SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •
Special Focus on Formation Control of Unmanned Systems

July 2017, Vol. 60 070205:1–070205:14
doi: 10.1007/s11432-016-9089-7

Simultaneous attack of a stationary target using multiple missiles: a consensus-based approach

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Received November 24, 2016; accepted April 24, 2017; published online June 9, 2017

Abstract This paper considers the simultaneous attack problem of multiple missiles against a stationary target. Built upon the classic proportional navigation structure, we propose a consensus-based approach to design the cooperative guidance law. Specifically, we present time-varying navigation ratios for the missiles, which exchange the time-to-go estimates between neighboring missiles via a communication network. For the cases where the communication topology is undirected or in the leader–follower structure with a missile acting as the leader whose navigation ratio cannot be tuned, we show that the proposed cooperative guidance law can solve the simultaneous attack problem. The effectiveness of the theoretical results is finally illustrated by numerical simulations.

Keywords simultaneous attack, consensus problem, cooperative control, distributed control, guidance law design

Citation Zhou J L, Yang J Y, Li Z K. Simultaneous attack of a stationary target using multiple missiles: a consensus-based approach. Sci China Inf Sci, 2017, 60(7): 070205, doi: 10.1007/s11432-016-9089-7

1 Introduction

In the past few years, simultaneous attack with multiple missiles against a single target has been an emerging research topic. When attacking a target, a group of well-organized low-cost missile attackers can be more effective than a single expensive missile attacker. The main advantage of multi-missile simultaneous attack is that it can enhance the survivability when penetrating a missile defense system and is more effective at destroying the target.

Simultaneous attack can be achieved in two ways, namely, individual homing and cooperative homing [1]. In the individual homing approach, a specific time is given in advance as the common impact time and each missile tries to arrive at the target on time independently. In this way, the many-to-one simultaneous attack problem can be considered as a one-to-one attack problem with impact time constraints. Impact-time control guidance laws were designed in [2–4] using optimal control theory. A sliding mode-based impact time and angle guidance law were presented in [5]. The drawback of the individual homing approach is that it requires the determination of a suitable common impact time before

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homing, which may cause the simultaneous attack to fail as some missiles may not be able to satisfy the impact time constraint due to limited speeds or accelerations.

The shortcomings of individual homing can be overcome by the cooperative homing approach, in which the missiles communicate among themselves to synchronize the arrival times. To the best of the authors' knowledge, studies on cooperative guidance law design for a simultaneous attack are few, but examples include [1, 6–9]. The main objective of [1, 6–8] is to drive the missiles to reach an agreement on their times-to-go. However, it is generally difficult to establish a mathematical expression of the time-to-go because the flight condition thereafter is unknown. In [6, 7], the time-to-go is approximated as the ratio of the distance between the missile and the target and the component of relative velocity along the line of sight (LOS). The limitation of [6, 7] is that it can only achieve the consensus of the approximated times-to-go, but not the real times-to-go. In [1], a cooperative proportional navigation (PN) guidance law was proposed, which reduces the variance of time-to-go estimates during the homing guidance. It is worth mentioning that the cooperative guidance law in [1] is centralized in the sense that it requires real-time updates of the global information of the whole missile system. Designing a distributed cooperative guidance law for the simultaneous attack problem is important and still needs further investigation.

Distributed control of multi-agent systems has been studied extensively over the past few decades, e.g., [10–14]. Owing to the spatial distribution of the agents and limited sensing capability of sensors, implementable control laws for multi-agent systems should be distributed, depending on only the local information of each agent and its neighbors. Of particular interest is distributed control design for the consensus problem, which means that the states of the agents reach agreement. Please refer to [15–20] and references therein for recent results. These results have been applied to several areas, such as spacecraft formation flying, sensor networks, and neural networks [14, 21–23]. However, few attempts have been made at application to simultaneous attack.

Motivated by the discussions above, in this paper we study the distributed cooperative guidance law design problem to achieve simultaneous attack of multiple missiles against a stationary target. With the use of a PN structure similarly as in [1], the cooperative guidance law design problem is reduced to the design of navigation ratios. Within the consensus framework, we present time-varying navigation ratios for the missiles, which exchange the time-to-go estimates between neighboring missiles via a communication network. For the cases where the communication topology is undirected or in the leader–follower structure with a missile acting as the leader whose navigation ratio cannot be tuned, we show that the proposed cooperative guidance law can solve the simultaneous attack problem. The main contributions of this paper lie in the following aspects. First, the guidance law proposed in this paper is distributed. Compared with the centralized guidance law in [1], which relies on global onboard information of time-to-go estimates, the cooperative guidance law depends only on time-to-go estimates of each missile and its neighbors, and thus can decrease the communication burden. Second, different from most existing time-to-go-based work, in which only the consensus of time-to-go estimates is considered, in this paper we further consider the consensus of real times-to-go. Therefore, the guidance law in this paper can ensure accurate simultaneous attack. Third, in comparison with the guidance law in [1], which only reduces the variance of time-to-go estimates, the guidance law in this paper can achieve finite-time consensus of real times-to-go. Furthermore, the guidance law designed in this paper can deal with not only undirected connected communication graphs, but also the case where a missile undergoes communication failure. Last but not the least, different from [1], which requires the small angle assumption of the heading error, this paper proves that such an assumption is not necessary, and thus is much more practical in engineering applications.

The rest of this paper is organized as follows. Useful mathematical preliminaries are introduced in Section 2. The simultaneous attack problem is formulated in Section 3. Distributed cooperative guidance laws are proposed in Section 4. Simulation results of a five-to-one engagement scenario are provided in Section 5 and conclusion is given in Section 6.

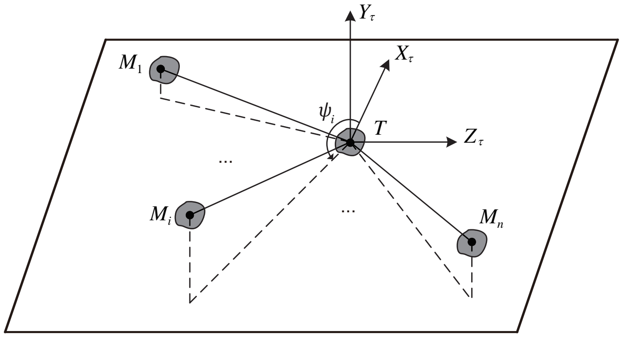

Figure 1 The 3D engagement geometry.

2 Mathematical preliminaries

A graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = 1, \ldots, n$ is the set of nodes (i.e., missiles), and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is the set of edges, in which an edge is represented by an ordered pair of distinct nodes. An edge (i, j) represents that node i is a neighbor of node j and node j can receive information from node i. A graph is undirected if for any $(i, j) \in \mathcal{E}$, $(j, i) \in \mathcal{E}$. A path from node i_1 to node i_l is a sequence of ordered edges of the form (i_k, i_{k+1}), $k = 1, \ldots, l-1$. An undirected graph is connected if for any $i \in \mathcal{V}$, there exists paths to all other nodes, while a directed graph contains a directed spanning tree if there exists a node called the root node such that the node has directed paths to all the other nodes.

Suppose there are n nodes in the graph $\mathcal{G}$. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is defined as $a_{ii} = 0$, $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and 0 otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ is defined as $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$.

Lemma 1 (Ren et al. [21])**.** Zero is an eigenvalue of $\mathcal{L}$ with $\mathbf{1}$ as a right eigenvector and all nonzero eigenvalues have positive real parts, where $\mathbf{1}$ denotes a column vector with all entries equal to 1. Moreover, zero is a simple eigenvalue of $\mathcal{L}$ if and only if $\mathcal{G}$ has a directed spanning tree.

Lemma 2 (Olfati-saber and Murray [24])**.** For a connected undirected graph $\mathcal{G}$, its Laplacian matrix $\mathcal{L}$ has the following properties: for any $x \in \mathbb{R}^n$ satisfying $\mathbf{1}^{\mathrm{T}}x = 0$, we have $x^{\mathrm{T}}\mathcal{L}x \geqslant \lambda_2 x^{\mathrm{T}}x$, where λ_2 denotes the smallest nonzero eigenvalue of $\mathcal{L}$.

Lemma 3 (Yu et al. [25])**.** For $x_i \in \mathbb{R}$, $i = 1, \ldots, p$, $0 < \alpha \leqslant 1$, then $\sum_{i=1}^{p} |x_i|^\alpha \geqslant (\sum_{i=1}^{p} |x_i|)^\alpha$.

Lemma 4 (Yu et al. [25])**.** If there exists a Lyapunov function $V(x)$ such that

$$\dot{V}(x) + aV(x) + bV^\alpha(x) \leqslant 0,$$

where $a > 0$, $b > 0$, and $0 < \alpha < 1$, then $V(x)$ will converge to the origin in finite time. In addition, the finite settling time T satisfies

$$T \leqslant \frac{1}{a(1-\alpha)} \ln \frac{aV^{1-\alpha}(x_0) + b}{b}.$$

3 Problem formulation

In this paper, we consider the scenario where n missiles cooperatively attack a stationary target in three-dimensional (3D) space. The task assigned to the multi-missile system is to hit the target simultaneously. The many-to-one 3D engagement geometry is illustrated in Figure 1, where M_i and T denote the ith missile and the target, respectively, $\mathcal{T}$ is an inertial coordinate system with $X_\mathcal{T}$–$Z_\mathcal{T}$ being the horizontal surface and $X_\mathcal{T}$–$Y_\mathcal{T}$ being the vertical surface, ψ_i is the angle between $X_\mathcal{T}$ and the projection of the LOS M_iT on the $X_\mathcal{T}$–$Z_\mathcal{T}$ plane, and $\psi_1, \ldots, \psi_n$ are different at initial time. For simplicity, assume that all the missiles have already laterally headed on the target, implying that the missiles' flight paths are on different vertical surfaces and therefore collisions can be avoided. For the case that the missiles do not laterally head on the target, we can always design the lateral accelerations to drive the missiles laterally

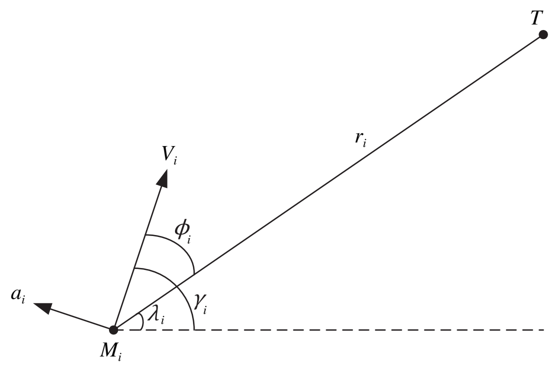

Figure 2 The planar engagement geometry.

head on the target, which is a very easy task. In this case, we only need to consider the planar engagement scenario on vertical surface with the longitudinal model of the missiles.

The planar engagement scenario of the ith missile and the target is depicted in Figure 2, where r_i is the relative range along the LOS, λ_i is angle between the LOS and the horizontal reference line, a_i, V_i, γ_i, and ϕ_i denote the acceleration, velocity, fight-path angle, and heading error of the ith missile, respectively. Similarly as in [1], the missile acceleration is assumed to be perpendicular to its velocity and the missile speed V_i remains constant during the entire process.

The planar missile-target engagement kinematics can be described by

$$\begin{aligned} \dot{r}_i =& - V_i \cos\phi_i, \quad \dot{\lambda}_i = -\frac{V_i \sin\phi_i}{r_i}, \\ a_i =& V_i \dot{\gamma}_i, \quad \phi_i = \gamma_i - \lambda_i, \quad i = 1, \ldots, n. \end{aligned} \tag{1}$$

The objective of the simultaneous attack problem is to design the cooperative guidance law for the multi-missile system such that the relative ranges $r_1, \ldots, r_n$ converge to zero simultaneously. The guidance law of the multi-missile system in this paper is designed based on PN, which has been widely used as the guidance scheme in the homing phase for most missile systems. In the PN scheme, the rate of turn of the missile is made proportional with the navigation ratio N to the rate of turn of the LOS. With the navigation ratio satisfying $N > 2$, the component of relative velocity between the missile and the target in the direction normal to the LOS can be driven to zero and the hit-to-kill attack can be achieved.

With the PN structure

$$a_i = N_i V_i \dot{\lambda}_i, \quad i = 1, \ldots, n, \tag{2}$$

where N_i denotes the navigation ratio of the ith missile, the kinematics equations (1) can be rewritten as

$$\dot{r}_i = -V_i \cos\phi_i, \quad \dot{\phi}_i = -\frac{(N_i - 1) V_i \sin\phi_i}{r_i}, \quad i = 1, \ldots, n. \tag{3}$$

Then, the simultaneous attack problem can be reformulated as designing the navigation ratios $N_1, \ldots, N_n$, for the multi-missile system described by (3), such that $r_1, \ldots, r_n$, reach zero simultaneously.

4 Main results

Similarly as in [1], the main idea of the cooperative guidance strategy here is that the missiles with longer times-to-go try to take shortcuts, whereas others with shorter times-to-go take detours to delay the arrival times, by designing of navigation ratios. Different from the conventional PN in which the navigation ratio is held fixed, the navigation ratio of each missile in this paper is adjusted using its onboard information and exchanging information with the other missiles. The information exchanges among the n missiles are conducted on a wireless network, whose topology is represented by a graph $\mathcal{G}$.

In the following subsections, we consider two cases where the communication graph $\mathcal{G}$ is undirected or of the leader-follower form.

4.1 Undirected communication graphs

In this subsection, the communication graph $\mathcal{G}$ is assumed to satisfy the following assumption.

Assumption 1. The communication graph $\mathcal{G}$ is undirected and connected.

Based on the local information of each missile and its neighbors, the navigation ratio of the ith missile is described by

$$N_i = N_s(1 - a_i r_i \xi_i - b_i r_i |\xi_i|^{\alpha}\mathrm{sgn}(\xi_i)), \quad i = 1, \ldots, n, \tag{4}$$

where

$$\xi_i = \sum_{j=1}^{n} a_{ij}(\hat{t}_{\mathrm{go},j} - \hat{t}_{\mathrm{go},i}), \tag{5}$$

$$\hat{t}_{\mathrm{go},i} = \frac{r_i}{V_i}\left(1 + \frac{\phi_i^2}{2(2N_s - 1)}\right), \tag{6}$$

and N_s, a_i, b_i, and α are real numbers satisfying $N_s > \frac{3\pi-4}{3\pi-8}$, $a_i > 0$, $b_i > 0$, and $0 < \alpha < 1$.

As shown in [1], the time-to-go of the ith missile $t_{\mathrm{go},i}$ can be calculated by $\frac{r_i}{V_i}(1 + \frac{\phi_i^2}{2(2N_i-1)})$ only if its navigation ratio N_i remains constant thereafter. Therefore, $\hat{t}_{\mathrm{go},i}$ in (6) can be understood as the time-to-go estimate of the ith missile. Note that the simultaneous attack is achieved if the times-to-go $t_{\mathrm{go},1}, \ldots, t_{\mathrm{go},N}$, reach agreement. This is done via communicating the time-to-go estimates between neighboring missiles and adjusting the navigation ratios for the missile which are now nonlinear and time-varying. Note that the control law (4) depends on only the local information of neighboring missiles, thereby it is distributed.

Before proceeding, the following assumption is introduced.

Assumption 2. We have $r_i \neq 0$, $\phi_i \neq 0$, $i = 1, \ldots, n$ before the consensus of $\hat{t}_{\mathrm{go},i}$, $i = 1, \ldots, n$.

Noting that $r_i = 0$, $\phi_i = 0$ only happens when the ith missile reaches the target, Assumption 2 is necessary as the simultaneous attack task fails if any missile arrives at the target before the consensus of times-to-go. Moreover, Assumption 2 can be satisfied by appropriately choosing the parameters N_s, a_i, b_i, and α, which will be discussed later.

Theorem 1. Suppose that Assumptions 1 and 2 hold, and $|\phi_i| \leqslant \frac{\pi}{2}$, $i = 1, \ldots, n$. For the multi-missile system (1) with the PN strategy (2) and the proposed navigation ratios (4), ξ_i, $i = 1, \ldots, n$ will converge to a neighborhood of the origin in finite time, and ϕ_i, $i = 1, \ldots, n$ will asymptotically converge to zero.

Proof. Let $\hat{t}_{\mathrm{go}} = [\hat{t}_{\mathrm{go},1}, \ldots, \hat{t}_{\mathrm{go},n}]^{\mathrm{T}}$ and $\xi = [\xi_1, \ldots, \xi_n]^{\mathrm{T}}$. Then, Eq. (5) can be written as $\xi = -\mathcal{L}\hat{t}_{\mathrm{go}}$, where $\mathcal{L}$ denotes the Laplacian matrix of $\mathcal{G}$. Since Assumption 1 holds, it follows that zero is a simple eigenvalue of $\mathcal{L}$ with $\mathbf{1}$ as the eigenvector. It is not difficult to see that $\xi = 0$ if and only if $\hat{t}_{\mathrm{go},1} = \cdots = \hat{t}_{\mathrm{go},n}$. Thus, ξ will be referred to as the consensus error of the time-to-go estimates.

Differentiating $\hat{t}_{\mathrm{go},i}$ with respect to time and substituting (3) into (6) yield

$$\begin{aligned}\dot{\hat{t}}_{\mathrm{go},i} &= \frac{\dot{r}_i}{V_i}\left(1 + \frac{\phi_i^2}{2(2N_s - 1)}\right) + \frac{r_i \phi_i \dot{\phi}_i}{V_i(2N_s - 1)} \\ &= -\cos\phi_i\left(1 + \frac{\phi_i^2}{2(2N_s - 1)}\right) - \frac{(N_i - 1)\phi_i \sin\phi_i}{2N_s - 1} \\ &= -1 + \frac{(N_s - N_i)\phi_i \sin\phi_i}{2N_s - 1} + \delta_i,\end{aligned} \tag{7}$$

where

$$\delta_i = 1 - \cos\phi_i - \frac{\phi_i^2\cos\phi_i}{2(2N_s - 1)} - \frac{(N_s - 1)\phi_i \sin\phi_i}{2N_s - 1}.$$

Note that $\delta_i(\phi_i)$ is an even function about the independent variable ϕ_i, and is monotone increasing in $\phi_i \in [0, \pi)$. Thus, we have $0 \leqslant \delta_i < 2 + \frac{\pi^2}{2(2N_s-1)}$, when $|\phi_i| < \pi$.

Substituting the protocol (4) into (7), we can obtain

$$\dot{\hat{t}}_{\mathrm{go},i} = -1 + \frac{N_s\phi_i \sin\phi_i (a_i r_i \xi_i + b_i r_i |\xi_i|^\alpha \mathrm{sgn}(\xi_i))}{2N_s - 1} + \delta_i. \tag{8}$$

Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{4}\sum_{(i,j)\in\mathcal{E}} a_{ij}(\hat{t}_{\mathrm{go},j} - \hat{t}_{\mathrm{go},i})^2 = \frac{1}{2}\hat{t}_{\mathrm{go}}^{\mathrm{T}}\mathcal{L}\hat{t}_{\mathrm{go}}. \tag{9}$$

The time derivative of V_1 along the trajectory of (8) is given by

$$\dot{V}_1 = \hat{t}_{\mathrm{go}}^{\mathrm{T}}\mathcal{L}\dot{\hat{t}}_{\mathrm{go}} = -\sum_{i=1}^{n}\left(\frac{N_s a_i r_i \phi_i \sin\phi_i}{2N_s - 1}\xi_i^2 + \frac{N_s b_i r_i \phi_i \sin\phi_i}{2N_s - 1}|\xi_i|^{1+\alpha} + \xi_i\delta_i\right), \tag{10}$$

where we have used the fact that $\mathcal{L}\mathbf{1} = 0$ and $\xi = -\mathcal{L}\hat{t}_{\mathrm{go}}$ to obtain the last equality, which can be further rewritten as the following two terms:

$$\dot{V}_1 = -\sum_{i=1}^{n}\left(\left(\frac{N_s a_i r_i \phi_i \sin\phi_i}{2N_s - 1} + \frac{\delta_i}{\xi_i}\right)\xi_i^2 + \frac{N_s b_i r_i \phi_i \sin\phi_i}{2N_s - 1}|\xi_i|^{1+\alpha}\right), \tag{11}$$

$$\dot{V}_1 = -\sum_{i=1}^{n}\left(\frac{N_s a_i r_i \phi_i \sin\phi_i}{2N_s - 1}\xi_i^2 + \left(\frac{N_s b_i r_i \phi_i \sin\phi_i}{2N_s - 1} + \frac{\delta_i}{|\xi_i|^\alpha \mathrm{sgn}(\xi_i)}\right)|\xi_i|^{1+\alpha}\right). \tag{12}$$

From (11), with Lemma 3 we can get

$$\dot{V}_1 \leqslant -k_1\xi^{\mathrm{T}}\xi - k_2(\xi^{\mathrm{T}}\xi)^{\frac{1+\alpha}{2}}, \tag{13}$$

where

$$k_1 = \min\{k_{11}, \ldots, k_{1n}\}, \quad k_2 = \min\{k_{21}, \ldots, k_{2n}\}$$

with

$$k_{1i} = \left(\frac{N_s a_i r_i \phi_i \sin\phi_i}{2N_s - 1} + \frac{\delta_i}{\xi_i}\right), \quad k_{2i} = \frac{N_s b_i r_i \phi_i \sin\phi_i}{2N_s - 1}, \quad i = 1, \ldots, n.$$

Since $\mathbf{1}^{\mathrm{T}}\mathcal{L}\mathbf{1} = (\mathcal{L}^{\frac{1}{2}}\mathbf{1})^{\mathrm{T}}(\mathcal{L}^{\frac{1}{2}}\mathbf{1}) = 0$, we have $\mathcal{L}^{\frac{1}{2}}\mathbf{1} = 0$. It follows that $\mathbf{1}^{\mathrm{T}}\mathcal{L}^{\frac{1}{2}}\hat{t}_{\mathrm{go}} = 0$. By Lemma 2, we have $\hat{t}_{\mathrm{go}}^{\mathrm{T}}\mathcal{L}\mathcal{L}\hat{t}_{\mathrm{go}} \geqslant \lambda_2\hat{t}_{\mathrm{go}}^{\mathrm{T}}\mathcal{L}\hat{t}_{\mathrm{go}}$, i.e., $\xi^{\mathrm{T}}\xi \geqslant 2\lambda_2 V_1$, in virtue of which, it follows from (18) that

$$\dot{V}_1 \leqslant -2\lambda_2 k_1 V_1 - (2\lambda_2)^{\frac{1+\alpha}{2}} k_2 V_1^{\frac{1+\alpha}{2}}, \tag{14}$$

if k_1, k_2 are positive. We can obtain from Lemma 4 that the finite-time convergence of V_1 is guaranteed with

$$T_1 \leqslant \frac{1}{\lambda_2 k_1(1-\alpha)} \ln\frac{2\lambda_2 k_1 V_1^{\frac{1-\alpha}{2}}(0) + (2\lambda_2)^{\frac{1+\alpha}{2}}k_2}{(2\lambda_2)^{\frac{1+\alpha}{2}}k_2}.$$

Note that k_1 and k_2 are positive if $|\xi_i| > \frac{(2N_s-1)\delta_i}{N_s a_i r_i \phi_i \sin\phi_i}$ for $i = 1, \ldots, n$. Then, the consensus errors can converge to the region $|\xi_i| \leqslant \frac{(2N_s-1)\delta_i}{N_s a_i r_i \phi_i \sin\phi_i}$ in finite time. From (12), we can similarly obtain the finite-time convergence region $|\xi_i| \leqslant (\frac{(2N_s-1)\delta_i}{N_s b_i r_i \phi_i \sin\phi_i})^{\frac{1}{\alpha}}$. Thus, the consensus errors can converge to the region $|\xi_i| \leqslant \min\{\frac{(2N_s-1)\delta_i}{N_s a_i r_i \phi_i \sin\phi_i}, (\frac{(2N_s-1)\delta_i}{N_s b_i r_i \phi_i \sin\phi_i})^{\frac{1}{\alpha}}\}$ in finite time.

Since $\frac{\delta_i}{\phi_i \sin\phi_i}$ is an even function about the independent variable ϕ_i and is monotone increasing in $\phi_i \in (0, \pi)$, we can obtain that $0 \leqslant \frac{\delta_i}{\phi_i \sin\phi_i} \leqslant \frac{2}{\pi} - \frac{N_s-1}{2N_s-1}$ when $|\phi_i| \leqslant \frac{\pi}{2}$. Thus, the convergence region of the consensus error is

$$\Delta_i = \left\{\xi_i : |\xi_i| \leqslant \min\left\{\frac{2N_s-1}{N_s a_i r_i}\left(\frac{2}{\pi} - \frac{N_s-1}{2N_s-1}\right), \left(\frac{2N_s-1}{N_s b_i r_i}\left(\frac{2}{\pi} - \frac{N_s-1}{2N_s-1}\right)\right)^{\frac{1}{\alpha}}\right\}\right\}. \tag{15}$$

Once the consensus error converges to the region, according to the proposed guidance law (4), we have

$$\begin{aligned} N_i \geqslant & N_s(1-|a_i r_i \xi_i| - |b_i r_i |\xi_i|^\alpha|) \\ \geqslant & N_s\left(1-2\times\frac{2N_s-1}{N_s}\left(\frac{2}{\pi}-\frac{N_s-1}{2N_s-1}\right)\right) \\ = & \left(3-\frac{8}{\pi}\right)N_s+\frac{4}{\pi}-2 \\ > & 1, \end{aligned}$$

where we have used the condition that $N_s > \frac{3\pi-4}{3\pi-8}$ to obtain the last inequality. In light of the dynamics of ϕ_i in (3), once $N_i > 1$, ϕ_i will asymptotically converge to zero.

Remark 1. In practice, the convergence region of the consensus error is much smaller than (15). The conservativeness of this region is introduced in tackling δ_i with (11) and (12). Thus, the demands of $|\phi_i| \leqslant \frac{\pi}{2}$, $i = 1,\ldots,n$ and $N_s > \frac{3\pi-4}{3\pi-8}$ are also conservative, which accords with the simulation results.

Theorem 2. Suppose that Assumptions 1 and 2 hold, and $|\phi_i| \leqslant \frac{\pi}{2}$, $i = 1,\ldots,n$. For the multi-missile system (1) with the PN strategy (2) and the proposed navigation ratios (4), the simultaneous attack problem can be solved.

Proof. From Theorem 1 we can obtain that the consensus error ξ_i will decrease to region Δ_i and ϕ_i will asymptotically converge to zero. When ϕ_i decreases to a small angle, we have $\sin\phi_i \approx \phi_i$, $\cos\phi_i \approx 1-\frac{\phi_2}{2}$. Then, Eq. (7) turns into

$$\dot{\hat{t}}_{\mathrm{go},i} = -1+\frac{(N_s-N_i)\phi_i^2}{2N_s-1}, \qquad i=1,\ldots,n, \tag{16}$$

and the time derivative of V_1 along the trajectory of (8) is given by

$$\dot{V}_1 = \hat{t}_{\mathrm{go}}^{\mathrm{T}}\mathcal{L}\dot{\hat{t}}_{\mathrm{go}} = -\frac{N_s}{2N_s-1}\left(\sum_{i=1}^{n} a_i r_i\phi_i^2\xi_i^2+\sum_{i=1}^{n} b_i r_i\phi_i^2|\xi_i|^{1+\alpha}\right). \tag{17}$$

Define $a_m = \min\{a_1,\ldots,a_n\}$, $b_m = \min\{b_1,\ldots,b_n\}$. By Assumption 2, there exist constants r_m and ϕ_m such that $r_i \geqslant r_m$, $|\phi_i| \geqslant \phi_m$, $i = 1,\ldots,n$. Then, we can obtain that

$$\begin{aligned} \dot{V}_1 &\leqslant -\frac{N_s}{2N_s-1}\left(\sum_{i=1}^{n} a_m r_m\phi_m^2\xi_i^2+\sum_{i=1}^{n} b_m r_m\phi_m^2|\xi_i|^{1+\alpha}\right) \\ &\leqslant -\frac{N_s r_m\phi_m^2}{2N_s-1}\left(a_m\xi^{\mathrm{T}}\xi+b_m(\xi^{\mathrm{T}}\xi)^{\frac{1+\alpha}{2}}\right), \end{aligned} \tag{18}$$

where we have used Lemma 3 to obtain the last inequality.

By Lemma 4, we have

$$\dot{V}_1 \leqslant -\frac{2\lambda_2 N_s a_m r_m\phi_m^2}{2N_s-1}V_1-\frac{(2\lambda_2)^{\frac{1+\alpha}{2}}N_s b_m r_m\phi_m^2}{2N_s-1}V_1^{\frac{1+\alpha}{2}}, \tag{19}$$

implying that V_1 converges to zero in finite time, which further implies that $\hat{t}_{\mathrm{go},i}$, $i = 1,\ldots,n$, reach consensus and ξ converges to zero in finite time. Moreover, the settling time T satisfies

$$T \leqslant T_1+\frac{2N_s-1}{N_s\lambda_2 a_m r_m\phi_m^2(1-\alpha)}\ln\frac{2\lambda_2 a_m V_1^{\frac{1-\alpha}{2}}(T_1)+(2\lambda_2)^{\frac{1+\alpha}{2}}b_m}{(2\lambda_2)^{\frac{1+\alpha}{2}}b_m}. \tag{20}$$

Note that once ξ converges to zero, all the navigation ratios stop varying and are equal to the constant N_s. Then, the cooperative guidance law of each missile turns into the conventional PN guidance law, implying that all the missiles will hit the target with zero LOS rates and zero heading errors at the final time. Moreover, $\hat{t}_{\mathrm{go},i}$, $i = 1,\ldots,n$, will represent the true values of times-to-go. Therefore, all missiles will hit the target at the same time. This completes the proof.

Remark 2. The settling time T depends on the initial conditions, the communication topology and the control parameters N_s, a_i, b_i, and α. To satisfy Assumption 2, the settling time T should be small so that the consensus of time-to-go estimates can be achieved fast enough before any missile arrives the target. According to (20), we can choose large a_i, b_i and small N_s, α to obtain a small settling time T. If the case that some missiles reach the target before consensus happens, these missiles are no longer in the communication group due to the impact, while the remaining communication channels keep working, under which the remaining missiles will still synchronize their time-to-go estimates with the proposed guidance law. The simultaneous attack of the whole group fails, but that of the remaining missiles may be achieved if their communication graph is still connected.

Remark 3. Different from [6, 7] in which only consensus of the time-to-go estimates is considered, this paper steps further and achieves consensus of the real times-to-go. Compared with the centralized guidance law in [1], which relies on global onboard information of time-to-go estimates, the cooperative guidance law proposed in this paper only requires the onboard time-to-go estimates of each missile and its neighbors, and thereby is fully distributed. Moreover, the guidance law in [1] can only make the variance of time-to-go estimates decrease, while in this paper the consensus of the real times-to-go can be achieved in finite time.

4.2 Leader-follower communication graphs

Theorem 1 shows that protocol (4) is applicable for the case that the communication graph is undirected and connected. However, the communication channels are not always reliable in practice. In this subsection, we consider the situation that there exists a missile which cannot obtain the information from its neighbors. As a result, this missile takes the role of the leader, and its navigation ratio cannot be adjusted and remains constant. Without loss of generality, let this missile be the nth one.

We assume that the communication graph $\mathcal{G}$ satisfies the following assumption.

Assumption 3. The communication topology $\mathcal{G}$ is a leader–follower graph containing a directed spanning tree with the nth node as the root, whose navigation ratio N_n remains constant satisfying $N_n > 2$, and the subgraph among the followers is undirected.

In this case, the navigation ratio of the nth missile N_n cannot be adjusted and remains unchanged. The assumption $N_n > 2$ is to ensure that it will hit the target, according to the property of the conventional PN guidance law. In addition, we can obtain its actual time-to-go $t_{\mathrm{go},n} = \frac{r_n}{V_n}(1 + \frac{\phi_n^2}{2(2N_n-1)})$, and its time derivative $\dot{t}_{\mathrm{go},n} = -1$. Let $\hat{t}_{\mathrm{go},n}$ in (4) be $t_{\mathrm{go},n}$. The following theorems show that the proposed navigation ratios (4) are still applicable to this leader-follower case.

Theorem 3. Suppose that Assumptions 2 and 3 hold, and $|\phi_i| \leqslant \frac{\pi}{2}$, $i = 1, \dots, n-1$. For the multi-missile system (1) with the PN strategy (2) and the proposed navigation ratios (4), ξ_i, $i = 1, \dots, n-1$ will converge to a neighborhood of the origin, and ϕ_i, $i = 1, \dots, n-1$ will converge to zero.

Proof. Since the graph $\mathcal{G}$ satisfies Assumption 3, the Laplacian matrix can be partitioned into

$$\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 \\ 0_{1\times(n-1)} & 0 \end{bmatrix},$$

where $\mathcal{L}_1 \in \mathbb{R}^{(n-1)\times(n-1)}$ is symmetric and $\mathcal{L}_2 \in \mathbb{R}^{n-1}$. In light of Lemma 1, it is obvious that $\mathcal{L}_1$ is positive definite. Let $\tilde{t}_{\mathrm{go}} = [\hat{t}_{\mathrm{go},1}, \dots, \hat{t}_{\mathrm{go},n-1}]^{\mathrm{T}}$ and $\tilde{\xi} = [\xi_1, \dots, \xi_{n-1}]^{\mathrm{T}}$. Noting that $\mathcal{L}_1\mathbf{1} = -\mathcal{L}_2$, we have

$$\tilde{\xi} = -[\mathcal{L}_1 \quad \mathcal{L}_2][\tilde{t}_{\mathrm{go}}^{\mathrm{T}} \quad t_{\mathrm{go},n}]^{\mathrm{T}} = -\mathcal{L}_1(\tilde{t}_{\mathrm{go}} - t_{\mathrm{go},n}\mathbf{1}). \tag{21}$$

Evidently, $\tilde{\xi} = 0$ if and only if $\hat{t}_{\mathrm{go},i} = t_{\mathrm{go},n}$, $i = 1, \dots, n-1$.

Consider the following Lyapunov function candidate:

$$V_2 = \frac{1}{2}(\tilde{t}_{\mathrm{go}} - t_{\mathrm{go},n}\mathbf{1})^{\mathrm{T}}\mathcal{L}_1(\tilde{t}_{\mathrm{go}} - t_{\mathrm{go},n}\mathbf{1}). \tag{22}$$

Note that $\dot{\hat{t}}_{\mathrm{go},i}$, $i=1,\ldots,n-1$, is given in (8). The time derivative of V_2 along (8) can be obtained as

$$\begin{aligned}\dot{V}_2 &= (\tilde{t}_{\mathrm{go}}-t_{\mathrm{go},n}\mathbf{1})^{\mathrm{T}}\mathcal{L}_1(\dot{\tilde{t}}_{\mathrm{go}}-\dot{t}_{\mathrm{go},n}\mathbf{1})\\ &= -\sum_{i=1}^{n-1}\left(\frac{N_s a_i r_i\phi_i\sin\phi_i}{2N_s-1}\xi_i^2+\frac{N_s b_i r_i\phi_i\sin\phi_i}{2N_s-1}|\xi_i|^{1+\alpha}+\xi_i\delta_i\right).\end{aligned} \tag{23}$$

Denote by $\lambda_1>0$ the smallest eigenvalue of $\mathcal{L}_1$. Observe that

$$\left(\mathcal{L}_1^{\frac{1}{2}}(\hat{t}_{\mathrm{go}}-\hat{t}_{\mathrm{go},n}\mathbf{1})\right)^{\mathrm{T}}\mathcal{L}_1\left(\mathcal{L}_1^{\frac{1}{2}}(\hat{t}_{\mathrm{go}}-\hat{t}_{\mathrm{go},n}\mathbf{1})\right)\geqslant\lambda_1\left(\mathcal{L}_1^{\frac{1}{2}}(\hat{t}_{\mathrm{go}}-\hat{t}_{\mathrm{go},n}\mathbf{1})\right)^{\mathrm{T}}\left(\mathcal{L}_1^{\frac{1}{2}}(\hat{t}_{\mathrm{go}}-\hat{t}_{\mathrm{go},n}\mathbf{1})\right),$$

which means that $\tilde{\xi}^{\mathrm{T}}\tilde{\xi}\geqslant 2\lambda_1 V_2$. Aware of this, taking similar steps as in the proof of Theorem 1, we can obtain that ξ_i, $i=1,\ldots,n-1$ will converge to the region Δ_i, $i=1,\ldots,n-1$ in finite time and ϕ_i, $i=1,\ldots,n-1$ will asymptotically converge to zero.

Theorem 4. Suppose that Assumptions 2 and 3 hold, and $|\phi_i|\leqslant\frac{\pi}{2}$, $i=1,\ldots,n-1$. For the multi-missile system (1) with the PN strategy (2) and the proposed navigation ratios (4), the simultaneous attack problem can be solved.

Proof. Taking similar steps as in the proofs of Theorems 2 and 3, we have

$$\dot{V}_2\leqslant-\frac{N_s r_m\phi_m^2}{2N_s-1}\left(\tilde{a}_m\tilde{\xi}^{\mathrm{T}}\tilde{\xi}+\tilde{b}_m(\tilde{\xi}^{\mathrm{T}}\tilde{\xi})^{\frac{1+\alpha}{2}}\right), \tag{24}$$

where $\tilde{a}_m=\min\{a_1,\ldots,a_{n-1}\}$ and $\tilde{b}_m=\min\{b_1,\ldots,b_{n-1}\}$. Then, it follows from (24) that

$$\dot{V}_2\leqslant-\frac{2\lambda_1 N_s\tilde{a}_m r_m\phi_m^2}{2N_s-1}V_2-\frac{(2\lambda_1)^{\frac{1+\alpha}{2}}N_s\tilde{b}_m r_m\phi_m^2}{2N_s-1}V_2^{\frac{1+\alpha}{2}} \tag{25}$$

implying that V_2 will converge to zero in finite time, which further implies that $\tilde{\xi}$ will converge to zero and $\hat{t}_{\mathrm{go},i}$, $i=1,\ldots,n-1$ will all equal to $t_{\mathrm{go},n}$.

Note that once $\tilde{\xi}$ converges to zero, the followers' navigation ratios stop varying and equal the constant N_s. Then, all the missiles are under the conventional PN guidance law, implying that all the missiles will hit the target with zero LOS rate and zero heading error at the final time. Moreover, the time-to-go estimates will represent the true values of times-to-go. Therefore, all missiles will hit the target at the same time. This completes the proof.

Remark 4. In this version, the followers' navigation ratios will converge to the value N_s, while the leader's navigation ratio retains a constant N_n. Although the navigation ratios of the whole group may not converge to the same value, the times-to-go of them can still achieve consensus in finite time.

5 Simulation

In this section, simulations are conducted on the engagement scenario that five missiles attack a single stationary target from different directions. The speed of the missiles, the initial ranges-to-go, the initial LOS angles, and the initial heading errors are given as $V_1=350$, $V_2=300$, $V_3=250$, $V_4=325$, and $V_5=275$ m/s^2; $r_1=31000$, $r_2=36000$, $r_3=25000$, $r_4=30000$, and $r_5=35000$ m; $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda_5=0$; and $\phi_1=0.3$, $\phi_2=0.15$, $\phi_3=0.25$, $\phi_4=0.175$, and $\phi_5=0.3$ rad.

We consider two cases where the communication topologies among missiles are an undirected connected graph as shown in Figure 3(a) and a directed graph in the leader–follower structure as shown in Figure 3(b), respectively. In Case 2, the 5th missile takes the role of the leader and its navigation ratio keeps the constant $N_5=5$.

Now, we use the novel cooperative guidance law as described in Theorems 2 and 4 for Cases 1 and 2, respectively. In both cases, N_s is designed to be three, as three is known as the energy-optimal navigation ratio when attacking a stationary target. Other control parameters for Cases 1 and 2 are as follows:

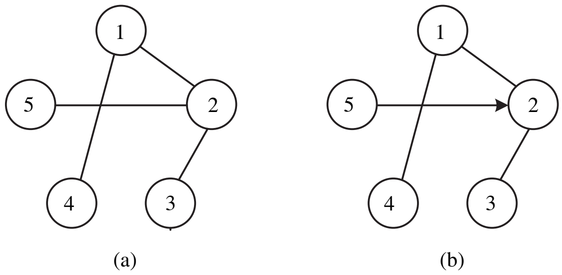

Figure 3 The communication topology among missiles. (a) Case 1; (b) Case 2.

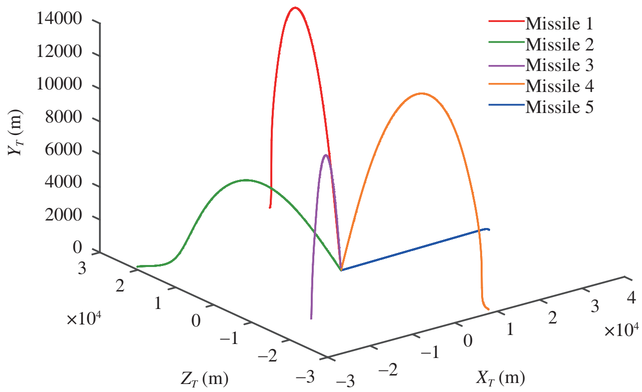

Figure 4 (Color online) Trajectories of missiles for Case 1.

Case 1: $\alpha = 0.8$, $a_1 = a_2 = 2 \times 10^{-5}$, $a_3 = a_4 = a_5 = 5 \times 10^{-5}$, $b_1 = b_2 = 2 \times 10^{-5}$, and $b_3 = b_4 = b_5 = 5 \times 10^{-5}$;

Case 2: $\alpha = 0.8$, $a_1 = 1.333 \times 10^{-5}$, $a_2 = 3.333 \times 10^{-5}$, $a_3 = a_4 = 5.128 \times 10^{-5}$, $b_1 = 10^{-5}$, $b_2 = 2.5 \times 10^{-5}$, and $b_3 = b_4 = 3.846 \times 10^{-5}$.

Simulation results for Case 1, including trajectories of missiles, ranges-to-go r_i, $i = 1, \ldots, 5$, estimates of time-to-go $\hat{t}_{\mathrm{go},i}$, $i = 1, \ldots, 5$, consensus errors of estimates of time-to-go ξ_i, $i = 1, \ldots, 5$, heading errors ϕ_i, $i = 1, \ldots, n$, navigation ratios N_i, $i = 1, \ldots, 5$, and missile accelerations a_i, $i = 1, \ldots, 5$, are presented in Figures 4–10. From Figures 4 and 5, we can see that the five missiles reach the target simultaneously. Figures 6 and 7 show that the consensus error ξ converges to zero and the estimates of times-to-go reach consensus. Figures 9 and 10 present the performance of the control inputs. It can be observed that all the navigation ratios converge to value $N_s = 3$ and all the missile accelerations are within the limits $|a_i| < 180$ m/s^2, $i = 1, \ldots, 5$. It should be noted that in the theorems, we demand $N_s > \frac{3\pi - 4}{3\pi - 8}$ and $|\phi_i| \leqslant \pi/2$, which are conservative. In the simulation, N_s is designed to be three, and ϕ_i exceed the limitation, the simultaneous attack task is still completed.

Simulation results for Case 2 are presented in Figures 11–17. From Figures 11 and 12, we can see that the five missiles can still attack the target simultaneously with the proposed guidance law, even when the navigation ratio of the 5th missile cannot be tuned. Figures 13 and 14 show that the consensus error ξ converges to zero and the estimates of times-to-go reach consensus. The navigation ratios are described by Figure 16, from which we can observe $N_5 = 5$ as we set, and $N_1, \ldots, N_4$ converge to $N_s = 3$. Figure 17 shows that the history of missile accelerations are all within the limits $|a_i| < 270$ m/s^2, $i = 1, \ldots, 5$.

6 Conclusion

In this paper, we studied the simultaneous attack problem of multiple missiles against a stationary target. We presented a consensus-based approach to design a cooperative guidance law that can achieve

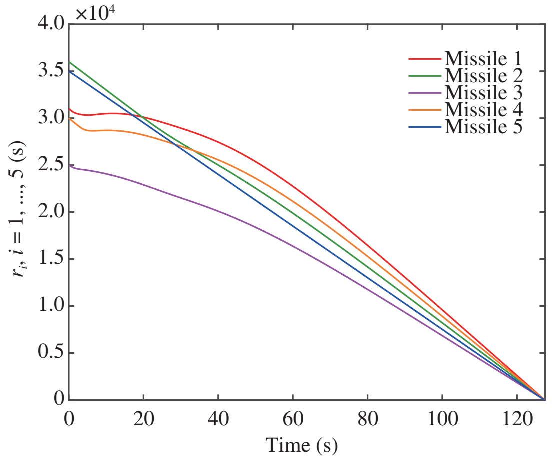

Figure 5 (Color online) Ranges-to-go histories for Case 1.

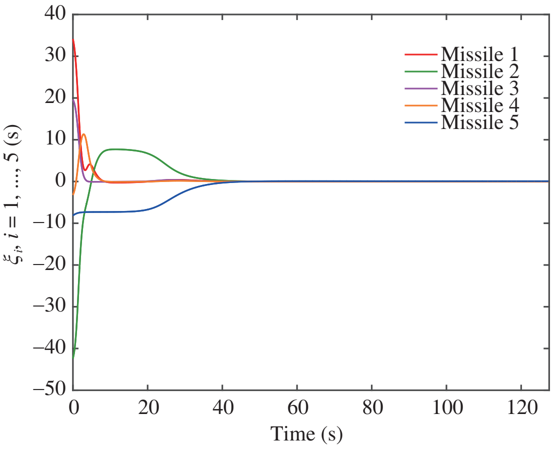

Figure 6 (Color online) Consensus errors of estimates of times-to-go for Case 1.

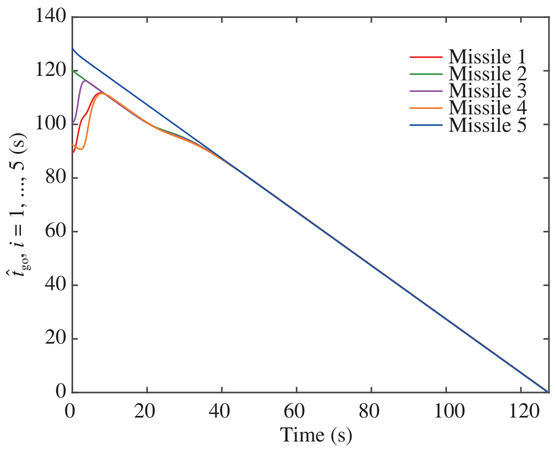

Figure 7 (Color online) Estimates of times-to-go for Case 1.

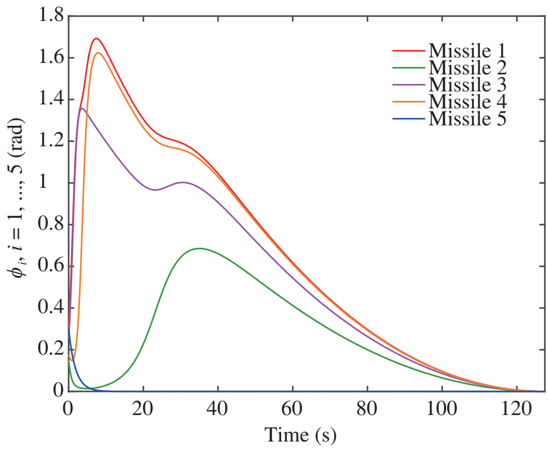

Figure 8 (Color online) Heading errors of missiles for Case 1.

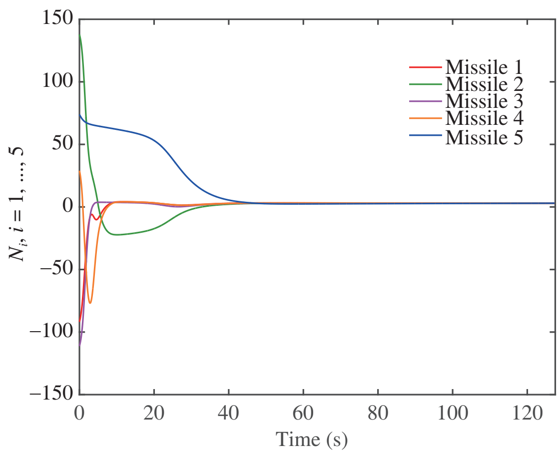

Figure 9 (Color online) Navigation ratios of missiles for Case 1.

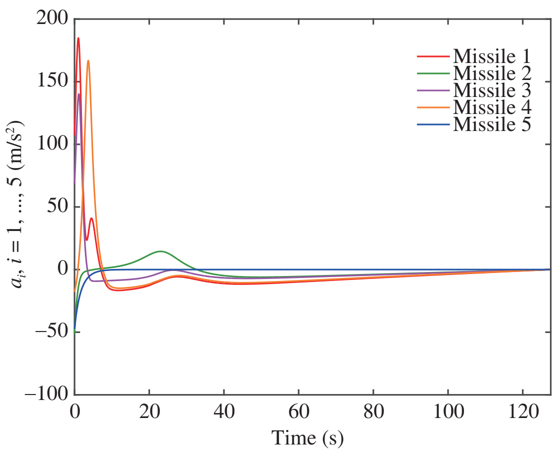

Figure 10 (Color online) Accelerations of missiles for Case 1.

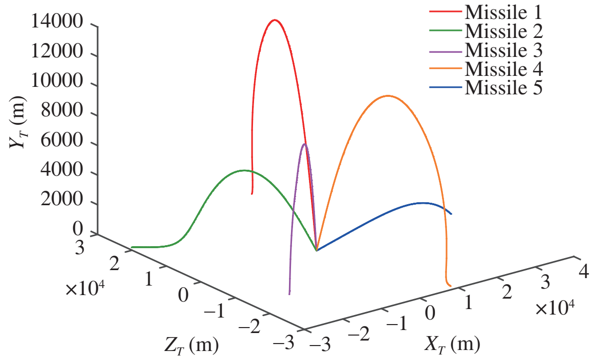

Figure 11 (Color online) Trajectories of missiles for Case 2.

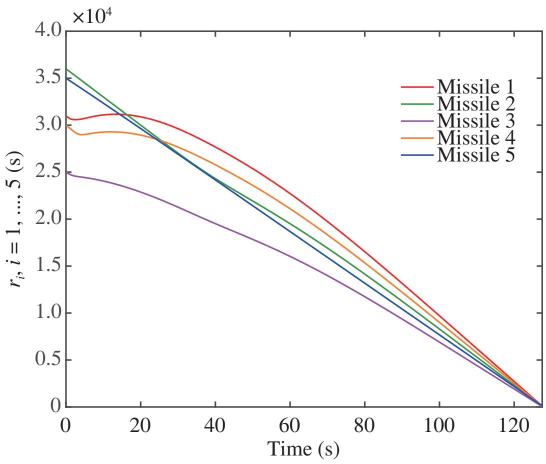

Figure 12 (Color online) Ranges-to-go histories for Case 2.

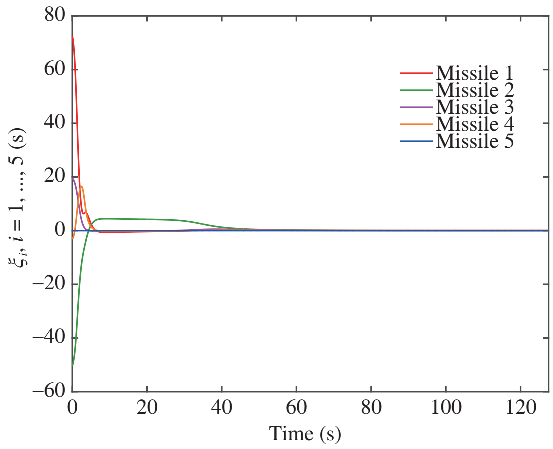

Figure 13 (Color online) Consensus errors of estimates of times-to-go for Case 2.

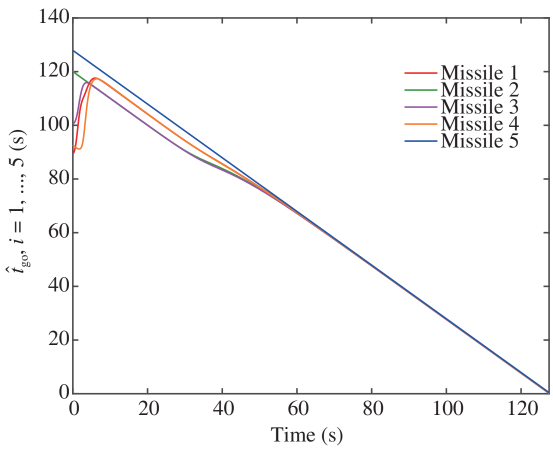

Figure 14 (Color online) Estimates of times-to-go for Case 2.

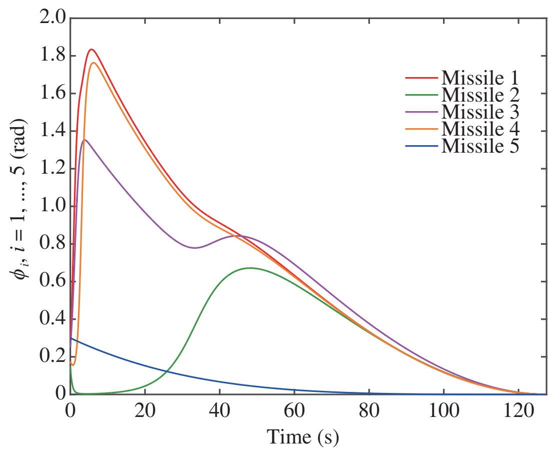

Figure 15 (Color online) Heading errors of missiles for Case 2.

simultaneous attack for both the case where the communication topology is undirected or in the leader–follower structure with a missile acting as the leader whose navigation ratio cannot be tuned. Compared with the existing related work, the main contribution of this paper is that the proposed cooperative guidance law depends on only the estimated times-to-go of the missile itself and its neighbors, and thereby

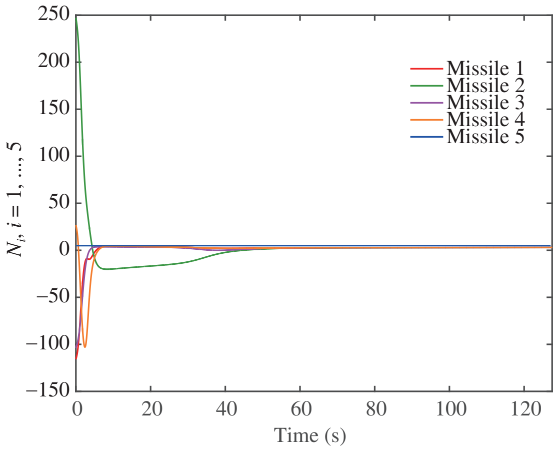

Figure 16 (Color online) Navigation ratios of missiles for Case 2.

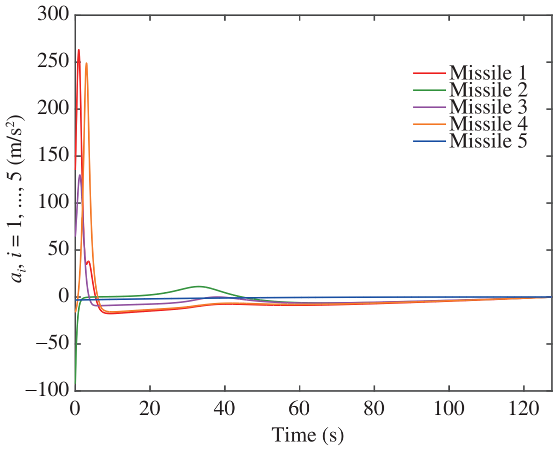

Figure 17 (Color online) Accelerations of missiles for Case 2.

is distributed, which can significantly improve the operational effectiveness and reduce the communication burden. Further study needs to be concerned on designing fully distributed adaptive guidance laws for general directed graph or the case that the target is not stationary but moving.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 11332001, 61473005, 91216304).

Conflict of interest The authors declare that they have no conflict of interest.

References

1 Jeon I S, Lee J I, Tahk M J. Homing guidance law for cooperative attack of multiple missiles. J Guidance Control Dyn, 2010, 33: 275–280

2 Jeon I S, Lee J I, Tahk M J. Impact-time-control guidance law for anti-ship missiles. IEEE Trans Contr Syst Tech, 2006, 14: 260–266

3 Lee J I, Jeon I S, Tahk M J. Guidance law to control impact time and angle. IEEE Trans Aerosp Electron Syst, 2007, 43: 301–310

4 Kim T H, Lee C H, Jeon I S, et al. Augmented polynomial guidance with impact time and angle constraints. IEEE Trans Aerosp Electron Syst, 2013, 49: 2806–2817

5 Harl N, Balakrishnan S N. Impact time and angle guidance with sliding mode control. IEEE Trans Contr Syst Tech, 2012, 20: 1436–1449

6 Snyder M G, Li C, Qu Z. A new parameterized guidance law for cooperative air defense. In: Proceedings of the 50th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition, Nashville, 2012

7 Zhang P, Liu H H T, Li X, et al. Fault tolerance of cooperative interception using multiple flight vehicles. J Franklin Institute, 2013, 350: 2373–2395

8 Wei X, Wang Y, Dong S, et al. A three-dimensional cooperative guidance law of multimissile system. Int J Aerospace Eng, 2015, 1: 1–8

9 Ou L, Wang Y, Liu L, et al. Cooperative control of multi-missile systems. IET Control Theory A, 2015, 9: 441–446

10 Olfati-Saber R, Murray R M. Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans Automat Contr, 2004, 49: 1520–1533

11 Olfati-Saber R, Fax J A, Murray R M. Consensus and cooperation in networked multi-agent systems. Proc IEEE, 2007, 95: 215–233

12 Li Z, Ren W, Liu X, et al. Distributed containment control of multi-agent systems with general linear dynamics in the presence of multiple leaders. Int J Robust Nonlinear Control, 2013, 23: 534–547

13 Zhao Y, Duan Z, Wen G. Distributed finite-time tracking of multiple Euler-Lagrange systems without velocity measurements. Int J Robust Nonlinear Control, 2015, 25: 1688–1703

14 Zhao Y, Duan Z, Wen G, et al. Distributed finite-time tracking control for multi-agent systems: an observer-based approach. Syst Control Lett, 2013, 62: 22–28

15 Li Z, Ren W, Liu X, et al. Distributed consensus of linear multi-agent systems with adaptive dynamic protocols. Automatica, 2013, 49: 1986–1995

16 Wen G, Duan Z, Yu W, et al. Consensus in multi-agent systems with communication constraints. Int J Robust Nonlinear Control, 2012, 22: 170–182

17 Wen G, Duan Z, Yu W, et al. Consensus of multi-agent systems with nonlinear dynamics and sampled-data information: a delayed-input approach. Int J Robust Nonlinear Control, 2013, 23: 602–619
18 Yu W, Ren W, Zheng W X, et al. Distributed control gains design for consensus in multi-agent systems with second-order nonlinear dynamics. Automatica, 2013, 49: 2107–2115
19 Zhang Y, Yang Y, Zhao Y. Finite-time consensus tracking for harmonic oscillators using both state feedback control and output feedback control. Int J Robust Nonlinear Control, 2013, 23: 878–893
20 Hu G. Robust consensus tracking of a class of second-order multi-agent dynamic systems. Syst Control Lett, 2012, 61: 134–142
21 Ren W, Beard R W, Atkins E M. Information consensus in multivehicle cooperative control. IEEE Control Syst Mag, 2007, 27: 71–82
22 Wang Q, Perc M, Duan Z, et al. Delay-induced multiple stochastic resonances on scale-free neuronal networks. Chaos, 2009, 19: 023112
23 Wang Q, Duan Z, Perc M, et al. Synchronization transitions on small-world neuronal networks: effects of information transmission delay and rewiring probability. Europhys Lett, 2008, 83: 50008
24 Olfati-Saber R, Murray R M. Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans Automat Contr, 2004, 49: 1520–1533
25 Yu S, Yu X, Shirinzadeh B, et al. Continuous finite-time control for robotic manipulators with terminal sliding mode. Automatica, 2005, 41: 1957–1964