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# Linear precoding based on a non-ideal Nakagami-$m$ channel in a massive MIMO system

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**Dear editor,**

Large scale multiple-input multiple-output (MIMO) is one of key technologies for next generation mobile communication [1–4]. However, in multiple user environments, multiple user interference (MUI) increases with the rise of the number of users. At present, one way to eliminate the MUI is to use precoding technology in the base station (BS), which involves BS preprocessing of the user signal for the downlink [5]. It is well known that dirty paper coding (DPC) can achieve capacity for the MIMO broadcast. But DPC is a nonlinear scheme and for most practical communication systems it is not feasible due to its very high computational complexity. In reality, there are channel state information (CSI) errors due to delays and other factors. Ref. [6] considered the constant envelope precoding method to minimize the MUI in a massive MIMO system. Ref. [7] proposed a dual-structured multi-user linear precoding, in which the subgrouping method based on co-polarization is additionally applied to the spatially grouped mobile stations in the preprocessing stage. However, those studies assumed Rayleigh fading, not Nakagami-$m$ fading. For more flexibility and accuracy in modelling the fading channel, massive MIMO systems have to be analyzed and tested regarding the effect of the Nakagami-$m$ fading channel, as it has demonstrated that Nakagami often yields a good fit with measured data in various land mobile and indoor mobile multipath propagation (or frequency selective) environments [8–10]. Motivated by this, the Nakagami-$m$ fading channel is considered in this paper, assuming that the CSI remains constant during the coherence time, and the different coherence times are orthogonal to each other in a time division duplex mode. We analyzed the effect of CSI estimation errors on the system performance, using the minimum mean square error (MMSE) criterion for the precoding matrix in a closed-form solution, and in the process we solved leakage equivalent interference to avoid using an iterative solution.

*Methodology.*Consider the single-cell scenario, where there are $K$ users. The BS has $N(N \geqslant 100)$ antennas, and the $k$-th user is equipped with $M_k$ antennas. $M = \sum_{k=1}^{K} M_k$ is defined as the total number of antennas among all users. The channel from the BS to the user is assumed to be the Nakagami-$m$ fading channel model. The CSI remains unchanged within the same coherence time, and users are distributed with the same power.

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The precoding matrix $\boldsymbol{W} \in \mathbb{C}^{N\times M}$ is defined for all users, and different precoding schemes have different precoding matrices. $s_k \in \mathbb{C}^{M_k\times L_k}$ is the $k$-th user's data and $L_k$ is the length of the signal vector. The actual channel matrix of the $k$-th user is defined as $\boldsymbol{H}_k \in \mathbb{C}^{M_k\times N}$, and each of its elements is subject to a Nakagami-$m$ distribution. The probability density function is given by [8]

$$f(x) = \frac{2x^{2m-1}}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m e^{-\frac{mx^2}{\Omega}}, \tag{1}$$

where $m$ is the Nakagami fading index, which reflects the fading situation, and $m \geqslant 0.5$ under normal circumstances. Delays and other factors result in a CSI estimation error at the BS. From [8], it is assumed that the BS estimates the channel matrix for the $k$-th user as

$$\hat{\boldsymbol{H}}_k = \boldsymbol{H}_k + \boldsymbol{E}_k, \tag{2}$$

where $\hat{\boldsymbol{H}}_k$ and $\boldsymbol{H}_k$ are the estimated and actual channel matrices of the $k$-th user respectively, $\boldsymbol{E}_k$ is the channel estimation error that is dependent on $\boldsymbol{H}_k$ , and its component has an independently and identically distributed complex Gaussian distribution with zero mean and a variance of $\sigma_\varepsilon^2$. In the presence of channel estimation errors it will affect the precoding matrix design. During the transmission from the BS to the user, the actual channel is different from the estimation's, so in addition to interference between users, there is interference caused by the estimation error. The signal from the $k$-th user is given by

$$\mathbf{y}_k = \beta\boldsymbol{H}_k\boldsymbol{W}_k s_k + \sum_{i=1,i\neq k}^{K}\beta\boldsymbol{H}_k\boldsymbol{W}_i s_i + \boldsymbol{n}_k, \tag{3}$$

where $\boldsymbol{W}_k \in \mathbb{C}^{N\times M_k}$ is the $k$-th user's precoding matrix designed by the estimation channel matrix, and the noise vector at the user is defined as $\boldsymbol{n}_k$ , which have identically Gaussian distribution with zero mean and a variance of $\sigma_n^2$. It is assumed that the signal vectors of different users are independent of each other. $\beta$ is the power factor, which meets $E\{\|\beta\boldsymbol{W}_k s_k\|^2\} = P_k$ and $P_k$ is the $k$-th user's transmit power, where $\|\bullet\|$ represents the Frobenius norm. We design the precoding matrix by minimizing the mean square error criterion when a channel estimation error exists at the BS, and signal leakage is used to replace the interface between users [10]. We can then obtain a precoding matrix of the closed-form solution through a noniterative method. From (3), the cost function of the precoding matrix is

$$\min_{W_k} E\{\|\beta^{-1}\mathbf{y}_k - \mathbf{s}_k\|^2\}, \text{ s.t. } \beta^2\|W_k\|^2 = P_k. \tag{4}$$

The key to obtain the precoding matrix is to solve (4). Defining $\Phi = E\{\|\beta^{-1}\mathbf{y}_k - \mathbf{s}_k\|^2\}$. From (3), we can get:

$$\Phi = E\left\{\left\|\boldsymbol{H}_k\sum_{i=1}^{K}\boldsymbol{W}_i\boldsymbol{s}_i - \boldsymbol{s}_k\right\|^2 + \beta^{-2}\|\boldsymbol{n}_k\|^2\right\}. \tag{5}$$

In (5), we know that $\sum_{i=1,i\neq k}^{K}\boldsymbol{H}_k\boldsymbol{W}_i s_i$ is the interference between other users and the $k$-th user, and the formula contains $K$ precoding matrix variables. For convenience in solving the equation, we may use the signal leakage to replace the interference between users. The received signal matrix can be gotten by all the users:

$$\boldsymbol{HWs} = \begin{bmatrix}\boldsymbol{H}_1\boldsymbol{W}_1 & \cdots & \boldsymbol{H}_1\boldsymbol{W}_K\\ \vdots & \ddots & \vdots\\ \boldsymbol{H}_K\boldsymbol{W}_1 & \cdots & \boldsymbol{H}_K\boldsymbol{W}_K\end{bmatrix}\begin{bmatrix}\boldsymbol{s}_1\\ \vdots\\ \boldsymbol{s}_K\end{bmatrix}. \tag{6}$$

If we can eliminate the interference between users, the signal coefficient matrix is a diagonal matrix in (6), and the nondiagonal elements are 0. In addition, $\sum_{i=1,i\neq k}^{K}\boldsymbol{H}_k\boldsymbol{W}_i s_i$ corresponds to each line's element in the signal coefficient matrix in addition to the $k$-th user; eliminating it means eliminating interference from the line vector angle, and ultimately makes the coefficient matrix for the diagonal matrix. To make $\sum_{i=1,i\neq k}^{K}\boldsymbol{H}_k\boldsymbol{W}_i s_i$ not contain $K$ precoding matrix variables, we eliminate interference from the angle of the column vector to get the diagonal's coefficient matrix. Then (5) can be rewritten as

$$\Phi = E\left\{\left\|\boldsymbol{W}_k\boldsymbol{s}_k\sum_{i=1}^{K}\boldsymbol{H}_i - \boldsymbol{s}_k\right\|^2 + \beta^{-2}\|\boldsymbol{n}_k\|^2\right\}. \tag{7}$$

From (4), we can get $\beta^2 = \frac{P_k}{\|\boldsymbol{W}_k\|^2}$. Defining $\boldsymbol{H}_{-k} = \sum_{i=1,i\neq k}^{K}\boldsymbol{H}_i$, $\boldsymbol{E}_{-k} = \sum_{i=1,i\neq k}^{K}\boldsymbol{E}_i$. Then the right side of (7) is expressed as

$$\begin{aligned}\operatorname{tr}\Big[&\boldsymbol{W}_k^{\mathrm{H}}\left(\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{H}_k + \boldsymbol{H}_{-k}^{\mathrm{H}}\boldsymbol{H}_{-k}\right)\boldsymbol{W}_k + \boldsymbol{I}_{M_k}\\ &- \boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{H}_k^{\mathrm{H}} - \boldsymbol{H}_k\boldsymbol{W}_k + \frac{M_k\sigma_n^2}{P_k}\boldsymbol{W}_k\boldsymbol{W}_k^{\mathrm{H}}\Big].\end{aligned} \tag{8}$$

In order to get $\boldsymbol{W}_k$, $\boldsymbol{W}_k$ is assumed to be dependent on $\boldsymbol{W}_k^{\mathrm{H}}$ [7]. We define

$$\begin{aligned}f(\boldsymbol{W}_k) = {}&\boldsymbol{W}_k^{\mathrm{H}}\left(\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{H}_k + \boldsymbol{H}_{-k}^{\mathrm{H}}\boldsymbol{H}_{-k}\right)\boldsymbol{W}_k\\ &- \boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{H}_k^{\mathrm{H}} - \boldsymbol{H}_k\boldsymbol{W}_k + \frac{M_k\sigma_n^2}{P_k}\boldsymbol{W}_k\boldsymbol{W}_k^{\mathrm{H}}.\end{aligned} \tag{9}$$

From (2), we can obtain:

$$\boldsymbol{H}_k = \hat{\boldsymbol{H}}_k - \boldsymbol{E}_k. \tag{10}$$

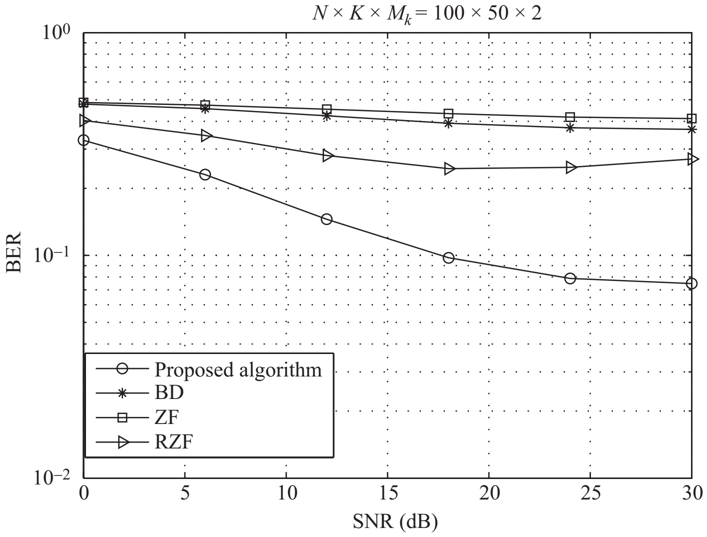


**Figure 1** BER Comparison of different pre-coding algorithms.

Substituting (10) into (9) and letting the first order derivatives of $f(\boldsymbol{W}_k)$ on $\boldsymbol{W}_k$ equal zero, the $k$-th user's precoding matrix is

$$\boldsymbol{W}_k = \left[\sum_{i=1}^{K}\left(\hat{\boldsymbol{H}}_i^{\mathrm{H}}\hat{\boldsymbol{H}}_i + \boldsymbol{E}_i^{\mathrm{H}}\boldsymbol{E}_i\right) + \frac{M_k\sigma_n^2}{P_k}\right]^{-1} \times \left(\hat{\boldsymbol{H}}_k - \boldsymbol{E}_k\right)^{\mathrm{H}}. \tag{11}$$

Eq. (11) shows that the precoding matrix is related to channel estimation error, power, and the number of receiving antennas. The algorithm does not have the process of singular decomposition, so it is simpler than BD's. We verify the performance of the proposed algorithm by simulation and compare with zero forcing (ZF), regularised zero forcing (RZF), and block diagonal (BD) precoding algorithms, when the BS has a channel estimation error. It is assumed that there are $K = 50$ users. Each user is equipped with 2 antennas, $\sigma_n^2 = 1$ and $\sigma_\varepsilon^2 = 0.02$. $m = 0.75$, $\Omega = 1$, and the bit error ratio (BER) is calculated based on quadrate phase shift keying (QPSK) modulation. Figure 1 shows the BER comparison among the different algorithms, where $N = 100$. It can be seen from the figure that the proposed algorithm performs well compared with the other three algorithms, and the performance gap increases with increasing the SNR when there are channel estimation errors at the BS, which compensate for the impact of channel estimation errors on the system performance.

*Conclusion.*In this letter, aiming at the problem of non-ideal Nakagami-$m$ fading channels, we obtained a closed-form solution to the precoding matrix by minimizing the mean square error criterion and substituting signal leakage for user interference.

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