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Mixed H_-/H_∞ fault detection filter design for the dynamics of high speed train

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Here we investigate the fault detection filter design problems of the dynamics of the high speed train (HST). Some of the researches have been addressed the modeling problems of HST [1, 2], but for handling convenience the nonlinear dynamic characters of the couplers are ignored, which is inevitably introducing some uncertainty factors to the dynamic model of HST. The fault diagnosis technology has been gaining great development recent years [3–6], but few researches have been conducted to extend its application to the dynamics of HST. In this article, the nonlinear characters of couplers are considered. And the disturbance attenuation conditions are considered as H_∞ norm formulation, while the fault sensitivity conditions are expressed as H_- index. We divide the design process of the fault detection filter into three major steps, which are respectively addressed as follows.

Dynamics of HST. Consider the dynamics of HST that are subject to rolling mechanical resistance, aerodynamic drag and wind gust in longitudinal motion as a cascade of cars connected with flexible couplers. The stiffness coefficient is defined as k, $k \in [k^-, k^+]$, where k^-, k^+ represent the minimal value and the maximal value of stiffness coefficient of the coupler, respectively. The dynamic equation of the motion of n-cars HST can be formulated as

$$\begin{cases} \dot{p}_i = v_i - v_{i+1},\ i = 1, 2, \ldots, n-1, \\ m_1\dot{v}_1 = u_1 - kp_1 - (c_0 + c_1 v_1)m_1 + w_1 \\ \quad -c_2\left(\sum_{i=1}^{n} m_i\right){v_1}^2, \\ m_i\dot{v}_i = u_i + kp_{i-1} - kp_i - (c_0 + c_1 v_1)m_i, \\ m_n\dot{v}_n = u_n + kp_{n-1} - (c_0 + c_1 v_1)m_n, \end{cases}$$

where p_i is the relative displacement between two adjacent cars i and $i+1$, m_i is the mass of the i car, v_i and u_i is the speed and effort of the i car respectively. w_1 is the wind gust. Apply linearization technique at a desired cruising speed where $\bar{v}_1 = \bar{v}_2 = \cdots = \bar{v}_n = v_r$ and $\dot{\bar{v}}_i = 0, \bar{p}_i = 0$. Define the control efforts in the equilibrium state as $\bar{u}_i$ and let $\hat{p}_i = p_i - \bar{p}_i, \hat{v}_i = v_i - \bar{v}_i, \hat{u}_i = u_i - \bar{u}_i$, $x(t) = [\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_{n-1}, \hat{v}_1, \hat{v}_2, \ldots, \hat{v}_n]^{\mathrm{T}}$, $u(t) = [\frac{\hat{u}_1(t)}{m_1}, \frac{\hat{u}_2(t)}{m_2}, \ldots, \frac{\hat{u}_n(t)}{m_n}]^{\mathrm{T}}$ and the faults $f(t)$ are supposed to occur in the actuators, then we attain the following linearized equations of the dynamics of HST:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_u u(t) + B_w w(t) + F_a f(t), \\ y(t) = Cx(t) + D_w w(t), \end{cases}$$

$$\begin{pmatrix} A & B_w & B_u \\ F_a & C & D_w \end{pmatrix} \in \left\{ \sum_{i=1}^{z} \alpha_i \begin{pmatrix} A_i & B_{w,i} & B_{u,i} \\ F_{a,i} & C_i & D_{w,i} \end{pmatrix} \alpha \in \Gamma \right\},$$

$$\Gamma := \left\{ (\alpha_1, \ldots, \alpha_z) : \sum_{i=1}^{z} \alpha_i = 1, \alpha_i \geqslant 0 \right\}.$$

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$x \in \Re^{n_p}$, $u \in \Re^{n_u}$, $y \in \Re^{n_y}$, $f \in \Re^{n_f}$, $n_y \geqslant n_f$. F_a is the distribution matrix of faults $f(t)$ and Γ is the unitary simplex.

Robustness and sensitivity conditions. We introduce a deconvolution filter based residual generator having the form of

$$F : \begin{cases} \dot{x}_F(t) = A_F x_F(t) + B_F \eta(t), \\ r(t) = L_F x_F(t) + H_F \eta(t), \end{cases} \tag{1}$$

and $x_F(t) \in \Re^{n_F}$, $r(t) \in \Re^p$, $\eta(t) = [y(t)^{\mathrm{T}}, u(t)^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{n_u+n_y}$. Define weighting system W_1 which has a low pass property in the following form: $W_1 : \left\{ \begin{smallmatrix} \dot{x}_l = A_l x_l + B_l \eta(t) \\ \eta_l(t) = C_l x_l \end{smallmatrix} \right.$ and $X_w \triangleq [x^{\mathrm{T}}\ x_l^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{n_p+n_w}$, then the system in the augment space is

$$\begin{cases} \dot{X}_w = A_w X_w + B_{lw} w(t) \\ \qquad\quad + B_{lu} u(t) + B_{lf} f(t), \\ \eta_l(t) = C_w X_w, \end{cases} \tag{2}$$

$$A_w \triangleq \begin{bmatrix} A & 0 \\ B_l \hat{I}_{n_y} C & A_l \end{bmatrix},\ B_{lw} \triangleq \begin{bmatrix} B_w \\ B_l \hat{I}_{n_y} D_w \end{bmatrix},$$
$$\tilde{I}_{n_u} = \begin{bmatrix} 0 \\ I_{n_u} \end{bmatrix},\ B_{lu} \triangleq \begin{bmatrix} B_u \\ B_l \tilde{I}_{n_u} \end{bmatrix},$$
$$C_w \triangleq \begin{bmatrix} 0 \\ C_l^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},\ \hat{I}_{n_y} \triangleq \begin{bmatrix} I_{n_y} \\ 0 \end{bmatrix}.$$

Define $X_{cl} \triangleq [X_w^{\mathrm{T}}\ x_F^{\mathrm{T}}]^{\mathrm{T}}$, then the composite system dynamics satisfies

$$\dot{X}_{cl} = \begin{bmatrix} A_w & 0 \\ B_F C_w & A_F \end{bmatrix} X_{cl} + \begin{bmatrix} B_{lu} \\ 0 \end{bmatrix} u + \begin{bmatrix} B_{lw} \\ 0 \end{bmatrix} w + \begin{bmatrix} B_{lf} \\ 0 \end{bmatrix} f, r(t) = \begin{bmatrix} H_F C_w & L_F \end{bmatrix} X_{cl}.$$

Theorem 1. Consider systems (1) and (2), for given $r_w > 0$, $r_u > 0$, the following conditions are equivalent:

(1) $\max\limits_{\alpha\in\Gamma} \bar{\sigma}_{w\in\Omega_1}(F_{rw}(jw)) < r_w$, $r_w > 0$, $\max\limits_{\alpha\in\Gamma} \bar{\sigma}_{u\in\Omega_1}(F_{ru}(jw)) < r_u$, $r_u > 0$.

(2) There exist symmetric matrices $X_{w,i} > 0$, $X_{u,i} > 0$, such that the following matrix inequalities hold:

$$[N_{s,t}^i]_{3\times 3} < 0,\ i = 1, \ldots, z, \tag{3}$$
$$\left[\mathbb{N}_{s,t}^i\right]_{3\times 3} < 0,\ i = 1, \ldots, z. \tag{4}$$

$$\mathcal{A}_i = \begin{bmatrix} A_{w,i} & 0 \\ B_F C_w & A_F \end{bmatrix},\ \mathcal{C} = \begin{bmatrix} C_w^{\mathrm{T}} H_F^{\mathrm{T}} \\ L_F^{\mathrm{T}} \end{bmatrix},$$

$$N_{1,1}^i = \mathcal{A}_i^{\mathrm{T}} X_{w,i} + X_{w,i} \mathcal{A}_i,\ N_{2,2}^i = -r_w I,$$
$$N_{3,3}^i = -r_w I,\ N_{1,2}^i = X_{w,i} \begin{bmatrix} B_{lw,i} \\ 0 \end{bmatrix},$$
$$N_{1,3}^i = \mathcal{C},\ N_{2,3}^i = 0.$$
$$\mathbb{N}_{1,1}^i = \mathcal{A}_i^{\mathrm{T}} X_{u,i} + X_{u,i} \mathcal{A}_i,\ \mathbb{N}_{2,2}^i = -r_u I,$$
$$\mathbb{N}_{3,3}^i = -r_u I,\ \mathbb{N}_{1,2}^i = X_{u,i} \begin{bmatrix} B_{lu,i} \\ 0 \end{bmatrix},$$
$$\mathbb{N}_{1,3}^i = \mathcal{C},\ \mathbb{N}_{2,3}^i = 0.$$

(3) There exist $\hat{V}, \hat{A}_F, \hat{B}_F, H_F, \hat{L}_F$, symmetric matrices $\hat{X}_{wi}, \hat{X}_{ui}$, and $\mu > 0$, where the first n_p columns of matrix $\hat{B}_F$ are zero vectors and

$$[M_{s,t}^i]_{5\times 5} < 0,\ i = 1, \ldots, z, \tag{5}$$
$$\left[\mathbb{M}_{s,t}^i\right]_{5\times 5} < 0,\ i = 1, \ldots, z. \tag{6}$$

$\mathcal{F} = \theta C_w^{\mathrm{T}} H_F^{\mathrm{T}} + \gamma \hat{L}_F^{\mathrm{T}}$, $\mathcal{E} = \hat{V}^{\mathrm{T}} \theta A_{w,i} \theta^{\mathrm{T}} + \Gamma^{\mathrm{T}} \hat{K}$, $\mathcal{D} = \mu(\hat{V} + \hat{V}^{\mathrm{T}})$, $M_{1,1}^i = \mathcal{D}$, $M_{1,2}^i = \hat{X}_{w,i} + \mathcal{E}$, $M_{1,3}^i = \hat{V}^{\mathrm{T}} \theta B_{lw,i}$, $M_{1,5}^i = \mu \hat{V}^{\mathrm{T}}$, $M_{2,2}^i = -\hat{X}_{w,i}$, $M_{2,4}^i = \mathcal{F}$, $M_{3,3}^i = -r_w I$, $M_{4,4}^i = -r_w I$, $M_{5,5}^i = -\hat{X}_{w,i}$, $\mathbb{M}_{1,1}^i = \mathcal{D}$, $\mathbb{M}_{1,2}^i = \hat{X}_{u,i} + \mathcal{E}$, $\mathbb{M}_{3,3}^i = -r_u I$, $\mathbb{M}_{4,4}^i = -r_u I$, $\mathbb{M}_{1,3}^i = \hat{V}^{\mathrm{T}} \theta B_{lu,i}$, $\mathbb{M}_{1,5}^i = \mu \hat{V}^{\mathrm{T}}$, $\mathbb{M}_{2,2}^i = -\hat{X}_{u,i}$, $\mathbb{M}_{2,4}^i = \mathcal{F}$, $\mathbb{M}_{5,5}^i = -\hat{X}_{u,i}$; $[M_{s,t}^i, \mathbb{M}_{s,t}^i] = 0$, otherwise. And $N = n_p + n_w$, $\Gamma \triangleq [I_N\ I_N]$, $\hat{K} \triangleq [\hat{B}_F\ \hat{A}_F]$, $\theta \triangleq \left[\begin{smallmatrix} I_N \\ 0_N \end{smallmatrix}\right]$, $\gamma \triangleq \left[\begin{smallmatrix} 0_N \\ I_N \end{smallmatrix}\right]$, $\hat{V} \triangleq \left[\begin{smallmatrix} \hat{V}_1 & \hat{V}_2 \\ \hat{V}_3 & \hat{V}_3 \end{smallmatrix}\right]$, and $A_F = \hat{A}_F \hat{V}_3^{-\mathrm{T}}$, $B_F = \hat{B}_F C_w^+$, $H_F = H_F$, $L_F = \hat{L}_F \hat{V}_3^{-\mathrm{T}}$, C_w^+ is the generalized inverse matrix of C_w.

(4) There exist matrices $\hat{L}, \hat{L}_0, \hat{C}, \hat{C}_0$ and symmetric matrices $X_{w,i} > 0$, $X_{w0,i} > 0$, $X_{u,i} > 0, X_{u0,i} > 0$ and

$$[H_{s,t}^i]_{4\times 4} < 0,\ i = 1, \ldots, z, \tag{7}$$
$$\left[\mathbb{H}_{s,t}^i\right]_{4\times 4} < 0,\ i = 1, \ldots, z. \tag{8}$$

$\tilde{A}_{w,i} \triangleq \left[\begin{smallmatrix} A_{w,i} & 0 \\ 0 & 0 \end{smallmatrix}\right]$, $\tilde{B}_{w,i} \triangleq \left[\begin{smallmatrix} B_{lw,i} \\ 0 \end{smallmatrix}\right]$, $\tilde{B}_{u,i} \triangleq \left[\begin{smallmatrix} B_{lu,i} \\ 0 \end{smallmatrix}\right]$, $\tilde{C}_w \triangleq \left[\begin{smallmatrix} -C_w & 0 \\ 0 & -I \end{smallmatrix}\right]$, $\hat{L} \triangleq \left[\begin{smallmatrix} 0 & 0 \\ B_F & A_F \end{smallmatrix}\right]$, $\hat{C} \triangleq \left[\begin{smallmatrix} H_F & L_F \end{smallmatrix}\right]$; $\mathcal{J}_{q1,i} = (X_{q,i} \tilde{A}_{w,i} + \tilde{A}_{w,i}^{\mathrm{T}} X_{q,i}) + (X_{q0,i} X_{q0,i} - X_{q,i} X_{q0,i} - X_{q0,i} X_{q,i}) + \tilde{C}_w^{\mathrm{T}} (\hat{L}_0^{\mathrm{T}} \hat{L}_0 - \hat{L}_0^{\mathrm{T}} \hat{L} - \hat{L}^{\mathrm{T}} \hat{L}_0) \tilde{C}_w, q = w, u$. $H_{1,1}^i = \mathcal{J}_{w1,i}$, $\mathbb{H}_{1,1}^i = \mathcal{J}_{u1,i}$, $H_{1,2}^i = X_{w,i} \tilde{B}_{w,i}$, $\mathbb{H}_{1,2}^i = X_{u,i} \tilde{B}_{u,i}$, $H_{1,3}^i = -(\hat{C} \tilde{C}_w)^{\mathrm{T}}$, $\mathbb{H}_{1,3}^i = -(\hat{C} \tilde{C}_w)^{\mathrm{T}}$, $H_{1,4}^i = (X_{w,i} - \hat{L} \tilde{C}_w)^{\mathrm{T}}$, $\mathbb{H}_{1,4}^i = (X_{u,i} - \hat{L} \tilde{C}_w)^{\mathrm{T}}$, $H_{2,2}^i = -r_w I$, $\mathbb{H}_{2,2}^i = -r_u I$, $H_{3,3}^i = -r_w I$, $\mathbb{H}_{3,3}^i = -r_u I$, $H_{4,4}^i = -I$, $\mathbb{H}_{4,4}^i = -I$, $H_{s,t}^i = 0$, $\mathbb{H}_{s,t}^i = 0$, otherwise.

Multiply the filter $F_m(s) = \mathrm{diag}\{(s+r)/r, \ldots, (s+r)/r\}$ to y to attain a modified output

$y_m(t) = (\frac{1}{r}CA+C)x(t)+\frac{1}{r}CF_af(t)+(\frac{1}{r}CB_u)u(t)+(\frac{1}{r}CB_w+D_w)w(t)$. Define a weighting system W_2 which has a high pass property with the form of $W_2 : \left\{ \begin{smallmatrix} \dot{x}_h = A_h x_h + B_h \eta_m \\ \eta_h = C_h x_h + D_h \eta_m \end{smallmatrix} \right.$. Let $X^*_{cl} \triangleq \left[\begin{smallmatrix} x \\ x_h \\ x_F \end{smallmatrix}\right]$, then the composite system dynamic satisfies

$$\begin{aligned} \dot{X}^*_{cl} = & \begin{bmatrix} A_f & 0 \\ B_F C_{hf} & A_F \end{bmatrix} X^*_{cl} + \begin{bmatrix} B_{hf} \\ B_F D_{hf} \end{bmatrix} f \\ & + \mathcal{P}(w,u), \\ \hat{r} = & H_F D_{hf} f + \mathcal{P}_d(w,u). \end{aligned} \tag{9}$$

$A_f \triangleq \left[\begin{smallmatrix} A & 0 \\ B_h \hat{I}_{ny}(\frac{1}{r}CA+C) & A_h \end{smallmatrix}\right]$, $B_{hf} \triangleq \left[\begin{smallmatrix} F_a \\ \frac{1}{r}B_h \hat{I}_{ny} C F_a \end{smallmatrix}\right]$, $C_{hf} \triangleq \left[\begin{smallmatrix} D_h \hat{I}_{ny}(C+\frac{1}{r}CA) & C_h \end{smallmatrix}\right]$, $D_{hf} \triangleq \frac{1}{r} D_h \hat{I}_{n_y} C F_a$.

Theorem 2. Assume Eq. (9) is asymptotically stable, then the following conditions are equivalent:

(1) $\min\limits_{\alpha\in\Gamma} \underline{\sigma}_{f\in\Omega_2}(F_{\hat{r}f}(jw)) > r_f,\ r_f > 0$.

(2) There exists matrix P, that $P = P^{\mathrm{T}}$, such that the following matrix inequalities hold

$$[G^i_{s,t}]_{2\times 2} > 0,\ i = 1,\ldots,z. \tag{10}$$

$G_{11,i} \triangleq P_{f,i}\left[\begin{smallmatrix} A_{f,i} & 0 \\ B_F C_{hf,i} & A_F \end{smallmatrix}\right] + \left[\begin{smallmatrix} A_{f,i} & 0 \\ B_F C_{hf,i} & A_F \end{smallmatrix}\right]^{\mathrm{T}} P_{f,i} + \left[\begin{smallmatrix} C^{\mathrm{T}}_{hf,i} H^{\mathrm{T}}_F \\ L^{\mathrm{T}}_F \end{smallmatrix}\right] \left[\begin{smallmatrix} H_F C_{hf,i} & L_F \end{smallmatrix}\right]$, $G_{12,i} \triangleq P_{f,i}\left[\begin{smallmatrix} B_{hf,i} \\ B_F D_{hf,i} \end{smallmatrix}\right] + \left[\begin{smallmatrix} C_{hf,i}{}^{\mathrm{T}} H^{\mathrm{T}}_F \\ L^{\mathrm{T}}_F \end{smallmatrix}\right] H_F D_{hf,i}$, $G_{22,i} \triangleq D_{hf,i}{}^{\mathrm{T}} H^{\mathrm{T}}_F H_F D_{hf,i} - r^2_f$.

(3) There exist matrices $\hat{L}, \hat{L}_0, \hat{C}, \hat{C}_0$ and symmetric matrices $P_{f,i}, P_{f0,i}$ such that

$$[W^i_{s,t}]_{5\times 5} > 0,\ i = 1,\ldots,z. \tag{11}$$

$\tilde{A}_{f,i} \triangleq \left[\begin{smallmatrix} A_{f,i} & 0 \\ 0 & 0 \end{smallmatrix}\right]$, $\tilde{B}_{hf,i} \triangleq \left[\begin{smallmatrix} B_{hf,i} \\ 0 \end{smallmatrix}\right]$, $\hat{C} \triangleq \left[\begin{smallmatrix} H_F & L_F \end{smallmatrix}\right]$, $\tilde{C}_{f,i} \triangleq \left[\begin{smallmatrix} -C_{hf,i} & 0 \\ 0 & -I \end{smallmatrix}\right]$, $\tilde{D}_{f,i} \triangleq \left[\begin{smallmatrix} D_{hf,i} \\ 0 \end{smallmatrix}\right]$, $\hat{L} \triangleq \left[\begin{smallmatrix} 0 & 0 \\ B_F & A_F \end{smallmatrix}\right]$, $G^i_{1,1} \triangleq (P_{f,i}\tilde{A}_{f,i} + \tilde{A}^{\mathrm{T}}_{f,i} P_{f,i}) + (2P_{f,i}P_{f0,i} + 2P_{f0,i}P_{f,i} - 2P_{f0,i}P_{f0,i}) + 2\tilde{C}^{\mathrm{T}}_{f,i}(\hat{C}^{\mathrm{T}}_0 \hat{C} + \hat{C}^{\mathrm{T}}\hat{C}_0 - \hat{C}^{\mathrm{T}}_0 \hat{C}_0)\tilde{C}_{f,i} + \tilde{C}^{\mathrm{T}}_{f,i}(\hat{L}^{\mathrm{T}}\hat{L}_0 + \hat{L}^{\mathrm{T}}_0 \hat{L} - \hat{L}^{\mathrm{T}}_0 \hat{L}_0)\tilde{C}_{f,i}$, $G^i_{2,2} \triangleq -r^2_f I + 2\tilde{D}^{\mathrm{T}}_{f,i}(\hat{C}^{\mathrm{T}}_0 \hat{C} + \hat{C}^{\mathrm{T}}\hat{C}_0 - \hat{C}^{\mathrm{T}}_0 \hat{C}_0)\tilde{D}_{f,i} + D^{\mathrm{T}}_{f,i}(\hat{L}^{\mathrm{T}}\hat{L}_0 + \hat{L}^{\mathrm{T}}_0 \hat{L} - \hat{L}^{\mathrm{T}}_0 \hat{L}_0)\tilde{D}_{f,i}$, $G^i_{1,2} = P_{f,i}\tilde{B}_{hf,i}$, $G^i_{1,3} = (P_{f,i} + \hat{L}\tilde{C}_{f,i})^{\mathrm{T}}$, $G^i_{1,4} = P_{f,i}$, $G^i_{1,5} = (\hat{C}\tilde{C}_{f,i})^{\mathrm{T}}$, $G^i_{2,4} = -(\hat{L}\tilde{D}_{f,i})$, $G^i_{2,5} = (\hat{C}\tilde{D}_{f,i})^{\mathrm{T}}$, $\mathrm{diag}\{G^i_{3,3}, G^i_{4,4}, G^i_{5,5}\} = \mathrm{diag}\{I,I,I\}; G^i_{s,t} = 0$, otherwise.

Algorithm design of fault detection filter. Let $\mu_1 > 0$, $\hat{\gamma} > 0$, $l = 0$.

Step 1. Choose a proper large $\mu > 0$; Minimize $\alpha_w r_w + \alpha_u r_u$ subject to (5) and (6). The solutions are denoted as $A^{\mathrm{opt}}_F, B^{\mathrm{opt}}_F, L^{\mathrm{opt}}_F, H^{\mathrm{opt}}_F$.

Step 2. (a) Substitute $A^{\mathrm{opt}}_F, B^{\mathrm{opt}}_F, L^{\mathrm{opt}}_F, H^{\mathrm{opt}}_F$ into (3), (4) and (10), minimize $\alpha_w r_w + \alpha_u r_u$. Denote the solution as $P^{\mathrm{opt}}_{f,i}, X^{\mathrm{opt}}_{w,i}, X^{\mathrm{opt}}_{u,i}, i = 1,\ldots,z$, $r^{\mathrm{opt}}_w, r^{\mathrm{opt}}_u$. And $\hat{L}^{\mathrm{opt}}, \hat{C}^{\mathrm{opt}}$ can be attained. Define $r^0_w = r^{\mathrm{opt}}_w$, $\hat{C}^0 = \hat{C}^{\mathrm{opt}}$, $\hat{L}^0 = \hat{L}^{\mathrm{opt}}$, $P^0_{f,i} = P^{\mathrm{opt}}_{f,i}, X^0_{w,i} = X^{\mathrm{opt}}_{w,i}$, $X^0_{u,i} = X^{\mathrm{opt}}_{u,i}$, $i = 1,\ldots,z$. (b) Put $l = l+1$, with $r^l_w := r^{l-1}_w + \mu_1$, $\hat{C}_0 := \hat{C}^{l-1}$, $\hat{L}_0 := \hat{L}^{l-1}, P_{f0,i} := P^{l-1}_{f,i}$, $X_{w0,i} := X^{l-1}_{w,i}$, $X_{u0,i} := X^{l-1}_{u,i}$, $i = 1,\ldots,z$, maximize r_f subject to (7), (8) and (11) and store $r^l_f, r^l_f/r^l_w, \hat{L}^l, \hat{C}^l, X^l_{w,i}, X^l_{u,i}, P^l_{f,i}, i = 1,\ldots,z$. (c) If $r^l_f/r^l_w < \hat{\gamma}$, repeat Step 2(b); Else, exit.

Step 3. Derive A_F, B_F, L_F, H_F from $\hat{L}^l, \hat{C}^l$; With A_F, B_F, L_F, H_F, minimize r_u subject to (4); $r_f = r^l_f, r_w = r^l_w$.

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