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Super-sensitive detection of quantum interferometer in atmospheric environment

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Abstract With squeezed states or entangled states being the source, quantum metrology, imaging and sensing can break the standard quantum limit (SQL), even reach the Heisenberg limit (HL), which is difficult to achieve by traditional methods. However, the photon loss or phase fluctuation caused by the atmospheric attenuation and turbulence cannot be ignored in the actual application. Atmospheric transmittance and phase fluctuation are related to the detection distance, and the phase sensitivity becomes worse as the distance increases. As the functions of distance, the photon loss and phase fluctuation are uniformly expressed according to the introduction of atmospheric attenuation coefficient, turbulence structure constant and receive aperture size in this paper. The density matrixes and phase sensitivities of N00N states and M&M′ states in the atmospheric environment are proposed in terms of distance variables. Then the quantitative computation of super-sensitive distance is carried out. SQL-contour is proposed to describe the super-sensitive ability of the quantum interferometer for the affection from both photon loss and phase fluctuation. The simulation results show that, in atmospheric environment the super-sensitive distance can reach hundreds of meters. M&M′ states with less total photon number are more likely to reflect the advantage of super-sensitivity. SQL-contour can provide references for interferometric source choosing.

Keywords quantum interferometer, standard quantum limit, entangled states, phase sensitivity, atmospheric transmission

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1 Introduction

With the continuous development of quantum technology, quantum enhanced sensing with non-classical quantum states is getting more and more attention [1–3]. The phase estimation error can achieve the Heisenberg limit by the use of Path-Number entangled states (such as N00N states), and this quantum super-sensitive detection system is a new direction for Radar and Lidar systems [4,5]. As the correlations of quantum states are fragile to the noise environment which typically collapses the quantum state and destroys the phase information, the system decoherence caused by noise environment is the core problem for quantum metrology. Moreover, because the interaction with atmospheric environment is inevitable,

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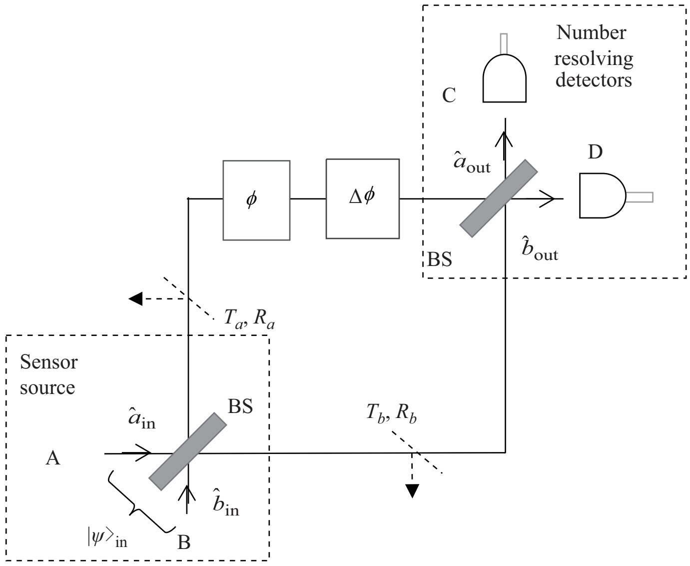

Figure 1 Interferometer model with photon loss and phase fluctuation.

photon loss and phase fluctuation noise are the main causes which lead to system decoherence and ultimately increase the error of phase estimation.

Several researches on N00N have been made under the condition of photon loss, and it turns out that photon loss makes N00N states decohere fast, and it is difficult to realize the actual application in the atmosphere environment [6–8]. In 2008, Huver et al. [9] proposed M&M′ states for robust quantum optical metrology, imaging and sensing by comparing N00N with M&M′ states under photon loss. Jiang et al. [10] gave the strategies for choosing path-entangled number states for robust quantum optical metrology in the presence of loss in 2012. And the phase fluctuation is researched by Bardhanh's papers in 2013, in which the effect of fluctuation to sensitivity, visibility and quantum Cramer-Rao bound is discussed [11, 12].

Since there is less research on quantitative description considering the two influences of photon loss and fluctuation in the properties of quantum interferometers, this paper does it. Based on the atmospheric attenuation coefficient and the atmospheric turbulence structure constant, this paper discusses the detection performance of the quantum interferometers with different input photon numbers in atmosphere to offer reference for supersensitive interferometric quantum radar.

2 Density matrixes with atmospheric attenuation and phase fluctuations

The atmospheric absorption, scattering and turbulence would cause photon loss and phase fluctuation for the entangled states. We set two fictitious beam splitters in the Mach-Zehnder interferometer (MZI) to mimic the loss of photon in Figure 1. T_i, R_i $(i = a, b)$ represent the atmospheric transmittance and attenuation rate for corresponding channel respectively. The sensor source provides path-entangled states, and the receivers are photon number resolving detectors. The upper beam passes through a phase-shifter ϕ, and the phase fluctuation $\Delta\phi$ is shown in the dotted box.

According to Beer Lambert law [13], the photon transmittance in the atmosphere can be expressed as

$$T = \exp(-\chi_\lambda \cdot L), \tag{1}$$

where χ_λ is the atmospheric attenuation coefficient at wavelength λ and L is the transmission distance. As the phase fluctuation caused by turbulence is related to the aperture size of the receiver [14], we have the phase fluctuation variance as

$$\sigma_\phi^2 = 1.03(D/r_0)^{5/3}, \tag{2}$$

where D is the aperture size of the receiver and r_0 is the atmospheric coherent radius with

$$r_0 = (0.423k_\lambda^2 C_n^2 L), \tag{3}$$

in which k_λ represents the wave number of the photons and C_n^2 is the turbulence structure constant. The phase fluctuation can be treated as a wiener process described by a zero mean Gaussian distribution with the variance σ_ϕ^2, that is

$$\langle \Delta\phi \rangle = 0, \quad \langle \Delta\phi^2 \rangle = \sigma_\phi^2 = 1.03(D/r_0)^{5/3} = 0.44D^{5/3}k_\lambda^2 C_n^2 L. \tag{4}$$

N00N state is the maximally number-path entangled state which can be produced by SPDC (spontaneous parametric down-conversion). It is a superposition of all photons in one path with none in the other. And for M&M′ states, there are photons in both paths. The expressions of N00N states and M&M′ state in Fock space are

$$|N00N\rangle_{a,b} = \frac{1}{\sqrt{2}}[|N0\rangle + |0N\rangle], \tag{5}$$

$$|M :: M'\rangle_{a,b} = \frac{1}{\sqrt{2}}[|MM'\rangle + |M'M\rangle], \tag{6}$$

where the subscript a, b indicate the two paths and the double-colon in $|A :: B\rangle$ refers to $A > B$.

We suppose that channel A is the signal photons in free space, and channel B is the idler photons in optical fiber. Therefore, the measured phase and phase fluctuation only exist in channel A and the photon loss exists in both channels with different attenuation coefficients, N00N states passing through the atmosphere can be defined as [1]

$$|N00N\rangle_\phi = \frac{1}{\sqrt{2}}\left[T_a^{\frac{N}{2}} \mathrm{e}^{\mathrm{i}N\phi} \mathrm{e}^{\mathrm{i}N\Delta\phi}|N0\rangle + T_b^{\frac{N}{2}} \mathrm{e}^{\mathrm{i}N\phi}|0N\rangle\right] + |\Phi\rangle, \tag{7}$$

where $|\Phi\rangle$ represents those states scattered into the atmosphere. Therefore the density matrix is

$$\begin{aligned}\rho_{|N00N\rangle} = &\frac{1}{2}[T_a^N|N0\rangle\langle N0| + T_b^N|0N\rangle\langle 0N| + (T_aT_b)^{\frac{N}{2}}\mathrm{e}^{-\mathrm{i}N\phi}\mathrm{e}^{-\mathrm{i}N\Delta\phi}|N0\rangle\langle 0N| \\ &+ (T_aT_b)^{\frac{N}{2}}\mathrm{e}^{\mathrm{i}N\phi}\mathrm{e}^{\mathrm{i}N\Delta\phi}|0N\rangle\langle N0| + \cdots].\end{aligned} \tag{8}$$

And similarly, M&M′ states passing through the atmosphere is given by

$$|M :: M'\rangle_\phi = \left(\sum_{k=0}^{M}\sum_{k'=0}^{M'} \alpha(k,k')|k,k'\rangle\right) + \sum_{k=0}^{M'}\sum_{k'=0}^{M} \beta(k,k')|k,k'\rangle, \tag{9}$$

with $M > M'$ and

$$\begin{aligned}\alpha(k,k') &= \frac{1}{\sqrt{2}}\left(\binom{M}{k}\binom{M'}{k'} T_a^k R_a^{M-k} T_b^{k'} R_b^{M'-k'}\right)^{\frac{1}{2}} \mathrm{e}^{\mathrm{i}k(\phi+\Delta\phi)}, \\ \beta(k,k') &= \frac{1}{\sqrt{2}}\left(\binom{M}{k'}\binom{M'}{k} T_a^k R_a^{M'-k} T_b^{k'} R_b^{M-k'}\right)^{\frac{1}{2}} \mathrm{e}^{\mathrm{i}k(\phi+\Delta\phi)}.\end{aligned} \tag{10}$$

So we can get the density matrix of M&M′ states as

$$\begin{aligned}\rho_{|M::M'\rangle} = &\sum_{k=0}^{M}\sum_{k'=0}^{M'} |\alpha(k,k')|^2|k,k'\rangle\langle k,k'| + \sum_{k=0}^{M'}\sum_{k'=0}^{M} |\beta(k,k')|^2|k,k'\rangle\langle k,k'| \\ &+ \sum_{k=0}^{M'}\sum_{k'=0}^{M'} \alpha(\Delta M + k, k')\beta^*(k, \Delta M + k')|\Delta M + k, k'\rangle\langle k, \Delta M + k'| \\ &+ \sum_{k=0}^{M'}\sum_{k'=0}^{M'} \alpha^*(\Delta M + k, k')\beta(k, \Delta M + k')|k, \Delta M + k'\rangle\langle \Delta M + k, k'|.\end{aligned} \tag{11}$$

3 Measured phase operator and phase sensitivity

In order to get more of the phase information, we choose the same operator used in [9], which sums up all the off-diagonal terms of the density matrix and provides sub-shot-noise sensitivity and can be implemented with number-resolving photon counting detectors,

$$\hat{A} = \sum_{r,s=0}^{M'} (|M-r, M'-s\rangle\langle M'-r, M-s| + |M'-r, M-s\rangle\langle M-r, M'-s|). \tag{12}$$

In the MZI phase measurement, the phase sensitivity can be described as

$$\delta\phi = \frac{\Delta\hat{A}}{|\partial\langle\hat{A}\rangle/\partial\phi|}, \tag{13}$$

where $\Delta\hat{A} = (\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2)^{\frac{1}{2}}$, $\langle\hat{A}\rangle = \mathrm{tr}[\hat{A}\rho]$.

Based on Taylor expansion, the term containing $\Delta\phi$ can be simplified as

$$\langle \mathrm{e}^{\mathrm{i}k\Delta\phi}\rangle = 1 + \mathrm{i}k\langle\Delta\phi\rangle - \frac{1}{2}k^2\langle\Delta\phi^2\rangle + \cdots \approx \mathrm{e}^{-0.22k^2D^{5/3}k_\lambda^2C_n^2L}. \tag{14}$$

Using $\hat{A}$ to measure N00N and M&M′ in the MZI system shown in Figure 1, we can obtain the phase sensitivity as

$$\delta\phi_{\mathrm{N00N}} = \frac{\sqrt{\frac{T_a^N+T_b^N}{2} - (T_aT_b)^N\mathrm{e}^{-0.44k^2D^{5/3}k_\lambda^2C_n^2L}\cos^2 N\phi}}{|(T_aT_b)^{N/2}\mathrm{e}^{-0.22k^2D^{5/3}k_\lambda^2C_n^2L}N\sin N\phi|}, \tag{15}$$

$$\delta\phi_{\mathrm{M\&M'}} = \frac{\sqrt{Q - S^2\mathrm{e}^{-0.44k^2D^{5/3}k_\lambda^2C_n^2L}\cos^2\Delta M\phi}}{|S\mathrm{e}^{-0.22k^2D^{5/3}k_\lambda^2C_n^2L}\Delta MN\sin\Delta M\phi|}, \tag{16}$$

where

$$\begin{aligned} Q &= \sum_{k,k'=0}^{M'} |\alpha(\Delta M+k,k')|^2 + |\beta(k,\Delta M+k')|^2, \\ S &= 2\mathrm{Re}\left(\sum_{k,k'=0}^{M'} \alpha^*(\Delta M+k,k')\beta(k,\Delta M+k')\right). \end{aligned} \tag{17}$$

In Eqs. (15) and (16), if the photon loss and phase fluctuation are ignored, the results of phase sensitivity are $\delta\phi_{\mathrm{N00N}} = 1/N$ and $\delta\phi_{\mathrm{M\&M'}} = 1/\Delta M$, which are the Heisenberg limit. When the phase shift $\phi = \frac{\pi}{2N}$ or $\frac{\pi}{2\Delta M}$, we get the minimum phase sensitivity. As the wavelength is short enough, we can obtain equaling integral multiple of $\phi = \frac{\pi}{2N}$ or $\frac{\pi}{2\Delta M}$ at arbitrary distance through adjusting the optical fiber delay, which makes the interferometer work at the optimal sensitivity.

4 Simulation analysis and SQL-contour

These above two entangle states can achieve super sensitivity detection in the MZI. As the atmospheric transmittance and the phase fluctuation variance are related to the distance, the phase sensitivity becomes worse as the distance increases. The distance where the sensitivity equals the standard quantum limit is defined as the super-sensitive detection distance.

We take 810 nm infrared photons produced by 405 nm photons through SPDC as the source, which is an typical wavelength for the generation of N00N state and was used by Afek in realizing high-N00N states [15]. The aperture size is set as 10 cm. According to the atmospheric turbulence vertical profiles model of spring in Hefei area, the turbulence structure constant is about $10^{-16}\mathrm{m}^{-2/3}$ at the height of 1 km [16]. Despite the photon loss and taking phase fluctuation into consideration, we get the relationship between the minimum sensitivity and distance shown in Figure 2.

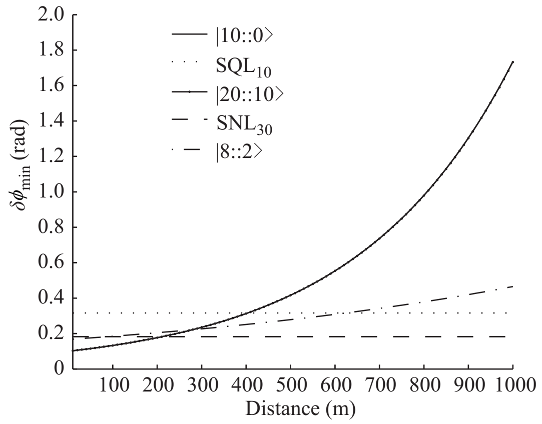

Figure 2 Relationship between minimum sensitivity and distance with phase fluctuation but no photon loss.

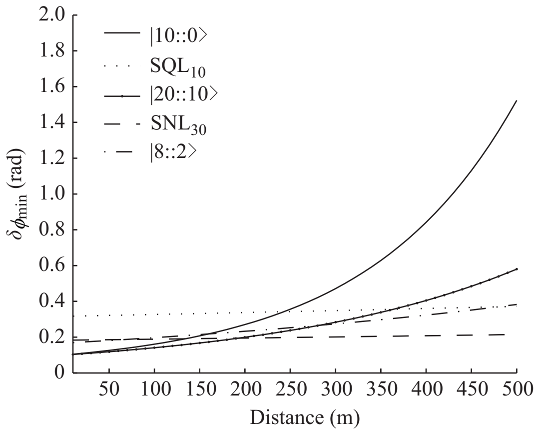

Figure 3 Relationship between minimum sensitivity and distance with both phase fluctuation and photon loss.

In Figure 2, we take N00N state $|10::0\rangle$, M&M′ states $|20::10\rangle$ and $|8::2\rangle$ as our analysis object. The corresponding SQL is $1/\sqrt{10}$ for $|10::0\rangle$ and $|8::2\rangle$, $1/\sqrt{30}$ for $|20::10\rangle$. As $N=\Delta M$ and the photon loss is ignored, we can get sensitivity curves of $|10::0\rangle$ and $|20::10\rangle$ coinciding with Eqs. (15) and (16). However, $|20::10\rangle$ has the lager total photon number and lower SQL. So the super-sensitive distance of N00N states is greater, 220 m for $|20::10\rangle$ and 420 m for $|10::0\rangle$, under the condition above. At the same time, although having the same total photon number with $|10::0\rangle$, the M&M′ state $|8::2\rangle$ gets the super-sensitivity detection distance of 630 m, which is much better than the other two states. This shows that the entangled Fock states with the same photon number difference are equally affected by phase fluctuation. With the same total photon number, M&M′ states perform better than N00N states. As SQL is inversely proportional to the total number, M&M′ states with less photons should be considered to reach further super-sensitive distance under the influence of only phase fluctuation.

Influenced by the atmospheric attenuation, the detected photons decrease and the SQL for both N00N and M&M′ states is no longer constant but increasing with the distance. Figure 3 shows that the sensitivity curves move upward obviously when the photon loss is involved and the super-sensitive distances for $|10::0\rangle$, $|20::10\rangle$ and $|8::2\rangle$ drop to 245 m, 200 m and 475 m, respectively. As the phase fluctuation affects states $|10::0\rangle$ and $|20::10\rangle$ equally, the perform difference in Figure 3 can be considered all originated from the photon loss. Although both of the super-sensitive distances reduce, the former is more affected. It is worthy noting that comparing with $|10::0\rangle$, $|8::2\rangle$ obtains better minimum phase sensitivity at about 300 m without loss (see in Figure 2), but better performs at 158 m with the loss (see in Figure 3). All of this shows the superiority of M&M′ states under photon loss.

In order to describe the influence of phase fluctuation and photon loss to the sensitivity of quantum interferometer more intuitively, we calculate the contour lines of the phase sensitivity with attenuation coefficient and atmospheric turbulence structure constant variables in a certain distance. Every point in this coordinate represents a working condition. And we define SQL-contour as the certain contour line which has the sensitivity equaling SQL. As the original point represents the ideal conditions, according to the dividing line of the SQL-contour of a certain state, region near the origin means this state could get super-sensitivity under those conditions.

With the SQL-contour, relationships between atmospheric environment and super-sensitivity of quantum interferometer could be analyzed. Figure 4 shows the SQL-contour of four different states in 200 m. All the contours decrease monotonously. Taking N00N state $|10::0\rangle$ and M&M′ states $|12::2\rangle$ and $|20::10\rangle$ which share the same photon number difference as research objects, N00N state covers the area with stronger phase fluctuation with low photon loss. $|20::10\rangle$ has the SQL contour lower than $|12::2\rangle$ state until the attenuation coefficient reaches 1.8 km^{-1}. Because of the low number of total photon number and tolerance to photon loss as an M&M′ state, $|8::2\rangle$ owns the highest SQL-contour in Figure 4, which means the best super-sensitive performance. The intersection of the SQL-contour for

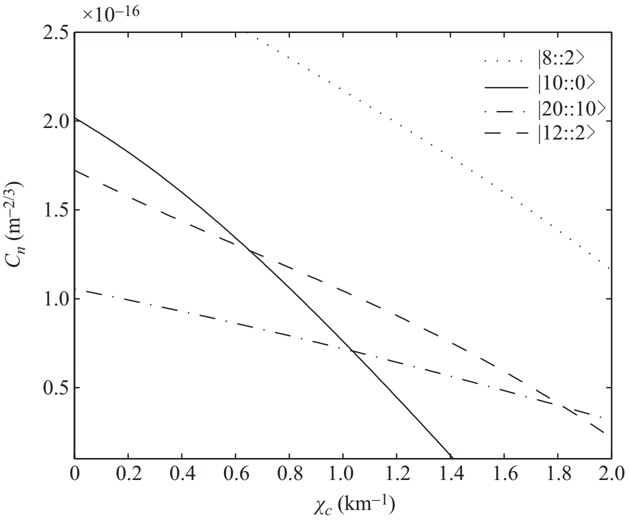

Figure 4 SQL-contour of different path entangled Fock states.

different states could be used as the reference point for source choosing. Taking the intersection (x, y) of the SQL-contours of $|10 :: 0\rangle$ and $|20 :: 10\rangle$ in Figure 4, as example, we may choose $|20 :: 10\rangle$, and choose $|10 :: 0\rangle$ when the attenuation coefficient is greater than x.

5 Conclusion

Phase sensitivity and super-sensitivity detection distance for path entangled Fock states (N00N and M&M′ states) affected by both photon loss and phase fluctuation is researched in this paper. Based on the atmospheric attenuation coefficient and the atmospheric turbulence structure constant, the relationship between phase sensitivity and detection distance is discussed. The SQL-contour is defined to describe the combined impact of photon loss and phase fluctuation, and provides references for interferometers' source choosing. Simulation shows that MZI with path-number entangled source gets super-sensitivity at the distance of hundreds of meters. States with the same photon number difference are equally affected by phase fluctuation. States with higher total photon number only perform better than the lower ones in high-loss environment. And super-sensitivity for M&M′ states with low photon number is the easiest to reach. For now, the entangled source with large photon number is still a technical problem. But the quantitative computation method of the super-sensitivity in this paper is still helpful for future quantum radar design.

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Conflict of interest The authors declare that they have no conflict of interest.

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