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Iterative learning control approach for consensus of multi-agent systems with regular linear dynamics

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Dear editor,

Iterative learning control (ILC) has a well-established research history, as shown in [1,2]. By generating a correct control signal from the previous control execution, it can achieve perfect tracking performance on a finite time interval. In the process of ILC design, the D-type [1] and P-type learning schemes [2] are often used to obtain good tracking performance and are applied, respectively, to irregular systems (without direct transmission from inputs to outputs) [1] and regular systems (with direct transmission from inputs to outputs) [2].

In the past decades, studies on distributed cooperative control of multi-agent systems (MASs) or networks have attracted widespread attention from many researchers [3,4]. The consensus problem, wherein every agent asymptotically approaches a common value, has become a hot issue in the distributed coordination of MASs. Ref. [3] discussed the necessary and sufficient conditions for the consensusability of linear MASs. In [4], the consensus control was designed for a class of MASs with double-integrator dynamics and time delays. Recently, the consensus-based iterative learning control (ILC) protocols for the MASs with linear or nonlinear dynamics have been studied in [5,6], and the obtained ILC laws can guarantee that every agent converges uniformly to a common function on the finite time interval along the iteration axis. Ref. [5] discussed the consensus-based ILC problems for the MASs with general linear models, and Ref. [6] considered the formation control problems for MASs by using the ILC approach. It is worth noticing that, hitherto, the ILC for MASs has focused mainly on the MASs with irregular dynamics, irrespective of what it is for the continuous-time dynamics in [5] or for the discrete-time dynamics in [6]. Ref. [7] studied the consensus-based ILC problem for the MASs with regular linear dynamics, and obtained the consensus conclusions at the terminal time of the finite time interval. However, please note that the iterative learning protocols in [7] only solve the finite-time consensus problem just at the desired terminal output. This observation motivates our present study. As is well known, regular dynamics play an important role in the field of ILC [8].

In this letter, based on the ILC technique, the consensus problem of the MASs with regular linear dynamics on the finite time interval is proposed, and all the agents in the considered systems are governed by the regular continuous-time linear dynamics or discrete-time linear dynamics.

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A distributed consensus-based P-type ILC law is proposed, and when the ILC law is applied to the MASs, consensus on the finite time interval can be reached for all the directed communication graphs with spanning trees.

The following notations are adopted. I_m denotes the $m \times m$ dimensional identity matrix. $\otimes$ denotes the Kronecker product. 1_N denotes an N dimensional column vector with all components 1. $\mathbb{Z}^+$ denotes the nonnegative integer. For a given vector or matrix X, X^{T} denotes its transpose, $\|X\|$ denotes its any generic norm, and X^{-1} denotes its inverse matrix if X is square and nonsingular. For vector $x(t) \in \mathbb{R}^n$, $t \in [0,T]$, the supreme norm of $x(t)$, is defined as $\|x\|_s = \sup_{t\in[0,T]} \|x(t)\|$. For the given $\lambda > 0$, the λ-norm of $x(t)$ is defined as $\|x\|_\lambda = \sup_{t\in[0,T]} \mathrm{e}^{-\lambda t} \|x(t)\|$. From [2], we know that $\|x\|_s$ and $\|x\|_\lambda$ are equivalent, i.e., either of the norms can be used to prove the convergence.

Let $G = (V, E, A)$ be a weighted digraph with the set of vertices $V = \{1, 2, \ldots, N\}$ and the set of edges $E \subseteq V \times V$. $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix of the graph G. If $(j, i) \in E$, then $a_{ij} > 0$; otherwise $a_{ij} = 0$. Moreover, we assume that $a_{ii} = 0$. The in-degree of vertex i is defined as $\deg_{\mathrm{in}}(i) = \sum_{j=1}^{N} a_{ij}$ and the Laplacian of the weighted digraph G is defined as $L_G = D - A$, where $D = \mathrm{diag}\,(\deg_{\mathrm{in}}(1), \ldots, \deg_{\mathrm{in}}(N))$. The next lemma shows an important property of Laplacian L_G associated with G.

Lemma 1 ([4])**.** L_G has at least one zero eigenvalue with 1_N as its eigenvector, and all the nonzero eigenvalues of L_G have positive real parts. Laplacian L_G has a simple zero eigenvalue if and only if graph G has a spanning tree.

Similar to [3], denote

$$\alpha = (a_{12}, a_{13}, \ldots, a_{1N})^{\mathrm{T}},$$

$$L_{22} = \begin{pmatrix} \deg_{\mathrm{in}}(2) & -a_{23} & \cdots & -a_{2N} \\ -a_{32} & \deg_{\mathrm{in}}(3) & \cdots & -a_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N2} & -a_{N3} & \cdots & \deg_{\mathrm{in}}(N) \end{pmatrix},$$

and take

$$S = \begin{pmatrix} 1 & 0 \\ 1_{N-1} & I_{N-1} \end{pmatrix}.$$

By the definition of Laplacian, we have

$$S^{-1} L_G S = \begin{pmatrix} 0 & -\alpha^{\mathrm{T}} \\ 0 & L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}} \end{pmatrix}.$$

Then, by Lemma 1, the next Lemma is given as follows:

Lemma 2 ([3])**.** If digraph G has a spanning tree, then the eigenvalues of $L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}$ have positive real parts.

Consider the following MASs with N agents labeled 1 through N:

$$\begin{cases} ox_{i,k}(t) = Fx_{i,k}(t) + Bu_{i,k}(t), \\ y_{i,k}(t) = Cx_{i,k}(t) + Du_{i,k}(t), \end{cases} \tag{1}$$

$t \in [0, T]$, $i = 1, 2, \ldots, N$, where $k \in \mathbb{Z}^+$ represents the iteration index, $x_{i,k}(t) \in \mathbb{R}^n$, $u_{i,k}(t) \in \mathbb{R}^r$, $y_{i,k}(t) \in \mathbb{R}^m$ represent the state, control input, and output of the ith agent at the kth iteration, respectively, F, B, C, and D are matrices with appropriate dimensions, $m \leqslant r$, and D is full row rank. o is either the differential operator in the continuous-time domain, i.e., $ox_{i,k}(t) = \dot{x}_{i,k}(t)$, or the forward shift operator in the discrete-time domain, i.e., $ox_{i,k}(t) = x_{i,k}(t + 1)$ (see [9]). Please note that a unified representation is given by (1) for both continuous-time and discrete-time systems. For the discrete-time systems, t denotes the discrete-time index, and $t \in [0, T]$ denotes the integer sequence $t = 0, 1, 2, \ldots, T$. The network topology at the kth iteration is assumed given and is represented by adjacency matrix A or Laplacian L_G.

Assumption 1. The digraph G has a spanning tree.

Assumption 2. For each iteration index k, the initial value of the ith agent is always set to fixed value τ_i, i.e., $x_{i,k}(0) = \tau_i$, $i = 1, 2, \ldots, N$.

This study is intended at determining an appropriate learning scheme to solve the consensus problem such that

$$\lim_{k\to\infty} \|y_{i,k} - y_{j,k}\|_s = 0, \quad \forall i, j \in \{1, 2, \ldots, N\}. \tag{2}$$

Construct the distributed P-type ILC law for the system (1) as follows:

$$u_{i,k+1}(t) = u_{i,k}(t) + \Gamma \sum_{j=1}^{N} a_{ij}(y_{j,k}(t) - y_{i,k}(t)), \quad i = 1, 2, \ldots, N, \tag{3}$$

where $\Gamma \in \mathbb{R}^{r\times m}$ is the gain matrix. Take Γ such that

$$\rho \left[I_{(N-1)m} - (I_{N-1} \otimes (D\Gamma)) \times ((L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}) \otimes I_m) \right] < 1 \tag{4}$$

holds, where ρ denotes the spectral radius.

Remark 1. If the eigenvalues of $L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}$ have positive real parts, the gain matrix Γ satisfying (4) exists.

Denote $\delta_{i,k}(t) = y_{i,k}(t) - y_{1,k}(t)$, $i = 1, 2, 3,$

$\dots, N$. Then, (2) is equivalent to $\|\delta_{i,k}\|_s \to 0$, $k \to \infty$, $i = 2, 3, \dots, N$.

From (1) and (3), we have

$$\begin{aligned}\delta_{i,k+1}(t) &= y_{i,k}(t) - y_{1,k}(t) + y_{i,k+1}(t) - y_{i,k}(t) \\ &\quad - (y_{1,k+1}(t) - y_{1,k}(t)) \\ &= \delta_{i,k}(t) + C\Delta x_{i,k}(t) + D\Gamma \sum_{j=1}^{N} a_{ij}(\delta_{j,k}(t) - \delta_{i,k}(t)) \\ &\quad - C\Delta x_{1,k}(t) - D\Gamma \sum_{j=1}^{N} a_{1j}\delta_{j,k}(t), \quad i = 2, 3, \dots, N,\end{aligned} \tag{5}$$

where $\Delta x_{i,k}(t) = x_{i,k+1}(t) - x_{i,k}(t)$, $i = 1, 2, \dots, N$. Denoting

$$\delta_k(t) = \left[\, \delta_{2,k}^{\mathrm{T}}(t) \ \delta_{3,k}^{\mathrm{T}}(t) \ \cdots \ \delta_{N,k}^{\mathrm{T}}(t) \,\right]^{\mathrm{T}},$$

$$\Delta x_k(t) = \left[\, \Delta x_{2,k}^{\mathrm{T}}(t) \ \Delta x_{3,k}^{\mathrm{T}}(t) \ \cdots \ \Delta x_{N,k}^{\mathrm{T}}(t) \,\right]^{\mathrm{T}},$$

we write (5) for all agents in the compact form and obtain

$$\begin{aligned}\delta_{k+1}(t) &= \left[I_{(N-1)m} - (I_{N-1} \otimes (D\Gamma)) \times \right. \\ &\qquad \left. ((L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}) \otimes I_m)\right] \delta_k(t) \\ &\quad + (I_{N-1} \otimes C)\left[\Delta x_k(t) - 1_{N-1} \otimes \Delta x_{1,k}(t)\right].\end{aligned} \tag{6}$$

It follows from (1) and (3) that

$$\begin{aligned}& o[\Delta x_{i,k}(t)] \\ &\quad = F\Delta x_{i,k}(t) + B(u_{i,k+1}(t) - u_{i,k}(t)) \\ &\quad = F\Delta x_{i,k}(t) + B\Gamma \sum_{j=1}^{N} a_{ij}(\delta_{j,k}(t) - \delta_{i,k}(t)), \\ &\quad\ i = 1, 2, \dots, N.\end{aligned}$$

Further we have

$$\begin{aligned}& o\left[\Delta x_{i,k}(t) - \Delta x_{1,k}(t)\right] = F\left[\Delta x_{i,k}(t) - \Delta x_{1,k}(t)\right] \\ &\quad + B\Gamma \sum_{j=1}^{N} a_{ij}(\delta_{j,k}(t) - \delta_{i,k}(t)) - B\Gamma \sum_{j=1}^{N} a_{1j}\delta_{j,k}(t), \\ &\quad i = 2, 3, \dots, N.\end{aligned}$$

The above equalities can be written in the compact form as

$$\begin{aligned}& o\left[\Delta x_k(t) - 1_{N-1} \otimes \Delta x_{1,k}(t)\right] = \\ & (I_{N-1} \otimes F)\left[\Delta x_k(t) - 1_{N-1} \otimes \Delta x_{1,k}(t)\right] \\ & - (I_{N-1} \otimes (B\Gamma))((L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}) \otimes I_m)\delta_k(t).\end{aligned} \tag{7}$$

Lemma 3 ([10])**.** For any given matrix $M \in \mathbb{R}^{n\times n}$, if spectral radius $\rho(M) < 1$, then there exists at least one matrix norm $\|\cdot\|_\gamma$ such that $\|M\|_\gamma < \rho(M) + \varepsilon < 1$.

Lemma 4 ([10])**.** For any matrix norm $\|\cdot\|_\gamma$, there exists at least one compatible vector norm $\|\cdot\|_\xi$ such that for any $F \in \mathbb{R}^{n\times n}$ and $x \in \mathbb{R}^n$, $\|Fx\|_\xi \leqslant \|F\|_\gamma \|x\|_\xi$.

In the sequel, we denote both the vector norm and its compatible matrix norm in Lemma 4 by $\|\cdot\|$.

For $ox_{i,k}(t) = \dot{x}_{i,k}(t)$, we have:

Theorem 1. Let system (1) satisfy Assumptions 1 and 2, and (4) hold true, then consensus is reached under the effect of learning scheme (3), i.e., $\lim_{k\to\infty} \|y_{i,k} - y_{j,k}\|_s = 0$, $\forall i, j \in \{1, 2, \dots, N\}$.

For $ox_{i,k}(t) = x_{i,k}(t+1)$, we have:

Theorem 2. Let system (1) satisfy Assumptions 1 and 2, then consensus is reached under the effect of learning scheme (3) if and only if (4) holds.

The proof of Theorems 1 and 2 and the simulation examples are included in supporting information.

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