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Characteristic model-based control of robotic manipulators with dynamic uncertainties

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Dear editor,

The extensive applications of robotic manipulators have spurred active research on its control problem during the past decades. Since parametric uncertainties are inevitable in many occasions, the development of effective adaptive controllers is of great importance for versatile applications of high-speed and high-precision robots.

To keep stability and compensate for unknown parameters, many adaptive control schemes for robotic manipulators have been proposed in the literature, and the majority of representative work focuses on the continuous-time domain. Nevertheless, in real implementations, the computers are usually utilized to produce digital control signals, proposing the necessity for controller design in discrete time. Characteristic modeling theory is a very effective discrete-time control method, bearing the simplicity of design, convenience of adjustment and strong robustness validated by practice. Different from the traditional modeling strategy which captures the plant dynamics as precise as possible, characteristic modeling is a control oriented modeling approach taking both the dynamic characteristics of the controlled plant and the performance specifications of the control system into account [1]. Characteristic modeling theory and characteristic model-based adaptive controller have already been applied successfully to more than 400 systems in the field of astronautics and industry, especially, in the reentry lift control of Shenzhou spacecraft [1], rendezvous and docking of Shenzhou spacecraft [2], and reentry guidance of Chang'e 5 tester [3], achieving the control accuracy at the world leading level.

In this letter, we deal with the discrete-time adaptive regulation problem of robotic manipulators via characteristic model-based control scenario. After characteristic modeling of dynamics of robot, the nonlinearity in the centripetal and Coriolis matrix of dynamics of robot makes the bounds of the characteristic model parameters rely on the states, resulting in great difficulties for controller design and stability analysis. To solve this, we propose a state-relied projection estimation algorithm, and construct an adaptive control law via introducing a discrete-time sliding vector, where the system is separated into two subsystems, i.e., the kinematic module and the dynamic module, cascaded by the sliding vector. Moreover, performance analysis for the complicated nonlinear closed-loop dynamics is performed by proposing a recursive induction technique, which shows that our scenario guarantees the asymptotic convergence of the joint position and the boundedness of the coefficients of the characteristic model as

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well. Finally, we conduct simulations to validate the effectiveness of our theoretical scheme.

Characteristic modeling of the robot. The dynamic model of the robotic manipulator ignoring gravitational forces can be written as [4]

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau, \tag{1}$$

where $q \in \Re^n$ is the joint position, $H(q) \in \Re^{n\times n}$ is the inertia matrix of the robot, $C(q,\dot{q}) \in \Re^{n\times n}$ is the coupled centripetal and Coriolis matrix and $\tau \in \Re^n$ is the exerted joint torque to the manipulator. The dynamics (1) bears the following property [4].

Property 1. For some positive constants k_{m1}, k_{m2} and k_C, we have $0 < k_{m1} \leqslant \|H(q)\| \leqslant k_{m2}$, $\|C(y_1,y_2)z\| \leqslant k_C\|y_2\|\|z\|$ for all vectors $y_1, y_2, z \in \Re^n$.

Now, we discretize (1) via the Euler discretization method [1] and by simple manipulation, we derive the typical two-order characteristic model as

$$\begin{aligned} q(k+1) = {} & F_{C1}(k)q(k) + F_{C2}(k)q(k-1) \\ & + G_{C0}(k)\tau(k), \end{aligned} \tag{2}$$

where $F_{C1}(k) \in \Re^{n\times n}$, $F_{C2}(k) \in \Re^{n\times n}$ and $G_{C0}(k) \in \Re^{n\times n}$ are coefficient matrices, described as

$$\begin{aligned} & F_{C1}(k) = 2I_n - T_sH^{-1}(q(k))C\left(q(k), \frac{e(k)}{T_s}\right), \\ & F_{C2}(k) = -I_n + T_sH^{-1}(q(k))C\left(q(k), \frac{e(k)}{T_s}\right), \\ & G_{C0}(k) = T_s^2H^{-1}(q(k)). \end{aligned} \tag{3}$$

Here, $e(k) = q(k) - q(k-1)$, and T_s is the sampling time. By (3) and Property 1, we get the properties of the coefficient matrices as follows.

Lemma 1. The coefficient matrix $G_{C0}(k)$ is symmetric and positive definite, and $b_0 \leqslant \|G_{C0}(k)\| = T_s^2\|H^{-1}(q(k))\| \leqslant b_1$, where $b_0 = k_{m2}^{-1}T_s^2$, $b_1 = k_{m1}^{-1}T_s^2$, and k_{m1} are k_{m2} are given in Property 1.

Lemma 2. The coefficient matrix $F_{C2}(k)$ satisfies

$$\|I_n + F_{C2}(k)\| \leqslant k_e\|e(k)\|, \tag{4}$$

where $k_e = \sqrt{n}k_{m1}^{-1}k_C$, k_C is introduced in Property 1, and I_n denote the $n \times n$ identity matrix.

Lemma 3. The addition of $F_{C1}(k)$ and $F_{C2}(k)$ satisfies

$$F_{C1}(k) + F_{C2}(k) = I_n. \tag{5}$$

As is known, the initial position and velocity of the robot are bounded, i.e., there exist constants $0 \leqslant Q^* < \infty$ and $0 \leqslant V^* < \infty$, such that

$$\|q(0)\| \leqslant Q^*, \quad \|v(0)\| \leqslant V^*, \tag{6}$$

where $v(0) = \frac{e(0)}{T_s}$ denotes the initial velocity.

Main result. Inspired by the continuous-time sliding vector design [5], we construct the kinematic part as

$$s(k+1) = q(k+1) - \epsilon q(k), \tag{7}$$

where $\epsilon = 1 - T_s$. Substituting (2) into (7), we get the dynamic module

$$\begin{aligned} s(k+1) = {} & -F_{C2}(k)(s(k) - T_sq(k-1)) \\ & + T_sq(k) + G_{C0}(k)\tau(k). \end{aligned} \tag{8}$$

The controller is then proposed as

$$\begin{aligned} \tau(k) = {} & \hat{G}_{C0}(k)^{-1}\big(-T_sq(k) \\ & + \hat{F}_{C2}(k)s(k) - T_s\hat{F}_{C2}(k)q(k-1)\big), \end{aligned} \tag{9}$$

where $\hat{F}_{C2}(k)$ and $\hat{G}_{C0}(k)$, respectively, denote the estimate of $F_{C2}(k)$ and $G_{C0}(k)$, updated by the estimation law given in the subsequent Steps 1–3. Before showing the updating algorithm, we first reformulate (2) as

$$e(k+1) = -F_{C2}(k)e(k) + G_{C0}(k)\tau(k). \tag{10}$$

Based on (10), we introduce $\Theta(k) = [F_{C2}(k) \;\; G_{C0}(k)]^{\mathrm{T}}$, $\hat{\Theta}(k) = [\hat{F}_{C2}(k) \;\; \hat{G}_{C0}(k)]^{\mathrm{T}}$, $\Phi(k) = [-e(k)^{\mathrm{T}} \;\; \tau(k)^{\mathrm{T}}]^{\mathrm{T}}$. Then, it is time for us to give the estimation steps as follows

Step 1. Utilize the classical gradient law to estimate the coefficients $F_{C2}(k)$ and $G_{C0}(k)$ based on (2), i.e.

$$\begin{aligned} \hat{\Theta}^o(k) = {} & \hat{\Theta}(k-1) \\ & + \frac{\lambda_1\Phi(k-1)(e(k) - \hat{\Theta}(k-1)^{\mathrm{T}}\Phi(k-1))^{\mathrm{T}}}{\lambda_2 + \Phi(k-1)^{\mathrm{T}}\Phi(k-1)}, \end{aligned} \tag{11}$$

where $0 < \lambda_1 < 1$, $\lambda_2 > 0$, and $\hat{\Theta}^o(k)$ is the output of the gradient law, which can be denoted as $\hat{\Theta}^o(k) = [\hat{F}_{C2}^o(k) \;\; \hat{G}_{C0}^o(k)]^{\mathrm{T}}$ with $\hat{F}_{C2}^o(k)$ and $\hat{G}_{C0}^o(k)$ being the estimate of $F_{C2}(k)$ and $G_{C0}(k)$, respectively.

Step 2. Formulate $\hat{\hat{G}}_{C0}(k)$ to be a symmetric matrix by setting

$$\hat{\hat{G}}_{C0}(k) = \frac{\hat{G}_{C0}^o(k) + \hat{G}_{C0}^o(k)^{\mathrm{T}}}{2}. \tag{12}$$

Step 3. Project $(\hat{F}_{C2}^o(k), \hat{\hat{G}}_{C0}(k))$ into the closed convex set

$$\begin{aligned} & D_s(k) = \\ & \quad \big\{(\hat{F}_{C2}(k), \hat{G}_{C0}(k)) \big| \|\hat{F}_{C2}(k) + I_n\| \leqslant k_e\|e(k)\|, \end{aligned}$$

$$B_0 I_n \leqslant \hat{G}_{C0}(k) \leqslant B_1 I_n \big\}, \tag{13}$$

where B_1 and B_0 are adjustable constants satisfying $b_0 \leqslant B_0 \leqslant \frac{1}{2}(b_0 + b_1) \leqslant B_1 \leqslant b_1$, and b_0, b_1 and k_e are given in Property 1 and Lemma 1.

Stability analysis. Now, we proceed to discuss the property of the closed-loop system based on the proposed controller and estimation law. Substituting (9) into (8), we get the closed-loop dynamics

$$\begin{aligned} s(k+1) &= -\Delta\bar{\theta}_2(k)s(k) + T_s\Delta\bar{\theta}_1(k)q(k) \\ &\quad + T_s\Delta\bar{\theta}_2(k)q(k-1), \end{aligned} \tag{14}$$

where $\Delta\bar{\theta}_1(k) = G_{C0}(k)[G_{C0}(k)^{-1} - \hat{G}_{C0}(k)^{-1}]$, and $\Delta\bar{\theta}_2(k) = G_{C0}(k)[G_{C0}(k)^{-1}F_{C2}(k) - \hat{G}_{C0}(k)^{-1}\hat{F}_{C2}(k)]$. By Lemma 1, Lemma 2 and (13), the properties of $\Delta\bar{\theta}_1(k)$ and $\Delta\bar{\theta}_2(k)$ can be summarized as below.

Lemma 4. If $B_0 = B_1 = \frac{1}{2}(b_0 + b_1)I_n$, then

$$\|\Delta\bar{\theta}_1(k)\| \leqslant b_2,$$

where $b_2 = \frac{b_1 - b_0}{b_1 + b_0}$, b_1 and b_0 are introduced in Lemma 1, and B_0 and B_1 are introduced in (13).

Lemma 5. $\Delta\bar{\theta}_2(k)$ satisfies

$$\|\Delta\bar{\theta}_2(k)\| \leqslant b_2 + b_3 k_e \|e(k)\|,$$

where $b_3 = 2 + b_2$, k_e and b_2 are defined in Lemma 2 and Lemma 4, respectively.

The proof of Lemma 4 and Lemma 5 are given in Appendix A and Appendix B.

Now, let us show the stability result.

Theorem 1. For the characteristic model of a robotic manipulator represented by (2), if $B_0 = B_1 = \frac{1}{2}(b_0 + b_1)I_n$, and

$$\frac{b_1}{b_0} < 1 + \mu, \tag{15}$$

$$T_s < \frac{\mu - (\mu + 2)b_2}{\mu + b_3 k_e (2\mu + 1)^2 (Q^* + V^*)}, \tag{16}$$

where k_e, b_i, $i = 0, \ldots, 3$, are given in Lemmas 1−5, respectively, $0 < \mu \leqslant 1$ is an adjustable constant, and Q^* and V^* are given in (6), then the adaptive control law (9) and the updating law (11)−(13) ensure

$$\|e(k)\| \leqslant T_s(\mu + 1 + T_s\mu)\|q(0)\| + T_s(\mu + c^*)\|v(0)\|,$$

where $c^* = \max\{T_s(1 + \mu), \mu\}$. Furthermore, the uniform ultimate boundedness (UUB) of $q(k)$ is guaranteed, and

$$\lim_{k\to\infty} \|q(k)\| \leqslant \mu[\|q(0)\| + \|v(0)\|]. \tag{17}$$

Theorem 2. For the robotic characteristic model (2), the controller (9) and the updating law (11)−(12) guarantee the asymptotic convergence of $q(k)$, i.e., $\lim_{k\to\infty} q(k) = 0$, provided that

$$\begin{aligned} B_0 = B_1 &= \tfrac{1}{2}(b_0 + b_1)I_n, \\ \mu &< \min\left\{1, \frac{1}{12k_e(Q^* + V^*)}\right\}, \end{aligned} \tag{18}$$

$$\frac{b_1}{b_0} < \min\left\{1 + \mu, 1 + T_s\frac{1 - 12\mu k_e(Q^* + V^*)}{1 + 6\mu T_s k_e(Q^* + V^*)}\right\}, \tag{19}$$

and the sampling time T_s satisfies (16).

The proof of Theorem 1 and Theorem 2 are demonstrated in Appendix C and Appendix D, respectively. Our control protocol is testified by numerical simulation, shown in Appendix E.

Conclusion. In this letter, we address the adaptive regulation problem of robotic manipulators with uncertain dynamics by use of characteristic model theory. Based on the multi-variable characteristic model of the robot, we propose a state-dependant projection estimation law, solving the problem caused by the nonlinear centripetal and Coriolis matrix. Then, we develop a characteristic model-based adaptive control law by constructing a discrete-time sliding vector. The stability analysis is also given and joint positions are guaranteed to be asymptotically convergent. Finally, numerical simulations are performed on a robotic manipulator to validate the effectiveness of the proposed strategy.

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Supporting information Appendixes A–E. The supporting information is available online at info.scichina.com and link.springer.com. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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