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Robust semi-global leader-following practical consensus of a group of linear systems with imperfect actuators

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Abstract This paper considers the problem of semi-global leader-following consensus of a multi-agent system whose agent dynamics are represented by linear systems. The input output characteristics of the follower agent actuators, such as those of saturation and dead-zone, are imperfect, not precisely known, and subject to the effect of disturbances. Two consensus control algorithms, of the low-and-high gain feedback type and the low gain based variable structure control type, are proposed for solving the consensus problem. It is shown that both of these control algorithms achieve semi-global leader-following practical consensus in the presence of the imperfectness of the actuators when the communication topology among the follower agents is represented by a strongly connected and detailed balanced directed graph and the leader agent is a neighbor of at least one follower agent. The theoretical results are illustrated by numerical simulation.

Keywords consensus, leader-following, actuator saturation, dead-zone, low-and-high gain feedback, low gain based variable structure control

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1 Introduction

Multi-agent systems, inspired by the phenomenon of cluster in nature, have found applications in many fields, including computer science, physics, biology, and control engineering. Through the information interaction among agents, cooperative control can be achieved in multi-agent systems. As a fundamental problem in cooperative control, the consensus problem has attracted a large amount of attention. Numerous results are available in the literature (see, e.g., [1–7]). Consensus means that all agents in the group converge to an agreement state by using only their neighbors' information. Among the many applications of the consensus problem are mobile robots [8, 9], autonomous underwater vehicles [10], unmanned air vehicles [11–13] and sensor networks [14].

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Because of the ubiquity of actuator nonlinearities and uncertainties in control systems, it is necessary to study the consensus problem in the presence of the imperfectness of the actuator input output characteristics. Indeed, much effort has been made to address this theoretical challenge and much progress has been made (see, e.g., [15–20]).

Ref. [16] studied the global leader-following consensus problem for two classes of linear systems in the presence of actuator saturation. For multi-agent systems whose agent dynamics are neurally stable or are those of double integrators, global leader-following consensus is shown to be achievable for both a fixed and a switching communication topology. The extension of the results in [16] to the discrete-time setting was made in [17]. The global leader-following consensus problem for a group of general linear systems was solved in [18]. The only assumption made on the linear systems is that they are asymptotically null controllable with bounded controls (ANCBC). A linear system is ANCBC if it is stabilizable and its poles are located in the closed left-half plane. Distributed nonlinear feedback control laws were constructed and shown to achieve global leader-following consensus as long as the communication topology among follower agents is a strongly connected and detailed balanced directed graph and the leader agent is a neighbor of at least one follower agent.

Ref. [15] considered the semi-global leader-following consensus problem of a group of general linear ANCBC systems in the presence of actuator saturation and on either a connected or jointly connected network. The low gain feedback design technique [21] was adopted to construct a consensus algorithm, parameterized in the low gain parameter, that achieves semi-global leader-following consensus. Ref. [19] investigated the semi-global leader-following consensus problem for a group of general linear ANCBC systems that are simultaneously subject to actuator position and rate saturation. Both a family of linear low gain state feedback control laws and a family of linear low gain output feedback control laws, both parameterized in the low gain parameter, were constructed for each agent. These feedback laws were shown to achieve semi-global leader-following consensus as along as the communication topology among follower agents is a connected undirected graph and the leader agent is a neighbor of at least one follower agent.

In the literature on the multi-agent consensus problem, actuator nonlinearity, when taken into consideration, is usually represented by a standard saturation function. In the real world, actuator nonlinearity is often more complex than a standard saturation function and is also often not precisely known. It could reflect the characteristics of both a saturation function and a deadzone function. The actuator might also be subject to disturbances that are superimposed on the input signal. As a result, it is important to account of such imperfectness of the actuator input output characteristics and input additive disturbances in considering the consensus problem. In fact, the imperfectness of the actuator input output characteristics and input additive disturbances have been considered in the context of semi-global stabilization [22]. As seen in [22], the main challenge in dealing with such imperfectness of the actuator input output characteristics is due to the possible co-existence of saturation nonlinearity, dead-zone nonlinearity and disturbances. To meet this challenge, two control design approaches, the low-and-high gain feedback design approach [23, 24] and the low gain based variable structure control design approach, were proposed to achieve robust semi-global practical stabilization, which requires the state of the system be driven, in a finite time, into a pre-specified arbitrarily small neighborhood of the origin and remain there. In the situation when the actuator nonlinearity is represented by a standard saturation and dead-zone functions and is precisely known, the low-and-high gain feedback design proposed in [22] was adopted to achieve semi-global leader-following consensus for a group of linear systems linked by an undirected graph [25]. Both semi-global stabilization and semi-global leader-following consensus require the systems to be asymptotically null controllable with bounded controls.

In this paper, we study the problem of semi-global leader-following consensus of a group of general linear ANCBC systems in the presence of imperfect actuator input output characteristics. Two consensus control algorithms, the low-and-high gain based consensus algorithms and the low gain based variable structure consensus algorithms, are proposed for each follower agent. We will show that these consensus algorithms achieve robust semi-global leader-following practical consensus when the communication topology among follower agents is a strongly connected and detailed balanced directed graph and the leader agent is a neighbor of at least one follower agent.

The organization of the remainder of the paper is as follows. Section 2 contains preliminaries, which include the basic definitions and notations in graph theory. Section 3 describes the properties of the imperfect actuator input output characteristics and states the problem of robust semi-global leader-following practical consensus. Sections 4 and 5 construct two different consensus algorithms for the follower agents and show that these consensus algorithms achieve robust semi-global leader-following practical consensus under certain assumptions on the communication topology. Section 6 contains simulation results for the two consensus algorithms proposed in Sections 4 and 5 to illustrate the theoretical results. Section 7 draws a brief conclusion to the paper.

2 Preliminaries

A multi-agent system consisting of N agents can be represented by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_N\}$ is a finite, non-empty set of nodes, each representing a follower agent, and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is a set of edges, each representing an ordered pair of nodes. An edge (ν_i, ν_j) in a directed graph represents that agent ν_j has access to the information of agent ν_i. Let $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ be the adjacency matrix associated with $\mathcal{G}$. If $(\nu_j, \nu_i) \in \mathcal{E}$, then $a_{ij} > 0$, otherwise $a_{ij} = 0$. We also assume that $a_{ii} = 0$ for all $i = 1, 2, \ldots, N$. Let $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ be the Laplacian matrix associated with $\mathcal{A}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$.

A directed graph is strongly connected if there exists a directed path between any pair of distinct nodes [26]. A directed graph is said to be detailed balanced if there exist some real numbers $\omega_i > 0$, $i = 1, 2, \ldots, N$, such that the coupling weights of the graph satisfy $\omega_i a_{ij} = \omega_j a_{ji}$ for all $i, j = 1, 2, \ldots, N$. We call these numbers $\omega_1, \omega_2, \ldots, \omega_N$ the detailed balance parameters associated with $\mathcal{G}$ [27].

In addition to the N follower agents, there is also a leader agent, denoted as ν_0. The communication between the follower agents and the leader agent is represented by a_{i0}, where $a_{i0} > 0$, if agent i has access to the information of the leader agent, and $a_{i0} = 0$ otherwise.

3 Problem statement

Consider a network of N agents, each described by the following linear system in the presence of actuator nonlinearity and input additive disturbances,

$$\dot{x}_i = Ax_i + B\sigma\left(u_i + d_i(t)\right), \ i = 1, 2, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state of agent i, $u_i \in \mathbb{R}^m$ is the control input of agent i, $d_i(t)$ is the disturbance superimposed on the input of agent i and σ is a function that represents actuator nonlinearities.

Assumption 1. The matrix pair (A, B) is asymptotically null controllable with bounded controls (ANCBC), that is, it is with all eigenvalues of A located on the closed left-half plane.

Assumption 1 is necessary for the solution of the robust semi-global consensus problem to be formulated below. Such an assumption is known to be necessary even for semi-global stabilization of an individual system subject to actuator saturation [21].

Next, we will list the properties of the function σ.

Definition 1. A function σ: $\mathbb{R}^m \to \mathbb{R}^m$ is called an actuator nonlinearity function if

1. $\sigma(u)$ is decentralized, i.e., $\sigma(u) = [\sigma_1(u_1), \sigma_2(u_2), \ldots, \sigma_m(u_m)]^{\mathrm{T}}$;

2. σ_j, $j = 1, 2, \ldots, m$, is locally Lipschitz;

3. $s\sigma_j(s) \geqslant 0$, $\forall s \in \mathbb{R}$;

4. $\liminf_{|s|\to\infty} |\sigma_j(s)| > 0$.

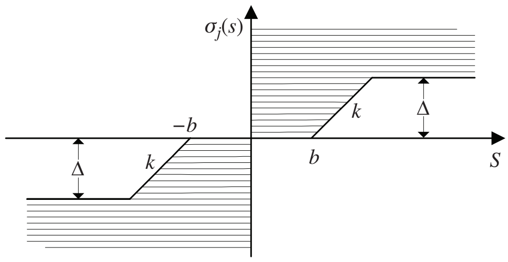

Figure 1 The actuator input-output characteristics.

Remark 1. 1. The actuator nonlinearity function $\sigma_j(s)$ lies in the shaded area in Figure 1, which is characterized by the constants $\Delta > 0$, $b \geqslant 0$ and $k > 0$. Among these three constants, Δ represents the saturation level, b the dead-zone break points and k is the slope. Without loss of generality, we assume that $k = 1$. Let $\mathrm{sat}_\Delta(s) = \mathrm{sgn}(s)\min\{\Delta, |s|\}$. Then, for each $\sigma_j(s)$,

$$s\left[\sigma_j(rs) - \mathrm{sat}_\Delta(s)\right] \geqslant 0, \tag{2}$$

whenever $|rs| \geqslant b + |s|,\ r \in \mathbb{R}^+,\ s \in \mathbb{R}$.

2. Because σ_j is locally Lipschitz, there exists a continuous and nondecreasing function $\varpi : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$|\sigma_j(s)| \leqslant \varpi(|s|),\ j = 1, 2, \ldots, m. \tag{3}$$

Remark 2. The standard saturation functions, the ideal deadzone functions, the standard saturations with ideal dead-zone characteristics and functions $\sigma(t) = t$, $\tanh(t)$ and $\arctan(t)$ all satisfy the definition of the actuator nonlinearity special cases of the actuator nonlinearity, as defined in Definition 1.

The function $d_i(t),\ i = 1, 2, \ldots, m$, is the input additive disturbance. Only an upper bound on the norm of the disturbance d_i is required.

Assumption 2. The $d_i(t), i = 1, 2, \ldots, N$, is a bounded piecewise continuous function, that is,

$$\|d_i(t)\| \leqslant D_0,\ t \geqslant 0, \tag{4}$$

where D_0 is a known nonnegative constant.

Definition 2. The set of all functions $\sigma : \mathbb{R}^m \to \mathbb{R}^m$ that satisfy properties (2) and (3) for some constants Δ, b and $k = 1$, and a function $\varpi(s) : \mathbb{R}_+ \to \mathbb{R}_+$ that satisfies $\varpi(s) \leqslant \varpi_0(s)$, $\forall s \in \mathbb{R}_+$ with a nondecreasing $\varpi_0 : \mathbb{R}_+ \to \mathbb{R}_+$, is denoted by $\mathscr{S}(\Delta, b, \varpi_0)$.

Definition 3. The set of data $(\Delta, b, \varpi_0, D_0, \chi, \chi_0)$ is said to be admissible if $\Delta > 0$, $b \geqslant 0$, $\varpi_0 : \mathbb{R}_+ \to \mathbb{R}_+$ is a nondecreasing function, $D_0 \geqslant 0$, $\chi \subset \mathbb{R}^n$ is a bounded set and the origin is an interior point of $\chi_0 \subset \mathbb{R}^n$.

The following autonomous linear system represents the dynamics of the leader agent,

$$\dot{x}_0 = Ax_0, \tag{5}$$

where the state $x_0 \in \mathbb{R}^n$.

The communication topology $\mathcal{G}$ among the follower agents satisfies the following assumption.

Assumption 3. The graph $\mathcal{G}$ is directed, strongly connected and detailed balanced, and $a_{i0} > 0$ for at least one i, $i = 1, 2, \ldots, N$.

Let $M = \mathcal{L} + \mathrm{diag}\{a_{10}, a_{20}, \ldots, a_{N0}\}$ and $\mathrm{diag}\{\omega\} = \mathrm{diag}\{\omega_1, \omega_2, \ldots, \omega_N\}$.

Lemma 1. Under Assumption 3, the matrix $\mathrm{diag}\{\omega\}M$ is symmetric and positive definite.

The above lemma can be easily derived based on the analysis given in the proof of Lemma 4 in [28]. Let $\lambda_1, \lambda_2, \ldots, \lambda_N$ be the eigenvalues of $\mathrm{diag}\{\omega\}M$, and $\underline{\lambda} = \min\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$. By Lemma 1, $\underline{\lambda} > 0$.

Let the error between follower agent i and the leader agent be denoted as $\tilde{x}_i = x_i - x_0$. Then, we have

$$\dot{\tilde{x}}_i = A\tilde{x}_i + B\sigma\left(u_i + d_i(t)\right).$$

Let $\tilde{x}=\left[\tilde{x}_1^{\mathrm{T}},\tilde{x}_2^{\mathrm{T}},\ldots,\tilde{x}_N^{\mathrm{T}}\right]^{\mathrm{T}}\in\mathbb{R}^{nN}$. Then,

$$\dot{\tilde{x}}=(I_N\otimes A)\,\tilde{x}+(I_N\otimes B)\,\sigma\,(u+d(t))\,,$$

where $u=[u_1^{\mathrm{T}},u_2^{\mathrm{T}},\ldots,u_N^{\mathrm{T}}]^{\mathrm{T}}$ and $d(t)=[d_1^{\mathrm{T}},d_2^{\mathrm{T}},\ldots,d_N^{\mathrm{T}}]^{\mathrm{T}}$.

We now state the problem to be studied in this paper as follows.

Problem 1 (Robust semi-global leader-following practical consensus)**.** Consider the multi-agent system that consists of N follower agents (1) and a leader agent (5). Let the communication topology satisfy Assumption 3. Given the admissible set of data $(\Delta,b,\varpi_0,D_0,\chi,\chi_0)$, construct, for each follower agent i, a state feedback law $u_i=F_i(x_0,x_1,\ldots,x_N)$, which uses the information of agent i's neighbors obtained through the communication network, such that, under these feedback laws, the trajectories of all error states $\tilde{x}_i$, $i=1,2,\ldots,N$, with any $\sigma\in\mathscr{S}(\Delta,b,\omega_0)$, any disturbances $d_i(t)$, $i=1,2,\ldots,N$, that satisfy Assumption 2 with D_0, and any initial conditions $x_i(0)\in\chi$, $i=0,1,\ldots,N$, will enter χ_0 and remain in it in a finite time.

4 Low-and-high gain feedback based consensus algorithms

In this section, we construct the following consensus algorithms based on the low-and-high gain feedback design technique for each of the follower agents,

$$u_i=-\omega_i(1+\rho)B^{\mathrm{T}}P(\varepsilon)\left(\sum_{j=1}^{N}a_{ij}\,(x_i-x_j)+a_{i0}\,(x_i-x_0)\right),\ i=1,2,\ldots,N,\tag{6}$$

in which ω_i is a detailed balance parameter, a_{ij} is the (i,j)th entry of the adjacency matrix $\mathcal{A}$, a_{i0} is the communication weight between follower agent i and the leader agent, $\varepsilon\in(0,1]$ and $\rho\geqslant 0$ are respectively the low gain parameter and the high gain parameter, and $P(\varepsilon)$ is the unique solution to the following parametric algebraic Riccati equation (ARE),

$$A^{\mathrm{T}}P(\varepsilon)+P(\varepsilon)A-2\gamma P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)+\varepsilon I=0,\tag{7}$$

where the low gain parameter $\varepsilon\in(0,1]$ and γ is any scalar such that $\gamma\in(0,\underline{\lambda}]$.

Lemma 2 ([21, Lemma 2.2.6])**.** Under Assumption 1, there exists a unique matrix $P(\varepsilon)>0$, for each $\varepsilon\in(0,1]$, that solves the ARE (7). Moreover, $\lim_{\varepsilon\to 0}P(\varepsilon)=0$.

The following result establishes that the low-and-high gain feedback consensus algorithms (6) solve Problem 1.

Theorem 1. There exists an $\varepsilon^*\in(0,1]$, and for any $\varepsilon\in(0,\varepsilon^*]$, there exists a $\rho^*(\varepsilon)>0$ such that, for any $\rho\geqslant\rho^*(\varepsilon)$, $\varepsilon\in(0,\varepsilon^*]$, the consensus algorithms (6) solve Problem 1.

Proof. Notice that, the controller (6) can be rewritten as

$$u_i=-(1+\rho)B^{\mathrm{T}}P(\varepsilon)\omega_i\left(\sum_{j=1}^{N}a_{ij}\,(\tilde{x}_i-\tilde{x}_j)+a_{i0}\tilde{x}_i\right),\ i=1,2,\ldots,N,$$

or, in a compact form, as

$$u=-(1+\rho)\left(\mathrm{diag}\{\omega\}M\otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x}.$$

Consequently, we have the following the closed-loop system,

$$\dot{\tilde{x}}=(I_N\otimes A)\,\tilde{x}+(I_N\otimes B)\,\sigma\left(-(1+\rho)\left(\mathrm{diag}\{\omega\}M\otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x}+d(t)\right).$$

Consider the following Lyapunov function

$$V(\tilde{x})=\tilde{x}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M\otimes P(\varepsilon)\right)\tilde{x},$$

which is positive definite since both $\mathrm{diag}\{\omega\}M$ and $P(\varepsilon)$ are positive definite matrices. Choose a $c>0$ such that

$$\sup_{\varepsilon\in(0,1],x_i\in\chi,i=0,1,2,\ldots,N} \tilde{x}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M\otimes P(\varepsilon)\right)\tilde{x}\leqslant c.$$

Such a c exists since $\lim_{\varepsilon\to 0}P(\varepsilon)=0$, by Lemma 2, and χ is a bounded set. Let $\varepsilon^*\in(0,1]$ be such that, for any $\varepsilon\in(0,\varepsilon^*]$, $\tilde{x}\in L_V(c):=\{\tilde{x}\in\mathbb{R}^{nN}:V(\tilde{x})\leqslant c\}$ implies that

$$\|\left(\mathrm{diag}\{\omega\}M\otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x}\|\leqslant\Delta.$$

Again, such an ε^* exists since $\lim_{\varepsilon\to 0}P(\varepsilon)=0$.

According to Lemma 1, $\mathrm{diag}\{\omega\}M$ is a positive definite matrix. Thus, there exists an orthogonal matrix $C\in\mathbb{R}^{N\times N}$ such that

$$\mathrm{diag}\{\omega\}M=C^{\mathrm{T}}\mathrm{diag}\{\lambda_1,\lambda_2,\ldots,\lambda_N\}C.$$

For notational brevity, we will hereafter denote $\Lambda=\mathrm{diag}\{\lambda_1,\lambda_2,\ldots,\lambda_N\}$.

The evaluation of the derivative of $V(\tilde{x})$ along the closed-loop trajectory inside the level set $L_V(c)$ is carried out as follows,

$$\begin{aligned}\dot{V}&=\dot{\tilde{x}}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M\otimes P(\varepsilon)\right)\tilde{x}+\tilde{x}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M\otimes P(\varepsilon)\right)\dot{\tilde{x}}\\&=\tilde{x}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M\otimes\left(P(\varepsilon)A+A^{\mathrm{T}}P(\varepsilon)\right)\right)\tilde{x}\\&\quad+2\tilde{x}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M\otimes P(\varepsilon)B\right)\sigma\left(-(1+\rho)\left(\mathrm{diag}\{\omega\}M\otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x}+d(t)\right)\\&=\tilde{x}^{\mathrm{T}}\left(\left(C^{\mathrm{T}}\Lambda C\right)\otimes\left(P(\varepsilon)A+A^{\mathrm{T}}P(\varepsilon)\right)\right)\tilde{x}\\&\quad+2\tilde{x}^{\mathrm{T}}\left(\left(C^{\mathrm{T}}\Lambda C\right)\otimes P(\varepsilon)B\right)\sigma\left(-(1+\rho)\left(\left(C^{\mathrm{T}}\Lambda C\right)\otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x}+d(t)\right)\\&=\tilde{x}^{\mathrm{T}}\left(C^{\mathrm{T}}\otimes I_n\right)\left(\Lambda\otimes\left(P(\varepsilon)A+A^{\mathrm{T}}P(\varepsilon)\right)\right)\left(C\otimes I_n\right)\tilde{x}\\&\quad+2\tilde{x}^{\mathrm{T}}\left(C^{\mathrm{T}}\otimes I_n\right)\left(\Lambda C\otimes P(\varepsilon)B\right)\sigma\left(-(1+\rho)\left(\left(C^{\mathrm{T}}\Lambda\right)\otimes B^{\mathrm{T}}P(\varepsilon)\right)\left(C\otimes I_n\right)\tilde{x}+d(t)\right).\end{aligned}$$

Let $\hat{x}=\left[\hat{x}_1^{\mathrm{T}},\hat{x}_2^{\mathrm{T}},\ldots,\hat{x}_N^{\mathrm{T}}\right]^{\mathrm{T}}=(C\otimes I_n)\tilde{x}$. Then, the evaluation of $\dot{V}$ is continued as

$$\begin{aligned}\dot{V}&=\hat{x}^{\mathrm{T}}\left(\Lambda\otimes\left(P(\varepsilon)A+A^{\mathrm{T}}P(\varepsilon)\right)\right)\hat{x}+2\hat{x}^{\mathrm{T}}\left((\Lambda C)\otimes P(\varepsilon)B\right)\sigma\left(-(1+\rho)\left(\left(C^{\mathrm{T}}\Lambda\right)\otimes B^{\mathrm{T}}P(\varepsilon)\right)\hat{x}+d(t)\right)\\&=\hat{x}^{\mathrm{T}}\left(\Lambda\otimes\left(P(\varepsilon)A+A^{\mathrm{T}}P(\varepsilon)\right)-2\Lambda^2\otimes P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\right)\hat{x}\\&\quad+2\hat{x}^{\mathrm{T}}\left(\Lambda C\otimes P(\varepsilon)B\right)\left(\sigma\left(-(1+\rho)\left(C^{\mathrm{T}}\Lambda\otimes B^{\mathrm{T}}P(\varepsilon)\right)\hat{x}+d(t)\right)+(C^{\mathrm{T}}\Lambda\otimes B^{\mathrm{T}}P(\varepsilon))\hat{x}\right)\\&=\hat{x}^{\mathrm{T}}\left(\Lambda\otimes I_n\right)\left(I_N\otimes\left(P(\varepsilon)A+A^{\mathrm{T}}P(\varepsilon)\right)-2\Lambda\otimes P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\right)\hat{x}\\&\quad+2\hat{x}^{\mathrm{T}}\left(\Lambda C\otimes P(\varepsilon)B\right)\left(\sigma\left(-(1+\rho)\left(C^{\mathrm{T}}\Lambda\otimes B^{\mathrm{T}}P(\varepsilon)\right)\hat{x}+d(t)\right)+(C^{\mathrm{T}}\Lambda\otimes B^{\mathrm{T}}P(\varepsilon))\hat{x}\right)\\&=\sum_{i=1}^{N}\lambda_i\hat{x}_i^{\mathrm{T}}\left(P(\varepsilon)A+A^{\mathrm{T}}P(\varepsilon)-2\lambda_iP(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\right)\hat{x}_i\\&\quad+2\hat{x}^{\mathrm{T}}\left(\Lambda C\otimes P(\varepsilon)B\right)\left(\sigma\left(-(1+\rho)\left(C^{\mathrm{T}}\Lambda\otimes B^{\mathrm{T}}P(\varepsilon)\right)\hat{x}+d(t)\right)+(C^{\mathrm{T}}\Lambda\otimes B^{\mathrm{T}}P(\varepsilon))\hat{x}\right).\end{aligned}$$

Since $\gamma\leqslant\underline{\lambda}$, according to (7), we have

$$\begin{aligned}&A^{\mathrm{T}}P(\varepsilon)+P(\varepsilon)A-2\lambda_iP(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\\&\qquad\leqslant A^{\mathrm{T}}P(\varepsilon)+P(\varepsilon)A-2\gamma P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)=-\varepsilon I,\ i=1,2,\ldots,N.\end{aligned}\tag{8}$$

Define

$$\Upsilon=-\left(\mathrm{diag}\{\omega\}M\otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x}=-\left(C^{\mathrm{T}}\Lambda\otimes B^{\mathrm{T}}P(\varepsilon)\right)\hat{x},\tag{9}$$

and $\Upsilon\in\mathbb{R}^{mN}$. Let υ_l and d_l, $l=1,2,\ldots,mN$, be the lth element of Υ and $d(t)$, respectively. Then, the evaluation of $\dot{V}$ is further continued as

$$\dot{V}\leqslant\sum_{i=1}^{N}\lambda_i\hat{x}_i^{\mathrm{T}}\left(A^{\mathrm{T}}P(\varepsilon)+P(\varepsilon)A-2\gamma P(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\right)\hat{x}_i-2\Upsilon^{\mathrm{T}}\left(\sigma\left((1+\rho)\Upsilon+d(t)\right)-\Upsilon\right)$$

$$= -\sum_{i=1}^{N} \varepsilon\lambda_i \hat{x}_i^{\mathrm{T}} \hat{x}_i - 2\sum_{l=1}^{mN} \upsilon_l \left(\sigma_l \left((1+\rho)\upsilon_l + d_l\right) - \upsilon_l\right).$$

If $|\rho\upsilon_l| \geqslant |d_l| + b$, we first consider the case $\upsilon_l > 0$. When $\upsilon_l > 0$, let $(1+\rho)\upsilon_l + d_l = r\upsilon_l$, and $r \geqslant 1$. Thus,

$$|r\upsilon_l| = r\upsilon_l = \upsilon_l + \rho\upsilon_l + d_l \geqslant \upsilon_l + |d_l| + b + d_l \geqslant \upsilon_l + b = |\upsilon_l| + b.$$

Inview of (2), a property of σ, we have

$$-\upsilon_l \left(\sigma_l \left((1+\rho)\upsilon_l + d_l\right) - \upsilon_l\right) = -\upsilon_l \left(\sigma_l \left((1+\rho)\upsilon_l + d_l\right) - \mathrm{sat}_\Delta(\upsilon_l)\right) \leqslant 0.$$

It can be shown in a similar way that the above inequality also holds when $\upsilon_l < 0$.

If $|\rho\upsilon_l| < |d_l| + b$, then, for all $\tilde{x} \in L_V(c)$ and $\varepsilon \in (0, \varepsilon^*]$,

$$|\upsilon_l \left(\sigma_l \left((1+\rho)\upsilon_l + d_l\right) - \upsilon_l\right)| \leqslant \frac{1}{\rho}\left(|d_l| + b\right)\left(\varpi_0(2|d_l| + b + \Delta) + \Delta\right).$$

Thus, we have, for all $\tilde{x} \in L_V(c)$ and $\varepsilon \in (0, \varepsilon^*]$,

$$\begin{aligned}\dot{V} &\leqslant -\sum_{i=1}^{N} \varepsilon\lambda_i \hat{x}_i^{\mathrm{T}} \hat{x}_i + 2\sum_{l=1}^{mN} \frac{1}{\rho}\left(|d_l| + b\right)\left(\varpi_0(2|d_l| + b + \Delta) + \Delta\right)\\ &\leqslant -\sum_{i=1}^{N} \varepsilon\lambda_i \hat{x}_i^{\mathrm{T}} \hat{x}_i + \frac{2mN}{\rho}\left(D_0 + b\right)\left(\varpi_0(2D_0 + b + \Delta) + \Delta\right).\end{aligned}$$

Choose a $c_0(\varepsilon) \in (0, c]$ such that $L_V(c_0(\varepsilon)) := \{\tilde{x} \in \mathbb{R}^{nN} : V(\tilde{x}) \leqslant c_0(\varepsilon)\} \subset \chi_0^N = \chi_0 \times \chi_0 \times \cdots \times \chi_0$. The existence of such a $c_0(\varepsilon)$ is due to the fact that the origin is an interior point of χ_0. Also, let

$$\rho^*(\varepsilon) = \frac{3mN\lambda_{\max}(P(\varepsilon))}{\varepsilon c_0(\varepsilon)}\left(D_0 + b\right)\left(\varpi_0(2D_0 + b + \Delta) + \Delta\right).$$

When $\rho \geqslant \rho^*$ and $c_0(\varepsilon) \leqslant V(\tilde{x}) \leqslant c$,

$$\begin{aligned}\dot{V} &\leqslant -\sum_{i=1}^{N} \varepsilon\lambda_i \hat{x}_i^{\mathrm{T}} \hat{x}_i + \frac{2\varepsilon c_0(\varepsilon)}{3\lambda_{\max}(P(\varepsilon))}\\ &\leqslant -\sum_{i=1}^{N} \varepsilon\lambda_i \hat{x}_i^{\mathrm{T}} \hat{x}_i + \frac{2\varepsilon}{3\lambda_{\max}(P(\varepsilon))}\sum_{i=1}^{N} \lambda_i \hat{x}_i^{\mathrm{T}} P(\varepsilon)\hat{x}_i\\ &\leqslant -\frac{\varepsilon}{\lambda_{\max}(P(\varepsilon))}\sum_{i=1}^{N} \lambda_i \hat{x}_i^{\mathrm{T}} \left(\lambda_{\max}(P(\varepsilon))I_n - \frac{2}{3}P(\varepsilon)\right)\hat{x}_i\\ &< 0.\end{aligned}$$

In summary, we have shown that, for any $\rho \geqslant \rho^*(\varepsilon)$, $\varepsilon \in (0, \varepsilon^*]$,

$$\dot{V} < 0,\ \tilde{x} \in L_V(c) \setminus L_V^o(c_0(\varepsilon)),$$

where $L_V^o(c_0(\varepsilon)) := \{\tilde{x} \in \mathbb{R}^{nN} : V(\tilde{x}) < c_0(\varepsilon)\}$. As a result, every trajectory that starts from $L_V(c)$ will enter $L_V(c_0(\varepsilon))$ and remain in it in a finite time.

Remark 3. It is clear in the proof above that, as χ_0 decreases to $\{0\}$, the high gain parameter ρ is required to increase to infinity. In the limit case when $\chi_0 = \{0\}$, robust semi-global leader-following consensus, instead of robust semi-global leader following practical consensus, will be achieved.

5 Low gain based variable structure consensus algorithms

In this section, low gain based variable structure consensus control algorithms for the follower agents will be constructed to solve Problem 1.

Recall the definition in (9),

$$\Upsilon = -\left(\mathrm{diag}\{\omega\}M \otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x},$$

where $\Upsilon \in \mathbb{R}^{mN}$, υ_l, $l = 1, 2, \ldots, mN$, is the lth element of Υ, $\varepsilon \in (0, 1]$, and $P(\varepsilon) > 0$ is the unique positive definite solution to the ARE (7). Then, the low gain based consensus control algorithms are given by

$$u = [u_1, u_2, \ldots, u_{mN}]^{\mathrm{T}}, \tag{10}$$

where

$$u_l = \begin{cases} \rho\frac{\upsilon_l}{|\upsilon_l|}, & \text{if } |\upsilon_l| > \mu, \\ \rho\frac{\upsilon_l}{\mu}, & \text{if } |\upsilon_l| \leqslant \mu, \end{cases}$$

with $\rho > 0$ and $\mu > 0$.

Remark 4. The variable structure consensus control algorithms (10) can be considered as soft switching consensus control laws since they are globally Lipschitz and reduce to the ideal switching consensus control laws as $\mu \to 0$. It will be illustrated below that, when χ_0 decreases to $\{0\}$, μ will decreases to 0. When $\mu = 0$, an infinitely fast switching results, and robust semi-global leader-following consensus, instead of robust semi-global leader-following practical consensus, will be achieved.

The following result establishes that the low gain based variable structure consensus control algorithms (10) solve Problem 1.

Theorem 2. There exists an $\varepsilon^* \in (0, 1]$ and a $\rho^* > 0$, and for any $\varepsilon \in (0, \varepsilon^*]$ and $\rho \geqslant \rho^*$, there exist a $\mu^*(\varepsilon, \rho) > 0$, such that, for any $\mu \in (0, \mu^*(\varepsilon, \rho)]$, $\varepsilon \in (0, \varepsilon^*]$, $\rho \geqslant \rho^*$, the consensus control algorithms (10) solve Problem 1.

Proof. Consider the following Lyapunov function

$$V(\tilde{x}) = \tilde{x}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M \otimes P(\varepsilon)\right)\tilde{x},$$

which is a positive definite function since $\mathrm{diag}\{\omega\}M$ and $P(\varepsilon)$ are both positive definite matrices. Let $c > 0$ be such that

$$\sup_{\varepsilon\in(0,1],x_i\in\chi,i=0,1,2,\ldots,N} \tilde{x}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M \otimes P(\varepsilon)\right)\tilde{x} \leqslant c.$$

The existence of such a c is guaranteed since $\lim_{\varepsilon\to 0} P(\varepsilon) = 0$, by Lemma 2, and χ is a bounded set. Let $\varepsilon^* \in (0, 1]$ be such that, for each $\varepsilon \in (0, \varepsilon^*]$, $\tilde{x} \in L_V(c)$ implies that

$$\|\left(\mathrm{diag}\{\omega\}M \otimes B^{\mathrm{T}}P(\varepsilon)\right)\tilde{x}\| \leqslant \Delta.$$

Such an ε^* exists since $\lim_{\varepsilon\to 0} P(\varepsilon) = 0$. For each $\varepsilon \in (0, \varepsilon^*]$, let $\rho^* > 0$ be such that $D_0 + \Delta + b < \rho^*$.

The evaluation of the derivative of $V(\tilde{x})$ along the closed-loop trajectory inside the level set $L_V(c)$ is then carried out as follows,

$$\begin{aligned}\dot{V} &= \tilde{x}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M \otimes \left(P(\varepsilon)A + A^{\mathrm{T}}P(\varepsilon)\right)\right)\tilde{x} + 2\tilde{x}^{\mathrm{T}}\left(\mathrm{diag}\{\omega\}M \otimes P(\varepsilon)B\right)\sigma\left(u + d(t)\right) \\ &= \sum_{i=1}^{N}\lambda_i\hat{x}_i^{\mathrm{T}}\left(A^{\mathrm{T}}P(\varepsilon) + P(\varepsilon)A - 2\lambda_iP(\varepsilon)BB^{\mathrm{T}}P(\varepsilon)\right)\hat{x}_i - 2\Upsilon^{\mathrm{T}}\left(\sigma\left(u + d(t)\right) - \Upsilon\right). \\ &\leqslant -\sum_{i=1}^{N}\varepsilon\lambda_i\hat{x}_i^{\mathrm{T}}\hat{x}_i - 2\sum_{l=1}^{mN}\upsilon_l\left(\sigma_l\left(u_l + d_l\right) - \upsilon_l\right),\end{aligned}$$

where υ_l and d_l, $l = 1, 2, \ldots, mN$, are the lth elements of Υ and $d(t)$, respectively.

When $|v_l| > \mu$, we first consider the case $v_l > \mu$. Because $v_l > \mu$, $u_l = \rho$. Then, for any $\tilde{x} \in L_V(c)$, $\rho \geqslant \rho^*$ and $\varepsilon \in (0, \varepsilon^*]$, by Property (2) of σ and Assumption 2, $v_l\left(\sigma_l\left(u_l + d_l\right) - v_l\right) > 0$. By similar reasoning, we can prove that the above inequality also holds when $v < -\mu$.

If $|v_l| \leqslant \mu$, by (3) and Definition 2, we have

$$-v_l\left(\sigma_l\left(u_l + d_l\right) - v_l\right) \leqslant \mu\left(\varpi_0(2\rho) + \Delta\right).$$

Thus, we have, for all $\tilde{x} \in L_V(c)$, $\varepsilon \in (0, \varepsilon^*]$ and $\rho \geqslant \rho^*$,

$$\dot{V} \leqslant -\sum_{i=1}^{N} \varepsilon \lambda_i \hat{x}_i^{\mathrm{T}} \hat{x}_i + 2mN\mu\left(\varpi_0(2\rho) + \Delta\right).$$

Choose a $c_0(\varepsilon) \in (0, c]$ such that $L_V(c_0(\varepsilon)) := \{\tilde{x} \in \mathbb{R}^{nN} : V(\tilde{x}) \leqslant c_0(\varepsilon)\} \subset \chi_0^N = \chi_0 \times \chi_0 \cdots \times \chi_0$. Such $c_0(\varepsilon)$ can be chosen since the origin is an interior point of χ_0. Also, let

$$\mu^*(\varepsilon, \rho) = \frac{\varepsilon c_0(\varepsilon)}{3mN\lambda_{\max}(P(\varepsilon))\left(\varpi_0(2\rho) + \Delta\right)}. \tag{11}$$

In summary, we have shown that, for any $\mu \in (0, \mu^*(\varepsilon, \rho)]$, $\varepsilon \in (0, \varepsilon^*]$ and $\rho \geqslant \rho^*$,

$$\dot{V} \leqslant -\sum_{i=1}^{N} \varepsilon \lambda_i \hat{x}_i^{\mathrm{T}} \hat{x}_i + \frac{2\varepsilon c_0(\varepsilon)}{3\lambda_{\max}(P(\varepsilon))} < 0,\ \tilde{x} \in L_V(c) \setminus L_V^o(c_0(\varepsilon)),$$

where $L_V^o(c_0(\varepsilon)) := \{\tilde{x} \in \mathbb{R}^{nN} : V(\tilde{x}) < c_0(\varepsilon)\}$.

Consequently, every trajectory starting from $L_V(c)$ will enter $L_V(c_0(\varepsilon))$ and remain in it in a finite time.

6 Numerical simulation

Consider a multi-agent system of 5 follower agents and a leader agent, whose dynamics are described respectively by (1) and (5) with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Clearly, the matrix pair (A, B) is asymptotically null controllable with bounded controls and hence Assumption 1 is satisfied.

The communication topology is represented by a directed graph as shown in Figure 2, with its adjacent matrix

$$\mathcal{A} = \begin{bmatrix} 0 & 8 & 0 & 0 & 0 \\ 10 & 0 & 12 & 0 & 0 \\ 0 & 16 & 0 & 4 & 0 \\ 0 & 0 & 6 & 0 & 10 \\ 0 & 0 & 0 & 20 & 0 \end{bmatrix},$$

and $a_{30} = 10$, $a_{10} = a_{20} = a_{40} = a_{50} = 0$. The detailed balance parameter matrix $\mathrm{diag}\{\omega\} = \mathrm{diag}\{0.5, 0.4, 0.3, 0.2, 0.1\}$. It is easy to verify that the directed graph is strongly connected and detailed balanced, and $\mathrm{diag}\{\omega\}M$ is a positive definite matrix. Thus, Assumption 3 holds. Pick $\gamma = 0.3 < \underline{\lambda} = 0.33161$.

For the purpose of running simulation, let

$$\sigma(s) = \begin{cases} \arctan(s + 0.5), & \text{if } s \leqslant -0.5, \\ 0, & \text{if } |s| < 0.5, \\ \arctan(s - 0.5), & \text{if } s \geqslant 0.5, \end{cases} \tag{12}$$

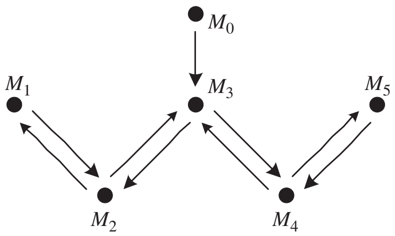

Figure 2 The directed graph that represents the communication topology.

and $d_i(t) = \sin(t)$. Thus, by (12), $\Delta = 1$, $b = 1.5$, $\varpi_0(s) = s$ and $D_0 = 1$. In the simulation, the initial states of the follower agents and the leader agent are set as

$$[x_0(0), x_1(0), x_2(0), x_3(0), x_4(0), x_5(0)] = \begin{bmatrix} 15 & 10 & 5 & -5 & 0 & 10 \\ -15 & -10 & -5 & 0 & -5 & -5 \\ 15 & 10 & 5 & -10 & 10 & 0 \end{bmatrix}.$$

6.1 Simulation for low-and-high gain feedback based consensus algorithms

In the simulation for low-and-high gain consensus algorithms, we adjust the values of the two parameters, the low gain parameter ε and the high gain parameter ρ, to verify the theoretical result in Section 4.

Let us consider two different values of the low gain parameter, $\varepsilon = 0.001$ and $\varepsilon = 0.0001$. For these two values of the low gain parameter ε, the solution of the ARE (7) results in

$$P(0.001) = \begin{bmatrix} 0.0070 & 0.0239 & 0.0408 \\ 0.0239 & 0.1262 & 0.2852 \\ 0.0408 & 0.2852 & 0.9759 \end{bmatrix}, \quad P(0.0001) = \begin{bmatrix} 0.0010 & 0.0051 & 0.0129 \\ 0.0051 & 0.0393 & 0.1314 \\ 0.0129 & 0.1314 & 0.6618 \end{bmatrix}.$$

Simulation results are shown in Figure 3 (a)–(c). As seen in Figure 3(a), leader-following consensus is not achieved with $\varepsilon = 0.001$, indicating that not all the agent initial states are within the region of initial conditions within which robust leader-following practical consensus occurs. On the other hand, as shown in Figure 3 (b) and (c), leader-following practical consensus is achieved with $\varepsilon = 0.0001$, indicting that the the region of the agent initial conditions where robust consensus occurs becomes bigger as the value of ε decreases. Figure 3 (b) and (c), which show the simulation results for two different values of the high gain parameters, $\rho = 50$ and $\rho = 200$, also demonstrate the theoretical conclusion that, for a given set χ and a given value of ε, a smaller set χ_o can be allowed by increasing the value of the high gain parameter ρ.

6.2 Simulation for low gain based variable structure consensus algorithms

We now simulate the multi-agent system under the low gain based variable structure consensus algorithms. In the simulation, we fix the parameter $\rho = 100$, and adjust the low gain parameter ε and the switching threshold value μ to achieve the robust consensus control.

For the same two values of the low gain parameter, $\varepsilon = 0.001$ and $\varepsilon = 0.0001$, the matrices $P(0.001)$ and $P(0.0001)$ have been solved in the simulation of the low-and-high gain based consensus algorithms.

Simulation results are shown in Figure 4 (a)–(c). The simulation results shown in Figures 4 (a) and (b) illustrate that the region of the agent initial conditions within which robust consensus occurs becomes bigger as the value of ε decreases.

Figure 4 (b) and (c), which show the simulation results for two different values of the switching parameter, $\mu = 1$ and $\mu = 0.1$, demonstrate that, for a given set χ and a given value of ε, a smaller χ_o can be allowed by decreasing the value of μ.

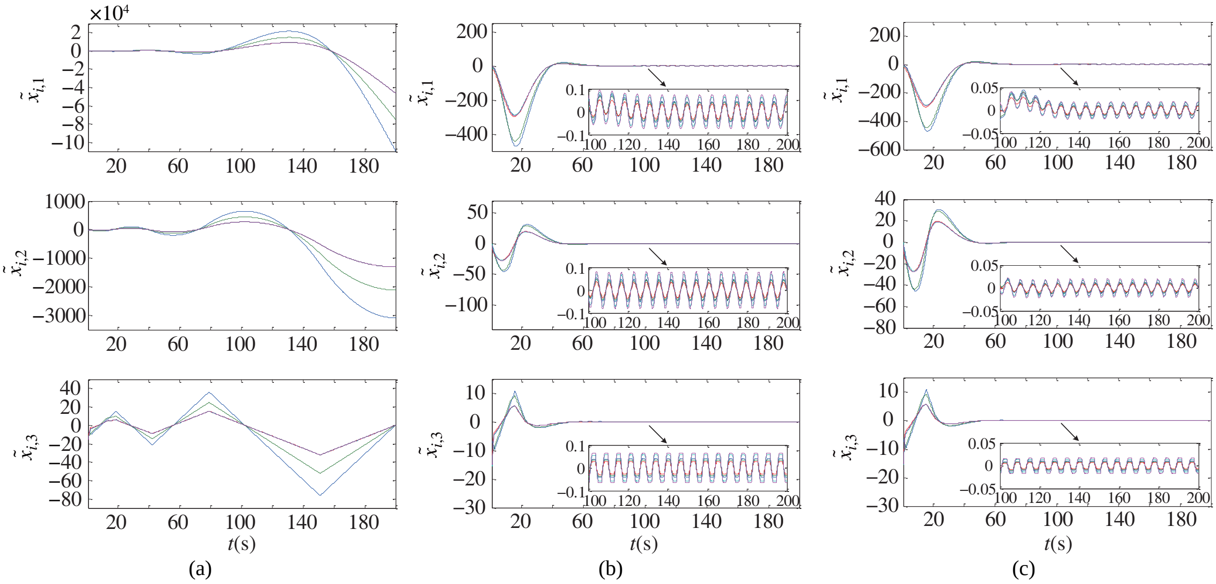

Figure 3 (Color online) The evolution of the agent dynamics under the low-and-high gain feedback based consensus algorithms. (a) $\varepsilon = 0.001$ and $\rho = 50$; (b) $\varepsilon = 0.0001$ and $\rho = 50$; (c) $\varepsilon = 0.0001$ and $\rho = 200$.

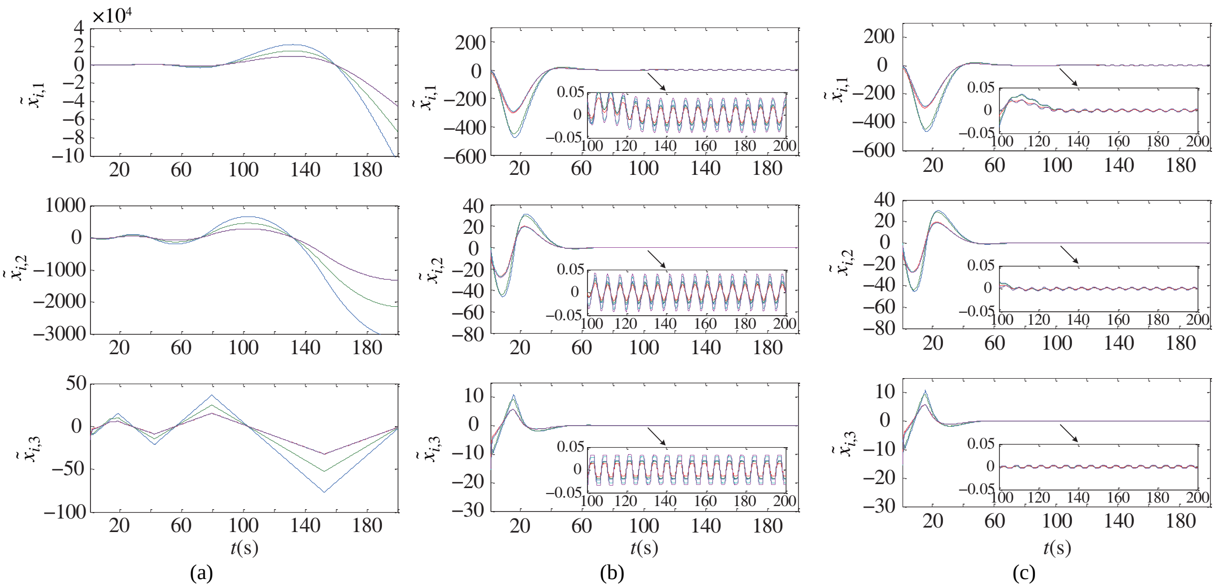

Figure 4 (Color online) The evolution of the agent dynamics under the low gain based variable structure consensus algorithms. (a) $\varepsilon = 0.001$ and $\mu = 1$; (b) $\varepsilon = 0.0001$ and $\mu = 1$; (c) $\varepsilon = 0.0001$ and $\mu = 0.1$.

7 Conclusion

We studied the problem of robust semi-global leader-following practical consensus of a group of general linear systems where the actuators of the follower agents are imperfect, that is, their input output characteristics are not precisely known and include a broad range of nonlinearities. The follower agents are also subject to the effect of input additive disturbances. We constructed two kinds of consensus algorithms, the low-and-high gain feedback consensus algorithms and the low gain based variable structure consensus algorithms, for the follower agents. These consensus algorithms were shown to achieve robust semi-global leader-following practical consensus when the communication topology among the follower agents is a strongly connected and detailed balanced directed graph and the leader agent is a neighbor of at least one follower agent. The generalization of the results to a more general directed graph seems to be a difficult task and is a topic of our on going research.

Conflict of interest The authors declare that they have no conflict of interest.

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