SCIENCE CHINA
Information Sciences

• **Supplementary File** •

Iterative learning control approach for consensus of multi-agent systems with regular linear dynamics

Qin FU*, Panpan GU, Xiangdong LI & Jianrong WU

School of Mathematics and Physics, Suzhou University of Science and Technology, Suzhou 215009*, China*

Appendix A Proof of Theorem 1

From (4) and Lemma 3, we have

$$\bar{\rho} = \left\| I_{(N-1)m} - (I_{N-1} \otimes (D\Gamma))((L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}) \otimes I_m) \right\| < 1. \tag{A1}$$

Integrating (7) and combining with Assumptions 2, it yields

$$\Delta x_k(t) - 1_{N-1} \otimes \Delta x_{1,k}(t) = \int_0^t (I_{N-1} \otimes F) \left[\Delta x_k(\eta) - 1_{N-1} \otimes \Delta x_{1,k}(\eta)\right] \mathrm{d}\eta$$
$$- \int_0^t (I_{N-1} \otimes (B\Gamma))((L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}) \otimes I_m)\delta_k(\eta)\mathrm{d}\eta.$$

Further we have

$$\left\| \Delta x_k - 1_{N-1} \otimes \Delta x_{1,k} \right\|_\lambda \leqslant O(\lambda^{-1}) \left\| \delta_k \right\|_\lambda, \tag{A2}$$

where

$$O(\lambda^{-1}) = \frac{c_1 \dfrac{1 - \mathrm{e}^{-\lambda T}}{\lambda}}{1 - c_2 \dfrac{1 - \mathrm{e}^{-\lambda T}}{\lambda}}, \quad c_1 = \left\| (I_{N-1} \otimes (B\Gamma))((L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}) \otimes I_m) \right\|, \quad c_2 = \left\| I_{N-1} \otimes F \right\|.$$

Therefore, by (6) and (A2), we obtain

$$\left\| \delta_{k+1} \right\|_\lambda \leqslant \hat{\rho} \left\| \delta_k \right\|_\lambda, \tag{A3}$$

where

$$\hat{\rho} = \bar{\rho} + c_3 O(\lambda^{-1}), \quad c_3 = \left\| I_{N-1} \otimes C \right\|.$$

Since $0 \leqslant \bar{\rho} < 1$ (by A1)), it is possible to choose λ sufficiently large such that $\hat{\rho} < 1$. Then, (A3) is a contraction for $\left\| \delta_k \right\|_\lambda$. Therefore

$$\lim_{k \to \infty} \left\| \delta_k \right\|_\lambda = 0.$$

This completes the proof.

Appendix B Proof of Theorem 2

Denote $\Omega = I_{(N-1)m} - (I_{N-1} \otimes (D\Gamma))((L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}) \otimes I_m)$ and $\tilde{\Delta} x_k(t) = \Delta x_k(t) - 1_{N-1} \otimes \Delta x_{1,k}(t)$. Taking $t = 0, 1, 2, \cdots, T$ in (6), we have

$$\begin{bmatrix} \delta_{k+1}(0) \\ \delta_{k+1}(1) \\ \vdots \\ \delta_{k+1}(T) \end{bmatrix} = \begin{bmatrix} \Omega & 0 & \cdots & 0 \\ 0 & \Omega & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Omega \end{bmatrix} \begin{bmatrix} \delta_k(0) \\ \delta_k(1) \\ \vdots \\ \delta_k(T) \end{bmatrix} + \begin{bmatrix} I_{N-1} \otimes C & 0 & \cdots & 0 \\ 0 & I_{N-1} \otimes C & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_{N-1} \otimes C \end{bmatrix} \begin{bmatrix} \tilde{\Delta} x_k(0) \\ \tilde{\Delta} x_k(1) \\ \vdots \\ \tilde{\Delta} x_k(T) \end{bmatrix}. \tag{B1}$$

* Corresponding author (email: fuqin925@sina.com)

Denoting $H = (I_{N-1} \otimes (B\Gamma))((L_{22} + 1_{N-1} \cdot \alpha^{\mathrm{T}}) \otimes I_m)$, then (7) can be written as

$$\tilde{\Delta} x_k(t+1) = (I_{N-1} \otimes F)\tilde{\Delta} x_k(t) - H\delta_k(t), \quad t = 0, 1, 2, \cdots, T-1.$$

Notice that $\tilde{\Delta} x_k(0) = 0$ by Assumptions 2, and write the above equalities in the following compact form:

$$\begin{bmatrix} \tilde{\Delta} x_k(0) \\ \tilde{\Delta} x_k(1) \\ \vdots \\ \tilde{\Delta} x_k(T) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ I_{N-1} \otimes F & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & I_{N-1} \otimes F & 0 \end{bmatrix} \begin{bmatrix} \tilde{\Delta} x_k(0) \\ \tilde{\Delta} x_k(1) \\ \vdots \\ \tilde{\Delta} x_k(T) \end{bmatrix} - \begin{bmatrix} 0 & 0 & \cdots & 0 \\ H & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & H & 0 \end{bmatrix} \begin{bmatrix} \delta_k(0) \\ \delta_k(1) \\ \vdots \\ \delta_k(T) \end{bmatrix}.$$

Further we have

$$\begin{aligned} \begin{bmatrix} \tilde{\Delta} x_k(0) \\ \tilde{\Delta} x_k(1) \\ \vdots \\ \tilde{\Delta} x_k(T) \end{bmatrix} &= - \begin{bmatrix} I_{(N-1)n} & 0 & \cdots & 0 \\ -I_{N-1} \otimes F & I_{(N-1)n} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & -I_{N-1} \otimes F & I_{(N-1)n} \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & \cdots & 0 \\ H & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & H & 0 \end{bmatrix} \begin{bmatrix} \delta_k(0) \\ \delta_k(1) \\ \vdots \\ \delta_k(T) \end{bmatrix} \\ &= - \begin{bmatrix} 0 & 0 & \cdots & 0 \\ H & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ (I_{N-1} \otimes F)^{T-1} H & \cdots & H & 0 \end{bmatrix} \begin{bmatrix} \delta_k(0) \\ \delta_k(1) \\ \vdots \\ \delta_k(T) \end{bmatrix}. \end{aligned}$$

Substituting the above equality into (B1), it yields

$$\begin{bmatrix} \delta_{k+1}(0) \\ \delta_{k+1}(1) \\ \vdots \\ \delta_{k+1}(T) \end{bmatrix} = \begin{bmatrix} \Omega & 0 & \cdots & 0 \\ -(I_{N-1} \otimes C)H & \Omega & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ -(I_{N-1} \otimes C)(I_{N-1} \otimes F)^{T-1} H & \cdots & -(I_{N-1} \otimes C)H & \Omega \end{bmatrix} \begin{bmatrix} \delta_k(0) \\ \delta_k(1) \\ \vdots \\ \delta_k(T) \end{bmatrix}.$$

So it can be concluded that

$$\lim_{k \to \infty} \begin{bmatrix} \delta_k(0) \\ \delta_k(1) \\ \vdots \\ \delta_k(T) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

holds if and only if $\rho(\Omega) < 1$. This completes the proof.

Appendix C Simulation examples

We consider the following four agents of the linear dynamics:

$$\begin{cases} o x_{i,k}(t) = F x_{i,k}(t) + b u_{i,k}(t) \\ y_{i,k}(t) = c x_{i,k}(t) + d u_{i,k}(t) \end{cases}, \quad t \in [0, 10], \quad i = 1, 2, 3, 4,$$

where

$$F = \begin{bmatrix} -10 & 0.2 \\ -0.3 & -8 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad c = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad d = 1.$$

Taking the adjacency matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

then it is obvious that the corresponding digraph G has a spanning tree. It is easily verify that (4) holds when $0 < \Gamma < 1$, therefore, we take $\Gamma = 0.5$. Furthermore, take initial values at the kth iteration: $x_{1,k}(0) = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}$, $x_{2,k}(0) = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix}$, $x_{3,k}(0) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, $x_{4,k}(0) = \begin{bmatrix} 0.6 \\ 0.6 \end{bmatrix}$, and take the initial controls: $u_{1,0}(t) = u_{2,0}(t) = u_{3,0}(t) = u_{4,0}(t) \equiv 0$. Under the effect of the learning scheme (3), we have the following:

Case 1: For the continuous-time systems, the simulation results of $\|\delta_{i,k}\|_s$ and $y_{i,k}(t)$ with the change of k are shown in Figs. C1-C4.

Case 2: For the discrete-time systems, the simulation results of $\|\delta_{i,k}\|_s$ and $y_{i,k}(t)$ with the change of k are shown in Figs. C5-C8.

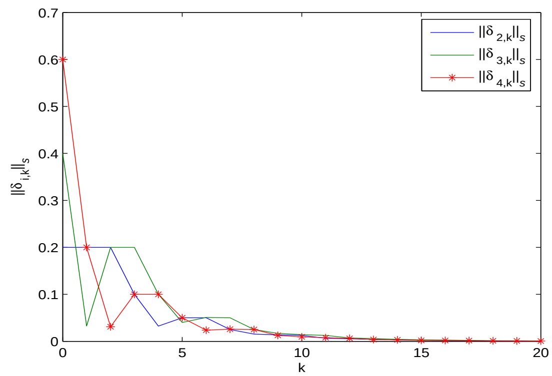

Figure C1 Simulation results with iteration $k = 20$.

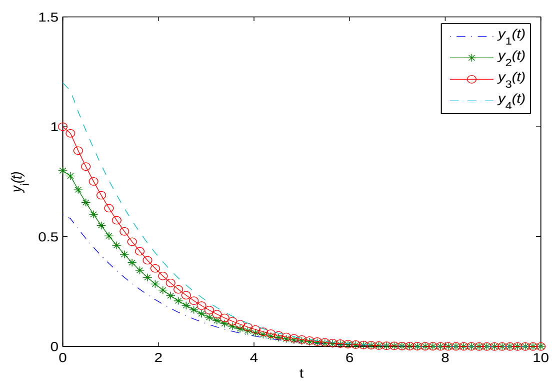

Figure C2 Output trajectories at iteration $k = 1$.

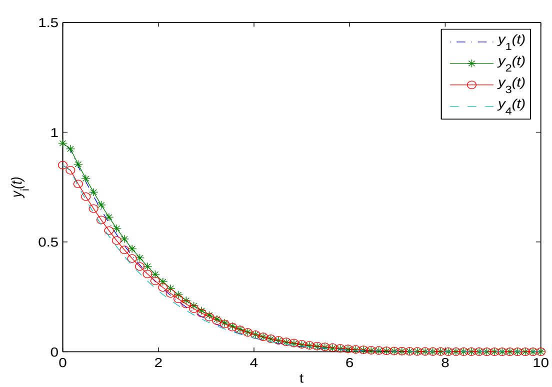

Figure C3 Output trajectories at iteration $k = 4$.

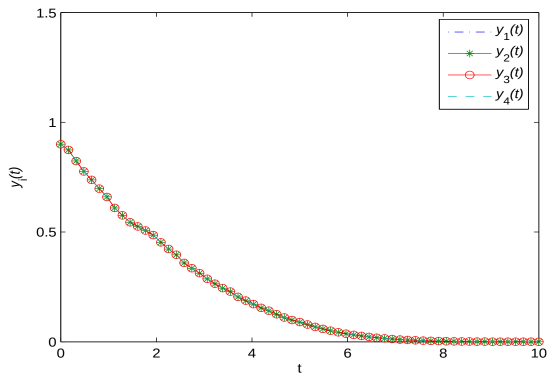

Figure C4 Output trajectories at iteration $k = 20$.

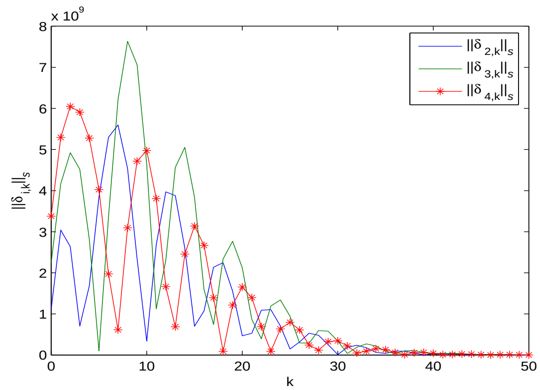

Figure C5 Simulation results with iteration $k = 50$.

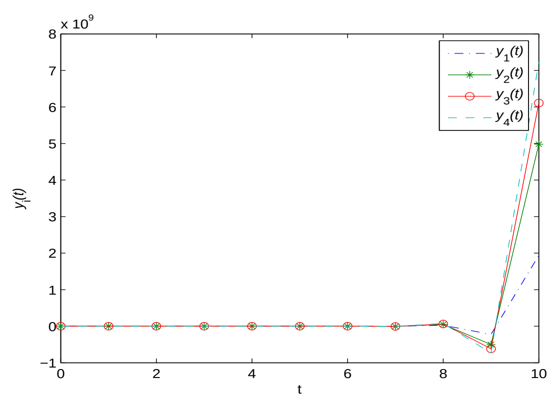

Figure C6 Output trajectories at iteration $k = 1$.

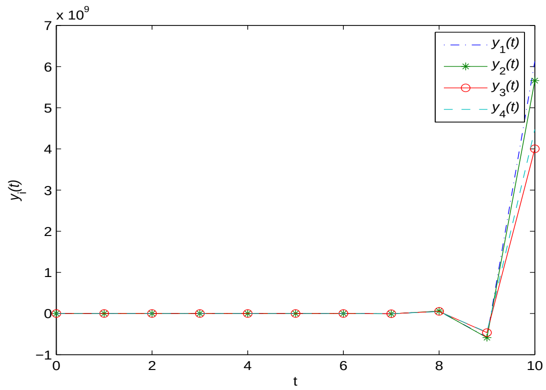

Figure C7 Output trajectories at iteration $k = 20$.

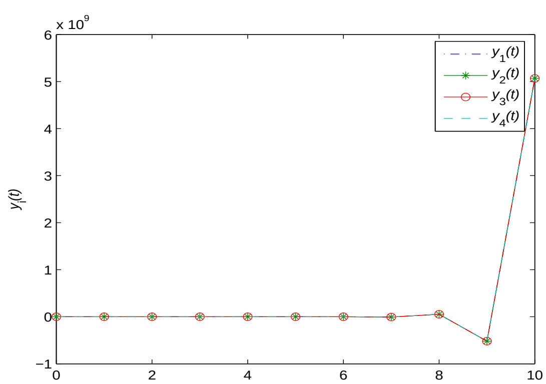

Figure C8 Output trajectories at iteration $k = 50$.