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Fixed-time synchronization of delayed memristor-based recurrent neural networks

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Abstract This paper focuses on the fixed-time synchronization control methodology for a class of delayed memristor-based recurrent neural networks. Based on Lyapunov functionals, analytical techniques, and together with novel control algorithms, sufficient conditions are established to achieve fixed-time synchronization of the master and slave memristive systems. Moreover, the settling time of fixed-time synchronization is estimated, which can be adjusted to desired values regardless of the initial conditions. Finally, the corresponding simulation results are included to show the effectiveness of the proposed methodology derived in this paper.

Keywords memristor, fixed-time synchronization, nonlinear control, master-slave systems, time delays

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1 Introduction

Based on the nonlinear relationship between charge q and magnetic flux φ, the concept of memristors was first theoretically postulated by Chua in 1971 [1], which was recognized as the first real-life understanding of the so-called missing fourth circuit element. Subsequently, Chua and Kang extended the idea of memristors to memristive systems and devices in 1976 [2]. In a seminal paper that appeared in late 2008, a two-terminal titanium dioxide nanoscale device that exhibited memristive characteristics was unveiled by Hewlett-Packard (HP), which was considered as the starting point for the design of a new class of high density processors, thus igniting renewed interest in memristors [3]. One immediate application is enabling low cost technology for nonvolatile memories where future computers will turn on instantly without the usual "booting time" that is currently required in all personal computers. Another important application is the construction of artificial memristive neural networks [4].

As described in [3], a memristor can be described by a nonlinear constitutive relation between the device terminal voltage v and the terminal current i given by

$$v = M(q)i \qquad \text{or} \qquad i = W(\varphi)v,$$

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where $M(q)$ and $W(\varphi)$ are the memristance and memductance, respectively, and are defined by

$$M(q) = \frac{\mathrm{d}\varphi(q)}{\mathrm{d}q} \geqslant 0 \quad \text{and} \quad W(\varphi) = \frac{\mathrm{d}q(\varphi)}{\mathrm{d}\varphi} \geqslant 0.$$

The slope of scalar functions $\varphi = \varphi(q)$ and $q = q(\varphi)$ are the memristor constitutive relation.

The memristor behavior can be summarized by the fact that its resistance depends on the current or the amount of charge that has passed through the device. When a sinusoidal or any bipolar periodic signal that assumes both positive and negative values is applied to the memristor, it exhibits a hysteresis loop in the $v - i$ plane that passes through the axis origin. This pinched hysteresis loop is considered as a characteristic of the memristor. In general, we call a two-terminal circuital element memristive if its typical $i - v$ curves are hysteretic passing through the axis origin.

Synchronization, which occurs when the dynamical behaviors of coupled systems achieve the same time spatial state, has been widely used in a variety of research fields such as secure communications, biology systems, linguistic networks, and information processing [5–16]. We know that there exist many different kinds of synchronization such as complete synchronization, anti-synchronization, lag synchronization, projective synchronization, as well as exponential synchronization. Many studies related to the memristive model have been conducted [17–25]. In [17], the stochastic exponential synchronization control of delayed memristive neural networks was discussed, and ref. [19] proposed finite-time synchronization for memristor-based neural networks. The Mittag-Leffler synchronization of fractional-order memristive neural networks was investigated in [21], and the corresponding lag quasi-synchronization for memristive model was addressed in [23]. The synchronization problem discussed above can be divided into two classes: infinite-time and finite-time synchronization. In many applications, synchronization may not be achieved in finite-time; for example, once exponential synchronization is realized, the external controllers should work continuously. If the controller is removed from the system, a small error will appear, which may cause a large difference between each node. In this case, finite-time synchronization may lead to better system performance.

For finite-time synchronization, the settling time depends heavily on the initial conditions of the system; therefore, in practical applications, the initial states of the networks must be given in advance, which limits practical applications since the knowledge of initial conditions may be hard to adjust or even impossible to estimate. To overcome this shortcoming, a new concept named fixed-time stability was proposed in [26]; future investigations on fixed-time algorithms can be found in [23–25]. Based on the sliding mode control technique, some fixed-time stability results were derived in [27]. In [28], a fixed-time control protocol for a specific formation control problem was designed. Furthermore, ref. [29] focused on the fixed-time average-consensus protocol for a weighted undirected network. Fixed-time work can avoid the inaccessibility of the final settling time and deterioration of system performance.

Fixed-time synchronization/stability implies that the system is globally finite-time stable and the settling time function is bounded for any initial values, which implies that the convergence settling time is independent of the initial states. Thus, this new technique possess important practical implications for practical engineering applications. However, in existing literature, very few papers concentrate on fixed-time synchronization [30–33], which inspired us to study the fixed-time synchronization of memristive neural networks.

The main objective of this paper is to design a new controller that provides a system with the fixed-time synchronization property. Under the derived protocols, all the states of the system converge to a common bounded settling time, which is independent of any initial values. Moreover, it can be estimated a priori by the control parameters.

The remainder of this paper is organized as follows. In Section 2, we formulate our problem and give some necessary definitions and lemmas; our assumptions are also presented in this section. Fixed-time synchronization criteria and their rigorous proofs are given in Section 3. Furthermore, some illustrative simulations are given in Section 4. Section 5 is the conclusion of this paper.

Notation. Throughout this paper, solutions of all systems are considered in Filippov's sense; specifically, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively.

For $r > 0$, $\mathcal{C}([-r,0];\mathbb{R}^n)$ denotes the family of continuous functions φ from $[-r,0]$ to $\mathbb{R}^n$. Let $\overline{a}_{ij} = \max\{\sup_{y\in\mathbb{R}}|a'_{ij}(y)|, \sup_{y\in\mathbb{R}}|a''_{ij}(y)|\}$ and $\overline{b}_{ij} = \max\{\sup_{y\in\mathbb{R}}|b'_{ij}(y)|, \sup_{y\in\mathbb{R}}|b''_{ij}(y)|\}$, for $i,j = 1,2,\ldots,n$.

2 Problem formulation and preliminaries

2.1 Problem formulation

By replacing memristors with resistors as connections in a circuit realization, the memristive system can be derived, which is a very large-scale integration (VLSI) circuit. For simplicity, we take the ith subsystem as the unit of analysis, which can be described by the following equation:

$$\begin{aligned}\dot{x}_i(t) = &-d_i x_i(t) + \sum_{j=1}^{n}[a_{ij}(f_j(x_j(t)) - x_i(t))f_j(x_j(t)) + b_{ij}(f_j(x_j(t-\tau_j)) - x_i(t))f_j(x_j(t-\tau_j))] \\ &+ I_i, \quad t \geqslant 0, \quad i = 1,2,\ldots,n, \end{aligned} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in \mathbb{R}^n$ is the neural state variable associated with the ith neuron, $D = \mathrm{diag}(d_1, d_2, \ldots, d_n)$ denotes the self-feedback connection weights with $d_i > 0$, and $A(x) = [a_{ij}(f_j(x_j(t)) - x_i(t))]_{n\times n}$ and $B(x) = [b_{ij}(f_j(x_j(t-\tau_j)) - x_i(t))]_{n\times n}$ are the feedback connection weights and the delayed feedback connection weights, respectively; furthermore, $\tau_j > 0$ is the transmission delay, $I = (I_1, I_2, \ldots, I_n)^{\mathrm{T}}$ is an input or bias vector, and $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^{\mathrm{T}}$ indicates the neuronal activation functions, which are subject to the following assumption.

(A_1) For $i = 1,2,\ldots,n$, $\forall x_1, x_2 \in \mathbb{R}$, and $x_1 \neq x_2$, the neural activation function f_i is bounded and satisfies the Lipschitz condition, i.e., there exist $l_i > 0$ and M_i such that

$$|f_i(x_1) - f_i(x_2)| \leqslant l_i|x_1 - x_2|, \quad |f_i(\cdot)| \leqslant M_i.$$

The initial value associated with system (1) is $x_i(t) = \phi_i(t) \in \mathcal{C}([-\tau,0];\mathbb{R})$, for $i = 1,2,\ldots,n$.

Since the interneuron connections are implemented by the memristor and its memductance depends on the voltage applied to it, the memristor-based weights a_{ij} and b_{ij} are a function of $f_j(x_j(t)) - x_i(t)$ and $f_j(x_j(t-\tau_j)) - x_i(t)$, respectively. Moreover, they are defined by

$$\varpi(\nu(t)) = \begin{cases} \varpi'(\nu(t)), & \nu(s)\downarrow, s \in (t-\sigma t, t], \\ \varpi''(\nu(t)), & \nu(s)\uparrow, s \in (t-\sigma t, t], \\ \lim_{s\to t^-}\varpi(\nu(s)), & \nu(s)\text{unchange}, s \in (t-\sigma t, t], \end{cases}$$

where σt is a sufficiently small positive constant; $\downarrow$ indicates "decrease", whereas $\uparrow$ indicates "increase". Moreover, $\lim_{s\to t^-}\varpi(\nu(s))$ implies that the memductance maintains the voltage value, which equals $\acute{\varpi}(\nu(t))$ or $\grave{\varpi}(\nu(t))$. Finally, $\varpi'(\cdot)$ and $\varpi''(\cdot)$ are assumed to be bounded.

Consider model (1) as the master system to investigate the fixed-time synchronization of memristive neural networks; the corresponding slave system is given by the following equation:

$$\begin{aligned}\dot{y}_i(t) = &-d_i y_i(t) + \sum_{j=1}^{n}[a_{ij}(f_j(y_j(t)) - y_i(t))f_j(y_j(t)) + b_{ij}(f_j(y_j(t-\tau_j)) - y_i(t))f_j(y_j(t-\tau_j))] \\ &+ I_i + u_i(t), \quad t \geqslant 0, \quad i = 1,2,\ldots,n, \end{aligned} \tag{2}$$

where $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^{\mathrm{T}}$ stands for the controller that will be appropriately designed for a certain synchronization objective.

For simplify, the above master/slave system can be rewritten in the following form:

$$\dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{n}[a_{ij}(x)f_j(x_j(t)) + b_{ij}(x)f_j(x_j(t-\tau_j))] + I_i, \tag{3}$$

and

$$\dot{y}_i(t) = -d_i y_i(t) + \sum_{j=1}^{n}[a_{ij}(y) f_j(y_j(t)) + b_{ij}(y) f_j(y_j(t-\tau_j))] + I_i + u_i(t), \tag{4}$$

where $a_{ij}(x) = a_{ij}(f_j(x_j(t)) - x_i(t))$ and $b_{ij}(x) = b_{ij}(f_j(x_j(t-\tau_j)) - x_i(t))$, for $i = 1, 2, \ldots, n$.

Note that the synchronization error signal is defined by $e_i(t) = y_i(t) - x_i(t)$; thus, the error dynamics system of (1) and (2) can be expressed as

$$\begin{aligned}\dot{e}_i(t) =& - d_i e_i(t) + \sum_{j=1}^{n}\Big[a_{ij}(y) f_j(y_j(t)) - a_{ij}(x) f_j(x_j(t))\Big] + \sum_{j=1}^{n}\Big[b_{ij}(y) f_j(y_j(t-\tau_j)) \\ & - b_{ij}(x) f_j(x_j(t-\tau_j))\Big] + u_i(t) \\ =& - d_i e_i(t) + \sum_{j=1}^{n}\Big[a_{ij}(y) g_j(e_j(t)) + (a_{ij}(y) - a_{ij}(x)) f_j(x_j(t))\Big] + \sum_{j=1}^{n}\Big[b_{ij}(y) g_j(e_j(t-\tau_j)) \\ & + (b_{ij}(y) - b_{ij}(x)) f_j(x_j(t-\tau_j))\Big] + u_i(t),\end{aligned} \tag{5}$$

where $g_j(e_j(t)) = f_j(y_j(t)) - f_j(x_j(t))$ and $g_j(e_j(t-\tau_j)) = f_j(y_j(t-\tau_j)) - f_j(x_j(t-\tau_j))$.

Remark 1. Based on Assumption A_1, we conclude that $g_j(\cdot)$ is bounded and satisfies

$$|g_j(e_j(t))| \leqslant l_j |e_j(t)|, \quad |g_j(\cdot)| \leqslant G_j,$$

where $G_j \geqslant 0$ is a constant.

2.2 Definitions and lemmas

Definition 1. The memristive neural network is said to be fixed-time synchronization if for any initial condition, there exists a settling time function $T(e_0(\theta))$ such that

$$\lim_{t \to T(e_0(\theta))} \|e(t)\| = 0, \quad e(t) = 0, \quad \forall t \geqslant T(e_0(\theta))$$

holds, where the settling-time function is bounded, i.e., there exists $T_{\max} > 0$ such that $T(e_0(\theta)) \leqslant T_{\max}$.

Definition 2. Network (1) is said to be completely synchronized onto (2) in finite time if by adding a suitable designed controller to system (2), there exists a constant $t_1 > 0$, where $t_1 > 0$ depends on the initial state vector value $x(0) = (x_1^{\mathrm{T}}(0), x_2^{\mathrm{T}}(0), \ldots, x_n^{\mathrm{T}}(0))^{\mathrm{T}}$, such that

$$\lim_{t \to t_1} \|e(t)\| = 0, \quad \|e(t)\| \equiv 0, \quad \forall t > t_1.$$

Definition 3 ([34]). Function $V(x): \mathbb{R}^n \to \mathbb{R}$ is C-regular if $V(x)$ is

(1) regular in $\mathbb{R}^n$;

(2) positive definite, i.e., $V(x) > 0$ for $x \neq 0$ and $V(0) = 0$;

(3) radially unbounded, i.e., $V(x) \to +\infty$ as $\|x\| \to +\infty$.

Note that a C-regular Lyapunov function $V(x)$ is not necessarily differentiable.

Lemma 1 ([35]). For any constant vector $z \in \mathbb{R}^n$ and $0 < r < l$, the following norm equivalence property holds

$$\left(\sum_{i=1}^{n}|z_i|^l\right)^{\frac{1}{l}} \leqslant \left(\sum_{i=1}^{n}|z_i|^r\right)^{\frac{1}{r}},$$

and

$$\left(\frac{1}{n}\sum_{i=1}^{n}|z_i|^l\right)^{\frac{1}{l}} \geqslant \left(\frac{1}{n}\sum_{i=1}^{n}|z_i|^r\right)^{\frac{1}{r}}.$$

Lemma 2 ([26]). If there exists a continuous radially unbounded function $V:\mathbb{R}^n\rightarrow\mathbb{R}_+\bigcup\{0\}$ such that

(1) $V(z)=0\Longleftrightarrow z=0$;

(2) for some $\alpha,\beta>0$, $0<p<1$, and $q>1$, any solution $z(t)$ satisfies the inequality

$$D^+V(z(t))\leqslant-\alpha V^p(z(t))-\beta V^q(z(t)),$$

then the origin is globally fixed-time stable and the following estimate holds

$$V(t)\equiv 0,\quad t\geqslant T(z_0),$$

with the settling time bounded by

$$T(z_0)\leqslant T_{\max}:=\frac{1}{\alpha(1-p)}+\frac{1}{\beta(q-1)},\quad \forall z_0\in\mathbb{R}^n.$$

Lemma 3 ([29]). If there exists a continuous radially unbounded function $V:\mathbb{R}^n\rightarrow\mathbb{R}_+\bigcup\{0\}$ such that

(1) $V(z)=0\Longleftrightarrow z=0$;

(2) for some $\alpha,\beta>0$, $p=1-\frac{1}{2\mu}$, $q=1+\frac{1}{2\mu}$, and $\mu>1$, any solution of $z(t)$ satisfies the inequality

$$D^+V(z(t))\leqslant-\alpha V^p(z(t))-\beta V^q(z(t)),$$

then the origin is globally fixed-time stable, and the following estimate of the settling time function holds

$$T(z_0)\leqslant T_{\max}:=\frac{\pi\mu}{\sqrt{\alpha\beta}},\quad \forall z_0\in\mathbb{R}^n.$$

Remark 2. The settling time function is bounded above by a priori value that relies on the design parameters instead of the initial system state; this implies that the convergence time can be guaranteed in a prescribed manner. Moreover, Lemma 2 presents a conservative settling time estimate compared with Lemma 3.

Lemma 4 ([36]). Suppose that function $V(x):\mathbb{R}^n\rightarrow\mathbb{R}$ is C-regular and that $x(t):[0,+\infty)\rightarrow\mathbb{R}^n$ is absolutely continuous on any compact interval $[0,+\infty)$. Let $v(t)=V(x(t))$. If there exists a continuous function $\gamma:[0,+\infty)\rightarrow\mathbb{R}$ with $\gamma(\sigma)>0$ for $\sigma\in(0,+\infty)$, such that

$$\dot{v}(t)\leqslant-\gamma(v(t)),$$

for all $t>0$, and $v(t)$ is differentiable at t and $\gamma(\cdot)$ satisfies the condition

$$\int_0^{v(0)}\frac{1}{\gamma(\sigma)}\mathrm{d}\sigma=t_1<+\infty,$$

then we have $v(t)=0$ for $t\geqslant t_1$. In particular, if $\gamma(\sigma)=Q\sigma^\mu$ for all $\sigma\in(0,+\infty)$, where $\mu\in(0,1)$ and $Q>0$, then the setting time is estimated by

$$t_1=\frac{v^{1-\mu}}{Q(1-\mu)}.$$

3 Main results

This section is dedicated to the problem of fixed-time synchronization control for the delayed memristive model in which some new discontinuous control strategies are proposed to reach the fixed-time synchronization goal. The finite-time synchronization criterion is provided as a corollary.

Theorem 1. Under Assumption A_1, the fixed-time synchronization between master-slave systems (1) and (2) can be achieved under the following controller:

$$u_i(t) = -\lambda_{1i}e_i(t) - \lambda_{2i}\text{sign}(e_i(t)) - \lambda_{3i}\text{sign}(e_i(t))|e_i(t)|^{\alpha} - \lambda_{4i}\text{sign}(e_i(t))|e_i(t)|^{\beta}, \tag{6}$$

where $0 < \alpha < 1$, $\beta > 1$ and

$$\begin{aligned} &\lambda_{1i} \geqslant -d_i + \sum_{j=1}^{n}\frac{1}{2}\bar{a}_{ij} + \sum_{j=1}^{n}\frac{1}{2}\bar{a}_{ji}l_i^2, \\ &\lambda_{2i} \geqslant \sum_{j=1}^{n}\bar{b}_{ij}G_j + \sum_{j=1}^{n}\Big(\sup_{u\in\mathbb{R}}(|a'_{ij}(u) - a''_{ij}(u)| + |b'_{ij}(u) - b''_{ij}(u)|)M_j\Big), \\ &\lambda_{3i} > 0, \quad \lambda_{4i} > 0. \end{aligned} \tag{7}$$

The fixed settling time T satisfies

$$T \leqslant T(e_0) \leqslant \frac{1}{\min(\lambda_{3i})2^{\frac{1+\alpha}{2}}(1-\alpha)} + \frac{1}{\min(\lambda_{4i})n^{\frac{1-\beta}{2}}2^{\frac{1+\beta}{2}}(\beta-1)}. \tag{8}$$

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{n}e_i^2(t). \tag{9}$$

Calculating the upper right Dini-derivative $D^+V(t)$ along the trajectory of (5) gives

$$\begin{aligned} D^+V(t) = &\sum_{i=1}^{n}e_i(t)\bigg\{ - d_ie_i(t) + \sum_{j=1}^{n}[a_{ij}(y)g_j(e_j(t)) + (a_{ij}(y) - a_{ij}(x))f_j(x_j(t))] \\ &+ \sum_{j=1}^{n}[b_{ij}(y)g_j(e_j(t-\tau_j)) + (b_{ij}(y) - b_{ij}(x))f_j(x_j(t-\tau_j))] + u_i(t)\bigg\} \\ \leqslant &\sum_{i=1}^{n}\bigg\{ - d_ie_i^2(t) + \sum_{j=1}^{n}\bar{a}_{ij}|e_i(t)||g_j(e_j(t))| + \sum_{j=1}^{n}\bar{b}_{ij}|e_i(t)||g_j(e_j(t-\tau_j))| \\ &+ \sum_{j=1}^{n}|e_i(t)||a_{ij}(y) - a_{ij}(x)||f_j(x_j(t))| + \sum_{j=1}^{n}|e_i(t)||b_{ij}(y) - b_{ij}(x)||f_j(x_j(t-\tau_j))| \\ &- \lambda_{1i}e_i^2(t) - \lambda_{2i}|e_i(t)| - \lambda_{3i}|e_i(t)|^{\alpha+1} - \lambda_{4i}|e_i(t)|^{\beta+1}\bigg\} \\ \leqslant &\sum_{i=1}^{n}\bigg\{ - d_ie_i^2(t) + \sum_{j=1}^{n}\bar{a}_{ij}|e_i(t)|l_j|e_j(t)| + \sum_{j=1}^{n}\bar{b}_{ij}|e_i(t)|G_j - \lambda_{1i}e_i^2(t) - \lambda_{2i}|e_i(t)| \\ &+ \sum_{j=1}^{n}|e_i(t)|\sup_{u\in\mathbb{R}}|a'_{ij}(u) - a''_{ij}(u)|M_j + \sum_{j=1}^{n}|e_i(t)|\sup_{u\in\mathbb{R}}|b'_{ij}(u) - b''_{ij}(u)|M_j \\ &- \lambda_{3i}|e_i(t)|^{\alpha+1} - \lambda_{4i}|e_i(t)|^{\beta+1}\bigg\}. \end{aligned} \tag{10}$$

Next, consider

$$\begin{aligned} \sum_{i=1}^{n}\sum_{j=1}^{n}\bar{a}_{ij}|e_i(t)|l_j|e_j(t)| &\leqslant \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{1}{2}\bar{a}_{ij}e_i^2(t) + \frac{1}{2}\bar{a}_{ij}l_j^2e_j^2(t)\right) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\bar{a}_{ij}e_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{2}\bar{a}_{ji}l_i^2e_i^2(t). \end{aligned} \tag{11}$$

Thus, along with (10) and (11), we obtain the following estimation for the right-hand side of (10):

$$\begin{aligned}D^+V(t) \leqslant& \sum_{i=1}^n \bigg\{ \bigg(-d_i + \sum_{j=1}^n \frac{1}{2}\bar{a}_{ij} + \sum_{j=1}^n \frac{1}{2}\bar{a}_{ji} l_i^2 - \lambda_{1i} \bigg) e_i^2(t) \\ &+ \bigg[-\lambda_{2i} + \sum_{j=1}^n \bar{b}_{ij} G_j + \sum_{j=1}^n \bigg(\sup_{u\in\mathbb{R}} (|a'_{ij}(u) - a''_{ij}(u)| \\ &+ |b'_{ij}(u) - b''_{ij}(u)|) M_j \bigg) \bigg] |e_i(t)| - \lambda_{3i}|e_i(t)|^{\alpha+1} - \lambda_{4i}|e_i(t)|^{\beta+1} \bigg\}. \end{aligned} \tag{12}$$

Considering the parameters defined in (7) and Lemma 1, it follows that

$$\begin{aligned}D^+V(t) \leqslant& -\min(\lambda_{3i}) \sum_{i=1}^n |e_i(t)|^{\alpha+1} - \min(\lambda_{4i}) \sum_{i=1}^n |e_i(t)|^{\beta+1} \\ \leqslant& -\min(\lambda_{3i}) \bigg(\sum_{i=1}^n e_i^2(t) \bigg)^{\frac{\alpha+1}{2}} - \min(\lambda_{4i}) \bigg(\sum_{i=1}^n e_i^2(t) \bigg)^{\frac{\beta+1}{2}} \\ =& -\min(\lambda_{3i}) 2^{\frac{1+\alpha}{2}} V(t)^{\frac{1+\alpha}{2}} - \min(\lambda_{4i}) n^{\frac{1-\beta}{2}} 2^{\frac{1+\beta}{2}} V(t)^{\frac{1+\beta}{2}}. \end{aligned} \tag{13}$$

Thus, according to Lemma 2, fixed-time synchronization is finally realized, and the settling time in (8) is obtained.

In particular, if α and β in Theorem 1 are chosen as $\alpha = 1 - \frac{1}{2\mu}$ and $\beta = 1 + \frac{1}{2\mu}$ with $\mu > 1$, then a more accurate estimation of the settling time can be obtained using Lemma 3; we state this fact without proof.

Corollary 1. Suppose all the conditions in Theorem 1 hold. Then the slave system in (2) can synchronize onto the master system in (1) in fixed-time under controller (6), and the fixed settling time T satisfies

$$T \leqslant T(e_0) \leqslant \frac{\pi\mu}{\sqrt{\min(\lambda_{3i})\min(\lambda_{4i}) 2^{\frac{2+\alpha+\beta}{2}} n^{\frac{1-\beta}{2}}}} \tag{14}$$

with $\mu > 1$.

Remark 3. Based on inequality (14), an extremely important conclusion can be derived, i.e., the system can be forced to have any a priori specified settling time by properly choosing parameters α, β, and μ.

Remark 4. The controller used in this paper is a discontinuous one, whereas in some cases, continuity is necessary; under these circumstances, we choose $\frac{e_i(t)}{e_i(t)+\sigma}$ to approximate the term $\mathrm{sign}(e_i(t))$, where $\sigma > 0$ is sufficiently small.

To make a comparison with fixed-time synchronization, we concentrate on finite-time synchronization of the target model.

Corollary 2. Under Assumption A_1, the master-slave memristive systems in (1) and (2) will realize finite-time synchronization in finite time t_1 under the following discontinuous controller:

$$u_i(t) = -\eta_{1i} e_i(t) - \eta_{2i}\mathrm{sign}(e_i(t)) - \eta_{3i}\mathrm{sign}(e_i(t))|e_i(t)|^{\alpha}, \tag{15}$$

where $0 < \alpha < 1$, and

$$\begin{aligned}&\eta_{1i} \geqslant -d_i + \sum_{j=1}^n \frac{1}{2}\bar{a}_{ij} + \sum_{j=1}^n \frac{1}{2}\bar{a}_{ji} l_i^2, \\ &\eta_{2i} \geqslant \sum_{j=1}^n \bar{b}_{ij} G_j + \sum_{j=1}^n \bigg(\sup_{u\in\mathbb{R}} (|a'_{ij}(u) - a''_{ij}(u)| + |b'_{ij}(u) - b''_{ij}(u)|) M_j \bigg), \\ &\eta_{3i} > 0.\end{aligned}$$

Moreover, the finite time t_1 can be chosen as

$$t_1 = \frac{V(0)^{\frac{1-\alpha}{2}}}{\min(\eta_{3i})2^{\frac{1+\alpha}{2}}\frac{1-\alpha}{2}}. \tag{16}$$

Remark 5. It can be shown that the settling time proposed in (16) depends on the initial condition $V(0)$; on the other hand, when the initial condition is very large, the settling time is impractical in some real applications. Settling times (8) and (14) are independent of initial states; thus, the pre-specified settling time can be obtained by appropriately selecting control parameters.

Remark 6. For finite-time synchronization, only one term such as $-V^p(t)$, $0 < p < 1$, can realize this goal. On the other hand, to realize fixed-time synchronization, an additional term $-V^q(t)$, $q > 1$, is necessary; its role can be regarded as pulling the system into the region with norm less than one in fixed-time.

As stated in [37], for digital computer applications requiring only two memory states, a memristor must exhibit only two sufficiently distinct equilibrium states R_{OFF} and R_{ON}. With $R_{\mathrm{OFF}} \gg R_{\mathrm{ON}}$, the high-resistance states can be easily switched to low resistance states. To better reflect this property of memristors, a more realistic mathematical description of the connection weights of memristor-based neural networks in (1) is defined; specifically,

$$a_{ij}(x) = \begin{cases} a'_{ij}, & f_j(x_j(s)) - x_i(s) \downarrow, s \in (t-\sigma t, t], \\ a''_{ij}, & f_j(x_j(s)) - x_i(s) \uparrow, s \in (t-\sigma t, t], \\ \lim_{s\to t^-} a_{ij}(f_j(x_j(s)) - x_i(s)), & f_j(x_j(s)) - x_i(s) \text{ unchange}, s \in (t-\sigma t, t], \end{cases}$$

and

$$b_{ij}(x) = \begin{cases} b'_{ij}, & f_j(x_j(s-\tau_j)) - x_i(s) \downarrow, s \in (t-\sigma t, t], \\ b''_{ij}, & f_j(x_j(s-\tau_j)) - x_i(s) \uparrow, s \in (t-\sigma t, t], \\ \lim_{s\to t^-} b_{ij}(f_j(x_j(s-\tau_j)) - x_i(s)), & f_j(x_j(s-\tau_j)) - x_i(s) \text{ unchange}, s \in (t-\sigma t, t]. \end{cases}$$

As a result, it is easy to reach the following conclusions:

$$\sup_{u\in\mathbb{R}} |a_{ij}(u) - a_{ij}(u)| = |a'_{ij} - a''_{ij}|, \quad \sup_{u\in\mathbb{R}} |b_{ij}(u) - b_{ij}(u)| = |b'_{ij} - b''_{ij}|.$$

The following corollaries match, mutatis mutandis, Theorem 1; we state them without proof.

Corollary 3. Suppose that Assumption A_1 holds for memristive systems (1) and (2). Then fixed-time synchronization of the master-slave system can be achieved under controller (6), and the control parameters to be determined are subjected to the following:

$$\begin{aligned} &\lambda_{1i} \geqslant -d_i + \sum_{j=1}^n \frac{1}{2}\bar{a}_{ij} + \sum_{j=1}^n \frac{1}{2}\bar{a}_{ji} l_i^2, \\ &\lambda_{2i} \geqslant \sum_{j=1}^n \bar{b}_{ij} G_j + \sum_{j=1}^n \Big[|a'_{ij} - a''_{ij}| + |b'_{ij} - b''_{ij}|\Big] M_j, \\ &\lambda_{3i} > 0, \quad \lambda_{4i} > 0. \end{aligned} \tag{17}$$

The fixed settling time T satisfies

$$T \leqslant T(e_0) \leqslant \frac{1}{\min(\lambda_{3i})2^{\frac{1+\alpha}{2}}(1-\alpha)} + \frac{1}{\min(\lambda_{4i})n^{\frac{1-\beta}{2}}2^{\frac{1+\beta}{2}}(\beta-1)}.$$

Corollary 4. Under the conditions of Corollary 1, for a given constant $\mu > 1$, fixed-time synchronization of memristive neural networks (1) and (2) can be achieved, and the fixed settling time T satisfies

$$T \leqslant T(e_0) \leqslant \frac{\pi\mu}{\sqrt{\min(\lambda_{3i})\min(\lambda_{4i})2^{\frac{2+\alpha+\beta}{2}}n^{\frac{1-\beta}{2}}}}.$$

Corollary 5. Suppose that all the conditions of Corollary 2 hold. Then slave system (2) can synchronize onto master system (1) in finite time t_1 under controller (15), where the control parameters satisfy

$$\begin{aligned}
&\eta_{1i} \geqslant -d_i + \sum_{j=1}^{n} \frac{1}{2}\bar{a}_{ij} + \sum_{j=1}^{n} \frac{1}{2}\bar{a}_{ji} l_i^2, \\
&\eta_{2i} \geqslant \sum_{j=1}^{n} \bar{b}_{ij} G_j + \sum_{j=1}^{n} [|a'_{ij} - a''_{ij}| + |b'_{ij} - b''_{ij}|] M_j, \\
&\eta_{3i} > 0,
\end{aligned}$$

and the finite time t_1 is defined according to (16).

4 Numerical examples

In this section, we perform two examples to demonstrate the validity and effectiveness of the proposed theoretical results derived above.

Example 1. Consider the following two-dimensional memristor-based recurrent neural networks:

$$\begin{cases}
\dot{x}_1(t) = -x_1(t) + a_{11}(t) f_1(x_1(t)) + a_{12} f_2(x_2(t)) + b_{11}(t) f_1(x_1(t-1)) + b_{12} f_2(x_2(t-1)) + I_1(t), \\
\dot{x}_2(t) = -x_2(t) + a_{21}(t) f_1(x_1(t)) + a_{22} f_2(x_2(t)) + b_{21} f_1(x_1(t-1)) + b_{22}(t) f_2(x_2(t-1)) + I_2(t),
\end{cases} \tag{18}$$

and

$$\begin{cases}
\dot{y}_1(t) = -y_1(t) + a_{11}(t) f_1(y_1(t)) + a_{12} f_2(y_2(t)) + b_{11}(t) f_1(y_1(t-1)) + b_{12} f_2(y_2(t-1)) \\
\qquad\qquad + I_1(t) + u_1(t), \\
\dot{y}_2(t) = -y_2(t) + a_{21}(t) f_1(y_1(t)) + a_{22} f_2(y_2(t)) + b_{21} f_1(y_1(t-1)) + b_{22}(t) f_2(y_2(t-1)) \\
\qquad\qquad + I_2(t) + u_2(t),
\end{cases} \tag{19}$$

with

$$a_{12} = -0.1, \quad a_{22} = 2.9, \quad b_{12} = -0.1, \quad b_{21} = -1.2,$$

$$a_{11}(t) = \begin{cases}
1, & f_{11}(s) \downarrow, s \in (t - \sigma t, t], \\
0.7, & f_{11}(s) \uparrow, s \in (t - \sigma t, t], \\
\lim_{s \to t^-} a_{11}(s), & f_{11}(s) \ \text{ unchange}, s \in (t - \sigma t, t],
\end{cases}$$

$$a_{21}(t) = \begin{cases}
-0.7, & f_{21}(s) \downarrow, s \in (t - \sigma t, t], \\
-1, & f_{21}(s) \uparrow, s \in (t - \sigma t, t], \\
\lim_{s \to t^-} a_{21}(s), & f_{21}(s) \ \text{ unchange}, s \in (t - \sigma t, t],
\end{cases}$$

$$b_{11}(t) = \begin{cases}
-2.5, & f_{11}(s-1) \downarrow, s \in (t - \sigma t, t], \\
-2.3, & f_{11}(s-1) \uparrow, s \in (t - \sigma t, t], \\
\lim_{s \to t^-} b_{11}(s), & f_{11}(s-1) \ \text{ unchange}, s \in (t - \sigma t, t],
\end{cases}$$

$$b_{22}(t) = \begin{cases}
-3.3, & f_{22}(s-1) \downarrow, s \in (t - \sigma t, t], \\
-3.6, & f_{22}(s-1) \uparrow, s \in (t - \sigma t, t], \\
\lim_{s \to t^-} b_{22}(s), & f_{22}(s-1) \ \text{ unchange}, s \in (t - \sigma t, t],
\end{cases}$$

where $f_{ij}(t) \triangleq f_j(x_j(t)) - x_i(t)$ and $f_{ij}(t-1) \triangleq f_j(x_j(t-1)) - x_i(t)$.

The activation function is taken as $f(s) = \tanh(s)$, and it is obvious that $l_1 = l_2 = 1$ and $M_1 = M_2 = 1$. The memristive neural network in (18) with the above conditions exhibits chaotic behavior, which is plotted in Figure 1. Moreover, Figure 2 gives the state trajectories of system (18).

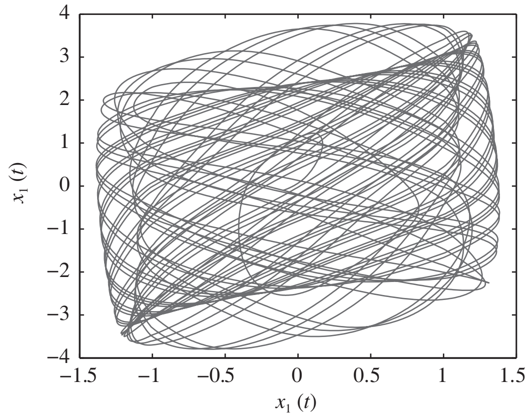

Figure 1 The chaotic attractor of system (18) with initial conditions $x_1(t) = -0.1$ and $x_2(t) = -0.1$, $t \in [-1, 0]$.

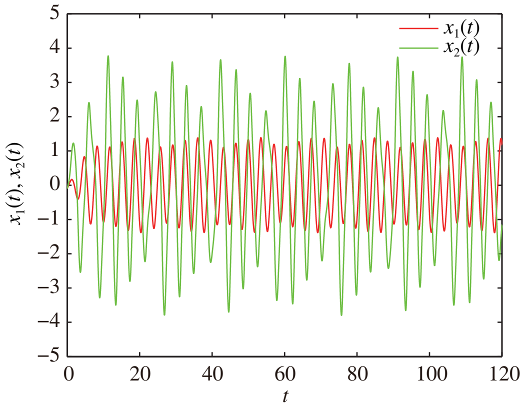

Figure 2 (Color online) Time evolution of states $x_1(t)$ and $x_2(t)$ in system (18) with initial conditions $x_1(t) = -0.1$ and $x_2(t) = -0.1$, $t \in [-1, 0]$.

First, our goal is to study the fixed-time synchronization problem of systems (18) and (19) with the above parameters. By simple computations, we obtain

$$\lambda_{11} \geqslant -d_1 + \sum_{j=1}^{2} \frac{1}{2}\bar{a}_{1j} + \sum_{j=1}^{2} \frac{1}{2}\bar{a}_{j1} l_1^2 = 0.55,$$

$$\lambda_{12} \geqslant -d_2 + \sum_{j=1}^{2} \frac{1}{2}\bar{a}_{2j} + \sum_{j=1}^{2} \frac{1}{2}\bar{a}_{j2} l_2^2 = 2.45,$$

$$\lambda_{21} \geqslant \sum_{j=1}^{2} \bar{b}_{1j} G_j + \sum_{j=1}^{2} [|a'_{1j} - a''_{1j}| + |b'_{1j} - b''_{1j}|] M_j = 3.1,$$

$$\lambda_{22} \geqslant \sum_{j=1}^{2} \bar{b}_{2j} G_j + \sum_{j=1}^{2} [|a'_{2j} - a''_{2j}| + |b'_{2j} - b''_{2j}|] M_j = 5.4,$$

thus, we choose $\lambda_{11} = 2$, $\lambda_{12} = 5$, $\lambda_{21} = 4$, and $\lambda_{22} = 6$; in addition, set $\lambda_{31} = \lambda_{32} = 0.2$, $\lambda_{41} = \lambda_{42} = 0.5$, $\alpha = 0.5$, and $\beta = 2$. According to Theorem 1 and the above parameters, the upper bound of the settling time $T_{\max}$ can be calculated as 6.9. Correspondingly, the controller can also be given in the following form:

$$\begin{aligned} u_1(t) &= -2e_1(t) - 4\text{sign}(e_1(t)) - 0.2\text{sign}(e_1(t))|e_1(t)|^{0.5} - 0.5\text{sign}(e_1(t))|e_1(t)|^2, \\ u_2(t) &= -5e_2(t) - 6\text{sign}(e_2(t)) - 0.2\text{sign}(e_2(t))|e_2(t)|^{0.5} - 0.5\text{sign}(e_2(t))|e_2(t)|^2. \end{aligned} \tag{20}$$

Under controller (20), the transient behaviors of the state variables of systems (18) and (19) with initial conditions $x(t) = (-0.6\sin 2t + 0.7, -0.1\cos t)^{\rm T}$ and $y(t) = (0.2\tanh 2t, 0.8\sin 5t)^{\rm T}$ for $t \in [-1, 0]$ are given in Figure 3. Moreover, Figure 4 shows the synchronization errors $e_i(t), i = 1, 2$. From these simulation results, note that master system (18) and slave system (19) are synchronized with the settling time $T_{\max}$, and the average convergence error remains zero thereafter. The simulation results are consistent with the theory derived above.

To future investigate the finite-time synchronization problem of memristive neural networks, we give the following example.

Example 2. Consider the following two-neuron memristor-based neural network with delay:

$$\dot{x}(t) = -x(t) + A(t)f(x(t)) + B(t)f(x_1(t-1)) + I(t), \tag{21}$$

and

$$\dot{y}(t) = -y(t) + A(t)f(y(t)) + B(t)f(y_1(t-1)) + I(t) + u(t), \tag{22}$$

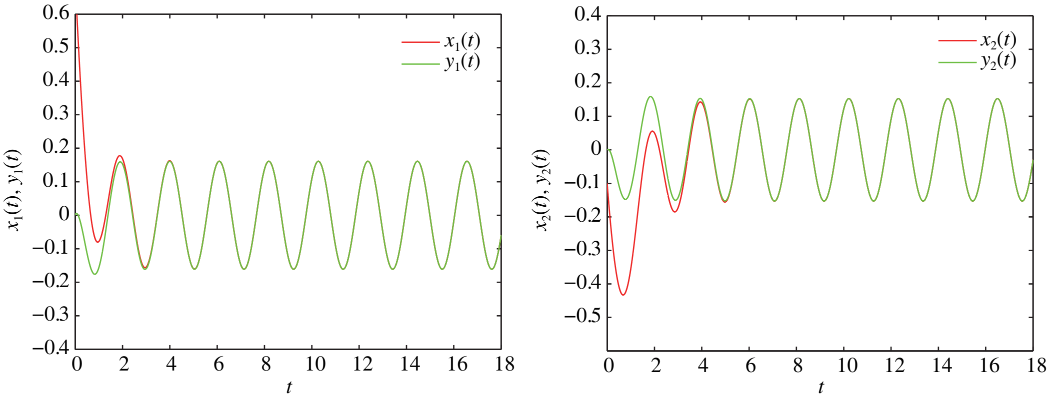

Figure 3 (Color online) Time-domain behavior of state variables $x(t)$ and $y(t)$ with initial conditions $x_1(t) = -0.6\sin(2t)+0.7$ and $y_1(t) = 0.2\tanh(2t)$, $t \in [-1, 0]$, under controller (20).

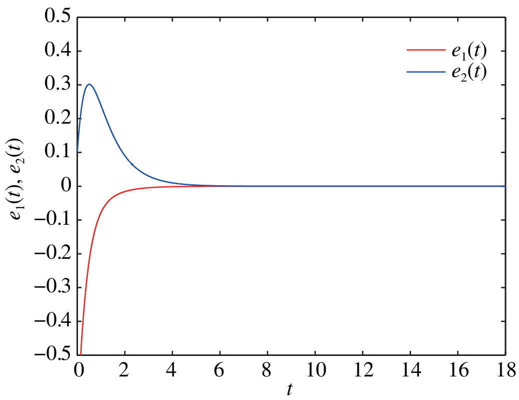

Figure 4 (Color online) Synchronization errors $e_1(t)$ and $e_2(t)$ under controller (20).

where

$$A(t) = \begin{pmatrix} a_{11}(t) & -0.1 \\ a_{21}(t) & 2.9 \end{pmatrix}, \qquad B(t) = \begin{pmatrix} b_{11}(t) & -0.1 \\ -0.2 & b_{22}(t) \end{pmatrix},$$

$$a_{11}(t) = \begin{cases} 1.9, & f_{11}(s) \downarrow, s \in (t-\sigma t, t], \\ 1.7, & f_{11}(s) \uparrow, s \in (t-\sigma t, t], \\ \lim_{s\to t^-} a_{11}(s), & f_{11}(s) \ \text{unchange}, s \in (t-\sigma t, t], \end{cases}$$

$$a_{21}(t) = \begin{cases} -4.7, & f_{21}(s) \downarrow, s \in (t-\sigma t, t], \\ -5, & f_{21}(s) \uparrow, s \in (t-\sigma t, t], \\ \lim_{s\to t^-} a_{21}(s), & f_{21}(s) \ \text{unchange}, s \in (t-\sigma t, t], \end{cases}$$

$$b_{11}(t) = \begin{cases} -1.5, & f_{11}(s-1) \downarrow, s \in (t-\sigma t, t], \\ -1.3, & f_{11}(s-1) \uparrow, s \in (t-\sigma t, t], \\ \lim_{s\to t^-} b_{11}(s), & f_{11}(s-1) \ \text{unchange}, s \in (t-\sigma t, t], \end{cases}$$

$$b_{22}(t) = \begin{cases} -2.3, & f_{22}(s-1) \downarrow, s \in (t-\sigma t, t], \\ -2.6, & f_{22}(s-1) \uparrow, s \in (t-\sigma t, t], \\ \lim_{s\to t^-} b_{22}(s), & f_{22}(s-1) \ \text{unchange}, s \in (t-\sigma t, t]. \end{cases}$$

The activation function is the same as that used in Example 1. Then the memristive neural networks with the above initial values exhibit chaotic behavior, as shown in Figure 5. The state trajectories are plotted in Figure 6.

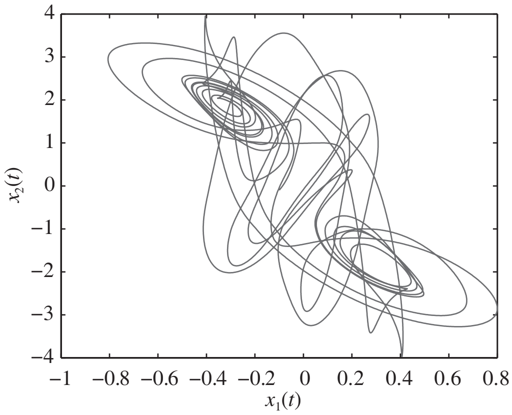

Figure 5 The chaotic attractor of system (21) with initial conditions $x_1(t) = -0.1$ and $x_2(t) = -0.1$, $t \in [-1, 0]$.

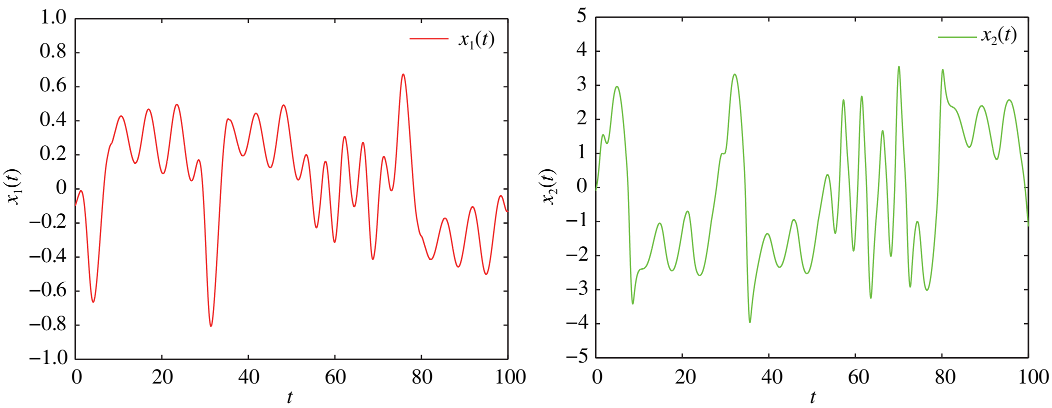

Figure 6 (Color online) Time evolution of states $x_1(t)$ and $x_2(t)$ in system (21) with initial conditions $x_1(t) = -0.1$ and $x_2(t) = -0.1$, $t \in [-1, 0]$.

Now, we investigate the finite-time synchronization problem of systems (21) and (22). According to the restrictions derived in Corollary 2, it follows that

$$\eta_{11} \geqslant -d_1 + \sum_{j=1}^{2} \frac{1}{2}\bar{a}_{1j} + \sum_{j=1}^{2} \frac{1}{2}\bar{a}_{j1} l_1^2 = 3.45,$$

$$\eta_{12} \geqslant -d_1 + \sum_{j=1}^{2} \frac{1}{2}\bar{a}_{2j} + \sum_{j=1}^{2} \frac{1}{2}\bar{a}_{j2} l_2^2 = 4.45,$$

$$\eta_{21} \geqslant \sum_{j=1}^{2} \bar{b}_{1j} G_j + \sum_{j=1}^{2} \left(\sup_{u\in\mathbb{R}} (|a'_{1j}(u) - a''_{1j}(u)| + |b'_{1j}(u) - b''_{1j}(u)|) M_j \right) = 2,$$

$$\eta_{22} \geqslant \sum_{j=1}^{2} \bar{b}_{2j} G_j + \sum_{j=1}^{2} \left(\sup_{u\in\mathbb{R}} (|a'_{2j}(u) - a''_{2j}(u)| + |b'_{2j}(u) - b''_{2j}(u)|) M_j \right) = 3.4,$$

thus, the corresponding controller can be derived as

$$\begin{aligned} u_1(t) &= -5e_1(t) - 2\mathrm{sign}(e_1(t)) - 0.2\mathrm{sign}(e_1(t))|e_1(t)|^{0.5} - 0.15\mathrm{sign}(e_1(t))|e_1(t)|^{0.5}, \\ u_2(t) &= -4.5e_2(t) - 4\mathrm{sign}(e_2(t)) - 0.2\mathrm{sign}(e_2(t))|e_2(t)|^{0.5} - 0.15\mathrm{sign}(e_2(t))|e_2(t)|^{0.5}. \end{aligned} \tag{23}$$

Under controller (23) and the initial conditions $x(t) = (-0.6\sin 2t + 0.7, -0.1\cos t)^{\mathrm{T}}$ and $y(t) = (0.2\tanh 2t, 0.8\sin 5t - 1)^{\mathrm{T}}$ for $t \in [-1, 0]$, the state trajectories of systems (21) and (22) are given in Figure 7. The evolution of the synchronization errors is described in Figure 8, which indicates that

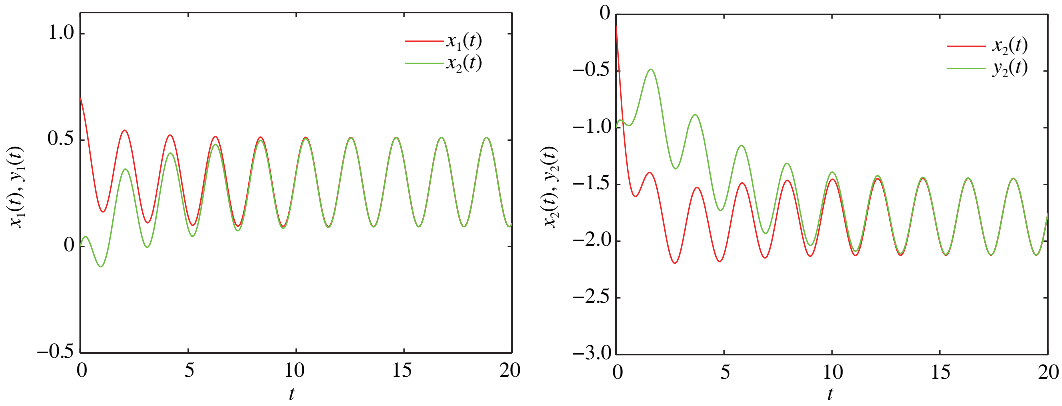

Figure 7 (Color online) Time-domain behavior of state trajectories $x(t)$ and $y(t)$ under controller (23).

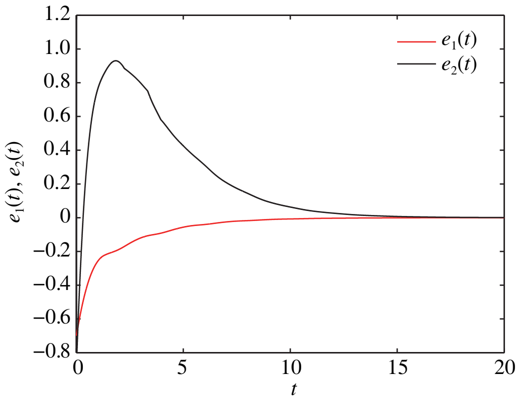

Figure 8 (Color online) Synchronization errors $e_1(t)$ and $e_2(t)$ under controller (23).

synchronization can be reached in finite time. Moreover, the settling time for synchronization is estimated by $t_1 \leqslant 14.2354$, which can also be verified in Figures 7 and 8.

5 Conclusion

This paper introduced fixed-time synchronization of a general class of delayed memristive neural networks. Some new control strategies were proposed to study the fixed-time synchronization problem of delayed memristive systems; in particular, we considered the concept of Lyapunov functions, analytical techniques, as well as a discontinuous controller. Some easy-verified conditions were established to ensure fixed-time synchronization for the target memristive neural networks within a settling time. Furthermore, the corresponding upper bound of the settling time was estimated. To help readers understand the differences and advantages of fixed-time synchronization over finite-time synchronization, a new criteria was added, which ensures that the target model can reach the finite-time synchronization goal. Finally, the effectiveness of our theoretical results were supported by numerical simulations.

Many factors may influence the dynamic behavior of memristive neural networks such as stochastic disturbance and Markovian jumps. Therefore, results corresponding to these factors will appear in the near future.

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Conflict of interest The authors declare that they have no conflict of interest.

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