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• **Supplementary File** •

Price-based time and energy allocation in cognitive radio multiple access networks with energy harvesting

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Appendix A Proof of Proposition 1

Inserting $E_i = \tau_i \left(\frac{\nu_i T}{g_i\lambda_i + \mu_i T} - \frac{\sigma^2}{h_i}\right)^+, i = 1, \ldots, N$ into P1, the problem at the CBS's side can be rewritten as

$$\max_{\boldsymbol{\lambda}\succeq 0,\boldsymbol{\mu}\succeq 0,\boldsymbol{\tau}\succeq 0} \sum_{i=1}^{N} \tau_i \left(\frac{g_i\lambda_i}{T} + \mu_i\right)\left(\frac{\nu_i T}{g_i\lambda_i + \mu_i T} - \frac{\sigma^2}{h_i}\right)^+ \tag{A1}$$

$$\text{s.t.} \sum_{i=1}^{N} \frac{g_i\tau_i}{T}\left(\frac{\nu_i T}{g_i\lambda_i + \mu_i T} - \frac{\sigma^2}{h_i}\right)^+ \leqslant Q_{max}, \tag{A2}$$

$$\sum_{i=1}^{N} \tau_i \left(\frac{\nu_i T}{g_i\lambda_i + \mu_i T} - \frac{\sigma^2}{h_i}\right)^+ \leqslant E_{max}, \tag{A3}$$

$$\sum_{i=1}^{N} \tau_i \leqslant T. \tag{A4}$$

If we fix the values of $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$, the problem (A1)-(A4) reduces to the following problem as

$$\max_{\boldsymbol{\tau}\succeq 0} \sum_{i=1}^{N} \left(\frac{1}{T} g_i\lambda_i + \mu_i\right) A_i\tau_i \tag{A5}$$

$$\text{s.t.} \sum_{i=1}^{N} \frac{1}{T} g_i A_i \tau_i \leqslant Q_{max}, \tag{A6}$$

$$\sum_{i=1}^{N} A_i\tau_i \leqslant E_{max}, \tag{A7}$$

$$\sum_{i=1}^{N} \tau_i \leqslant T, \tag{A8}$$

where $A_i = \left(\frac{\nu_i T}{g_i\lambda_i + \mu_i T} - \frac{\sigma^2}{h_i}\right)^+$. Since $\ln(x)$ is a monotonically increasing function with respect to x, the following problem has the same solution as the one for the above problem:

$$\max_{\boldsymbol{\tau}} \ln\left(\sum_{i=1}^{N} \left(\frac{1}{T} g_i\lambda_i + \mu_i\right) A_i\tau_i\right) \tag{A9}$$

$$\text{s.t.} \sum_{i=1}^{N} \frac{1}{T} g_i A_i \tau_i \leqslant Q_{max}, \tag{A10}$$

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$$\sum_{i=1}^{N} A_i\tau_i \leqslant E_{max}, \tag{A11}$$

$$\sum_{i=1}^{N} \tau_i \leqslant T, \tag{A12}$$

$$\tau_i \geqslant 0, \forall i \tag{A13}$$

It can be verified that the above problem is convex and thus can be solved by convex optimization approach. Define the Lagrangian as

$$L = \ln\left(\sum_{i=1}^{N}\left(\frac{1}{T}g_i\lambda_i + \mu_i\right)A_i\tau_i\right) - \alpha_1\left(\sum_{i=1}^{N}\frac{1}{T}g_iA_i\tau_i - Q_{max},\right)$$
$$- \alpha_2\left(\sum_{i=1}^{N}A_i\tau_i - E_{max}\right) - \alpha_3\left(\sum_{i=1}^{N}\tau_i - T\right) + \sum_{i=1}^{N}\alpha_4^i\tau_i, \tag{A14}$$

where α_1, α_2, α_3 and α_4^i are the non-negative Lagrange multipliers with respect to the constraints in (A10), (A11), (A12) and (A13), respectively. According to the KKT conditions [1], the optimal solution of the problem (A9)-(A13) must satisfy the following equations

$$\frac{\left(\frac{1}{T}g_j\lambda_j + \mu_j\right)A_j}{\sum_{i=1}^{N}\left(\frac{1}{T}g_i\lambda_i + \mu_i\right)A_i\tau_i} - \frac{1}{T}\alpha_1 g_jA_j - \alpha_2A_j - \alpha_3 + \alpha_4^j = 0, \forall j \tag{A15}$$

$$\alpha_4^j\tau_j = 0, \forall j \tag{A16}$$

$$\tau_j \geqslant 0, \forall j \tag{A17}$$

$$\alpha_4^j \geqslant 0, \forall j \tag{A18}$$

Assuming that there are two SUs n and m with $\tau_n > 0$ and $\tau_m > 0$, then we have $\alpha_4^n = \alpha_4^m = 0$ from (A16). Thus, inserting $\alpha_4^n = 0$ and $\alpha_4^m = 0$, respectively, into (A15), we have

$$\frac{\left(\frac{1}{T}g_n\lambda_n + \mu_n\right)A_n}{\sum_{i=1}^{N}\left(\frac{1}{T}g_i\lambda_i + \mu_i\right)A_i\tau_i} - \frac{1}{T}\alpha_1 g_nA_n - \alpha_2A_n - \alpha_3 = 0, \tag{A19}$$

$$\frac{\left(\frac{1}{T}g_m\lambda_m + \mu_m\right)A_m}{\sum_{i=1}^{N}\left(\frac{1}{T}g_i\lambda_i + \mu_i\right)A_i\tau_i} - \frac{1}{T}\alpha_1 g_mA_m - \alpha_2A_m - \alpha_3 = 0, \tag{A20}$$

from which we have

$$\frac{\left(\frac{1}{T}g_n\lambda_n + \mu_n\right)A_n}{\frac{1}{T}\alpha_1 g_nA_n + \alpha_2A_n + \alpha_3} = \frac{\left(\frac{1}{T}g_m\lambda_m + \mu_m\right)A_m}{\frac{1}{T}\alpha_1 g_mA_m + \alpha_2A_m + \alpha_3}. \tag{A21}$$

Supposing that $A_n = 0$ and $A_m = 0$, then the solution $\tau_n = 0$ and $\tau_m = 0$ provides the same objective function in (A9) as the solution $\tau_n > 0$ and $\tau_m > 0$ does. This contradicts our assumption that the solution $\tau_n > 0$ and $\tau_m > 0$ is optimal. Thus, we have $A_n = \frac{\nu_nT}{g_n\lambda_n+\mu_nT} - \frac{\sigma^2}{h_n} > 0$ and $A_m = \frac{\nu_mT}{g_m\lambda_m+\mu_mT} - \frac{\sigma^2}{h_m} > 0$. Since $\{h_i\}$ and $\{g_i\}$ are sets of independent random variables, the equality in (A21) is satisfied with a zero probability. Thus, it can be concluded that there is at most one SU k, $k \in \{1,\ldots,N\}$, with $\tau_k > 0$.

This completes the proof.

Appendix B Proof of Proposition 2

Based on Proposition 1, we conclude that the optimal solution to P1 with $E_i = E_i^*$ has zero or only one SU k, $k \in \{1,\ldots,N\}$, with $\tau_k > 0$. Supposing that the optimal solution to P1 with $E_i = E_i^*$ has all $\tau_i, \forall i$ being zero. Then, the objective function in (A1) is zero. However, if we randomly pick one τ_k, $k \in \{1,\ldots,N\}$, let it equal to 1, and decrease the values of λ_k and μ_k to let $\left(\frac{\nu_kT}{g_k\lambda_k+\mu_kT} - \frac{\sigma^2}{h_k}\right)^+ > 0$, then it is easy to see that the objective function in (A1) is larger than zero. This contradicts the assumption that all $\tau_i, \forall i$ being zero is optimal. Therefore, the optimal solution to P1.2 has only one SU k, $k \in \{1,\ldots,N\}$, with $\tau_k > 0$.

This completes the proof.

Appendix C Proof of Theorem 1

Supposing that $\tau_i > 0$ and $\tau_j = 0, \forall j \neq i$, then it is easy to observe from the objective function of P1 with $E_i = E_i^*$ that the optimal λ_i and μ_i must satisfy $\frac{\nu_iT}{g_i\lambda_i+\mu_iT} - \frac{\sigma^2}{h_i} > 0$ and P1 reduces to the following problem as

$$\max_{\lambda_i\geqslant 0,\mu_i\geqslant 0,\tau_i>0} \tau_i\left(\nu_i - \frac{\sigma^2}{h_iT}(g_i\lambda_i + \mu_iT)\right) \tag{C1}$$

$$\text{s.t.}\ \frac{1}{T}g_i\tau_i\left(\frac{\nu_iT}{g_i\lambda_i + \mu_iT} - \frac{\sigma^2}{h_i}\right) \leqslant Q_{max}, \tag{C2}$$

$$\tau_i \left(\frac{\nu_i T}{g_i \lambda_i + \mu_i T} - \frac{\sigma^2}{h_i} \right) \leqslant E_{max}, \tag{C3}$$

$$\frac{\nu_i T}{g_i \lambda_i + \mu_i T} - \frac{\sigma^2}{h_i} > 0 \tag{C4}$$

$$\tau_i \leqslant T. \tag{C5}$$

To solve the above problem, we first fix the value of τ_i to optimize the values of λ_i and μ_i. Combining the constraints in (C2), (C3), (C4), we have

$$\frac{\nu_i T}{\frac{1}{\tau_i} \min\left(\frac{Q_{max}T}{g_i}, E_{max}\right) + \frac{\sigma^2}{h_i}} \leqslant g_i \lambda_i + \mu_i T < \frac{\nu_i T h_i}{\sigma^2}. \tag{C6}$$

It is easy to observe that the objective function in (C1) is a decreasing function of $g_i \lambda_i + \mu_i T$. Thus, the optimal values of λ_i^* and μ_i^* must satisfy

$$g_i \lambda_i^* + \mu_i^* T = \frac{\nu_i T}{\frac{1}{\tau_i} \min\left(\frac{Q_{max}T}{g_i}, E_{max}\right) + \frac{\sigma^2}{h_i}}. \tag{C7}$$

Inserting the above expression into the problem (C1)-(C5), we have

$$\max_{0<\tau_i \leqslant T} \left(\frac{\nu_i \min\left(\frac{Q_{max}T}{g_i}, E_{max}\right)}{\frac{1}{\tau_i} \min\left(\frac{Q_{max}T}{g_i}, E_{max}\right) + \frac{\sigma^2}{h_i}} \right). \tag{C8}$$

It is observed that the objective function in (C8) is an increasing function of τ_i. Thus, the optimal solution of the above problem is $\tau_i^* = T$. Inserting $\tau_i^* = T$ into (C7), we have

$$g_i \lambda_i^* + \mu_i^* T = \frac{\nu_i T}{\min\left(\frac{Q_{max}}{g_i}, \frac{E_{max}}{T}\right) + \frac{\sigma^2}{h_i}}. \tag{C9}$$

Inserting $\tau_i^* = T$ and (C9) into (C1), we have the maximum revenue of the CBS as $\frac{\nu_i \min\left(\frac{Q_{max}T}{g_i}, E_{max}\right)}{\min\left(\frac{Q_{max}}{g_i}, \frac{E_{max}}{T}\right) + \frac{\sigma^2}{h_i}}$.

This completes the proof.

References

1 Boyd S, Vandenberghe L. Convex Optimization. Cambridge, U.K.: Cambridge Univ Press, 2004