

# Price-based time and energy allocation in cognitive radio multiple access networks with energy harvesting

Ding XU\* & Qun LI

*Wireless Communication Key Lab of Jiangsu Province, Nanjing University of Posts and Telecommunications, Nanjing 210003, China*

Received January 17, 2017; accepted February 6, 2017; published online April 27, 2017

**Citation** Xu D, Li Q. Price-based time and energy allocation in cognitive radio multiple access networks with energy harvesting. *Sci China Inf Sci*, 2017, 60(10): 108302, doi: 10.1007/s11432-016-9016-4

Cognitive radio (CR), a concept that can solve the conflict between spectrum scarcity and low spectrum utilization, has been researched widely recently [1]. There are mainly three CR models: interweave, overlay and underlay [1]. Among the three CR models, the underlay model is the simplest and the easiest to implement in practice, where the secondary users (SUs) can transmit simultaneously on the same spectrum band with the primary users (PUs) as long as the interference power from the former to the later is under a certain threshold. Meanwhile, energy harvesting is a promising way to support continuous green energy supply to communication systems by continually harvesting energy from nature. There are a lot of works focusing on investigating the energy harvesting CR networks. Specifically, the works in [2,3] studied the harvesting-sensing-throughput tradeoff problems in the energy harvesting interweave CR networks, and the works in [4,5] studied the energy-data cooperation problems in the energy harvesting overlay CR networks. However, very few work focused on the energy harvesting underlay CR networks. Notably, under the energy causality constraint, ref. [6] used a geometric water-filling power allocation to maximize the SU throughput with the peak transmit power constraint, and ref. [7] proposed a robust power al-

location scheme to maximize the SU throughput with the interference power constraint.

In this article, we consider an underlay CR multiple access network where  $N$  SUs communicate to a cognitive base station (CBS) that shares a narrow spectrum band with the PU. The communications of the SUs are time-slotted with duration  $T$  and the inter-SU interference is avoided by allocating each SU a fraction of time in each time slot. The CBS is assumed to harvest energy from nature and then transfer the energy to the SUs through wired power lines. The CBS prices the interference power and allocated energy to regulate the interference and energy consumption of the SUs. The price-based joint time and energy allocation problem subject to the interference power and the energy causality constraints is studied. We formulate the problem as a Stackelberg game to jointly maximize the revenue of the CBS (the leader) and the individual utilities of the SUs (the followers). The Stackelberg equilibrium is then investigated. We derive the optimal interference price, the optimal energy price, the optimal time allocation at the CBS's side, and the optimal energy allocation at the SUs' side. It is shown that one SU occupying the entire time slot is optimal. It is also shown that the optimal interference price and energy price are not unique. It is noted that, to

\* Corresponding author (email: xuding.bupt@gmail.com)

The authors declare that they have no conflict of interest.

our best knowledge, resource allocation problems in energy harvesting underlay CR multiple access network have not been studied yet in literature.

*System model and problem formulation.* The channel power gains from SU  $i$  to the CBS and SU  $i$  to the PU are denoted by  $h_i$  and  $g_i$ , respectively. The noises power is denoted by  $\sigma^2$ . The total energy consumed by the SUs is limited by the amount of harvested energy in the CBS. For simplicity, we assume that energy is consumed only for wireless transmission and the total energy available for the SUs in a time slot is denoted by  $E_{\max}$ . The time fraction allocated to SU  $i$  is denoted as  $\tau_i$ , and the energy allocated to SU  $i$  is denoted as  $E_i$ . The energy causality constraint requires that  $\sum_{i=1}^N E_i \leq E_{\max}$ . Let  $p_i = \frac{E_i}{\tau_i}$  denote the transmit power of SU  $i$ . To protect the PU transmission, the interference power constraint requires that the interference power in each time slot from the SUs to the PU is restricted as  $\frac{1}{T} \sum_{i=1}^N \tau_i p_i g_i = \frac{1}{T} \sum_{i=1}^N E_i g_i \leq Q_{\max}$ , where  $Q_{\max}$  is the predefined interference power limit.

We assume that the CBS complies with the interference power and the energy causality constraints by pricing the interference power and the energy, and allocating proper time fractions to the SUs. The CBS aims to maximize its revenue and the SUs aim to maximize their individual utilities. We formulate the price-based resource allocation problem using the hierarchical Stackelberg game. At each time slot, the SUs (the followers) know the prices charged and time fraction allocated by the CBS (the leader) and the CBS knows the SUs' energy consumption.

The CBS decides the price  $\lambda_i$  per unit of interference power and the price  $\mu_i$  per unit of energy for SU  $i$  to maximize the revenue given by  $R(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\tau}, \mathbf{E}) = \frac{1}{T} \sum_{i=1}^N E_i g_i \lambda_i + \sum_{i=1}^N E_i \mu_i$ , where  $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_N]$ ,  $\boldsymbol{\mu} = [\mu_1, \dots, \mu_N]$ ,  $\boldsymbol{\tau} = [\tau_1, \dots, \tau_N]$  and  $\mathbf{E} = [E_1, \dots, E_N]$  are the interference power price vector, the energy price vector, the time fraction allocation vector and the energy allocation vector, respectively. The aim of the CBS is to find the optimal  $\boldsymbol{\lambda}$ ,  $\boldsymbol{\mu}$ ,  $\boldsymbol{\tau}$  such that its revenue is maximized under the interference power and the energy causality constraints. The problem at the CBS's side is formulated as (P1)

$$\max_{\boldsymbol{\lambda} \geq 0, \boldsymbol{\mu} \geq 0, \boldsymbol{\tau} \geq 0} R(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\tau}, \mathbf{E}) \quad (1)$$

$$\text{s.t. } \frac{1}{T} \sum_{i=1}^N E_i g_i \leq Q_{\max}, \quad (2)$$

$$\sum_{i=1}^N E_i \leq E_{\max}, \quad (3)$$

$$\sum_{i=1}^N \tau_i \leq T. \quad (4)$$

At the SUs' side, the utility of SU  $i$  is defined as  $U_i(E_i) = \nu_i \tau_i \ln(1 + \frac{E_i h_i}{\tau_i \sigma^2}) - \frac{E_i g_i \lambda_i}{T} - E_i \mu_i$ , where  $\nu_i$  denotes the unit rate utility gain for SU  $i$ . The utility of SU  $i$  consists of three parts. The first part is the profit for successfully transmission, the second and the third parts are the costs for causing interference power to the PU and consuming energy from the CBS, respectively. The problem at the side of SU  $i$  is formulated as (P2)

$$\max_{E_i \geq 0} U_i(E_i), i = 1, \dots, N. \quad (5)$$

P1 and P2 together form the Stackelberg game and we aim to find the Stackelberg equilibrium (SE) where neither the CBS nor the SUs have incentives to deviate. Since the utility of SU  $i$  does not depend on other SUs' energy consumption, the best response of each SU can be found by solving P2. Then, the best response of the CBS can be found by solving P1 given the best responses of the SUs. Thus, the SE for the formulated Stackelberg game can be found by solving P2 for given  $\boldsymbol{\lambda}$ ,  $\boldsymbol{\mu}$ ,  $\boldsymbol{\tau}$ , and then solving P1 with the obtained best responses of the SUs.

*Stackelberg equilibrium solution.* At the SUs' side, it can be easily verified that P2 is convex and thus the optimal solution of P2 is obtained by setting the first derivative of  $U_i(E_i)$  to zero as [8]

$$E_i^* = \tau_i \left( \frac{\nu_i T}{g_i \lambda_i + \mu_i T} - \frac{\sigma^2}{h_i} \right)^+, i = 1, \dots, N, \quad (6)$$

where  $(\cdot)^+ = \max(\cdot, 0)$ . It is observed from the above expression that if  $\lambda_i$  or  $\mu_i$  is too high, the SU's transmit power is zero, i.e., the SU is not allowed to transmit.

With the obtained best responses of the SUs given in (6), we solve P1 by exploring its special structure. The following two propositions give important properties of the solution of  $\boldsymbol{\tau}$  to P1 with  $E_i = E_i^*$ .

**Proposition 1.** For fixed values of  $\boldsymbol{\lambda}$  and  $\boldsymbol{\mu}$ , the optimal solution to P1 with  $E_i = E_i^*$  has at most one SU  $k$ ,  $k \in \{1, \dots, N\}$ , with  $\tau_k > 0$ .

*Proof.* Refer to Appendix A.

**Proposition 2.** The optimal solution to P1 with  $E_i = E_i^*$  has only one SU  $k$ ,  $k \in \{1, \dots, N\}$ , with  $\tau_k > 0$ .

*Proof.* Refer to Appendix B.

Based on Proposition 2, we can solve P1 with  $E_i = E_i^*$  by finding  $N$  optimal solutions, that is, one for each of the  $N$  SUs with allocated time fraction larger than zero, and then choosing the SU that maximizes the objective function in (1).

**Theorem 1.** Supposing that SU  $i$  is allocated with nonzero time fraction, i.e.,  $\tau_i > 0$  and  $\tau_j = 0, \forall j \neq i$ , then the optimal value of  $\tau_i$  to P1 with  $E_i = E_i^*$  is  $\tau_i^* = T$ , the optimal values of  $\lambda_i$  and  $\mu_i$  must satisfy

$$g_i \lambda_i^* + \mu_i^* T = \frac{\nu_i T}{\min\left(\frac{Q_{\max}}{g_i}, \frac{E_{\max}}{T}\right) + \frac{\sigma^2}{h_i}}, \quad (7)$$

and the maximum revenue of the CBS is  $\frac{\nu_i \min(\frac{Q_{\max} T}{g_i}, E_{\max})}{\min(\frac{Q_{\max}}{g_i}, \frac{E_{\max}}{T}) + \frac{\sigma^2}{h_i}}$ .

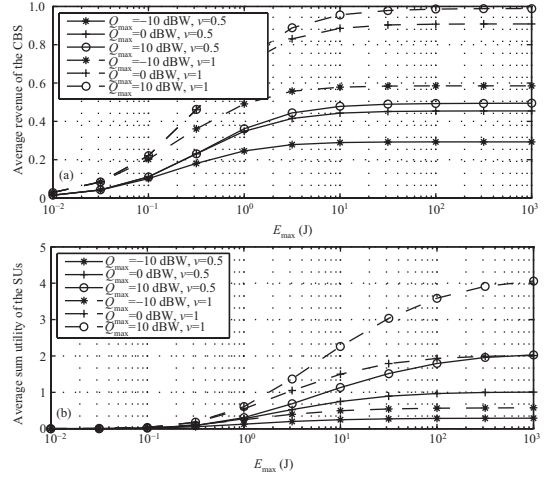
*Proof.* Refer to Appendix C.

Theorem 1 indicates that the optimal interference price and energy price are not unique and must satisfy the equality in (7). Based on Theorem 1, the SU indexed by  $k$  given as

$$k = \arg \max_{i \in \{1, \dots, N\}} \frac{\nu_i \min\left(\frac{Q_{\max} T}{g_i}, E_{\max}\right)}{\min\left(\frac{Q_{\max}}{g_i}, \frac{E_{\max}}{T}\right) + \frac{\sigma^2}{h_i}}, \quad (8)$$

is selected for allocating the entire time slot and the prices must satisfy the equality in (7).

*Simulation results.* All the channels involved are assumed to follow Rayleigh fading with unit mean. For simplicity, we assume that the unit rate utility gains for all the SUs are the same, i.e.,  $\nu_i = \nu, \forall i$ . In addition, we set  $\sigma^2 = 1$  and  $T = 1$ . Figure 1 illustrates the average revenue of the CBS and the average sum utility of the SUs against the energy causality constraint. It is seen that both the average revenue of the CBS and the average sum utility of the SUs increase as  $E_{\max}$  increases and then saturate when  $E_{\max}$  is large. It is also seen that the curves for different values of  $Q_{\max}$  almost overlap when  $E_{\max}$  is small. When  $E_{\max}$  is large, it is seen that higher value of  $Q_{\max}$  leads to higher average revenue of the CBS and higher average sum utility of the SUs. This indicates that the revenue of the CBS and the utilities of the SUs are constrained by  $E_{\max}$  when  $E_{\max}$  is small, and constrained by  $Q_{\max}$  when  $E_{\max}$  is large. In addition, it is seen that higher value of  $\nu$  leads to higher average revenue of the CBS and higher average sum utility of the SUs.



**Figure 1** (a) Average revenue of the CBS and (b) average sum utility of the SUs vs.  $E_{\max}$ .

**Acknowledgements** This work was supported by National Natural Science Foundation of China (Grant Nos. 61401218, 61571238), National Science and Technology Major Project (Grant No. 2017ZX030107001), and Natural Science Foundation of the Higher Education Institutions of Jiangsu Province (Grant No. 16KJA510005).

**Supporting information** Appendixes A–C. The supporting information is available online at [info.scichina.com](http://info.scichina.com) and [link.springer.com](http://link.springer.com). The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

## References

- Goldsmith A, Jafar S A, Marić I, et al. Breaking spectrum gridlock with cognitive radios: an information theoretic perspective. *Proc IEEE*, 2009, 97: 894–914
- Yin S X, Qu Z W, Li S F. Achievable throughput optimization in energy harvesting cognitive radio systems. *IEEE J Sele Areas Commun*, 2015, 33: 407–422
- Chung W, Park S, Lim S, et al. Spectrum sensing optimization for energy-harvesting cognitive radio systems. *IEEE Trans Wirel Commun*, 2014, 13: 2601–2613
- Yin S X, Zhang E Q, Qu Z W, et al. Optimal cooperation strategy in cognitive radio systems with energy harvesting. *IEEE Trans Wirel Commun*, 2014, 13: 4693–4707
- Zhai C, Liu J, Zheng L. Cooperative spectrum sharing with wireless energy harvesting in cognitive radio network. *IEEE Trans Veh Tech*, 2016, 65: 5303–5316
- He P, Zhao L. Optimal power control for energy harvesting cognitive radio networks. In: *Proceedings of IEEE International Conference on Communications*, London, 2015. 92–97
- Gong S, Duan L, Wang P. Robust optimization of cognitive radio networks powered by energy harvesting. In: *Proceedings of IEEE Conference on Computer Communications*, Toronto, 2015. 612–620
- Boyd S, Vandenberghe L. *Convex Optimization*. Cambridge: Cambridge University Press, 2004