Mixed $H_–/H_\infty$ fault detection filter design for the dynamics of high speed train

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Here we investigate the fault detection filter design problems of the dynamics of the high speed train (HST). Some of the researches have been addressed the modeling problems of HST [1, 2], but for handling convenience the nonlinear dynamic characters of the couplers are ignored, which is inevitably introducing some uncertainty factors to the dynamic model of HST. The fault diagnosis technology has been gaining great development recent years [3–6], but few researches have been conducted to extend its application to the dynamics of HST. In this article, the nonlinear characters of couplers are considered. And the disturbance attenuation conditions are considered as $H_\infty$ norm formulation, while the fault sensitivity conditions are expressed as $H_–$ index. We divide the design process of the fault detection filter into three major steps, which are respectively addressed as follows.

Dynamics of HST. Consider the dynamics of HST that are subject to rolling mechanical resistance, aerodynamic drag and wind gust in longitudinal motion as a cascade of cars connected with flexible couplers. The stiffness coefficient is defined as $k$, $k \in [k^-, k^+]$, where $k^-$, $k^+$ represent the minimal value and the maximal value of stiffness coefficient of the coupler, respectively. The dynamic equation of the motion of n-cars HST can be formulated as

$$\begin{align*}
\dot{p}_i &= v_i - v_{i+1}, \quad i = 1, 2, \ldots, n-1, \\
m_1\dot{v}_1 &= u_1 - kp_1 - (c_0 + c_1 v_1)m_1 + v_1 \\
&- c_2 \left(\sum_{i=1}^{n-1} m_i\right)v_1^2, \\
m_i\dot{v}_i &= u_i - kp_{i-1} - kp_i - (c_0 + c_1 v_1)m_i, \\
m_n\dot{v}_n &= u_n + kp_{n-1} - (c_0 + c_1 v_1)m_n,
\end{align*}$$

where $p_i$ is the relative displacement between two adjacent cars $i$ and $i+1$, $m_i$ is the mass of the $i$ car, $v_i$ and $u_i$ is the speed and effort of the $i$ car respectively. $w_i$ is the wind gust. Apply linearization technique at a desired cruising speed where $\ddot{v}_1 = \ddot{v}_2 = \cdots = \ddot{v}_n = v_r$ and $\dot{v}_1 = 0, \dot{p}_1 = 0$. Define the control efforts in the equilibrium state as $\dot{u}_i$ and let $\dot{p}_i = p_i - \dot{u}_i$, $\dot{v}_i = v_i - \dot{v}_i$, $\dot{u}_i = u_i - \ddot{u}_i$, $x(t) = [\dot{p}_1, \dot{p}_2, \ldots, \dot{p}_{n-1}, \dot{v}_1, \dot{v}_2, \ldots, \dot{v}_n]^T$, $u(t) = [\dot{u}_1, \dot{u}_2, \ldots, \dot{u}_n]^T$ and the faults $f(t)$ are supposed to occur in the actuators, then we attain the following linearized equations of the dynamics of HST:

$$\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(u(t) + B_ww(t)) + F_\alpha f(t), \\
y(t) &= Cx(t) + D_ww(t),
\end{align*}$$

$$\begin{align*}
\begin{bmatrix}
A & B_w & B_u \\
F_a & C & D_w
\end{bmatrix} &\in \left\{ \sum_{i=1}^{z} \alpha_i \begin{bmatrix}
A_{i} & B_{w,i} & B_{u,i} \\
F_{a,i} & C_{i} & D_{w,i}
\end{bmatrix}_\alpha \in \Gamma \right\}, \\
\Gamma := \left\{ (\alpha_1, \ldots, \alpha_z) : \sum_{i=1}^{z} \alpha_i = 1, \alpha_i \geq 0 \right\}.
\end{align*}$$

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Theorem 1. Consider systems (1) and (2), for given $r_w > 0$, $r_u > 0$, the following conditions are equivalent:

1. $\max_{\omega \in \Omega} \omega_{u,i} \theta(F \omega_{u,i}(jw)) < r_w$, $r_u > 0$.
2. $\max_{\omega \in \Omega} \omega_{u,i} \theta(F \omega_{u,i}(jw)) < r_u$, $r_u > 0$.

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There exist symmetric matrices $X_{w,i} > 0$, $X_{u,i} > 0$, such that the following matrix inequalities hold:

\[
\begin{bmatrix}
N_{1,1}^i & [A] \quad [B] \quad [C] \\
A & 0 & 0 & 0 \\
B & 0 & 0 & 0 \\
C & 0 & 0 & 0
\end{bmatrix}_{3x3} < 0, \quad i = 1, \ldots, z,
\]

\[
\begin{bmatrix}
N_{1,1}^i & [A] \quad [B] \quad [C] \\
A & 0 & 0 & 0 \\
B & 0 & 0 & 0 \\
C & 0 & 0 & 0
\end{bmatrix}_{3x3} < 0, \quad i = 1, \ldots, z,
\]

\[
A_i = \begin{bmatrix}
A_{w,i} & 0 \\
B_{w}C_{w} & A_F
\end{bmatrix}, \quad C = \begin{bmatrix}
C_{w}^T H_F^T \\
L_F^T
\end{bmatrix},
\]

\[
N_{1,1}^{w,i} = A_{w,i} X_{w,i} + X_{w,i} A_{i}, \quad N_{1,2}^{w,i} = -r_u I,
\]

\[
N_{1,1}^{w,i} = -r_w I, \quad N_{1,2}^{w,i} = X_{w,i} \begin{bmatrix}
B_{w}\end{bmatrix},
\]

\[
N_{1,1}^{w,i} = C, \quad N_{1,2}^{w,i} = 0.
\]

(3) There exist $\hat{V}, \hat{A}_F, B_F, H_F, L_F$, symmetric matrices $X_{w,i}, X_{u,i}$, and $\mu > 0$, where the first $n_p$ columns of matrix $B_F$ are zero vectors and

\[
[M_i, \mu]_{5x5} < 0, \quad i = 1, \ldots, z,
\]

\[
[M_i, \mu]_{5x5} < 0, \quad i = 1, \ldots, z.
\]

Define $X_{c,d} \doteq [X_{c}^T, x_F^T]^T$, then the composite system dynamics satisfies

\[
\dot{X}_{c,d} = \begin{bmatrix}
A_{w} & 0 \\
B_{w}C_{w} & A_F
\end{bmatrix} X_{c,d} + \begin{bmatrix}
B_{tu} & 0 \\
0 & 0
\end{bmatrix} u + \begin{bmatrix}
B_{w} & 0 \\
0 & 0
\end{bmatrix} w
\]

\[
+ \begin{bmatrix}
B_{f} & 0
\end{bmatrix} f, \quad r(t) = [H_F C_w L_F] X_{c,d}.
\]

\[
\begin{bmatrix}
A_{w} & 0 \\
B_{w}C_{w} & A_F
\end{bmatrix} X_{c,d} + \begin{bmatrix}
B_{tu} & 0 \\
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\end{bmatrix} u + \begin{bmatrix}
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\]

\[
+ \begin{bmatrix}
B_{f} & 0
\end{bmatrix} f, \quad r(t) = [H_F C_w L_F] X_{c,d}.
\]
which has a high pass property with the form of
\[ y_n(t) = \frac{1}{\tau} CA + Cx(t) + \frac{1}{\tau} CF_0 f(t) + \frac{1}{\tau} CB u(t) + \frac{1}{\tau} CB w(t). \]
Define a weighting system \( W_2 \) which has a high pass property with the form of
\[ W_2 : \begin{cases}
  \hat{z}_n = \hat{A}_n \hat{z}_n + \hat{B}_n \hat{v}_n \\
  \hat{v}_n = C_n \hat{z}_n + D_n \hat{w}(t).
\end{cases} \]
Let \( X_{cl} \triangleq \begin{bmatrix} \hat{r}_e \\ \hat{r}_p \end{bmatrix} \), then the composite system dynamic satisfies
\[
X_{cl} = \begin{bmatrix} A_f \\ B_f Ch_f Af \end{bmatrix} X_{cl} + \begin{bmatrix} B_hf \\ B_f Dh_f \end{bmatrix} f + \mathcal{P}(w, u), \quad \hat{r} = H_f D_h f + \mathcal{P}(d, u).
\]

Algorithm design of fault detection filter. Let \( \mu_1 > 0, \gamma > 0, \lambda = 0 \).

Step 1. Choose a proper large \( \mu > 0 \); Minimize \( \omega u r w + \alpha u r_u \) subject to (5) and (6). The solutions are denoted as \( A_F^{\text{opt}}, B_F^{\text{opt}}, L_F^{\text{opt}}, H_F^{\text{opt}} \).

Step 2. (a) Substitute \( A_F^{\text{opt}}, B_F^{\text{opt}}, L_F^{\text{opt}}, H_F^{\text{opt}} \) into (3), (4) and (10), minimize \( \omega u r w + \alpha u r_u \). Denote the solution as \( P_F^{\text{opt}}, X^{\text{opt}}, X_u^{\text{opt}}, i = 1, \ldots, z, r_u^{i, \text{opt}}, u_r^{i, \text{opt}} \).

Step 3. Derive \( A_F, B_F, L_F, H_F \) from \( \tilde{L}, \tilde{C} \); With \( A_F, B_F, L_F, H_F \) minimize \( r_u \) subject to (4); \( r_f = r_f^1, u_r = r_1^w \).

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