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Observable liveness of Petri nets with controllable and observable transitions

Guanjun LIU[1,2,3*] & Changjun JIANG[2,3]

[1]*Department of Computer Science, Tongji University, Shanghai* 201804*, China;*
[2]*Key Laboratory of Embedded System and Service Computing of Ministry of Education, Tongji University, Shanghai* 201804*, China;*
[3]*E-commerce Transactions and Information Services Collaborative Innovation Center of Shanghai University Knowledge Service Platform, Tongji University, Shanghai* 201804*, China*

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Observability and/or controllability are paid much attention to by researchers not only from the control science field [1–3] but also from the computer science field [4–9].

Recently, Desel and Kilinc [4] proposed the notion of observable liveness for Petri nets with (un-) observable and (un-) controllable transitions. The purpose of defining observable liveness is to represent that "a user can always enforce the occurrence of any observable transition only by stimulating the net by choosing appropriate enabled controllable transition" [4]. An important assumption of defining observable liveness is that "the uncontrollable part of a net proceeds, i.e., the net behaves weakly fair with respect to uncontrollable transitions" [4]. Our understanding to this assumption is that any controllable activity cannot influence the progress of an uncontrollable activity when this uncontrollable activity can be executed by a system (except that the system destroys the condition of executing this uncontrollable activity by executing another uncontrollable activity). It should be easily understood according to their words [4] as follows:

Assumption 1. There might be states in which controllable activities and uncontrollable ones are enabled, i.e., both the machine and the user can do something. In such a state, we cannot expect that the user is able to do his controllable activity first. This means that, in the case of competition between activities, the user only has full control if only controllable activities are involved.

However, the definition of observable liveness proposed by Desel and Kilinc is not completely coincident with their assumption. Let us consider the Petri net in Figure 1(a). Notice that in a Petri net graph controllable transitions are illustrated as the black filled rectangles, observable but uncontrollable transitions are drawn as the bold rectangles, while an unobservable transition is illustrated by a plain rectangle. A controllable transition is also observable. At the initial marking, only controllable transition t_1 is enabled. Firing t_1 yields a new marking $p_3 + p_4 + p_7 + p_{11} + p_{12}$ at which unobservable transition t_6 and controllable one t_5 are both enabled. By their assumption, it is t_6, not t_5, to be fired firstly. Then, t_8, t_3, and t_9 are fired in turn and marking $p_1 + p_3 + p_7 + p_{12}$ is reached. At this marking only transitions t_2 and t_5 are enabled. They are two controllable transitions and may be concurrently fired (if a user can execute them concurrently). After firing them and then firing t_7, t_4, and t_9, the system reaches its initial

* Corresponding author (email: liuguanjun@tongji.edu.cn)
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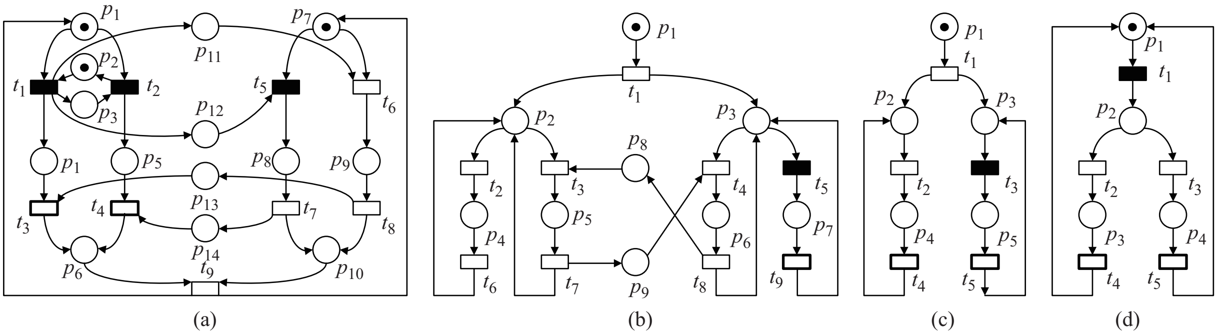

Figure 1 Four PN-cots: (a) L_3-o-live, (b) L_1-o-live, (c) and (d) L_2-o-live.

marking again. Therefore, this Petri net is observably live by their assumption. However, this Petri net is not observably live by their definition.

One of the possible reasons resulting in the above inconsistence is that under their assumption the modelling power of Petri nets with (un-) controllable and (un-) observable transitions is equal to that of Turing machines, but they hope to represent/characterise the observable behaviors of these Petri nets through the behaviors of traditional Petri nets whose modelling power is lower than Turing machines.

In order to coincide with the assumption proposed by Desel and Kilinc, we define Petri net with controllable and observable transitions (PN-cot) and propose the new rules of enabling and firing (un-) controllable and (un-) observable transitions. For some basic knowledge of Petri net, one can refer to [10].

Definition 1 (PN-cot)**.** (P, T, F) is a PN-cot if it satisfies the following conditions:

1. P is a non-empty and finite set of places, T is a non-empty and finite set of transitions, and $P \cap T = \emptyset$;
2. $F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs;
3. T is partitioned into two parts: T_O, the set of observable transitions, and T_U, the set of unobservable transitions. Furthermore, T_O is partitioned into two parts: T_C, the set of controllable transitions, and T_E, the set of observable but uncontrollable transitions.

In Definition 1, an observable but uncontrollable transition may be viewed as an export or output of a system, i.e., its occurrence is decided by the system but cannot be controlled directly by a user. One of the aims of observable liveness is that the occurrence of every observable uncontrollable transition depends on the occurrences of some controllable transitions. Therefore, all unobservable transitions are uncontrollable (from the user's point of view), which is coincident with the constraint in [4].

For convenience, a PN-cot is denoted as $(P, T_C \cup T_E \cup T_U, F)$. When no ambiguity is yielded, we use T to represent the set of all transitions in a PN-cot. A marking of PN-cot $(P, T_C \cup T_E \cup T_U, F)$ is a mapping M: $P \to \mathbb{N}$. A PN-cot with an initial marking is called a PN-cot system.

Definition 2 (Firable)**.** Given a PN-cot $(P, T_C \cup T_E \cup T_U, F)$ and a marking M:

1. An uncontrollable transition $t \in T_E \cup T_U$ is firable at M if $\forall p \in {}^\bullet t$: $M(p) > 0$. This is denoted as $M[t\rangle$. Notice that ${}^\bullet t$ and $t^\bullet$ represent the set of all input and output places of t, respectively.
2. A controllable transition $t \in T_C$ is firable at M if $\forall p \in {}^\bullet t$: $M(p) > 0$ and $\forall t' \in T_E \cup T_U$: $M[t'\rangle \Rightarrow {}^\bullet t \cap {}^\bullet t' = \emptyset$. This is also denoted as $M[t\rangle$.

Firing a firable transition t yields a new marking M', which is denoted as $M[t\rangle M'$, such that $M'(p) = M(p) - 1$ if $p \in {}^\bullet t \setminus t^\bullet$; $M'(p) = M(p) + 1$ if $p \in t^\bullet \setminus {}^\bullet t$; and $M'(p) = M(p)$ otherwise.

In order to characterize the observable interactional behaviors between a system and its users and to guarantee a system divergence-free and/or controllable, we propose three levels of observable liveness. For convenience, we use o-liveness and o-live to replace observable liveness and observably live, respectively. Additionally, we assume that a PN-cot has no self-loop in order to describe concurrence. The following notations are used in the definitions of three levels of observable liveness:

T^* is the set of all sequences of transitions in T, that includes the empty sequence and all infinite sequences. $|\sigma|$ is the length of σ where $\sigma \in T^*$. Especially, $|\sigma| = \infty$ if σ is an infinite sequence. $\text{Pre}(\sigma)$ is the set of all finite prefixes of σ, including the empty sequence. $\mathbb{T}(\sigma)$ is the set of all transitions occurring in σ. $\mathbb{O}(\sigma)$ is the set of all observable transitions occurring in σ. $\mathbb{C}(\sigma)$ is the set of all controllable transitions occurring in σ. Given a PN-cot system and a reachable marking M, an infinite sequence σ is firable at M if $\forall \sigma' \in \text{Pre}(\sigma)$: $M[\sigma'\rangle$, i.e., each finite prefix of σ is firable at M. This is still denoted by $M[\sigma\rangle$. Given a PN-cot system and a reachable marking M, a (finite or infinite) sequence σ and a transition t is concurrently firable at M, denoted by $M[\sigma \parallel t\rangle$ or $M[t \parallel \sigma\rangle$, if $\forall \sigma' \in \mathrm{Pre}(\sigma)$: $M[\sigma' t \sigma''\rangle$ where $\sigma = \sigma'\sigma''$.

Definition 3 (L_1-o-liveness)**.** A PN-cot system $(N, M_0) = (P, T_C \cup T_E \cup T_U, F, M_0)$ is L_1*-o-live* if $\forall t \in T_C \cup T_E$, $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$: $M'[t\rangle$.

L_1-o-liveness means that every observable transition has a (potential) firable chance at any reachable marking. For example, Figure 1(b) is L_1-o-live. We usually do not hope a system is sinked into an unobservable infinite loop, which is called *divergence* in the theory of process algebra [9]. But L_1-o-liveness allows the existence of divergence. For example, $t_2 t_6 t_2 t_6 \cdots$ in Figure 1(b) becomes an infinite unobservable firable sequence after t_1 is fired. Therefore, we define L_2-o-liveness in order to avoid divergence.

Definition 4 (L_2-o-liveness)**.** $(N, M_0) = (P, T_C \cup T_E \cup T_U, F, M_0)$ is L_2-o-live if

1. (N, M_0) is L_1-o-live;
2. $\forall M \in R(N, M_0)$, $\exists k \in \mathbb{N}$, $\forall \sigma \in T^*$: $(M[\sigma\rangle \wedge |\sigma| = k) \Rightarrow \mathbb{O}(\sigma) \neq \emptyset$.

L_2-o-liveness means that no matter which state a system is in, an observable event can always occur in finite steps. For example, the PN-cot systems in Figures 1 (c) and (d) are both L_2-o-live. The system in (d) may be thought of as a (nondeterministic) vending machine with tea.

After a user inserts a coin (which is represented by the controllable transition t_1), the machine will randomly select one from two cases (which are represented by the unobservable transitions t_2 and t_3). If it selects t_2, then a cup of tea is output (which is represented by the observable uncontrollable transition t_4). If it selects t_3, then the coin is returned (which is represented by the observable uncontrollable transition t_5).

According to the notion of weak bisimulation [8], however, the unobservable transitions t_2 and t_3 in Figure 1(d) are viewed as two internal nondeterministic activities. This will result in the nondeterminism of interactional behaviors between the system and its users, e.g., the machine chooses $t_3 t_5$ to return the coin while the user wants a cup of tea. In order to represent that observable activities of a system are always controllable by users and nondeterministic unobservable activities are avoided, we therefore define L_3-o-liveness.

Definition 5 (L_3-o-liveness)**.** $(N, M_0) = (P, T_C \cup T_E \cup T_U, F, M_0)$ is L_3-o-live if

1. (N, M_0) is L_2-o-live;
2. $\forall M \in R(N, M_0)$, $\exists k \in \mathbb{N}$, $\forall \sigma \in T^*$: $(M[\sigma\rangle \wedge |\sigma| = k) \Rightarrow \mathbb{C}(\sigma) \neq \emptyset$;
3. $\forall M \in R(N, M_0)$, $\exists k \in \mathbb{N}$, $\forall \sigma \in T^*$: $(M[\sigma\rangle \wedge |\sigma| = k \wedge \mathbb{O}(\sigma) \neq T_O) \Rightarrow ((\exists t \in T_C \setminus \mathbb{C}(\sigma), \exists \sigma' \in \mathrm{Pre}(\sigma)$: $M[\sigma' t\rangle) \vee (\exists t \in T \setminus \mathbb{T}(\sigma), \exists \sigma'' \in \mathrm{Pre}(\sigma)$: $M'[\sigma''' \parallel t\rangle))$, where $M[\sigma''\rangle M'$ and $\sigma = \sigma''\sigma'''$.

The second condition in Definition 5 implies that a system always requires an external input/control in finite steps, i.e., infinitely many occurrences of any uncontrollable transition must be dependent on infinitely many occurrences of some controllable transitions. For example, Figure 1(a) is L_3-o-live. We can prove that the observable liveness problem is undecidable and the proof is omitted due to the space limitation.

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