SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

April 2017, Vol. 60 042303:1–042303:18
doi: 10.1007/s11432-016-0249-9

Energy-aware deployment of dense heterogeneous cellular networks with QoS constraints

Qimei CUI[1]*, Zhiyan CUI[1], Wei ZHENG[1], Riku JÄNTTI[2] & Weiliang XIE[3]

[1]*National Engineering Laboratory for Mobile Network Security, Beijing University of Posts and Telecommunications, Beijing* 100876*, China;*
[2]*Department of Communications and Networking, Aalto University, Espoo* 02150*, Finland;*
[3]*China Telecom Corporation Limited, Beijing* 100140*, China*

Received July 15, 2016; accepted August 11, 2016; published online November 16, 2016

Abstract The base station (BS) configuration is a key factor to improve energy efficiency (EE). In this paper, we focus on designing the network deployment parameters (i.e., BS densities) for biased K-tier heterogeneous cellular network (HCN) with quality of service (QoS) provisioning. Using appropriate approximations, we derive the closed-form expressions of optimal BS density across all tiers to minimize the area power consumption (APC) by applying the stochastic geometry theory, while satisfying the users' QoS requirements. These results are used to obtain some new insights into the EE performance of biased HCN deployment. With the aid of this approach, the best type of BSs to be deployed or switched off for energy saving purposes can be identified from the perspectives of BS transmission power. More precisely, if the BS transmission power ratio between an arbitrary pair of tiers of K-tier HCN, e.g., the small cell BS and macro BS tiers, is higher than a threshold which is a function of path loss exponent, bias factor and power consumption, the small cell BSs are preferred. The opposite situation takes place otherwise. Furthermore, it is also shown that, compared to the unbiased HCN scenario, significant energy savings are possible by appropriately biasing the HCN and optimizing the BS density, subject to the QoS constraints among all tiers.

Keywords heterogeneous cellular networks, stochastic geometry, energy efficiency, BS density, energy saving

Citation Cui Q M, Cui Z Y, Zheng W, et al. Energy-aware deployment of dense heterogeneous cellular networks with QoS constraints. Sci China Inf Sci, 2017, 60(4): 042303, doi: 10.1007/s11432-016-0249-9

1 Introduction

The last ten years have witnessed explosive growth in mobile data traffic due to the rapid proliferation of Internet-connected smart devices [1, 2]. One of the challenges is how to provide high quality of service (QoS) for the explosive data traffic that is expected. To deal with this situation, a new multi-tier heterogeneous cellular network (HCN) architecture has been proposed as a promising solution as link efficiency has almost approached its fundamental limits [3]. However, densely deployed small cells in a HCN are usually lightly loaded because they have low transmit power, when compared to macro BSs. In order to take full advantage of small cells, a biasing technique known as cell range expansion (CRE)

* Corresponding author (email: cuiqimei@bupt.edu.cn)

can be effectively used to serve more users and offload more traffic from the macro cell tier to small cell tiers [4–6].

Besides, the highly dense deployment of HCN will cause a significant increase in energy consumption. It is reported that 3% of the worldwide electric energy consumption and 2% of total carbon emissions, of which mobile networks represent about 0.2%, are caused by the information and communication technology industry. Moreover, it is expected that this share will even grow in the next forthcoming years [7, 8]. Pushed by the necessity of energy reduction, various energy-efficient methods have been lately proposed to exploit different aspects of HCN deployments, such as high-efficiency power amplifier, various cooperative communications techniques, energy-efficient network resource management methods, as well as advanced network deployment strategies [9–12]. Among all these alternatives, energy-efficient network deployments have attracted increasing interest lately.

Different studies have been carried out to design energy-efficient deployment strategies in realistic scenarios [13, 14]. For example, the authors of [13] investigated three cellular network deployment strategies for variable traffic conditions over a real district of central London and revealed that a network composed by a large number of low power BSs that were properly deployed to utilize the space optimally was the most energy efficient topology for both downlink and uplink cases. Similarly, different deployment strategies for both indoor and outdoor scenarios with extreme densification levels were studied in [14]. The obtained results indicated that when the systems were pushed to their capacity limits, dedicated indoor solutions with densely deployed femtocells were more spectrum and energy-efficient than the solution that densified the outdoor macro layer only. There are also some research works that rely on stochastic geometry models to address the energy saving problem with BS sleeping strategies [15–18]. In [15], BS density configurations for achieving the optimal energy efficiency (EE) in two-tier cellular networks were analyzed using a dynamic gradient based iterative algorithm, based on the fact that the equivalent optimization problem was not necessarily convex. Similarly, the authors of [16] extended the previous results to optimal BS density design for cellular networks with traffic-aware sleeping strategies in both single and two-tier cellular networks. In [17], the EE of cellular networks through the deployment of sleeping strategy as well as small cells was analyzed. The authors in [18] evaluated a distributed sleep-mode strategy for cognitive small access points and analyzed the trade-off between traffic offloading from the macro cell and the energy consumption of the small cells. Unfortunately, none of the above works focuses on the optimization of BS densities in the context of a biased multi-tier HCN with QoS constraints. Whereas, it is now well established (both empirically and theoretically) that biasing user equipments (UEs) towards small cells leads to not only significant improvement in downlink throughput [6] but also potential EE enhancement in HCN. Ref. [19] indicated that both the network throughput and EE had a locally optimal point as the cell association bias of the small cell tier initially increased from 0 dB, but afterwards decreased and then increased to a constant value that may be higher than the previous local optimum.

On the other hand, to the best of our knowledge, the energy saving purposes make sense only when the desired QoS requirements of users are guaranteed, which means that when optimizing EE with BS sleeping strategies, the user QoS experience could not be destroyed [20]. To save energy while maintaining acceptable QoS, refs. [21–23] investigated the BS deployment strategies with consideration of delay, average spatial spectrum efficiency and signal-to-interference plus noise power ratio (SINR) coverage performance, respectively. The authors of [21] jointly designed the traffic-aware BS sleeping control and power matching schemes for a single BS to achieve the energy saving with consideration of delay. The obtained results indicated that power matching had a wider range of adaptability to the traffic variation while sleeping control was more energy-efficient when the traffic load was relatively low. In [22], the heterogeneity of large-scale user behavior was quantitatively characterized and exploited to study the EE in HCN. An optimization problem was then formulated for energy efficient two-tier deployment and configuration, where the BS density and average spatial spectrum efficiency were taken into account. In [23], the authors optimized both the BS densities and transmit powers in a two-tier HCN to minimize the area power consumption (APC) subject to a SINR coverage constraint. However, almost all of the above results ignore the optimization of BS densities in a biased multi-tier HCN with the rate coverage performance constraint, which is one key performance indicator impacting call dropping rate from user perspective. In [24], the authors proposed a framework that consisted in combining BS densities in a two-tier HCN to minimize the cost of energy, subject to the rate coverage constraint of the macro BS tier. Nevertheless, it does not focus on the rate coverage performance of each tier. Though a lower APC subject to the rate coverage constraint of only macro BS tier may be achieved, the QoS requirements of each specific tier may not be satisfied. In other words, the energy saving is achieved through the sacrifice of the performance experience of user equipments in some tiers of the HCN. Thus, in order to ensure fairness, the rate coverage performance of each tier should be guaranteed at both system planning and network deployment stage.

In this paper, we study the EE performance of biased multi-tier HCN with the aid of tools borrowed from stochastic geometry. Using appropriate approximations, we derive the closed-form expressions of the rate coverage probability and optimal BS density for each tier. With the aid of these expressions, we systematically determine what is the best type of BSs to be deployed or switched off to save energy from the perspectives of BS transmission power. Specifically, if the BS transmission power ratio between an arbitrary two-tier HCN (e.g., the small cell BS tier and the macro BS tier) is higher than a threshold which is a function of path loss exponent, bias factor and power consumption, the small cell BSs are preferred. In other words, it is more convenient to deploy more small cell BSs or switch off certain macro BSs for energy saving. The opposite situation takes place otherwise. Furthermore, we consider rate coverage constraints for each tier and demonstrate their necessity in different environments and QoS targets. Numerical results show that significant energy saving is possible by appropriately biasing the HCN deployment.

2 System model

2.1 K-tier HCN model

A downlink HCN consisting of K tiers of BSs is considered, where the locations of the BSs of the i-th tier are assumed to form a homogeneous poisson point process (HPPP) Φ_i with intensity λ_i. Without loss of generality, let us assume that macro cell BSs constitute tier 1 and that small cell BSs with different characteristics represent the other tiers of the HCN. All the BSs in the i-th tier transmit a constant power P_i [25] and adopt identical bias factor B_i, which is always a positive value. α_i is the path loss exponent factor in the i-th tier. Therefore, each tier can be uniquely defined by the tuple $\{\lambda_i, P_i, B_i, \alpha_i\}$. Moreover, the bias factors verify $1 \leqslant B_1 \leqslant B_2 \leqslant \cdots \leqslant B_i \leqslant \cdots \leqslant B_K$ for the sake of simplicity. The spatial distribution of UEs is another HPPP Φ_u, with constant intensity λ_u. We assume that the density λ_u is very high, such that almost surely every BS in the network has at least one UE to serve. In other words, the BSs are always active to trasmit power. Moreover, Φ_i and Φ_u are also considered independent.

Universal frequency reuse is adopted, which means that a typical user receives interference from every active BS in the HCN. Each user is always associated to the strongest BS in terms of long-term average biased-received-power (BRP) [5]. For example, the typical user is associated to an i-th tier BS if the BRP received from the i-th tier BS $P_{r,i}$ is larger than that received from the j-th tier BS $P_{r,j}$, i.e. $P_{r,i} > P_{r,j}, \forall j \in K, j \neq i$. The association probability of the i-th tier, which is denoted as A_i [5], is expressed as

$$A_i = 2\pi\lambda_i \int_0^\infty r \exp\left(-\pi \sum_{j=1}^{K} \lambda_j \left(\frac{P_j B_j}{P_i B_i}\right)^{2/\alpha_j} r^{2\alpha_i/\alpha_j}\right) \mathrm{d}r. \tag{1}$$

If all tiers have the same path loss exponent, i.e. $\{\alpha_i\} = \alpha$, the probability is simplified to

$$A_i = \frac{\lambda_i (P_i B_i)^{2/\alpha}}{\sum_{j=1}^{K} \lambda_j (P_j B_j)^{2/\alpha}}. \tag{2}$$

Lemma 1. In the underlaid K-tier HCN with the BRP based association strategy, the activation probability of a typical BS in the i-th tier P_{ai} is expressed as

$$P_{ai} = 1 - \left(1 + M^{-1}\frac{\lambda_u A_i}{\lambda_i}\right)^{-M}, \tag{3}$$

where M is a predefined constant, which is related to the cell-size probability density function (PDF) of a Poisson-Voronoi tessellation and is known to be accurately predicted by a gamma distribution [6] with $M = 3.5$. According to (3), when $\lambda_u A_i \geqslant 4.68\lambda_i$, more than 95% of BSs are activated, which means that almost surely every BS in the network has at least one UE to serve under this condition.

Proof. See Appendix A.

Wireless channel gain is modeled as a standard path-loss factor multiplied by an independent Rayleigh fading coefficient. Specifically, we consider a reference user located at the origin with a distance r_i from its serving BS. The fading coefficient between the reference user and its serving BS is denoted as h_{r_i}, which is assumed to be i.i.d exponentially distributed with mean 1, i.e. $h_{r_i} \sim \exp(1)$. The standard path loss function is given by $l\left(r_i\right) = \|r_i\|^{-\alpha_i}$. Therefore, the BRP that this reference user experiences from serving BS (belonging to i-th tier) is $P_i B_i h_{r_i} \|r_i\|^{-\alpha_i}$. Then, the resulting SINR γ attends the form

$$\gamma\left(r_i\right) = \frac{P_i B_i h_{r_i} \|r_i\|^{-\alpha_i}}{I_{r_i} + \sigma^2}, \tag{4}$$

where $I_{r_i} = \sum_{j=1}^{K} \sum_{r_j \in \Phi_j \backslash r_i} P_j B_j h_{r_j} \|r_j\|^{-\alpha_j}$ represents the interference, and σ^2 is constant additive white gaussian noise (AWGN) power.

The SINR coverage for a target SINR threshold T is $C\left(T\right) = \mathbb{P}\left(\gamma \geqslant T\right)$, and the rate coverage for a target rate threshold ρ is $R\left(\rho\right) = \mathbb{P}\left(\beta \geqslant \rho\right)$.

2.2 BS power consumption model

We adopt the power consumption model used in [23]. That is, we assume that power consumption for each BS is fixed, which can also be interpreted as a constant average BS power consumption since a large number of BSs is considered. Based on this model, the power consumption for the i-th BSs during downlink transmission is given by

$$P_{i,\text{total}} = a_i P_i + b_i, \tag{5}$$

where coefficient a_i accounts for the power consumption that scales linearly with the transmit power, whereas term b_i represents the static power consumption due to signal processing, battery backup, site cooling, among other functionalities. Note that power consumption model in (5) reflects the fact that the average power consumption of a BS comprises both transmit power and static power.

Thus, the network APC per unit area E_P can be treated as the weighted sum of the BS densities across all tiers, which is written as follows:

$$E_P = \sum_{i=1}^{K} \lambda_i P_{ai} P_{i,\text{total}}. \tag{6}$$

In this paper, we use the E_P as energy efficiency metric, which is measured in W/km^2. Note that the E_P is also equivalent to the network energy consumption, which is the product of E_P, constant area and time [23]. For convenience, the mathematical symbols used in this paper are summarized in Table 1.

3 Performance analysis and optimal deployment of K-tier biased HCN

This section derives the SINR coverage and the mean load approximation, which are subsequently used for deriving the rate coverage. Furthermore, rate constraint of each tier is also characterized to determine the optimal energy-aware deployment strategy (i.e., optimal BS density across all tiers) that minimizes the E_P of the HCN.

Table 1 Summary of notation

Symbol	Meaning
λ_i	Intensity of BSs in the i-th tier
P_i	Transmission power of BSs in the i-th tier
B_i	Bias factor of the i-th tier
λ_u	Intensity of UEs
$P_{r,i}$	The BRP received from the i-th tier BS
$P_{r,j}$	The BRP received from the j-th tier BS
A_i	The association probability of the i-th tier
α_i	Path loss exponent of the i-th tier
P_{ai}	The activation probability of a typical BS in the i-th tier
M	Constant value 3.5
γ, T	SINR and SINR threshold
β, ρ_i	Rate and the target rate of users in the i-th tier
$P_{i,\text{total}}$	The power consumption for the i-th BS
a_i	Power parameters of BS in the i-th tier
b_i	Power parameters of BS in the i-th tier
E_P	Area power consumption
W	System bandwidth
η	Rate coverage threshold
N_i	The cell load of the i-th tier
$\overline{N}_i$	The mean cell load of the i-th tier
$\lambda_{i,\max}$	Max. available BS density of the i-th tier
$\lambda_{i,\min}$	Min. available BS density of the i-th tier

3.1 SINR coverage analysis

3.1.1 *General case*

Lemma 2. The SINR coverage of the i-th tier in a K-tier biased HCN can be expressed as

$$C_i\left(T\right) = \frac{2\pi\lambda_i}{A_i}\int_{r_i=0}^{\infty} r_i \exp\Bigg\{-\frac{\|r_i\|^{\alpha_i} T\sigma^2}{P_i} - \pi r_i^{2\alpha_i/\alpha_j}\left[\sum_{j=1}^{K}\lambda_j\left(\frac{P_jB_j}{P_iB_i}\right)^{2/\alpha_j}\left(1+P_{a\,j}\rho\left(\frac{B_i}{B_j}T,\alpha\right)\right)\right]\Bigg\}\,\mathrm{d}r_i. \quad (7)$$

Proof. See Appendix B.

Although Lemma 2 does not give a closed-from expression, the integral can be computed to any desired accuracy using numerical integration methods. Moreover, several insightful results arise for certain special cases of the above result. For example, in the interference-limited case, where the effect of the AWGN noise may be ignored in the presence of the strong interferences from the BSs, the simplified result is obtained as shown in the following section.

3.1.2 *Special case: interference-limited HCN with equal path loss exponent*

Since small cell BSs will be more densely deployed in future, to boost network capacity, which means that the interference power easily dominates AWGN noise [6]. AWGN noise can often therefore be neglected, i.e. $\sigma^2 = 0$, as we do in the rest of the paper. Moreover, by assuming identical path loss exponent in all tiers, i.e. $\{\alpha_i\} = \alpha$, we can benefit from more compact and useful expressions of SINR coverage and rate coverage probabilities, which will be crucial to derive the closed-form expressions of optimal BS density across all tiers to minimize the APC. Furthermore, the value of path loss exponent for urban area cellular radio environment typically lies within a small range. Therefore, the approximation of $\{\alpha_i\} = \alpha$ does not cause much loss in accuracy and it has already been adopted in many previous literatures [16,24]. For simplicity, in the following of the paper, we focus on the interference-limited case with equal path loss exponent.

Lemma 3. The SINR coverage of the i-th tier in an interference-limited HCN can be expressed as

$$C_i\left(T\right)=\frac{\sum_{j=1}^{K}\left(\frac{P_jB_j}{P_iB_i}\right)^{2/\alpha}\lambda_j}{\sum_{j=1}^{K}\lambda_j\left(\frac{P_jB_j}{P_iB_i}\right)^{2/\alpha}\left(1+P_{a\,j}\rho\left(\frac{B_i}{B_j}T,\alpha\right)\right)},\tag{8}$$

where $\rho\left(r,\alpha\right)=r^{2/\alpha}\int_{r^{-2/\alpha}}^{\infty}\frac{1}{1+x^{\alpha/2}}\mathrm{d}x$. Furthermore, the overall SINR coverage can be calculated as $C=\sum_{i=1}^{K}A_iC_i$ in a biased network. In an unbiased network, where the bias factor of each tier is equal to 1, it is possible to show that the overall SINR coverage becomes $C=\frac{1}{\sum_{i=1}^{K}A_i[1+P_{ai}\rho(T,\alpha)]}$, which is equal to the SINR coverage of each tier. Furthermore, when the user density λ_u is high enough, i.e., $\{P_{a\,j}\}=1$ is satisfied, the overall SINR coverage reduces to a simple form, i.e. $C=\frac{1}{1+\rho(r,\alpha)}$, which is independent from BS density and transmission power.

Proof. Setting $\sigma^2=0$ and $\{\alpha_i\}=\alpha$, the SINR coverage of the i-th tier in an interference-limited HCN can be obtained from (7).

3.2 Rate coverage analysis

3.2.1 *Load characterization*

In this section, we first analyze the load, which is crucial to get a handle on the rate coverage. The probability mass function of the users associated with the tagged BS of a PPP Φ_i, which is denoted as $p_i\left(n\right)$ [6, 26], is expressed as

$$p_i\left(n\right)=\mathbb{P}\left(N_i=n\right)=\frac{3.5^{3.5}}{n!}\frac{\Gamma\left(n+3.5\right)}{\Gamma\left(3.5\right)}\left(\frac{\lambda_uA_i}{\lambda_i}\right)^{n-1}\left(3.5+\frac{\lambda_uA_i}{\lambda_i}\right)^{-(n+3.5)}\left(n\geqslant 1\right),\tag{9}$$

where $\Gamma\left(M\right)=\int_0^{\infty}t^{M-1}\exp\left(-t\right)\mathrm{d}t$ is gamma function.

3.2.2 *Main results*

Lemma 4. The rate coverage of the i-th tier in a K-tier biased HCN is given by

$$\begin{aligned}R_i&=\mathbb{E}_{N_i}\left[C_i\left(2^{\frac{\rho_iN_i}{W}}-1\right)\right]\overset{(a)}{=}\sum_{n\geqslant 1}p_i\left(n\right)C_i\left(2^{\frac{\rho_in}{W}}-1\right)\\&=\sum_{n\geqslant 1}\frac{3.5^{3.5}}{n!}\frac{\Gamma\left(n+3.5\right)}{\Gamma\left(3.5\right)}\left(\frac{\lambda_uA_i}{\lambda_i}\right)^{n-1}\left(3.5+\frac{\lambda_uA_i}{\lambda_i}\right)^{-(n+3.5)}C_i\left(2^{\frac{\rho_in}{W}}-1\right),\end{aligned}\tag{10}$$

where (a) ignores the dependence between the load and the SINR coverage for tractability of the analysis, as in [26]. W is the system bandwidth, N_i is to characterize the load at the tagged BS and ρ_i represents for the target rate of the users that belong to the i-th tier.

Proof. Using the rate coverage expression $R\left(\rho\right)=\mathbb{P}\left(\beta\geqslant\rho\right)$, the rate coverage of the i-th tier is

$$R_i=\mathbb{P}\left(\beta_i\geqslant\rho_i\right)=\mathbb{P}\left(\frac{W}{N_i}\log_2\left(1+\gamma\right)\geqslant\rho_i\right)=\mathbb{P}\left(\gamma\geqslant 2^{\frac{\rho_iN_i}{W}}-1\right).\tag{11}$$

Combining $C\left(T\right)=\mathbb{P}\left(\gamma\geqslant T\right)$, (9) and (10), we obtain the result presented in (11).

3.2.3 *Mean load approximation*

The rate coverage expression can be further simplified (sacrificing accuracy) if the load at each BS is assumed to be its mean. Therefore, instead of considering the cell size distribution and the user distribution in each cell like in [17], we adopt the mean load approximation proposed in [26]. Note that the mean cell load for the i-th tier, is given by

$$\overline{N}_i = 1 + \frac{1.28\lambda_u A_i}{\lambda_i}. \tag{12}$$

Lemma 5. Based on the approximated cell load, the rate coverage of the i-th tier in an interference-limited HCN is given by

$$R_i = \mathbb{E}_{N_i}\left[C_i\left(2^{\frac{\rho_i N_i}{W}} - 1\right)\right] = C_i\left(2^{\frac{\rho_i \mathbb{E}[N_i]}{W}} - 1\right) = C_i\left(2^{\frac{\rho_i \bar{N}_i}{W}} - 1\right). \tag{13}$$

Proof. Please refer to the Lemma 4.

3.3 Optimal deployment strategy

(1) QoS requirements. The user QoS requirement specifies that the rate coverage across all tiers should be more than a threshold $1-\eta$, where η is the probability that the instantaneous downlink rate for a user in the i-th tier is lower than the predefined value ρ_i for $i = 1, \ldots, K$. The QoS constraints can be then written as follows:

$$R_i = \mathbb{P}\left(\beta_i \geqslant \rho_i\right) \geqslant 1 - \eta, \quad i = 1, \ldots, K. \tag{14}$$

(2) Problem formulation. In order to pursue a unified study on rate coverage and energy efficiency performances, we formulate a theoretical framework that determines the optimal BS density across all tiers to minimize the APC while satisfying the users' QoS requirements. This optimization problem can be formally written as follows:

$$\min_{\lambda_k} \quad E_P \quad \text{s.t.} \quad R_k \geqslant 1-\eta, \quad \lambda_{k,\min} \leqslant \lambda_k \leqslant \lambda_{k,\max}, \quad k = 1, \ldots, K, \quad (\text{Optimization Problem } P_0),$$

where k is an arbitrary tier of the K-tier HCN. Note that Optimization Problem (P_0) has a unique solution, since the left-hand side of the constraints is a strictly monotone increasing function (detailed proof is omitted). Thus, the optimal solution can be achieved numerically using the binary search algorithm. However, to analyze the impact of BS density on APC, we now approximate the optimal BS density by forcing equality in the K constraints of Optimization Problem (P_0).

Lemma 6. The optimal BS density of each tier in an interference-limited HCN, subject to the rate coverage constraint of the k-th tier, can be expressed as follows:

$$\begin{cases} \lambda_k = \dfrac{1}{\sum_{i=1}^K \tau_i b_i c_i} \cdot \dfrac{1.28\lambda_u}{\frac{W}{\rho_k}\log_2\left[1 + \frac{\eta(\alpha-2)}{2(1-\eta)}\sum_{j=k+1}^K \frac{B_j}{B_k}\right] - 1}, \\[3ex] \lambda_m = \dfrac{\tau_m}{\sum_{i=1}^K \tau_i b_i c_i} \cdot \dfrac{1.28\lambda_u}{\frac{W}{\rho_k}\log_2\left[1 + \frac{\eta(\alpha-2)}{2(1-\eta)}\sum_{j=k+1}^K \frac{B_j}{B_k}\right] - 1}, \end{cases} \tag{15}$$

where $k, m = 1, \ldots, K, m \neq k$, $b_i = (\frac{B_i}{B_k})^{2/\alpha}$, $c_i = (\frac{P_i}{P_k})^{2/\alpha}$ and $\tau_i = (\frac{\lambda_i}{\lambda_k})^{2/\alpha}$.

Proof. See Appendix C.

For a K-tier HCN that guarantees the user QoS requirement for the k-th tier, the Optimization Problem for the APC becomes

$$\begin{aligned} &\min_{\lambda_k} \frac{\sum_{i=1}^K \tau_i P_{ai} P_{i,\text{total}}}{\sum_{i=1}^K \tau_k b_i c_i} \cdot \frac{1.28\lambda_u}{\frac{W}{\rho_k}\log_2\left[1 + \frac{\eta(\alpha-2)}{2(1-\eta)}\sum_{j=k+1}^K \frac{B_j}{B_k}\right] - 1} \\ &\text{s.t.} \quad \lambda_{k,\min} \leqslant \frac{1}{\sum_{i=1}^K \tau_i b_i c_i} \cdot \frac{1.28\lambda_u}{\frac{W}{\rho_k}\log_2\left[1 + \frac{\eta(\alpha-2)}{2(1-\eta)}\sum_{j=k+1}^K \frac{B_j}{B_k}\right] - 1} \leqslant \lambda_{k,\max}, \ k = 1, \ldots, K, \end{aligned} \tag{P_k}$$

$$\lambda_{m,\min} \leqslant \frac{\tau_m}{\sum_{i=1}^{K}\tau_i b_i c_i} \cdot \frac{1.28\lambda_u}{\frac{W}{\rho_k}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\sum_{j=k+1}^{K}\frac{B_j}{B_k}\right]-1} \leqslant \lambda_{m,\max},\ m=1,\ldots,K, m\neq k.$$

Note that $\lambda_k(\tau_m)$ is a monotonically decreasing function of τ_m, and that $\lambda_m(\tau_m) = \tau_m\lambda_k(\tau_m)$ is an increasing function of τ_m. Therefore, by defining $\tau_{k,\min}$ and $\tau_{k,\max}$ as the values of τ_m that verifies constraints $\lambda_{k,\min}$ and $\lambda_{k,\max}$ with equality, respectively, the original optimization problem can be simplified as follows:

$$\begin{aligned} &\min_{\lambda_k} \ \frac{\sum_{i=1}^{K}\tau_i P_{ai}P_{i,\text{total}}}{\sum_{i=1}^{K}\tau_k b_i c_i} \cdot \frac{1.28\lambda_u}{\frac{W}{\rho_k}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\sum_{j=k+1}^{K}\frac{B_j}{B_k}\right]-1} \\ &\text{s.t.}\ \ \max\{\tau_{m,\min},\tau_{k,\max}\} \leqslant \tau_m \leqslant \min\{\tau_{k,\min},\tau_{m,\max}\}, \qquad k,m=1,\ldots,K, m\neq k. \end{aligned} \tag{16}$$

The monotonicity of the APC objective function with respect to τ_m is shown in the following lemma.

Lemma 7. In a K-tier HCN, while satisfying $\sum_{j=k+1}^{K}\frac{B_j}{B_k} > \frac{2(1-\eta)}{\eta(\alpha-2)}(2^{\frac{\rho_k}{W}}-1)$, the APC objective function is a monotonically increasing function of τ_m when $\frac{B_m}{B_k} \leqslant (\frac{P_{am}P_{m,\text{total}}\sum_{i=1,i\neq m}^{K}\tau_i b_i c_i}{c_m\sum_{i=1,i\neq m}^{K}\tau_i P_{ai}P_{i,\text{total}}})^{\alpha/2}$ is verified. Otherwise, the APC objective function is a monotonically decreasing function of τ_m.

Proof. See Appendix D.

Remark. From Lemma 7, it is possible to conclude that APC objective function is a monotonically increasing function of τ_m when $\frac{P_m}{P_k}$ is smaller than $(\frac{P_{am}P_{m,\text{total}}\sum_{i=1,i\neq m}^{K}\tau_i b_i c_i}{b_m\sum_{i=1,i\neq m}^{K}\tau_i P_{ai}P_{i,\text{total}}})^{\alpha/2}$; therefore, the optimal value for τ_m should be the smallest possible; if the opposite situation takes place, τ_m should be as large as possible. In other words, from an energy saving perspective, if $\frac{P_m}{P_k}$ is smaller than $(\frac{P_{am}P_{m,\text{total}}\sum_{i=1,i\neq m}^{K}\tau_i b_i c_i}{b_m\sum_{i=1,i\neq m}^{K}\tau_i P_{ai}P_{i,\text{total}}})^{\alpha/2}$, then it is more convenient to deploy more BSs on the k-th tier and switch off certain BSs on the m-th tier; otherwise, the optimal deployment strategy is the opposite.

4 Example of optimal deployment for two-tier biased HCN

In order to simplify the analysis, we consider the conventional two-tier HCN consisting of a macro cell tier and a small cell tier. This is often the existing HCN case, where the macro BSs are mainly responsible for providing coverage, and the small cell BSs are deployed for coverage and capacity enhancement. In the two-tier HCN, the optimal solution should satisfy both rate coverage constraints across each tier. Thus, we separate the optimization problem into two separate optimization sub-problems, one constrained by the rate coverage probability on the first tier and the other one constrained by the rate coverage probability of the second tier.

4.1 Rate coverage constraint of the first tier

The APC optimization problem for the two-tier HCN that guarantees the user QoS requirement on the first tier can be formally written as

$$\begin{aligned} &\min_{\lambda_1^{(1)},\lambda_2^{(1)}} \ \lambda_1^{(1)}P_{a1}P_{1,\text{total}} + \lambda_2^{(1)}P_{a2}P_{2,\text{total}} \\ &\text{s.t.}\ \ \lambda_{1,\min} \leqslant \lambda_1^{(1)} \leqslant \lambda_{1,\max}, \quad \lambda_{2,\min} \leqslant \lambda_2^{(1)} \leqslant \lambda_{2,\max}. \end{aligned} \tag{P_1}$$

It is possible to show that Optimization Problem (P_1) is equivalent to (17), where $b=(\frac{B_2}{B_1})^{2/\alpha}$, $c=(\frac{P_2}{P_1})^{2/\alpha}$, and $\tau=(\frac{\lambda_2}{\lambda_1})^{2/\alpha}$. Note that function $\lambda_1^{(1)}(\tau)$ in Optimization Problem (P_1) is decreasing, and $\lambda_2^{(1)}=\tau\lambda_1^{(1)}$ is increasing with τ. Therefore, by denoting the values of τ that make constraints $\lambda_{k,\min}$ and $\lambda_{k,\max}$ hold with equalitly by $\tau_{k,\min}^{(1)}$ and $\tau_{k,\max}^{(1)}$ $(k=1,2)$, respectively, the

$$\begin{aligned}
&\min_{\lambda_1^{(1)},\lambda_2^{(1)}} \quad \frac{P_{a1}P_{1,\text{total}}+\tau P_{a2}P_{2,\text{total}}}{1+\tau bc}\cdot\frac{1.28\lambda_u}{\frac{W}{\rho_1}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\cdot\frac{B_2}{B_1}\right]-1}\\
&\text{s.t.}\quad \lambda_{1,\min}\leqslant\frac{1}{1+\tau bc}\cdot\frac{1.28\lambda_u}{\frac{W}{\rho_1}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\cdot\frac{B_2}{B_1}\right]-1}\leqslant\lambda_{1,\max},\lambda_{2,\min}\\
&\qquad\leqslant\frac{\tau}{1+\tau bc}\cdot\frac{1.28\lambda_u}{\frac{W}{\rho_1}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\cdot\frac{B_2}{B_1}\right]-1}\leqslant\lambda_{2,\max},
\end{aligned}\tag{17}$$

Optimization Problem (P_1) then can be simplified as follows:

$$\begin{aligned}
&\min_{\lambda_1^{(1)},\lambda_2^{(1)}} \quad \frac{P_{a1}P_{1,\text{total}}+\tau P_{a2}P_{2,\text{total}}}{1+\tau bc}\cdot\frac{1.28\lambda_u}{\frac{W}{\rho_1}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\cdot\frac{B_2}{B_1}\right]-1}\\
&\text{s.t.}\quad \max\left(\tau_{1,\max}^{(1)},\tau_{2,\min}^{(1)}\right)\leqslant\tau\leqslant\min\left(\tau_{1,\min}^{(1)},\tau_{2,\max}^{(1)}\right).
\end{aligned}\tag{18}$$

From Lemma 7, we can conclude that when $\frac{2(1-\eta)}{\eta(\alpha-2)}(2^{\frac{\rho_1}{W}}-1)<\frac{B_2}{B_1}\leqslant(\frac{P_{a2}P_{2,\text{total}}}{cP_{a1}P_{1,\text{total}}})^{\alpha/2}$, the objective function of Optimization Problem (P_1) is an increasing function; therefore, optimal $\tau^{*(1)}$ should be as small as possible. If the opposite situation takes place, then $\tau^{*(1)}$ should be as large as possible. In other words,

$$\tau^{*(1)}=\begin{cases}\max\left(\tau_{1,\max}^{(1)},\tau_{2,\min}^{(1)}\right), & \max\left\{\frac{2(1-\eta)}{\eta(\alpha-2)}\left(2^{\frac{\rho_1}{W}}-1\right),1\right\}<\frac{B_2}{B_1}\leqslant\left(\frac{P_{a2}P_{2,\text{total}}}{cP_{a1}P_{1,\text{total}}}\right)^{\alpha/2},\\ \min\left(\tau_{1,\min}^{(1)},\tau_{2,\max}^{(1)}\right), & \frac{B_2}{B_1}>\left(\frac{P_{2,\text{total}}}{cP_{1,\text{total}}}\right)^{\alpha/2},\end{cases}\tag{19}$$

where

$$\tau_i^{(1)}=\begin{cases}\dfrac{1.28\lambda_u}{\lambda_i\left(\frac{W}{\rho_1}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\cdot\frac{B_2}{B_1}\right]-1\right)bc}-\dfrac{1}{bc}, & i=1,\min\text{ or }1,\max,\\ \dfrac{1}{\dfrac{1.28\lambda_u}{\lambda_i\left(\frac{W}{\rho_1}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\cdot\frac{B_2}{B_1}\right]-1\right)}-bc}, & i=2,\min\text{ or }2,\max.\end{cases}\tag{20}$$

Specifically, for energy saving, if $\max\{\frac{2(1-\eta)}{\eta(\alpha-2)}(2^{\frac{\rho_1}{W}}-1),1\}<\frac{B_2}{B_1}\leqslant(\frac{P_{a2}P_{2,\text{total}}}{cP_{a1}P_{1,\text{total}}})^{\alpha/2}$, i.e. if $\frac{P_{2,\text{total}}}{P_{1,\text{total}}}\geqslant\frac{cP_{a1}}{P_{a2}}(\frac{B_2}{B_1})^{2/\alpha}$ holds, meanwhile, $\frac{B_2}{B_1}$ in its feasible interval, it is better to deploy more macro BSs and switch off certain small cell BSs, while if $\frac{P_{2,\text{total}}}{P_{1,\text{total}}}<\frac{cP_{a1}}{P_{a2}}(\frac{B_2}{B_1})^{2/\alpha}$, the optimal deployment strategy is to deploy small cell BSs and switch off macro BSs. It should be noted that $\frac{B_2}{B_1}\leqslant\frac{2(1-\eta)}{\eta(\alpha-2)}\left(2^{\frac{1.28\lambda_u\rho_1+\lambda_{1,\min}\rho_1}{\lambda_{1,\min}W}}-1\right)$ is necessary in order to ensure $\tau_i^{(1)}\geqslant 0$.

4.2 Rate coverage constraint of the second tier

The APC optimization problem for the two-tier HCN that guarantees the user QoS requirement on the second tier can be formally written as

$$\begin{aligned}
&\min_{\lambda_1^{(2)},\lambda_2^{(2)}} \quad \lambda_1^{(2)}P_{a1}P_{1,\text{total}}+\lambda_2^{(2)}P_{a2}P_{2,\text{total}}\\
&\text{s.t.}\quad \lambda_{1,\min}\leqslant\lambda_1^{(2)}\leqslant\lambda_{1,\max},\quad \lambda_{2,\min}\leqslant\lambda_2^{(2)}\leqslant\lambda_{2,\max}.
\end{aligned}\tag{P_2}$$

It is possible to show that Optimization Problem (P_2) is equivalent to (21),

$$\begin{aligned}
&\min_{\lambda_1^{(2)},\lambda_2^{(2)}} \quad \frac{P_{a1}P_{1,\text{total}}+\tau P_{a2}P_{2,\text{total}}}{\tau+1/bc}\cdot\frac{1.28\lambda_u}{\frac{W}{\rho_2}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\right]-1}\\
&\text{s.t.}\ \ \lambda_{1,\min}\leqslant\frac{1}{\tau+1/bc}\cdot\frac{1.28\lambda_u}{\frac{W}{\rho_2}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\right]-1}\leqslant\lambda_{1,\max},\lambda_{2,\min}\\
&\qquad\leqslant\frac{\tau}{\tau+1/bc}\cdot\frac{1.28\lambda_u}{\frac{W}{\rho_2}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\right]-1}\leqslant\lambda_{2,\max}.
\end{aligned}\tag{21}$$

The function $\lambda_1^{(2)}(\tau)$ in (P_2) is decreasing, whereas $\lambda_2^{(2)}=\tau\lambda_1^{(2)}$ is increasing with τ. Therefore, by denoting the values of τ that make constraints $\lambda_{k,\min}$ and $\lambda_{k,\max}$ $(k=1,2)$ hold with equality by $\tau_{k,\min}^{(2)}$ and $\tau_{k,\max}^{(2)}$ $(k=1,2)$, respectively, the Optimization Problem (P_2) then can be simplified as

$$\begin{aligned}
&\min_{\lambda_1^{(2)},\lambda_2^{(2)}} \quad \frac{P_{a1}P_{1,\text{total}}+\tau P_{a2}P_{2,\text{total}}}{\tau+1/bc}\cdot\frac{1.28\lambda_u}{\frac{W}{\rho_2}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\right]-1}\\
&\text{s.t.}\quad \max\left(\tau_{1,\max}^{(2)},\tau_{2,\min}^{(2)}\right)\leqslant\tau\leqslant\min\left(\tau_{1,\min}^{(2)},\tau_{2,\max}^{(2)}\right).
\end{aligned}\tag{22}$$

From Lemma 7, we can conclude that when $\frac{2(1-\eta)}{\eta(\alpha-2)}(2^{\frac{\rho_1}{W}}-1)<\frac{B_2}{B_1}\leqslant(\frac{P_{2,\text{total}}}{cP_{1,\text{total}}})^{\alpha/2}$, the objective function of Optimization Problem (P_2) is an increasing function. Therefore, optimal $\tau^{*(2)}$ should be as small as possible; otherwise, $\tau^{*(2)}$ should be as large as possible. To sum up,

$$\tau^{*(2)}=\begin{cases}\max\left(\tau_{1,\max}^{(2)},\tau_{2,\min}^{(2)}\right), & \max\left\{\frac{2(1-\eta)}{\eta(\alpha-2)}\left(2^{\frac{\rho_1}{W}}-1\right),1\right\}<\frac{B_2}{B_1}\leqslant\left(\frac{P_{a2}P_{2,\text{total}}}{cP_{a1}P_{1,\text{total}}}\right)^{\alpha/2},\\ \min\left(\tau_{1,\min}^{(2)},\tau_{2,\max}^{(2)}\right), & \frac{B_2}{B_1}>\left(\frac{P_{a2}P_{2,\text{total}}}{cP_{a1}P_{1,\text{total}}}\right)^{\alpha/2},\end{cases}\tag{23}$$

where

$$\tau_i^{(2)}=\begin{cases}\frac{1.28\lambda_u}{\lambda_i\left(\frac{W}{\rho_2}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\right]-1\right)}-\frac{1}{bc}, & i=1,\min\text{ or }1,\max,\\ \frac{1/bc}{\frac{1.28\lambda_u}{\lambda_i\left(\frac{W}{\rho_2}\log_2\left[1+\frac{\eta(\alpha-2)}{2(1-\eta)}\right]-1\right)}-1}, & i=2,\min\text{ or }2,\max.\end{cases}\tag{24}$$

Specifically, for energy saving purposes, if $\max\{\frac{2(1-\eta)}{\eta(\alpha-2)}(2^{\frac{\rho_1}{W}}-1),1\}<\frac{B_2}{B_1}\leqslant(\frac{P_{a2}P_{2,\text{total}}}{cP_{a1}P_{1,\text{total}}})^{\alpha/2}$ holds, i.e. if $\frac{P_{2,\text{total}}}{P_{1,\text{total}}}\geqslant\frac{cP_{a1}}{P_{a2}}(\frac{B_2}{B_1})^{2/\alpha}$ holds, meanwhile, $\frac{B_2}{B_1}$ in its feasible interval, it is better to deploy more macro BSs and switch off certain small cell BSs; on the other hand, if $\frac{P_{2,\text{total}}}{P_{1,\text{total}}}<\frac{cP_{a1}}{P_{a2}}(\frac{B_2}{B_1})^{2/\alpha}$, the optimal deployment strategy consists in deploying small cell BSs and switch off macro BSs. It should be noted that $\frac{B_2}{B_1}\geqslant(\frac{\lambda_{1,\min}(W\log 2(1+\frac{\eta(\alpha-2)}{2(1-\eta)})-\rho_2)}{1.28\lambda_u\rho_2 c})^{\alpha/2}$ should be verified in order to ensure $\tau_i^{(2)}\geqslant 0$.

4.3 Overall optimization of the two-tier HCN

The overall optimal solution combines $\tau^{*(1)}$ and $\tau^{*(2)}$ is shown as follows:

$$\tau^{*}=\begin{cases}\max\left\{\tau^{*(1)},\tau^{*(2)}\right\}, & \max\left\{\frac{2(1-\eta)}{\eta(\alpha-2)}\left(2^{\frac{\rho_1}{W}}-1\right),1\right\}<\frac{B_2}{B_1}\leqslant\left(\frac{P_{a2}P_{2,\text{total}}}{cP_{a1}P_{1,\text{total}}}\right)^{\alpha/2},\\ \min\left\{\tau^{*(1)},\tau^{*(2)}\right\}, & \frac{B_2}{B_1}>\left(\frac{P_{a2}P_{2,\text{total}}}{cP_{a1}P_{1,\text{total}}}\right)^{\alpha/2}.\end{cases}\tag{25}$$

Table 2 Simulation parameters

Parameters	Value
System bandwidth	$W = 10$ MHz
Path loss exponent	$\alpha = 4$
SINR threshold	$T = -3$ dB
Power parameters of Macro BS	$a_1 = 22.6$, $b_1 = 412.4$
Macro BS transmission power	$P_1 = 10$ W
Small cell BS transmission power	$P_2 = 0.1$ W
Macro BS density	$\lambda_1 = 1/\text{km}^2$
Small cell BS density	$\lambda_2 = 10/\text{km}^2$
User density	$\lambda_u = 100/\text{km}^2$
Rate coverage threshold	$\eta = 0.3$
Target rate of macro cell users	$\rho_1 = 300$ kbps
Target rate of small cell users	$\rho_2 = 1200$ kbps
Macro BS bias	$B_1 = 1$

Remark. Optimal solution (25) identifies the most convenient deployment strategy of a two-tier HCN while guaranteeing user rate coverage requirements in both tiers. Specifically, for energy saving purposes, it is better to deploy more macro BSs and switch off certain small cell BSs if $\frac{P_{2,\text{total}}}{P_{1,\text{total}}} \geqslant \frac{cP_{a1}}{P_{a2}}(\frac{B_2}{B_1})^{2/\alpha}$ holds, meanwhile, $\frac{B_2}{B_1}$ in its feasible interval; otherwise, if $\frac{P_{2,\text{total}}}{P_{1,\text{total}}} < \frac{cP_{a1}}{P_{a2}}(\frac{B_2}{B_1})^{2/\alpha}$, the optimal deployment strategy consists in deploying small cell BSs and switch off macro BSs. It should be noted that $(\frac{\lambda_{1,\min}(W\log 2(1+\frac{\eta(\alpha-2)}{2(1-\eta)})-\rho_2)}{1.28\lambda_u\rho_2 c})^{\alpha/2} \leqslant \frac{B_2}{B_1} \leqslant \frac{2(1-\eta)}{\eta(\alpha-2)}(2^{\frac{1.28\lambda_u\rho_1+\lambda_{1,\min}\rho_1}{\lambda_{1,\min}W}}-1)$ needs to be verified in order to ensure $\tau^* \geqslant 0$.

5 Numerical results and discussion

5.1 Simulation setup

The numerical simulations that presented in this section are used to validate the presented theoretical analysis. As metric of energy efficiency, APC measured in W/km^2 is used. Results are presented for a two-tier network consisting of macro cells and small cells, respectively. The effect of AWGN noise is neglected, because the HCN works in the interference-limited regime. Furthermore, the network deployment parameters (i.e., BS densities λ_1 and λ_2) identify the average number of BSs that are randomly deployed per square kilometer. The baseline of this paper includes two aspects. On one hand, since the optimal APC of the K-tier biased HCN in our energy-aware scheme is a function of bias factor ratio, the baseline of it is naturally such a scheme that the APC is optimized in a unbiased HCN. On the other hand, the baseline of our scheme that the APC is optimized with rate coverage constraints of all tiers is such a scheme that the APC is optimized with only rate coverage constraint of certain tier. It should be noted that all the presented results can be extended in a quite straightforward way to an arbitrary number of tiers. We first investigate the effect of bias factor and BS density ratio on the SINR and rate coverage, respectively. Then, we verify the APC performance by comparing the unbiased and biased network scenarios under diverse configurations pertaining to BS power consumption ratio, BS density ratio, and bias factor ratio. Unless it is explicitly mentioned, simulation parameters are listed in Table 2, which refers to 3GPP LTE standard specifications.

5.2 Effect of bias factor and BS density ratio

Figure 1 shows the effect of bias factor and BS density on SINR and rate coverage. Unbiased association results in the same SINR coverage of each tier. The overall SINR coverage that is obtained for different BS densities verifies the Lemma 3 that was derived. As the small cell bias factor increases, more macro users with low SINR start to become associated with small cell BSs. As result of this phenomenon, the SINR

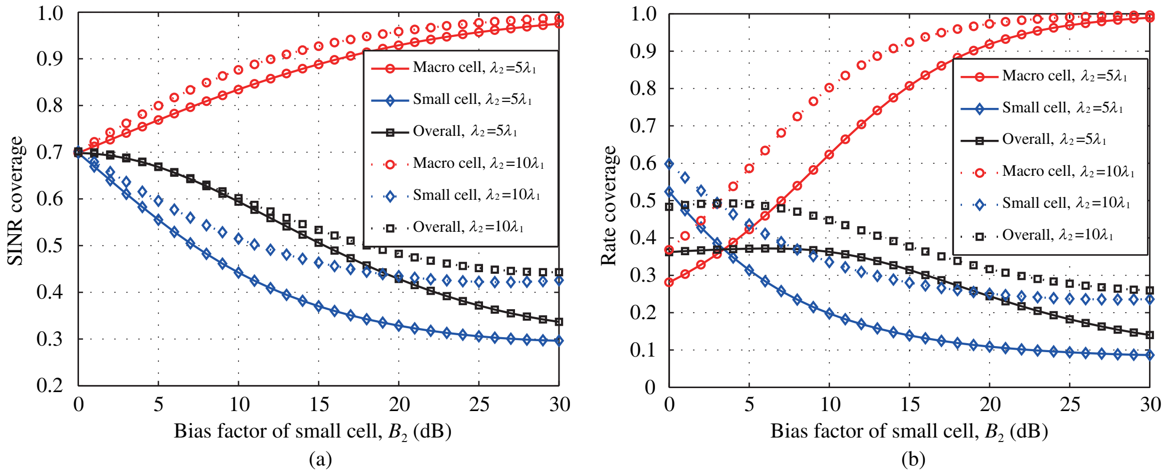

Figure 1 (Color online) SINR and rate coverage versus varying association bias B_2 ($B_1 = 1$, $\lambda_1 = 1/\text{km}^2$). (a) Effect of association bias on SINR coverage; (b) effect of association bias on rate coverage.

and rate coverage of macro cell improves, degrading the SINR and rate coverage of small cell as well as the one that corresponds to the overall network. From the small cell and overall network perspective, unbiased cell association always outperforms biased cell association since, in the latter case, some users are forced to become associated to a BS that provides a lower received signal strength reducing the SINR of those users [5]. However, SINR and rate coverage degradation in this situation can be tackled by increasing the density of small cells in the HCN. Intuitively, the network energy consumption is an increasing function of BS density. Nevertheless, when $\frac{P_{2,\text{total}}}{P_{1,\text{total}}} < \frac{cP_{a1}}{P_{a2}}(\frac{B_2}{B_1})^{2/\alpha}$, the APC becomes a decreasing function of BS density ratio τ and, due to that, small cell BSs represent the optimal type for energy saving purposes.

From the perspective of an optimal deployment strategy, when the small cell bias factor B_2 is low, the rate coverage of small cells outperforms the one experienced by macro cells. This means the QoS requirement of small cells is easier to be verified when compared to macro cell case. When B_2 is further increased, however, the rate coverage of macro cells becomes gradually better than the one experienced by small cells. As a consequence, the optimal APC with rate coverage constraint of only macro cell decreases as the B_2 increases, whereas the rate coverage constraint of only small cells increases. In this situation, a larger density of small cells is needed in order to tackle the rate coverage degradation that small cells experience. In other words, as B_2 increases, the optimal APC subject to the rate coverage constraints of both tiers becomes gradually dominated by the small cell tier, which verifies our optimal deployment strategy. That is, when $\frac{P_{2,\text{total}}}{P_{1,\text{total}}}$ is lower than a pre-defined threshold, it is better to deploy more small cell BSs and switch off certain macro BSs; the opposite situation takes place otherwise. It should be noted that when ratio $\frac{B_2}{B_1}$ becomes large enough, the optimal APC almost converges to a fixed value ultimately; this is because the rate coverage of small cells nearly converges to a fixed value. Specially, in the intersection of two curves, we have the minimum point of optimal APC. The reason behind this is that the rate coverage constraints of both tiers, i.e. $R_i = \mathbb{P}(\beta_i \geqslant \rho_i) \geqslant 1 - \eta \, (i = 1, 2)$, is equivalent to the constraint of $\min\{R_1, R_2\} \geqslant 1 - \eta$. Then, since the rate coverage in the macro (small) cell tier is a monotonically increasing (decreasing) function of B_2, $\min\{R_1, R_2\}$ achieves its maximum value in the intersection of two curves, thus leading to the minimum point of optimal APC.

Figure 2 shows that in both biased and unbiased networks without noise, the SINR and rate coverages approach a constant value. Specially, in an unbiased network without noise, the overall coverage probability is independent from $\{\lambda_i, P_i, B_i\}$, and equal to coverage probability of each tier. However, the SINR and rate coverages are no longer invariant to $\{\lambda_i, P_i, B_i\}$ in a biased network, since some users becomes associated to small BSs which may be providing lower signal strength than the corresponding macro BSs.

In a biased HCN, when the BS density is small, the improvement of small cell BS density can enhance the received signal strength, which leads to the raising of the SINR and rate coverages of both tiers. When the small cell density is further increased, the improvement in received signal power by adding

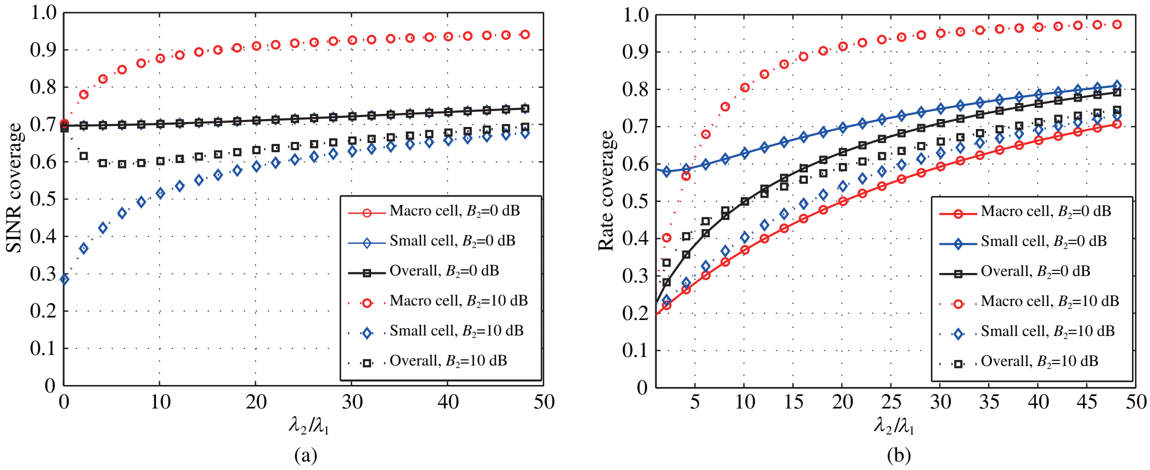

Figure 2 (Color online) SINR and rate coverage versus varying BS density ratio ($B_1 = 1$, $\lambda_1 = 1/\text{km}^2$). (a) Effect of BS density ratio on SINR coverage; (b) effect of BS density ratio on rate coverage.

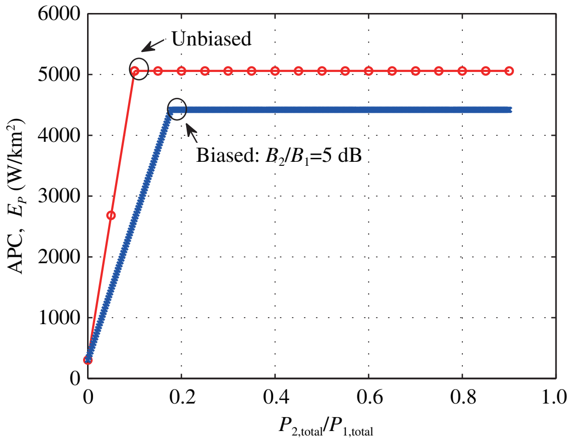

Figure 3 (Color online) Optimal APC as function of BS power consumption ratio.

more BSs will also introduce more interference. This is the main reason why the SINR and rate coverages ultimately converge to a fixed value. Beyond this point, deploying extra small BSs does not bring any improvement on both SINR or rate coverage, although they consume more energy. However, it must be noted that they still contribute to the total network rate if they cover users.

5.3 APC performance analysis and comparison

In this section, we analyze and compare the APC performance between the unbiased and biased network scenario as well as the rate coverage constraint of one and two tiers with different configurations. The optimal deployment strategy and the corresponding APC results are shown in Figures 3–5, respectively.

Figure 3 shows the effect of BS power consumption ratio and bias factor on optimal APC. The parameter setting is as follows: $B_2 = 0$ dB and 5 dB in the unbiased and biased network scenario, respectively, $\lambda_u = 100$ per unit area, $\lambda_{1,\min} = 0.4770$ per unit area, $\lambda_{2,\min} = 0$ per unit area, and $\lambda_{1,\max}$, $\lambda_{2,\max} \to \infty$. Note that Figure 3 allow us to visualize the most convenient rule for BS type selection. Specifically, when $e = P_{2,\text{total}}/P_{1,\text{total}} \leqslant 0.1$, the optimal BS type is the small cell BSs, and the optimal APC is linear with e; the opposite situation takes place otherwise. Similarly, in biased network scenario with $\frac{P_{2,\text{total}}}{P_{1,\text{total}}} < \frac{cP_{a1}}{P_{a2}}(\frac{B_2}{B_1})^{2/\alpha}$, the optimal BS type is the small cell BSs; otherwise, the optimal BS type is the macro BSs. Furthermore, results show that the biasing of the HCN provides significant energy savings compared to the unbiased case.

Figure 4 shows the effect of BS density ratio on APC. Comparing Figure 4(a) and (b), we see that

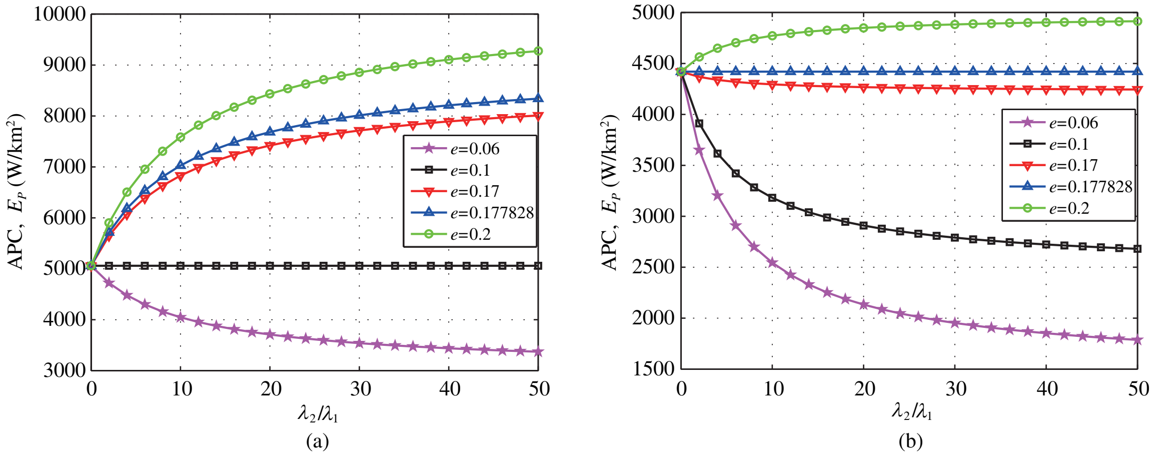

Figure 4 (Color online) APC versus BS density ratio. (a) Unbiased network scenario; (b) biased network scenario ($B_2 = 5$ dB).

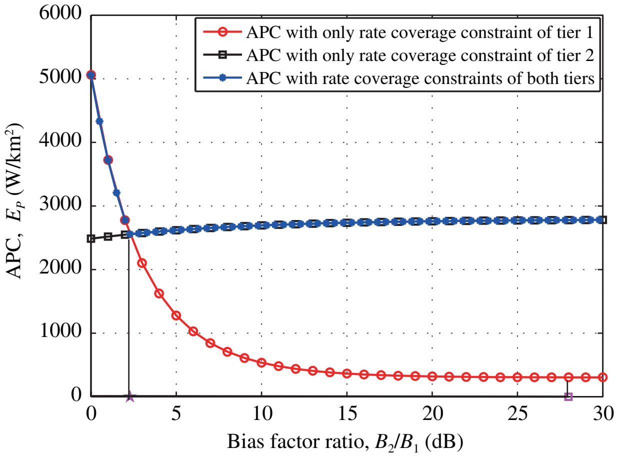

Figure 5 (Color online) Optimal APC as function of bias factor ratio ($e = 0.1$).

significant energy savings can be achieved by appropriately biasing the HCN regardless the value that BS consumption ratio e takes. Furthermore, in the unbiased network scenario, the APC is a decreasing (increasing) function of τ when $e \leqslant 0.1$ ($e > 0.1$), while in the biased network case with $B_2 = 5$ dB, the APC is a decreasing (increasing) function of τ when $e \leqslant 0.177828$ ($e > 0.177828$). The different transition points for the monotonicity of optimal APC function with respect to BS density ratio τ verify the rule in (25) for different scenarios. Specifically, when $\frac{P_{2,\mathrm{total}}}{P_{1,\mathrm{total}}} < \frac{cP_{a1}}{P_{a2}}(\frac{B_2}{B_1})^{2/\alpha}$, the optimal BS type is the small cell BSs since the APC is a decreasing function of τ; otherwise, the optimal BS type is the macro BSs.

Figure 5 shows the effect of bias factor ratio on optimal APC. According to (25), the feasible range of B_2/B_1 is the one that makes $\tau^* \geqslant 0$ and guarantee the QoS requirements of users in both tiers. In this case, the feasible interval for B_2/B_1 is $[0, 27.987]$ dB, which is highlighted in bold on the x-axis of Figure 5 for the sake of clarity. As expected, significant energy savings can be achieved by appropriately biasing the HCN compared to the unbiased HCN. For instance, at the lowest point of optimal APC, where $B_2/B_1 = 2.29$ dB (indicated by pentagram), a power saving of 2502.4 W/km^2 (about 49.5%) is achieved when compared to the unbiased HCN. Therefore, by using the analysis in this paper, an energy efficient BS deployment configuration can be determined. Moreover, as the small cell bias factor B_2 increases, optimal APC with only rate coverage constraint of tier 2 is firstly lower but then, it becomes larger than that of optimal APC with only rate coverage constraint of tier 1 according to Figure 5. This is because the rate coverage of macro cells actually improve and those of small cells degrade, due to the limited transmit power of small cell BSs and the increasing average distance that UEs experience when B_2 grows. In other words, the QoS requirement of tier 2 is much easier to be satisfied than tier 1 when B_2/B_1 <2.29 dB; otherwise, the QoS requirement of tier 1 is much easier to be satisfied. Therefore, the QoS requirement of both tiers should be guaranteed in order to ensure fairness in different environments and QoS targets. Furthermore, it is possible to observe that when $e = 0.1$ and $B_2/B_1 = 5$ dB, the optimal APC with rate coverage constraints of both tiers is 2618 W/km^2, which is in accordance with Figure 3.

6 Conclusion

In this paper, we used stochastic geometry theory to characterize the most convenient network deployment parameters (i.e. BS density) for K-tier biased HCN. We derived closed-form expressions of optimal BS density for each tier which, later on, were used to systematically determine what is the best type of BSs to be deployed or switched off for energy saving purposes. Moreover, significant energy saving was achieved by appropriately biasing the HCN scenario compared to the unbiased HCN scenario. Furthermore, results also demonstrated that it is necessary to consider rate coverage constraints among all tiers in order to guarantee the desired QoS requirements of each tier.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant No. 61471058), Beijing Nova Program (Grant No. xx2012037), International Cooperation NSFC Program (Grant No. 61461136002), and Hong Kong, Macao and Taiwan Science and Technology Cooperation Projects (Grant No. 2014DFT10320).

Conflict of interest The authors declare that they have no conflict of interest.

References

1 Ma Z, Zhang Z Q, Ding Z G, et al. Key techniques for 5G wireless communications: network architecture, physical layer, and MAC layer perspectives. Sci China Inf Sci, 2015, 58: 041301
2 Cui Q M, Wang H, Hu P X, et al. Evolution of limited feedback CoMP systems from 4G to 5G. IEEE Veh Tech Mag, 2014, 9: 94–103
3 Liu Y J, Lu L, Geoffrey L, et al. Joint user association and spectrum allocation for small cell networks with wireless backhauls. IEEE Wirel Commun Lett, 2016, doi: 10.1109/LWC.2016.2593465
4 Fei Z S, Ding H C, Xing C W, et al. Performance analysis for range expansion in heterogeneous networks. Sci China Inf Sci, 2014, 57: 082305
5 Jo H S, Sang Y J, Xia P, et al. Heterogeneous cellular networks with flexible cell association: a comprehensive downlink SINR analysis. IEEE Trans Wirel Commun, 2012, 11: 3484–3495
6 Singh S, Dhillon H S, Andrews J G. Offloading in heterogeneous networks: modeling, analysis, and design insights. IEEE Trans Wirel Commun, 2013, 12: 2484–2497
7 Fettweis G, Zimmermann E. ICT energy consumption-trends and challenges. In: Proceedings of the 11th International Symposium on Wireless Personal Multimedia Communications, Lapland, 2008. 8–11
8 Hasan Z, Boostanimehr H, Bhargava V K. Green cellular networks: a survey, some research issues and challenges. IEEE Commun Surv Tut, 2011, 13: 524–540
9 Fikadu M, Sofotasios P, Muhaidat S, et al. Error rate and power allocation analysis of regenerative networks over generalized fading channels. IEEE Trans Commun, 2016, 64: 1751–1768
10 Zhou M, Cui Q M, Jantti R, et al. Energy-efficient relay selection and power allocation for two-way relay channel with analog network coding. IEEE Commun Lett, 2012, 16: 816–819
11 Cui Q M, Yang X J, Jyri H, et al. Optimal energy-efficient relay deployment for the bidirectional relay transmission schemes. IEEE Trans Veh Tech, 2014, 63: 2625–2641
12 Cui Q M, Huang X Q, Luo B, et al. Capacity analysis and optimal power allocation for coordinated transmission in MIMO-OFDM systems. Sci China Inf Sci, 2012, 55: 1372–1387
13 Koutitas G, Karousos A, Tassiulas L. Deployment strategies and energy efficiency of cellular networks. IEEE Trans Wirel Commun, 2012, 11: 2552–2563
14 Yunas S F, Valkama M, Niemela J. Spectral and energy efficiency of ultra-dense networks under different deployment strategies. IEEE Commun Mag, 2015, 53: 90–100
15 Li L, Peng M G, Yang C Q, et al. Base station density optimization for high energy efficiency in two-tier cellular networks. In: Proceedings of IEEE Global Communications Conference, Austin, 2014. 1804–1809
16 Li L, Peng M, Yang C Q, et al. Optimization of base station density for high energy efficient cellular networks with sleeping strategies. IEEE Trans Veh Tech, 2015, 65: 7501–7514

17 Yong S S, Quek T Q S, Kountouris M, et al. Energy efficient heterogeneous cellular networks. IEEE J Sel Area Commun, 2013, 31: 840–850
18 Wildemeersch M, Quek T Q S, Slump C H, et al. Cognitive small cell networks: energy efficiency and trade-offs. IEEE Trans Commun, 2013, 61: 4016–4029
19 Yu P S, Lee J, Quek T, et al. Traffic Offloading in heterogeneous networks with energy harvesting personal cells-network throughput and energy efficiency. IEEE Trans Wirel Commun, 2015, 15: 1146–1161
20 Du Q H, Zhang X. Statistical QoS provisionings for wireless unicast/multicast of multi-layer video streams. IEEE J Sel Area Commun, 2010, 28: 420–433
21 Wu J, Zhou S, Niu Z S. Traffic-aware base station sleeping control and power matching for energy-delay tradeoffs in green cellular networks. IEEE Trans Wirel Commun, 2013, 12: 4196–4209
22 Huang Y, Zhang X, Zhang J X, et al. Energy-efficient design in heterogeneous cellular networks based on large-scale user behavior constraints. IEEE Trans Wirel Commun, 2014, 13: 4746–4757
23 Peng J L, Hong P L, Xue K P. Energy-aware cellular deployment strategy under coverage performance constraints. IEEE Trans Wirel Commun, 2015, 14: 69–80
24 Cao D X, Zhou S, Niu Z S. Optimal combination of base station densities for energy-efficient two-tier heterogeneous cellular networks. IEEE Trans Wirel Commun, 2015, 53: 90–100
25 Du Q H, Song H B, Xu Q, et al. Interference-controlled D2D routing aided by knowledge extraction at cellular infrastructure towards ubiquitous CPS. Pers Ubiquit Comput, 2015, 19: 1033–1043
26 Singh S, Jeffrey G A. Joint resource partitioning and offloading in heterogeneous cellular networks. IEEE Trans Wirel Commun, 2014, 13: 888–901

Appendix A Proof of Lemma 1

The cell size probability density function (PDF) of a Poisson-Voronoi tessellation is known to be accurately predicted by a gamma distribution [6] with $M = 3.5$,

$$f_{X_i}(x) = \frac{M^M}{\Gamma(M)}\left(\frac{\lambda_i}{A_i}\right)^M x^{M-1}\exp\left(-M\frac{\lambda_i}{A_i}x\right), \tag{A1}$$

where x denotes the cell size, λ_i is the BS density, and $\Gamma(M) = \int_0^\infty t^{M-1}\exp(-t)\,\mathrm{d}t$ is gamma function.

Since the users are located according to a PPP, the number of users in a cell with cell size x follows the Poisson distribution:

$$P_{x_i}(n) = \frac{(\lambda_u x)^n}{n!}\exp(-\lambda_u x). \tag{A2}$$

Denote the number of users associated with a typical BS in the i-th tier as N_i, of which the PDF $\mathbb{P}[N_i = n]$ is derived as

$$\begin{aligned}\mathbb{P}[N_i = n] &= \mathbb{E}_{X_i}[N_i = n] = \int_0^\infty \mathbb{P}[N_i = n|X_i = x]\, f_{X_i}(x)\mathrm{d}x\\ &= \frac{\lambda_u^n M^M}{n!\Gamma(M)}\left(\frac{\lambda_i}{A_i}\right)^M\int_0^\infty x^{n+M-1}\exp\left\{-\left(\lambda_u + M\frac{\lambda_i}{A_i}\right)x\right\}\mathrm{d}x\\ &= \frac{M^M\Gamma(n+M)\left(\frac{\lambda_u A_i}{\lambda_i}\right)^n}{n!\Gamma(M)\left(\frac{\lambda_u A_i}{\lambda_i} + M\right)^{n+M}}.\end{aligned} \tag{A3}$$

The activation probability of BSs in the i-th tier is the complementary probability of $\mathbb{P}[N_i = n]$, which is expressed as

$$P_{ai} = 1 - \mathbb{P}[N_i = 0] = 1 - \left(1 + \frac{\lambda_u A_i}{M\lambda_i}\right)^{-M}. \tag{A4}$$

Lemma 1 is proved.

Appendix B Proof of Lemma 2

Let γ be the SINR, then the SINR coverage of the i-th tier is given as

$$C_i(T) = \int_{r_i=0}^\infty \mathbb{P}[\gamma(r_i) \geqslant T] f_{r_i}(r_i)\,\mathrm{d}r_i, \tag{B1}$$

$$f_{r_i}(r_i) = \frac{2\pi\lambda_i}{A_i} r_i \exp\left(-\pi\sum_{j=1}^K \lambda_j\left(\frac{P_j B_j}{P_i B_i}\right)^{2/\alpha_j} {r_i}^{2\alpha_i/\alpha_j}\right), \tag{B2}$$

where $f_{r_i}(r_i)$ is the PDF of r_i and the expression of $f_{r_i}(r_i)$ follows from Lemma 2 of [5]. The complementary cumulative distribution function (CCDF) of the user SINR at distance r_i from its associated BS in i-th tier attains the form

$$\begin{aligned}\mathbb{P}[\gamma(r_i) \geqslant T] &= \mathbb{P}\left[h_{r_i} \geqslant \frac{\|r_i\|^{\alpha_i} T\left(I_{r_i} + \sigma^2\right)}{P_i}\right] \overset{(a)}{=} \mathbb{E}_{I_{r_i}}\left\{\exp\left(-\frac{\|r_i\|^{\alpha_i} T\left(I_{r_i} + \sigma^2\right)}{P_i}\right)\right\}\\ &= \exp\left(-\frac{\|r_i\|^{\alpha_i} T\sigma^2}{P_i}\right)\prod_{j=1}^K \mathcal{L}_{I_{r_i}}\left(\frac{\|r_i\|^{\alpha_i} T}{P_i}\right),\end{aligned} \tag{B3}$$

where (a) follows from $h_{r_i} \sim \exp(1)$. Here $\mathcal{L}_{I_{r_i}}(s)$ is the Laplace transform of interference I_{r_i}. Therefore, we can derive

$$\begin{aligned}\mathcal{L}_{I_{r_i}}(s) &= \mathbb{E}_{I_{r_i}}\{\exp(-sI_{r_i})\} = \mathbb{E}_{\Phi_j}\left\{\exp\left(-s\sum_{r_j\in\Phi_j\backslash r_i} P_j h_{r_j}\|r_j\|^{-\alpha_j}\right)\right\} \\ &= \mathbb{E}_{\Phi_j}\left\{\prod_{r_j\in\Phi_j\backslash r_i}\mathbb{E}_{h_{r_j}}\left(\exp\left(-sP_j h_{r_j}\|r_j\|^{-\alpha_j}\right)\right)\right\} \\ &\overset{(b)}{=} \mathbb{E}_{\Phi_j}\left\{\prod_{r_j\in\Phi_j\backslash r_i}\frac{1}{1+sP_j\|r_j\|^{-\alpha_j}}\right\},\end{aligned} \tag{B4}$$

where (b) follows from $h_{r_j} \sim \exp(1)$. Based on the probability generating functional of HPPP[1)], we have

$$\mathbb{E}\left\{\prod_{\Phi} f(x)\right\} = \exp\left(-\lambda\int_{\mathbb{R}^2}(1-f(x))\mathrm{d}x\right). \tag{B5}$$

By substituting $s = \frac{\|r_i\|^{\alpha_i}T}{P_i}$, the Laplace transform (B4) can be rewritten as follows:

$$\begin{aligned}\mathcal{L}_{I_{r_i}}\left(\frac{\|r_i\|^{\alpha_i}T}{P_i}\right) &= \exp\left(-2\pi\lambda_j p_{a\,j}\int_{z_j}^{\infty}\left(1-\mathcal{L}_{h_{r_j}}\left(T\left(\frac{P_j}{P_i}\right)\left(\frac{\|r_i\|^{\alpha_i}}{\|r_j\|^{\alpha_j}}\right)\right)\right)r_j\mathrm{d}r_j\right) \\ &= \exp\left(-2\pi\lambda_j p_{a\,j}\int_{z_j}^{\infty}\left(1-\frac{1}{1+T\left(\frac{P_j}{P_i}\right)\left(\frac{\|r_i\|^{\alpha_i}}{\|r_j\|^{\alpha_j}}\right)}\right)r_j\mathrm{d}r_j\right).\end{aligned} \tag{B6}$$

The integration limits are from z_j to ∞ since the closest interferer in j-th tier is at least at a distance $z_j = (\frac{P_jB_j}{P_iB_i})^{1/\alpha_j} r_i^{\alpha_i/\alpha_j}$. By using variable substitution $x = \|r_i\|^{-2\alpha_i/\alpha_j}(\frac{P_j}{P_i}T)^{-2/\alpha_j}\|r_j\|^2$,

$$\mathcal{L}_{I_{r_i}}\left(\frac{\|r_i\|^{\alpha_i}T}{P_i}\right) = \exp\left(-\pi\|r_i\|^{2\alpha_i/\alpha_j}\lambda_j p_{a\,j}\left(\frac{P_j}{P_i}T\right)^{2/\alpha_j}\int_{\left(\frac{B_iT}{B_j}\right)^{-2/\alpha_j}}^{\infty}\left(\frac{1}{1+x^{\alpha_j/2}}\right)\mathrm{d}x\right), \tag{B7}$$

Then, plugging (B7) into (B3) gives

$$\begin{aligned}\mathbb{P}[\gamma(r_i)\geqslant T] &= \exp\left(-\frac{\|r_i\|^{\alpha_i}T\sigma^2}{P_i}\right)\prod_{j=1}^{K}\mathcal{L}_{I_{r_i}}\left(\frac{\|r_i\|^{\alpha_i}T}{P_i}\right) \\ &= \exp\left(-\frac{\|r_i\|^{\alpha_i}T\sigma^2}{P_i}\right)\exp\left(-\pi\|r_i\|^{2\alpha_i/\alpha_j}\sum_{j=1}^{K}\lambda_j p_{a\,j}\left(\frac{P_j}{P_i}T\right)^{2/\alpha_j}\int_{\left(\frac{B_iT}{B_j}\right)^{-2/\alpha_j}}^{\infty}\left(\frac{1}{1+x^{\alpha_j/2}}\right)\mathrm{d}x\right).\end{aligned} \tag{B8}$$

Combining (B1) and (B8), we obtain the SINR coverage of the i-th tier in (7).

Appendix C Proof of Lemma 6

Based on (8) we have,

$$\begin{aligned}C_k(T) &= \frac{\sum_{i=1}^{K}\left(\frac{P_iB_i}{P_kB_k}\right)^{2/\alpha}\lambda_i}{\sum_{i=1}^{K}\lambda_i\left(\frac{P_iB_i}{P_kB_k}\right)^{2/\alpha}\left[1+p_{ai}\rho\left(\frac{B_k}{B_i}T,\alpha\right)\right]} \\ &\geqslant \frac{\sum_{i=1}^{K}\left(\frac{P_iB_i}{P_kB_k}\right)^{2/\alpha}\lambda_i}{\sum_{i=1}^{K}\lambda_i\left(\frac{P_iB_i}{P_kB_k}\right)^{2/\alpha}\left[1+\left(\frac{B_k}{B_i}T\right)^{2/\alpha}\int_{\left(\frac{B_k}{B_i}T\right)^{-2/\alpha}}^{\infty}x^{-\alpha/2}\mathrm{d}x\right]} \\ &= \frac{\sum_{i=1}^{K}\lambda_ib_ic_i}{\sum_{i=1}^{K}\lambda_ib_ic_i\left[1+\frac{2}{\alpha-2}\frac{B_k}{B_i}T\right]}.\end{aligned} \tag{C1}$$

Let $T = T_k = 2^{\frac{\rho_k\overline{N}_k}{W}} - 1$ and $R_k \geqslant 1-\eta$. Then,

$$\left(2^{\frac{\rho_k\overline{N}_k}{W}}-1\right) \leqslant \frac{\eta(\alpha-2)}{2(1-\eta)}\frac{\sum_{i=1}^{K}\lambda_ib_ic_i}{\sum_{i=1}^{K}\lambda_ib_ic_i\frac{B_k}{B_i}}. \tag{C2}$$

1) Chiu S N, Stoyan D, Kendall W S, et al. Stochastic Geometry and Its Applications. Hoboken: John Wiley and Sons, 2013.

For the sake of simplicity, we assume that the bias factor for macro (tier 1) BSs is the smallest and, the higher the tier order is, the bias factor for corresponding tier becomes larger, i.e. $B_1 \leqslant \cdots \leqslant B_k \leqslant \cdots \leqslant B_K, k \in K$, then

$$\left(2^{\frac{\rho_k \overline{N}_k}{W}} - 1\right) \leqslant \frac{\eta\left(\alpha-2\right)}{2\left(1-\eta\right)} \frac{\sum_{i=1}^{K} \lambda_i b_i c_i}{\sum_{i=1}^{K} \lambda_i b_i c_i \frac{B_k}{B_i}} \leqslant \frac{\eta\left(\alpha-2\right)}{2\left(1-\eta\right)} \frac{\sum_{i=1}^{K} \lambda_i b_i c_i}{\sum_{j=k+1}^{K} \frac{B_k}{B_j} \sum_{i=1}^{K} \lambda_i b_i c_i} = \frac{\eta\left(\alpha-2\right)}{2\left(1-\eta\right)} \frac{1}{\sum_{j=k+1}^{K} \frac{B_k}{B_j}}. \tag{C3}$$

Plugging (12) into (C3), final expression (15) is obtained.

Appendix D Proof of Lemma 7

By taking partial derivative to the APC, we have

$$\frac{\mathrm{d}E_{\text{total}}}{\mathrm{d}\tau_m} = \frac{p_{am} P_{m,\text{total}} \sum_{i=1}^{K} \tau_i b_i c_i - b_m c_m \sum_{i=1}^{K} \tau_i p_{ai} P_{i,\text{total}}}{\left[\sum_{i=1}^{K} \tau_i b_i c_i\right]^2} \frac{1.28\lambda_u}{\frac{W}{\rho_k} \log_2 \left[1 + \frac{\eta(\alpha-2)}{2(1-\eta)} \sum_{j=k+1}^{K} \frac{B_j}{B_k}\right] - 1}, \tag{D1}$$

where $\frac{1}{\left[\sum_{i=1}^{K} \tau_i b_i c_i\right]^2} \frac{1.28\lambda_u}{\frac{W}{\rho_k} \log_2 [1 + \frac{\eta(\alpha-2)}{2(1-\eta)} \sum_{j=k+1}^{K} \frac{B_j}{B_k}] - 1} > 0$ when $\sum_{j=k+1}^{K} \frac{B_j}{B_k} > \frac{2(1-\eta)}{\eta(\alpha-2)} (2^{\frac{\rho_k}{W}} - 1)$.

Let $p_{am} P_{m,\text{total}} \sum_{i=1}^{K} \tau_i b_i c_i - b_m c_m \sum_{i=1}^{K} \tau_i p_{ai} P_{i,\text{total}} > 0$, we derive $\frac{B_m}{B_k} \leqslant (\frac{p_{am} P_{m,\text{total}} \sum_{i=1, i \neq m}^{K} \tau_i b_i c_i}{c_m \sum_{i=1, i \neq m}^{K} \tau_i p_{ai} P_{i,\text{total}}})^{\alpha/2}$, i.e., $\frac{\mathrm{d}E_{\text{total}}}{\mathrm{d}\tau_m} > 0$ is satisfied. Similarly, we can also derive $\frac{dE_{\text{total}}}{d\tau_m} < 0$ when $\frac{B_m}{B_k} > (\frac{p_{am} P_{m,\text{total}} \sum_{i=1, i \neq m}^{K} \tau_i b_i c_i}{c_m \sum_{i=1, i \neq m}^{K} \tau_i p_{ai} P_{i,\text{total}}})^{\alpha/2}$.