol{S}(y)\boldsymbol{S}^{\mathrm{T}}(y) - \hat{\theta}\psi(y)\tanh\left(\frac{y^3\hat{\theta}\psi(y)}{\kappa}\right)\right), \\ &v = \hat{\lambda}_{\mathrm{D}}\alpha_v, \quad \alpha_v = -c_2 z_2 - 2\hat{x}_1 + 2\hat{x}_2 + 2\left(\hat{x}_1 - y\right) - \left[\hat{x}_1 - 0.5(\hat{x}_1 - y)\right]^3 + \left[\hat{x}_2 + (\hat{x}_1 - y)\right]^5. \end{aligned} \tag{48}$$

In this experiment, the initial conditions are $\boldsymbol{x}(0) = [0.5, -0.5]^{\mathrm{T}}, \hat{\lambda}(0) = 0.1, \hat{\theta}(0) = 0.1, \hat{\lambda}_{\mathrm{D}} = 0.2$. The Figures 1–4 present the comparison results. In Figure 1, the proposed algorithm is compared with the scheme in [24]. When there exists no the dead zone nonlinearity, the system response could be stabilized effectively with the favorable performance. However, for the opposite case, the performance is perturbed severely by the dead zone. Comparing with the work in [24], the proposed algorithm could eliminate the adverse effect due to the time-varying delay and dead zone. The control inputs $v(t), u(t)$

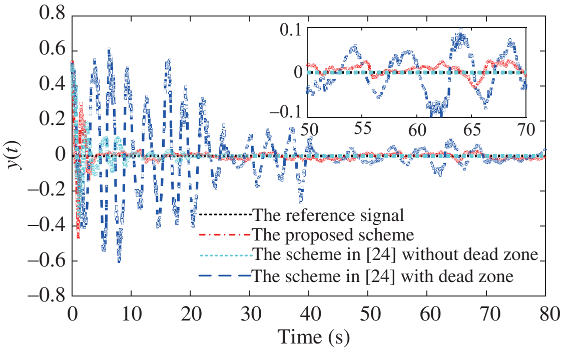

Figure 1 (Color online) Comparison of system responses between the proposed scheme and the scheme in [24] without and with dead zone.

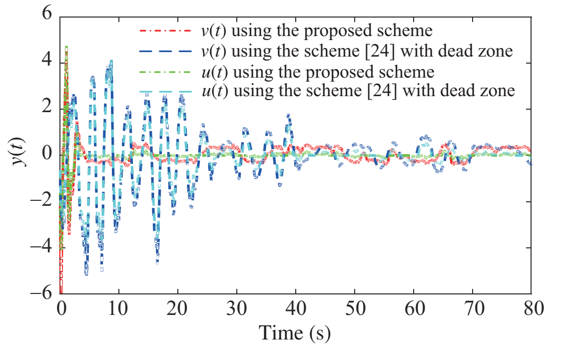

Figure 2 (Color online) Comparison of control inputs $v(t), u(t)$ between the proposed scheme and the scheme in [24] with dead zone.

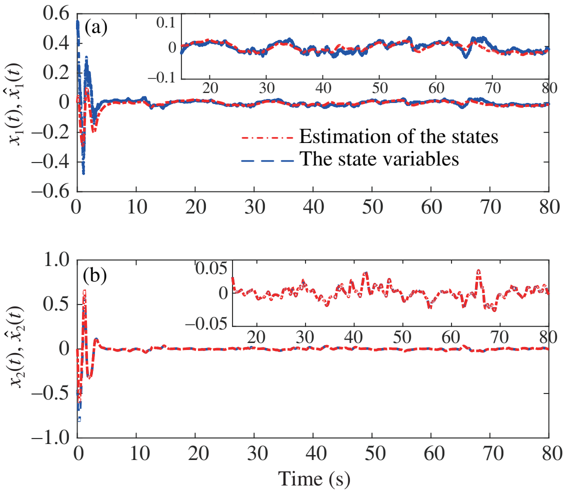

Figure 3 (Color online) Responses of the state variables and their estimation under the proposed scheme. (a) $x_1, \hat{x}_1$; (b) $x_2, \hat{x}_2$.

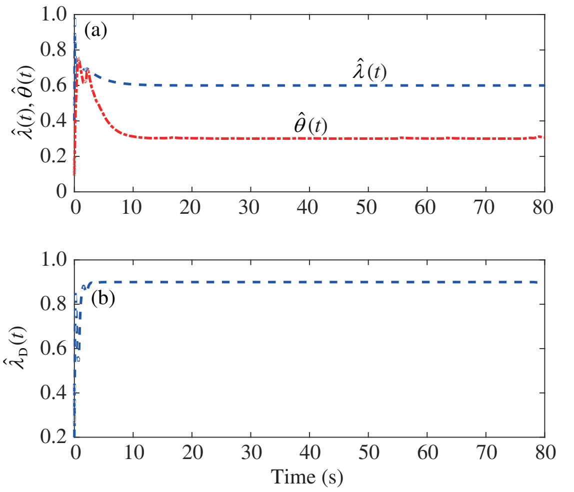

Figure 4 (Color online) The adaptive parameters under the proposed scheme. (a) $\hat{\lambda}, \hat{\theta}$; (b) $\hat{\lambda}_{\rm D}$.

shown in Figure 2 also verify the conclusion. Figure 3 gives responses of the state variables and their estimations, which has illustrated the effectiveness of the observer. For the controller (47), (48), only three parameters are required to be updated online and that is fixed for the high-order process. That may produce the burden-some superiority than the existed NNs based algorithm. Figure 4 is the adaptive parameters $\hat{\lambda}, \hat{\theta}, \hat{\lambda}_{\rm D}$ under the control law proposed in this article. Based on the above analysis, the proposed algorithm could efficiently compensate the effect of time-varying delay and dead zone and stabilize the system output with the favorable performance.

5 Conclusion

A robust neural output-feedback control algorithm, using the circle criterion, the NNs approximation and the MLP technique, has been developed to address the stabilization of the stochastic nonlinear process with the time-varying delay and dead zone. It is concluded that the proposed algorithm is with the concise structure, the favorable robustness and the burden-some superiority, which could improve its practicability in the engineering practice. By virtue of the Lyapunov criteria, the resulting closed-loop system is guaranteed to be mean-square semiglobally uniformly ultimately bounded and the stabilizing error can be stabilized to a arbitrary small compact set. The comparative experiment has demonstrated that the robust neural algorithm is effective and with the improved performance.

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Conflict of interest The authors declare that they have no conflict of interest.

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